Integrated Circuits and Applications

Prof. Shaik Rafi Ahamed
Department of Electronics and Electrical Engineering Indian Institute of Technology, Guwahati

Problem Solving
Lecture – 12
Examples on Transfer Function Computation

Ok. In the last lecture, we have discussed some of the examples or some practice problems. Today, we will discuss some more practice problems. So, determine the gain of following circuit for $v_i > 0$, as well as for $v_i > 0$ as well as$v_i < 0$. Assume diode is ideal. This is the basic circuit. This is v_i. This is diode. This is the output. This is R_1, R_2, R_3. This diode is ideal diode. So, to find out the gain which is $\frac{v_o}{v_i}$. Gain is equal to $\frac{v_o}{v_i}$. First let us consider $v_i > 0$. Because, this is inverting amplifier, if $v_i > 0$, implies $v_o < 0$. So, that for the diode, this is the diode. Here, this voltage is v_i, which is greater than 0; this voltage is v_0, which is less than 0. So, what happens to this diode? Diode will be forward bias and then acts as a and hence acts as a short circuit. Therefore, what will be equivalent circuit? See the equivalent circuit. So, this R_2, this will be short circuited and there is one more resistance here. This is v_i, R_1, R_3 and R_2.

Now, R_3 and R_2 will be in parallel that will be the feedback resistance and this is standard inverting amplifier whose gain is minus of feedback resistance divided by the input resistance that is $\frac{R_3||R_2}{R_1}$. This is equal to $-\frac{R_3R_2}{R_1(R_3+R_2)}$. See the gain of the amplifier if $v_i > 0$.

Case 2, if $v_i < 0$, this implies $v_o > 0$. So, what happens to this diode? This is less than 0 at anode, at cathode voltage is greater than 0, implies diode is reversed biased and hence acts as a open circuit.

So, now what will be equivalent circuit? So, this is R_1, v_i, this will be grounded. So, this is R_2, but here this is open circuited, this is R_3. So, because of this open circuit, this part will not be there; simply, R_3 will be there in the feedback resistance. Therefore, what is gain? Simply $-\frac{R_3}{R_1}$. This is the gain with $v_i > 0$ and this is the gain with $v_i < 0$.

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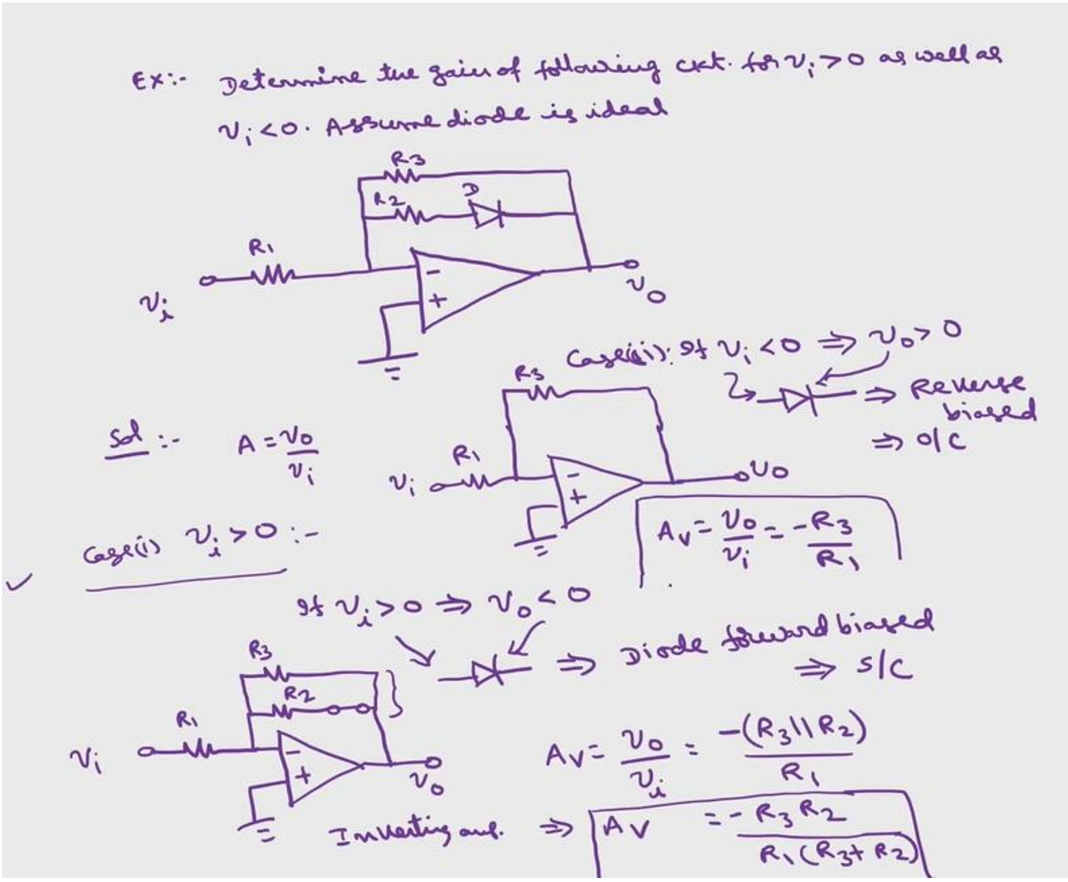

