Integrated Circuits and Applications
Prof. Shaik Rafi Ahamed
Department of Electronics and Electrical Engineering Indian Institute of Technology, Guwahati

Problem Solving
Lecture – 13
Examples on Instrumentation Amplifiers

Ok. In the last lecture, we have discussed some of the problems, today, we will discuss some more practice problems. The next example is determine the output of the following circuit. This is basically a double integrator circuit. I have discussed about the single integrator, where this acts as a double integrator. This is inverting terminal, here we have capacitor, this is input voltage, this is the output voltage v_0, this is grounded. These values are *R*, *R*, *C*, this is $\frac{C}{2}$, $\frac{C}{2}$, $\frac{R}{2}$.

So, you have to find out the expression for the output of the circuit. Let us call this current as I_i, this current as I_1, this current as I_2, this current as I_3. If I assume that the op-amp is ideal, then this also will be at a same ground potential of this positive terminal. So, this will acts as a virtual ground.

So, if I consider the circuitry from v_i to this ground, this will be something like this is v_i, this is I_i, this is capacitor and then this resistor also will be grounded. These two points are grounded, because this point is this point which is virtual ground, this is virtual ground. This is v_i, this is I_i, I want the expression for I_i in terms of v_i. So, you can take this total impedance as *Z*. So, *Z* is nothing but these two will be in parallel in series with this.

This is $Z = R + R||\frac{1}{SC} = R + \frac{R\times\frac{1}{SC}}{R+\frac{1}{SC}} = R + \frac{R}{1+SRC} = \frac{R+SR^2C+R}{1+SRC} = \frac{SR^2C+2R}{1+SRC}$. This is the total impedance of the circuit and we know that $v_i = ZI_i \Rightarrow I_i = \frac{v_i}{Z} = v_i\frac{[1+SRC]}{[SR^2C+2R]}$. This is expression for I_i. Now, what is the expression for I_1? This I_1, there is a current division. This total current is I_i; a part of this current flows through the capacitor, and the remaining part will through the resistor.

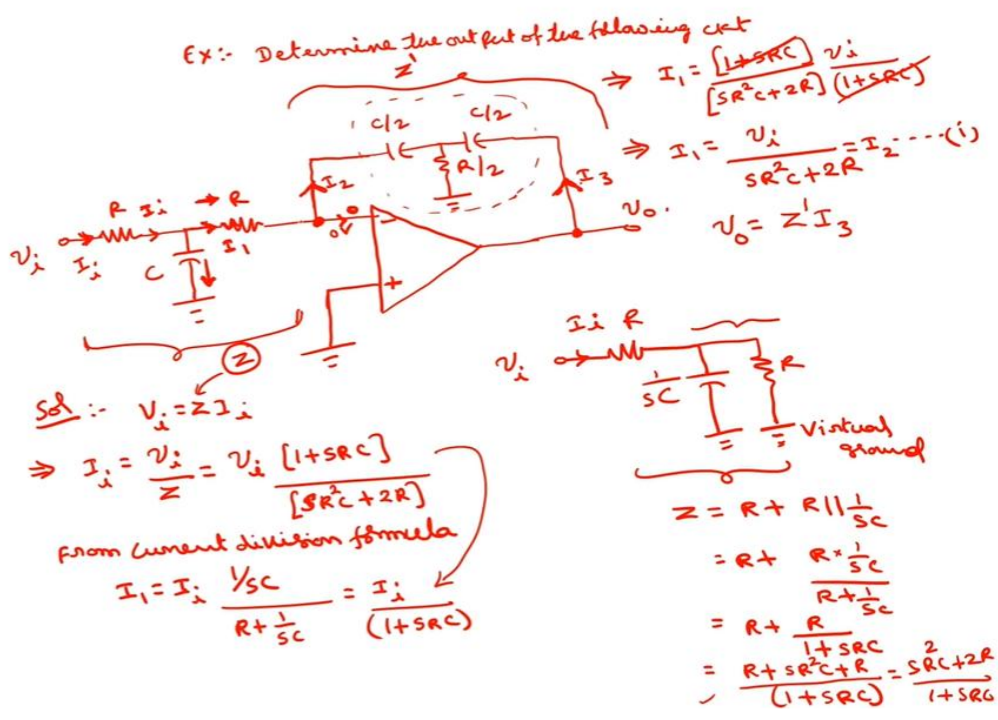
So, we know the expression for the current division circuit. So, the output through this resistor I_1 is equal to total current I_i into opposite impedance divided by this impedance R resistance R. So, from current division formula $I_1 = I_i\frac{\frac{1}{SC}}{R+\frac{1}{SC}} = \frac{I_i}{(1+SRC)}$. This is equal to I_i