So, next example is determine the gain of the following circuit. See this circuit in the feedback; instead of having single resistance, there will be a delta network. Assume that op-amp is ideal unless otherwise it is mentioned, we can assume that op-amp is ideal. This is say R_1, R_2, R_3, R_4 let us call this one as I_1, I_2, I_3, I_4. Now, to basically find out the gain, closed loop gain, which is $\frac{v_o}{v_i}$.

So, first, we will assume that because the op-amp is ideal, this is 0V, because this is ground point and the current here is 0, current here is 0. So, what is the expression for I_1? From here to here this is v_i, this is 0V. So, $I_1 = \frac{v_i - 0}{R_1} \Rightarrow I_1 = v_i R_1$. But what about I_2? So, this current is I_1, then the entire I_1 will flows through the this feedback path because the bias current is 0. So, imply this is also equal to I_2.

Let us call this voltage as say v_x. Then what is expression for v_x in terms of v_i? So, basically, here are the currents and voltages I want to express in terms of v_i and v_0 so that, finally, we can take the ratio of v_0 to v_i. So, in order to find out v_x in terms of v_i, if I consider between this point and this point, the current is I_2 in this direction. So, I_2 is given by this is 0V. So, $I_2 = \frac{0 - v_x}{R_2} \Rightarrow v_x = -R_2 I_2 = -\frac{R_2}{R_1} v_i$, this is equation 2.

This is expression for v_x. This is an expression for I_1 and I_2. Then, coming for the I_3, I_3 expression is this is the current direction. This is ground to v_x. So, $I_3 = \frac{0 - v_x}{R_3} = -\frac{v_x}{R_3} = \frac{R_2}{R_1 R_3} v_i$.

So, this I_1 is also function of v_i, I_2 and v_x also function of v_i, I_3 also function of v_i. I_4 is nothing, but this is the direction of the current. this voltage v_x this voltage is v_0. So, $I_4 = \frac{v_x - v_o}{R_4} \Rightarrow I_4 R_4 = v_x - v_o \Rightarrow v_o = v_x - I_4 R_4$.

So, here we have to substitute this v_x, I_4 in terms of v_i. So, what is I_4? If I apply at this point, KCL Kirchhoff's current law, I_2 is entering, I_3 is entering, I_4 is leaving. So, $I_4 = I_2 + I_3$. So, from equation 1, we have $I_2 = \frac{v_i}{R_1}$ and from equation 3, $I_3 = \frac{R_2}{R_1 R_3} v_i$. Now, we substitute 5 and 4.

So, it will be v_0, $v_o = v_x$, v_x is nothing, but from 2, $v_x = -\frac{R_2}{R_1} v_i$, this, then minus I_4 is this times R_4. So, $-[\frac{v_i}{R_1} + \frac{v_i R_2}{R_1 + R_3}] R_4$. So, if you take v_i as common, what is $\frac{v_o}{v_i}$? This is $\frac{v_o}{v_i} = -[\frac{R_2}{R_1} + \left(\frac{1}{R_1} + \frac{R_2}{R_1 R_3}\right)] R_4$ or otherwise you can express this in a simplified form as gain $A_v = \frac{v_o}{v_i} = -[\frac{R_2}{R_1} + \left(\frac{1}{R_1} + \frac{R_2}{R_1 R_3}\right)] R_4$. So, we can call as $\frac{R_4}{R_1}(1 + \frac{R_2}{R_3})$. This is the expression for the gain of the given circuit.

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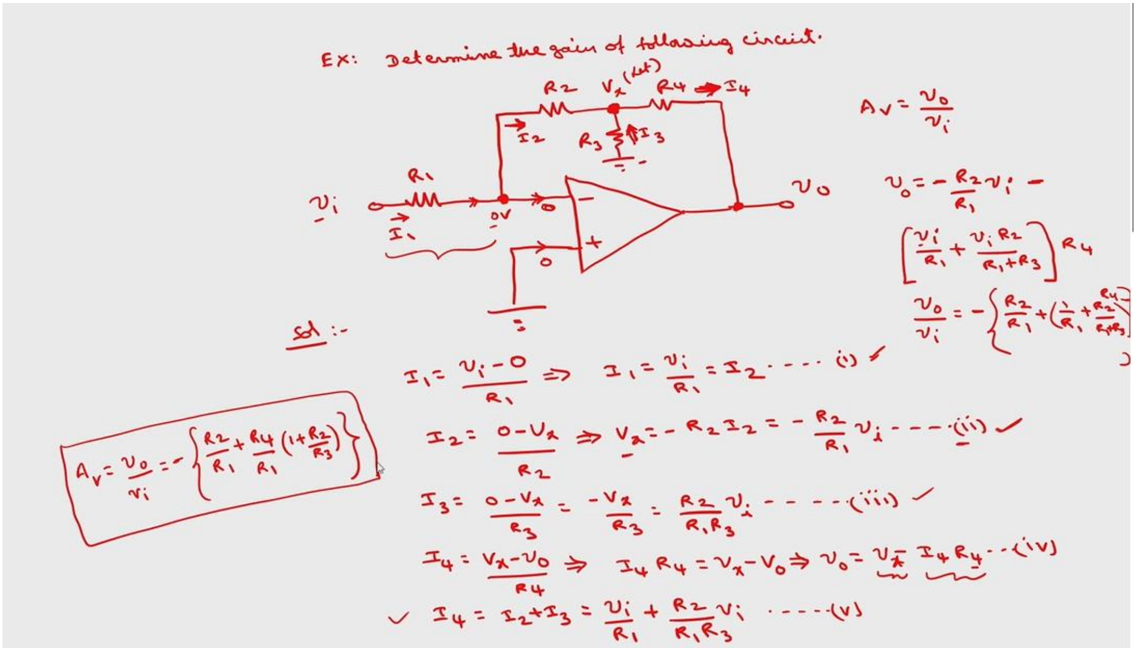

We will discuss some more problems on finding the input resistance or input impedance of inverting amplifier. I have discussed about the voltage gain, but not the input resistance of the inverting amplifier. So, the inverting amplifier is basically, is this is R_1, R_F.

The gain we have obtained as $-\frac{R_F}{R_1}$, but the input impedance is if I look into here the input resistance. If it is a non-resistive network consisting of the inductors and capacitors also then it is called impedance. Basically, R_i is defined as this input voltage divided by input current. If I assume that input current is I_i, R_i is basically, $\frac{v_i}{I_i}$. Here, what is I_i? This voltage is 0V.

So, $I_i = \frac{v_i - 0}{R_1} \Rightarrow I_i = \frac{v_i}{R_1}$. If you substitute this here, $R_i = \frac{v_i}{v_i/R_i} = R_i$.