times SC SC will get cancel $\frac{1}{SRC}$. If I substitute this I_i here what will be I_1? $I_1 = \frac{[1+SRC]}{[SR^2C+2R]}\frac{v_i}{(1+SRC)} = \frac{v_i}{SR^2C+2R} = I_2$. You can see that this I_1 flows through the feedback path also because the current here is 0 because the operational amplifier is ideal this current is 0. So, the entire I_1 will flows through this. So, this is also equal to I_2. Yes, I will call as equation 1.

Then I will consider the circuitry at the output side from here to this virtual ground. So, what will be this I_3 now? Similar to this I_i we have considered from input side from here to here this was ground. Now, from here to this output side this output side if I consider. So, v_o is the voltage, and this I_3 we have to find out. So, you have to find out this v_o in terms of I_3 if I compute this impedance of this entire feedback path.

What is the impedance of this path? If I know this impedance, simply $v_o = Z'I_3$.

(Refer to the slide at 08:41)



Here this Z' I am calling as this impedance. This you call as Z here, you have used similarly I will call this one as Z'. So, what is Z'? Z' is this is at ground potential. So, what will be this equivalent circuit? This is the output v_o. This is the current I_3, and here you have one capacitance, one resistor, and one capacitance. This is also almost at the ground. This is virtual ground now.

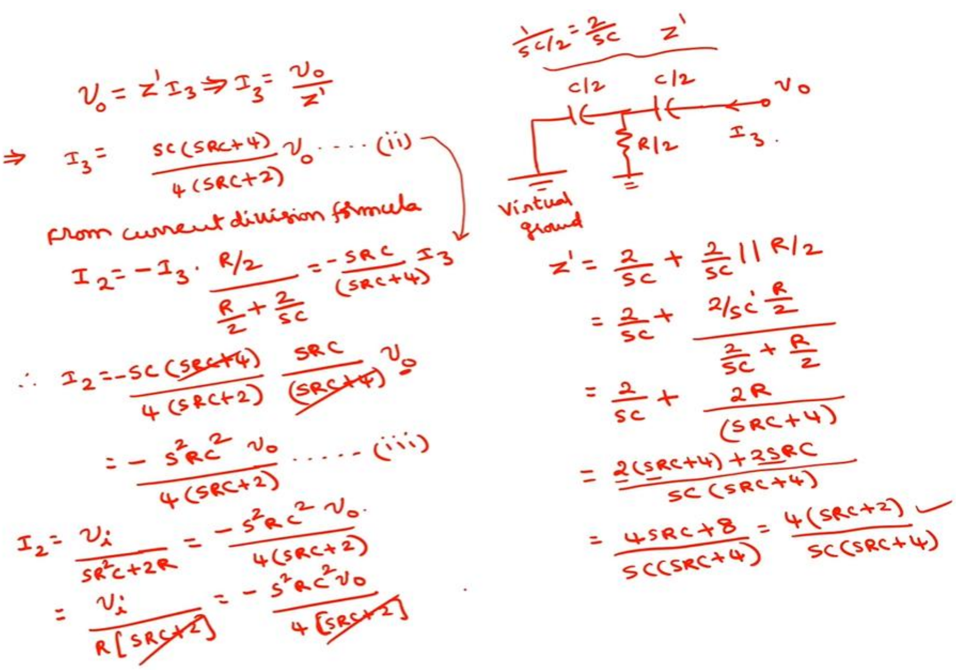
This is $\frac{R}{2}, \frac{C}{2}, \frac{C}{2}$, but in the S domain, this $\frac{C}{2}$ becomes $\frac{1}{SC/2} = \frac{2}{SC}$. So, this entire impedance we are calling as Z'. So, what is Z'? $Z' = \frac{2}{SC} + \frac{2}{SC} || \frac{R}{2} = \frac{2}{SC} + \frac{\frac{2}{SC} \times \frac{R}{2}}{\frac{2}{SC} + \frac{R}{2}} = \frac{2}{SC} + \frac{2R}{(SRC+4)} = \frac{2(SRC+4)+2SRC}{SC(SRC+4)} = \frac{4SRC+8}{SC(SRC+4)} = \frac{4(SRC+2)}{SC(SRC+4)}$.