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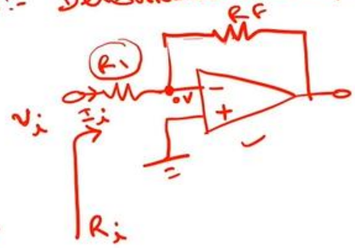
Ex:- Determine the input resistance of inverting amp.



sol.

$$R_i = \frac{v_i}{I_i}$$

$$I_i = \frac{v_i - 0}{R_1} \Rightarrow I_i = \frac{v_i}{R_1}$$

$$R_i = \frac{v_i}{v_i/R_1} = R_1$$

$$\boxed{R_i = R_1}$$

So, the input impedance of this inverting amplifier is basically this resistance. That is why in the design, you have to choose R_1 value very large; otherwise, input resistance will be less, and it is going to load the input source. So, we will take some other example on finding the input resistance input resistance of following circuit.

This is somewhat complicated circuit. This is v_i, this is R_1, this is feedback resistance. This is v_0; there is one more feedback resistance, v_2; this is R_3, this is v_i. If you take this ratio, you will get input resistance. This is R_1, this is R_2, this is also R_2, this is say $2R_1$.

Then what is the input resistance if I look into R_i here? What is the expression for this R_i? Let us call this node as A node, this is B node, this is C node. This is input voltage v_i itself, and this is output voltage v_0, and if I call this current as I_1, I_2, I_3, then you call this one as I_4. So, what is the expression for I_1? $I_1 = \frac{v_i - v_a}{R_1} = \frac{v_i}{R_1}$, since $v_a = 0$. Now, what about I_2? This I_2 here because current is 0, this current is I_1.

So, $I_2 = I_1$. This is equal from here to here. This is 0V that is v_a in fact, $\frac{-v_b}{R_2}$, but this is equal to $v_b = \frac{-R_2}{R_1} v_i$, because v_a is 0. So, if I equate these two, this is also I_1, this is also I_1 that implies $\frac{v_i}{R_1} = \frac{v_o}{R_2}$. This is one expression. Now, from B to C, what is the expression for I_3? $\frac{v_b - v_c}{R_2} = \frac{v_b}{R_2}$, because $v_c = 0$, because of this virtual ground concept.

Similarly this here the current is 0. So, the entire I_3 will flows through this one also. So, if I write down another expression for I_3 between C to output v_0, $\frac{v_c - v_o}{2R_1}$. This is equal to $\frac{-v_o}{2R_1} = \frac{v_b}{R_2}$.

So, implies what is v_0? $v_o = \frac{2R_1}{R_2} v_b$. This is expression 2. From this equation 1, what is v_b minus v_b by R_2? This is v_b, this is v_b by R_2, this is v_i by R_1 I_1, this is also I_1 v_b by R_2. So, implies what is v_b minus R_2 by R_1 times v_i. If I substitute this here, what is v_0? $v_o = \frac{2R_1}{R_2}\left(\frac{-R_2}{R_1}\right) v_i \Rightarrow v_o = 2v_i$.

This is expression for v_0, but we are interested in input resistance. This is one expression, which is third expression. The relation between the v_0 and v_i is $v_o = 2v_i$, but we are interested in input resistance R_i. So, if I take this I_i as total current, $I_i = I_1 + I_4$. What is I_1 is v_i minus v_a, which is, of course, 0 divided by R_1 plus this is v_i, this is v_0, and the resistance is R_3.

So, v_i minus v_0 divided by R_3 this is equal to I_i, I will keep I_i as it is. This I am going to express in terms of v_i so, that input resistance $R_i = \frac{v_i}{I_i}$, I want this expression. So, I_i this side, this side I have to express in terms of v_i. So, what will be this? Then this is equal to v_i, v_i is 0, $\frac{v_i}{R_1} +$ v_i from this 3, $v_o = 2v_i$. So, this is equal to $\frac{v_i - 2v_i}{R_3}$.

If I take LCM, what will be this one? So, this will be R_1 R_3 LCM, v_i is of course, common. So, R_3 plus this becomes $v_i - 2v_i = -v_i$, of course, so minus R_1 times this one, this is I_i. Therefore, what is $R_i = \frac{v_i}{I_i} = \frac{R_1 R_3}{R_3 - R_1}$. This is expression for the input impedance of the given circuit. If want to have larger input impedance, you have to properly choose R_1 and R_3, you have to choose large values of R_1 R_2, you have to properly choose the values of R_1 and R_3.