This is only Z'. What is v_0? $v_o = Z' I_3 \Rightarrow I_3 = \frac{SC(SRC+4)}{4(SRC+2)} v_o$. You can call this as equation 2. Now, we have to find out this I_2 because I have already derived the expression for the I_2 this is equation 1.

Now, from this I_3 also I will derive the expression for this I_2. So, that I can equate these two to find out the output v_0. Again, here, this current division this I_3, this is I_3, is divided into this resistor as well as this capacitor, but the current through this capacitor is in opposite direction. This is I_2 we are taking. So, we will take minus sign. So, if you want I_2, this is equal to I_3 into because we are going to find out the current through the capacitor $\frac{C}{2}$. You have to take the opposite impedance or resistance $\frac{R}{2}$ divided by total impedance.
What will be expression for I_2 in terms of I_3? From current division formula, I_2 is equal to minus I_3 because now the direction is opposite to that of the current flow through the capacitor times you have to find out the current through this capacitor this one. So, you have to take the opposite impedance or resistance $\frac{\frac{R}{2}}{\frac{R}{2}+\frac{2}{SC}}$, here is equal to $-\frac{SRC}{SRC+4} \times I_3$. If I substitute this I_3 here, what will be I_2? $I_2 = \frac{-SC(SRC+4)}{4(SRC+2)} \times \frac{SRC}{(SRC+4)} v_o = \frac{-S^2RC^2}{4(SRC+2)} v_o$. This is the expression for I_2, I call as equation 3 and I have already derived the expression for the I_2 here.

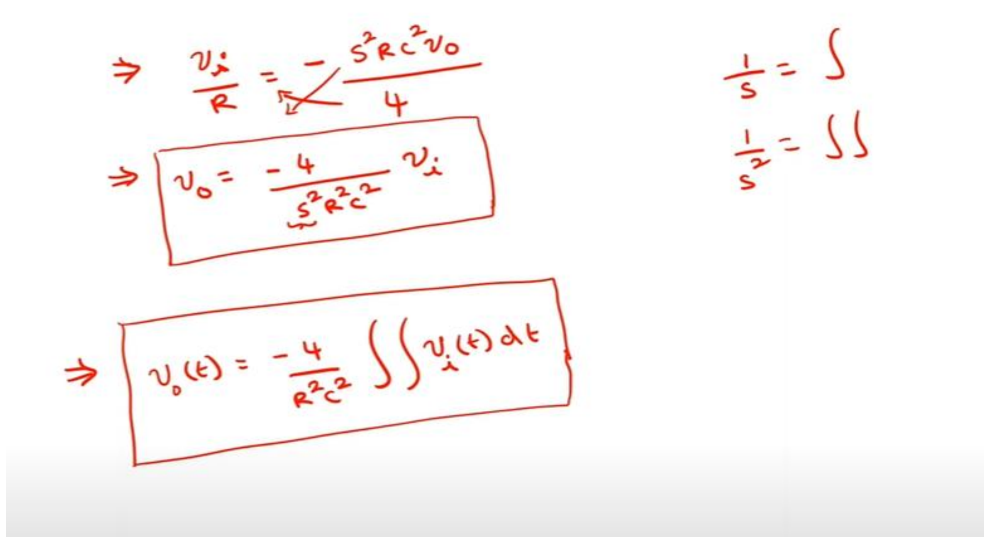
Now, we can equate this equation 1 and equation 3, $I_2 = \frac{v_i}{SR^2C+2R} = \frac{-S^2RC^2}{4(SRC+2)} v_o$, v_0 is here also. So, now, here, if I take in this denominator or if I take as common, $\frac{v_i}{R(SRC+2)} = \frac{-S^2RC^2}{4(SRC+2)} v_o$. So, this SRC plus 2 SRC plus 2 get cancelled.

(Refer to the slide at 16:01)



So, what will be resultant? $\frac{v_i}{R} = -\frac{S^2RC^2}{4} v_o \Rightarrow v_o = \frac{-4}{S^2RC^2} v_i$. This is expression for the output of the circuit. So, we know that in the S domain, if you have $\frac{1}{S}$, this is integration. If I have $\frac{1}{S^2}$, this will be double integration. So, if I remove this S^2 here in the S domain in the time domain, it becomes a double integration. So, in terms of the output, $v_o(t) = \frac{-4}{R^2C^2} \iint v_i(t)dt$. So, this given circuit will acts as double integrator ok.

(Refer to the slide at 17:31)



This is one of the important circuit. The next example is to find out the output voltage of a instrumentation amplifier. Consider the instrumentation amplifier given below with a common mode voltage of plus 3V DC and the differential voltage of 80mV peak sinusoidal wave. So, the given circuit is this instrumentation amplifier. This is $50k\Omega$. This is also $50k\Omega$, this is $1k\Omega$.

And, here we have a differential amplifier. This is the final output v_0, these are all input voltage. Now, here the input voltage is given as a differential voltage of 80mV peak and common mode voltage of 3V DC. This is a 40mV peak; this is another 40mV peak. Both together will be 80mV peak, and this is a common mode signal. So, these two are differential signals.

So, this is the given circuit. So, the problem is determine the voltages at different nodes. Let us call this as node A, this as node B, node C, node D and then finally, the output voltage v_0. Hence, you have to find out what is v_A, v_B, v_C, v_D and v_0. If I call this input voltage here as v_{01} voltage, this as v_{02} voltage. Then what is the expression for v_{01}? If I go with this loop, so, this voltage will be this is minus 2 plus if I take as plus voltage.