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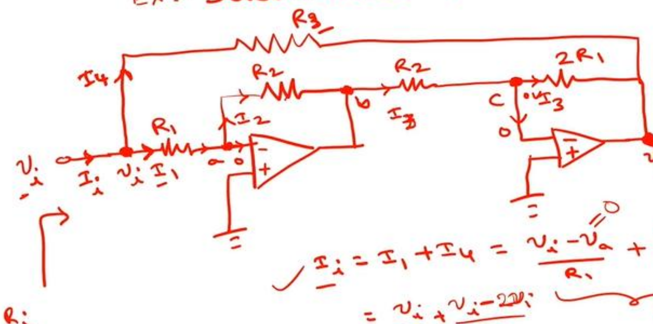
Ex:- Determine the input resistance of following ckt



$I_i = \frac{(R_3 - R_1)\, v_i}{R_1 R_3}$

$\boxed{R_i = \frac{v_i}{I_i} = \frac{R_1 R_3}{R_3 - R_1}}$

$I_i = I_1 + I_4 = \frac{v_i - \overset{=0}{v_a}}{R_1} + \frac{v_i - v_o}{R_3}$

$= \frac{v_i}{R_1} + \frac{v_i - 2v_i}{R_3}$

$R_i = \frac{v_i}{I_i}$

Sol: $I_1 = \frac{v_i - v_a}{R_1} = \frac{v_i}{R_1}$ since $v_a = 0$; $I_2 = I_1 = \frac{v_a - v_b}{R_2} = \frac{-v_b}{R_2}$

$\Rightarrow \frac{v_i}{R_1} = \frac{-v_b}{R_2}$ . . . (i)

$I_3 = \frac{v_b - v_c}{R_2} = \frac{v_b}{R_2}$ since $v_c = 0$

$I_3 = \frac{v_c - v_o}{2R_1} = \frac{-v_o}{2R_1} = \frac{v_b}{R_2}$

$v_b = -\frac{R_2}{R_1} v_i$

$\Rightarrow v_o = -\frac{2R_1}{R_2} v_b$ - - - (ii)

$v_o = -\frac{2\cancel{R_1}}{\cancel{R_2}} \cdot \left(-\frac{\cancel{R_2}}{\cancel{R_1}}\right) v_i \Rightarrow \boxed{v_o = 2v_i}$ . . . . (iii)

Next, another interesting circuit is called negative impedance converter. Then, the input impedance of following circuit. Here is the input and feedback. There is the impedance; here, there is impedance; here, there is another impedance. Let this is Z_P, this is Z, this is Z_N, this is v_i, this is v_0. I have to find out the input impedance Z_i here.

So, let this current is I_i, then the ratio of v_i by Z_i is Z_i. So, Z_i is nothing but $\frac{v_i}{I_i}$. So, if I assume that this is I_N, this is I_P, this is I_z, assuming that op-amp is ideal. So, this $I_i = I_n$ because here the current is 0. So, the entire this I_i will flows through, I_N and what is the expression for this one? This is v_i, v_i, and this is v_0. $\frac{v_i - v_o}{Z_N} \Rightarrow v_i - v_o = I_i Z_N$, or what is v_0? $v_i - I_i Z_N$ this is one expression.

Now, if I take this I_P expression, I_P is this, this voltage minus this voltage divided by Z_P. This is V because this is of virtual ground concept, this is because the op-amp is ideal, if this voltage is v_i inverting terminal voltage is equal to non-inverting terminal voltage. So, this is also at v_i. So, $I_P = \frac{v_o - v_i}{Z_P}$. What is I_Z? This is v_i, this is 0, $I_Z = \frac{v_i - 0}{Z}$. What is the relation between I_P and I_Z? $I_P = I_Z$ because this is the current entering here as I_P; this current is 0.