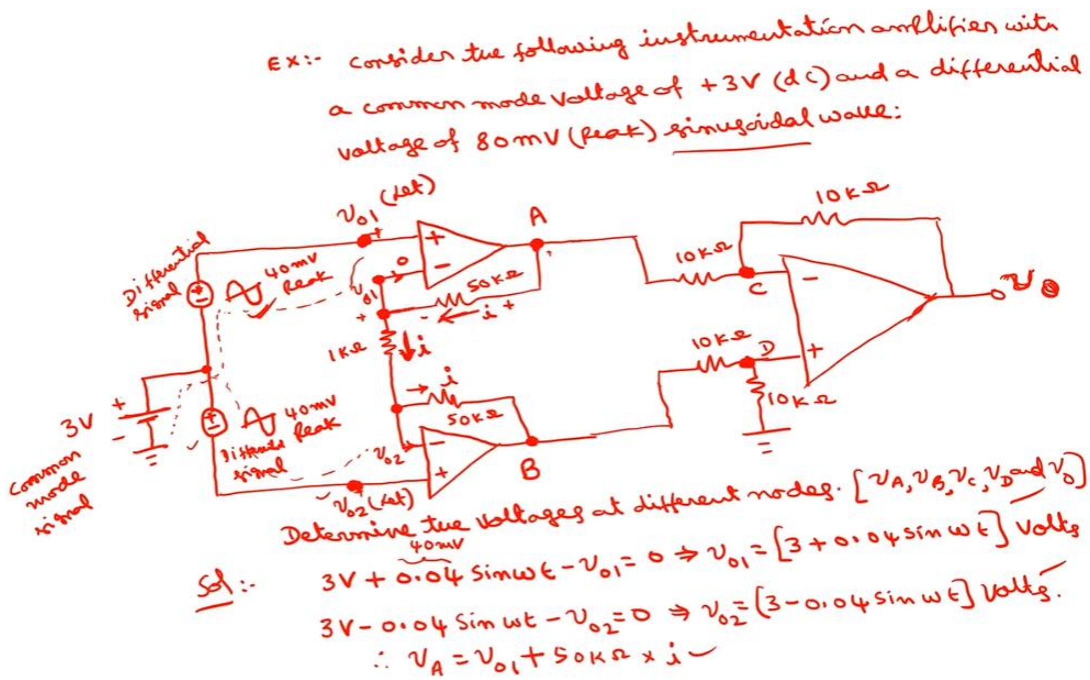
So, 3V this is also minus 2 plus. This is peak value of 40mV means $+0.04sin\omega t$ is this voltage, peak value is 40mV, this is 40mV, and it is given as sinusoidal. So, $sin\omega t$ and here also this will be plus with respect to ground. This is place we are going to measure this with respect to the ground.

So, plus 2 minus this is minus 2 plus minus 2 plus this is plus 2 minus. So, minus v_{01} is equal to 0. This is plus means with respect to ground. So, ground is minus. So, this will be plus 2 minus implies what is v_{01}? $v_{01} = [3 + 0.04sin\omega t]V$; this is so many volts.

Similarly, if I take this KVL in this loop, what is v_{02}? v_{02} will be this will be place with respect to ground. This is minus 2 plus. If I take it as plus, this is plus 2 minus. This is also plus 2 minus. These two will be minus this is $3V - 0.04sin\omega t - v_{01} = 0$, implies what is v_{02}? $v_{01} = [3 - 0.04sin\omega t]Volts$. So, we have determined this v_{01} and v_{02}, but we have to find out v_A, v_B, v_C, v_D and final output v_0. How to find out v_A? If I assume that this current is I flows in this direction, this is I because here the current is 0 because you are assuming that the op-amp is ideal. So, whatever the current that is coming from here, this I, the entire I will flows through this $1k\Omega$ resistor because the current here is 0. Similarly, whatever the current that is coming here this entire current flows through this 50k resistor because this current is 0. So, what will be expression for v_A now? This is v_{01} means because of the ideal op-amp, the voltage at inverting terminal is equal to the voltage at non inverting terminal. This is also v_{01}.

This is v_{02}, means this is also v_{02}. This current is 0, but voltage here is v_{02}. So, this voltage here is v_{01} here or here and this I want to find out v_A, this current direction is this one is plus minus. This v_{01} is plus with respect to the ground, which is minus. So, you are coming from minus to plus v_{01} minus to plus this i into 50k and this is plus to minus. So, what is v_A? $v_A = v_{01} + 50k\Omega \times i$, but what is i? i is equal to this is v_{01}, this is v_{02} current direction is this.

(Refer to the slide at 26:51)



So, $i = \frac{v_{01} - v_{02}}{1k\Omega}$. What is v_{01} is this v_{02} this? So, $i = \frac{3 + 0.04 sin\omega t - 3 + 0.04 sin\omega t}{1k\Omega} = 0.08 sin\omega t\ mA$.