So, this entire I_P will flows through the I_Z. So, implies if I equate these two $\frac{v_o - v_i}{Z_P} = \frac{v_i}{Z}$ implies what is v_0? $v_o - v_i = \frac{Z_P v_i}{Z} \Rightarrow v_o = v_i + \frac{Z_P v_i}{Z}$. This is the second expression. The first expression is also v_0, second expression also v_0. We can equate 1 is equal to 2 implies

$$v_i - I_i Z_N = v_i + \frac{Z_P v_i}{Z}$$

.

This v_i v_i get cancelled. So, I want the impedance as $Z_i = \frac{v_i}{I_i}$. So, what is $\frac{v_i}{I_i}$? $Z_i = \frac{v_i}{I_i}$, that is equal to $-\frac{Z_N}{Z_P}Z$. So, if I take the condition as $Z_N = Z_P$, then what happens Z_i is simply $-Z$. This Z_i is simply whatever the Z is there minus of this Z if $Z_N = Z_P$. That is why this is also called as negative impedance converter called NIC.

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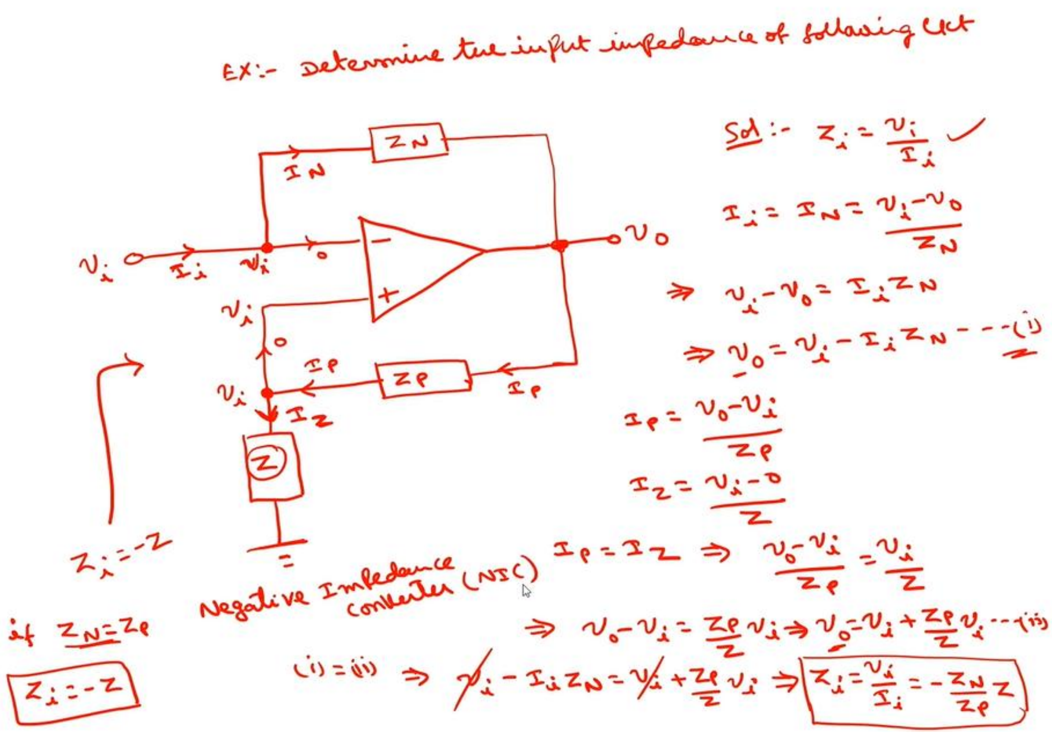

So, this can be easy to realize the inductors also. So, if I take another example based on this result, determine the input impedance of following circuit. This is input. So, this is input. Let us call this is R R R C, and this is also R; this is v_i. basically, I want this input impedance Z_i.

There will be some input impedance at this point also. This you call as Z_i'. If I consider this circuit, this is same as the previous circuit exactly same here $Z_N = Z_P = Z$. So, what is the input impedance of this entire circuit? Because $Z_N = Z_P$, $Z_i = -Z$, that is whatever the connected here that is minus of that which is R. So, this Z_i' is nothing, but $-R$ from the previous example.