This kilo I have taken to numerator that becomes milli amps this is the current. Therefore, what is $v_A = v_{01} + 50 \times 10^3 \times 0.08 sin\omega t \times 10^{-3} = v_{01} + 4 sin\omega t = 3 + 0.04 sin\omega t + 4 sin\omega t = [3 + 4.04 sin\omega t] volts$. These two get cancelled 50 into 0.01 becomes 4.

So, implies $v_A = [3 + 4.04 sin\omega t] volts$. This is the voltage at node A. Similarly, node B, what is v_B? This is v_{02} and this current direction is this. This is minus 2 plus, this is plus 2 minus, this is plus 2 minus. So, this $v_B = v_{02} - 50k\Omega \times i$, i is this, and v_{02} is this. This is equal to $v_{02} = 3 - 0.04 sin\omega t - 50 \times 10^3 \times 0.08 sin\omega t \times 10^{-3} = [3 - 4.04 sin\omega t] volts$. This is mA. So, many volts. This is v_B. Now, what is v_C? What is v_D? What is v_o? What is v_D? From here, this is v_B, and this is voltage divider. This is

v_B, and here, 10k, 10k, and this is the point where we are taking v_D. So, what is v_D? $v_D = v_B \times \frac{10k}{20k} = \frac{v_B}{2} = 1.5 + 2.02 sin\omega t$. This is v_D and also v_C because this half amp is ideal. The voltage at inverting and non inverting are the same.

So, v_C and v_D both are same. So, implies $v_C = v_D = 1.5 + 2.02 sin\omega t$. Now, the last one is final output v_0.

We can see that what is this circuit from here to here. This we have already studied in the previous lectures. So, because this, all the 4 resistors are equal. So, output v_0 is given by this voltage minus this voltage. This is positive is v_B negative is v_A. So, this is $v_0 = v_0 = v_B - v_A$ because these resistors are equal R_2 by R_1 into v_B minus v_A, but R_2 is equal to R_1.

(Refer to the slide at 33:41)

$$i = \frac{v_{01} - v_{02}}{1K\Omega} = \frac{3 + 0.04\, sin\omega t - \underbrace{3 + 0.04\, sin\omega t}}{1K\Omega}$$

$$= 0.08\, sin\omega t \;\; mA$$

$$\therefore v_A = v_{01} + 50 \times \cancel{10}^{3} \times 0.08\, sin\omega t \times \cancel{10}^{-3}$$

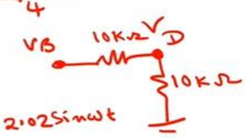
$$= v_{01} + 4\, sin\,\omega t = 3 + 0.04\, sin\omega t + 4\, sin\omega t$$

$$\Rightarrow \boxed{v_A = [3 + 4.04\, sin\omega t]\, volts}$$

$$v_B = v_{02} - 50K\Omega \times i$$

$$= 3 - 0.04\, sin\omega t - \underbrace{50 \times \cancel{10}^{3} \times 0.08\, sin\omega t \times \cancel{10}^{-3}}_{4}$$

$$\Rightarrow \boxed{v_B = [3 - 4.04\, sin\omega t]\, volts}$$



$$V_D = V_B \cdot \frac{10K}{20K} = \frac{V_B}{2} = 1.5 + 2.02\, sin\omega t$$

$$\Rightarrow \boxed{v_C = v_D = [1.5 + 2.02\, sin\omega t]\, volts} \Rightarrow v_0 = v_B - v_A$$

$$\therefore \boxed{v_0 = -8.08\, sin\omega t\; volts}$$

So, this is $v_B - v_A$ simply. This is different circuit or subtractor, which we have already studied subtractor. So, what is v_0 implies $v_0 = v_B - v_A$. So, if I subtract v_A from this v_B, 3 3 get cancelled will get minus therefore, $v_0 = -8.08 sin\omega t\ volts$. So, this is about this circuit. Next, we will discuss some of the examples on finding the CMRR of a given circuit. Thank you.