Now, this is R and R. So, this total impedance is equal to minus of this resistance, here this is resistance. So, what will be the equivalent circuit now? I have v_i, this is minus plus, and I have capacitor here. Of course, we are taking the output v_o, this is R, then here we have the equivalent resistance of $-R$. This is C R v_i and this is I_i. So, what is total Z_i? This is again the same circuit with $Z_i = -\frac{Z_N}{Z_P} Z$.

Here Z_N is this, Z_P is this and Z is this. So, in this example Z_N is R, $Z = -R$, $Z_P = \frac{1}{SC}$. So, if I substitute these values here this is equal to minus Z_N is minus R divided by Z_P is $\frac{1}{SC}$ into Z is minus R. This is equal to SR^2C. So, if I take the impedance of a inductor $Z_L = j\omega L = SL$.

So, if I compare these two, so this will acts as a inductor. This is one of the interesting result. So, here, $L = R^2C$. Even though we do not have any inductor here, we can use this circuit to simulate the inductor because it is difficult to fabricate the inductor in the ICs. So, we can fabricate the inductor in terms of its equivalent circuit.

If I choose $R = 10k\Omega$, $C = 0.01\mu F$. So, you will get R square is this microton get cancel. So, what will be L? $100 \times 10^6 \times 0.01 \times 10^{-6} = 1H$. So, we can implement by choosing $R = 10k\Omega$, $C = 0.01\mu F$. This circuit will simulate the inductance of 1 Henry.

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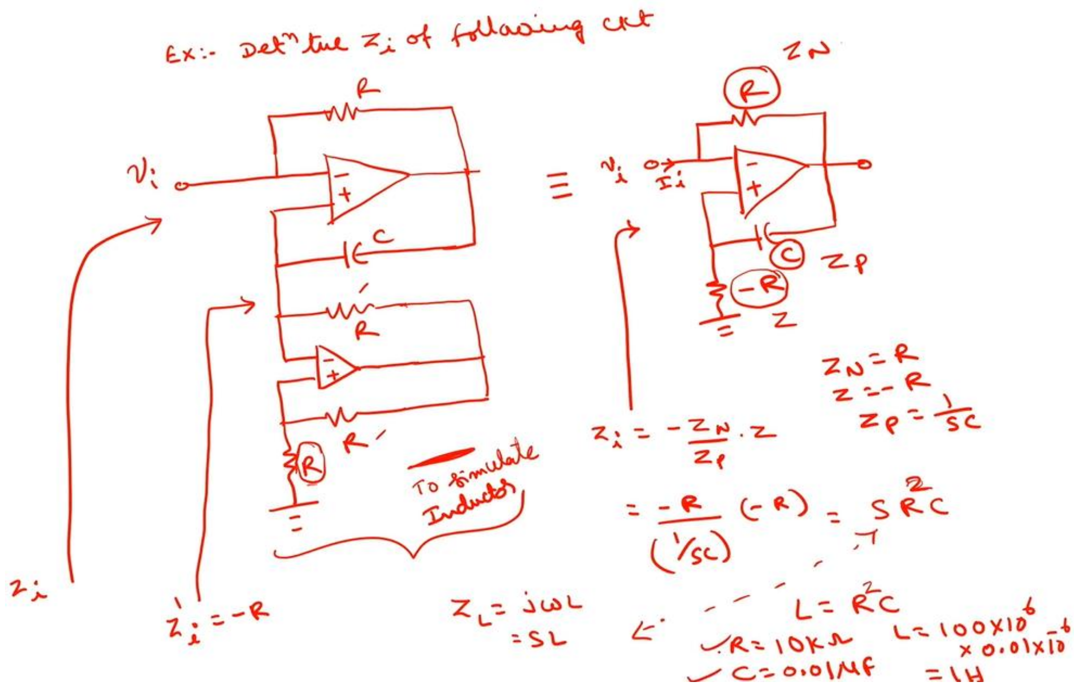

This is one of the interesting circuit. So, we will discuss some more examples in the next lecture. Thank you.