

Transducers For Instrumentation
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Lecture - 05

Thermal Sensors: Equilibrium and Predictive measurements

Hello, welcome to the course Transducers for Instrumentation. Today we are going to discuss a new topic the Thermal Sensors. As the name says thermal sensors means we are trying to measure the temperature of a certain body. The temperature of a body is the measure of how much thermal energy a body has. So, while measuring the thermal energy or the temperature we need to discuss certain points. The very first thing is whenever we measure the temperature of a body which we assume is in equilibrium the first body is in equilibrium and as soon as we connect our sensor to measure the temperature we are disturbing that equilibrium.

So, body was in equilibrium let us say at 50 degree centigrade it was uniform throughout the time as soon as we connect a sensor to measure its temperature no matter how small the that temperature sensor is this is going to disturb that equilibrium. How it happens? Let us say we have a thermal body. So, we have this ok. So, we have a body which is at certain temperature let us say TB this temperature is in equilibrium at time equal to 0.

As soon as we connect a second body to this , this is in close contact with this body this is a point of contact. This is the point of contact as soon as we make this connection there is a certain amount of thermal energy which transfers from the body to the temperature sensor. This is our second body which is a sensor and initial let us say temperature sensor is TS. So, because of the thermal difference TB minus TS there is going to be a transfer of thermal energy. So, this measurement actually requires certain amount of energy which transfers from body to sensor.

No matter how small it is that depends on what is the temperature difference and what is the thermal capacity of the second body as well as the first body. So, it depends on multiple factors, but as soon as we connect two things together which are at different temperature there is going to be a energy exchange no matter how small it is, it is going to disturb the equilibrium. So, this sensor is going to warm up or cool down depending upon the temperature difference. If the TB is lesser than TS then the sensor temperature will go down and if temperature of body is greater than the temperature of sensor then the sensor temperature will go up. So, this rise or fall in the sensor temperature is dependent on the heat exchange whether heat is flowing from the body to sensor or from sensor to the body.

So, there is a heat exchange involved which is disturbing our equilibrium. Now how small this sensor is no matter how small it is, it is going to disturb the equilibrium because this is

let us say very big body with thermal capacity very high this sensor is a small, but having a thermal capacity which is very less, but still there will be a thermal exchange no matter how small this sensor is and it is going to disturb the equilibrium. And this happens in all the modes of conduction. For example, here we have a metallic body here and a sensor which is physically in contact. So, we have conduction form of heat transfer, we can have a convection where kind of air is flowing from higher to lower temperature.

We can have this equilibrium disturbance even in radiative sensor. For example, a body is radiating and there is a sensor which is far apart which is taking the measurement of this radiating body even in that case the equilibrium will be disturbed because at time t equal to 0 or at the starting of measurement we assume that the body is in equilibrium with its surrounding. So, if a body is radiative let us say we have a body here which is radiating, this is right now in equilibrium with the surroundings as soon as we bring the temperature closer to that we bring the temperature sensor closer to this body. This is let us say our probe of temperature sensor as soon as we bring it closer to this body, this is going to disturb the equilibrium because the current environment is not equal to the environment which we had earlier. The surrounding is disturbed so, the equilibrium is also disturbed.

So, this equilibrium disturbance this happens even in the radiative sensors and this happens in conduction and convection already. So, this means whatever we do whatever measurement we are doing for the temperature sensor there is inherently there is an error this is inevitable which we discussed even in error propagations. This small amount of error or big amount of error depending upon amount of heat exchange this error we cannot eliminate completely we cannot make it 0. So, we have to minimize this error in temperature measurement, and this is the engineering task how to minimize this error during the temperature measurement and we apply multiple techniques to do this. So, now depending upon the heat exchange we have two methods of measuring the temperature.

First is the equilibrium-based temperature measurement and the other technique is predictive temperature measurement. So, we have equilibrium and the second is predictive. In equilibrium method a thermal equilibrium is reached between the sensor and the body and we wait enough amount of time so, that this equilibrium is reached and the temperature readings are stable. It means the heat exchange between the body and the sensor is negligible it may not be 0 because this is exponential function and the steady state value will come only in infinite time. Practically there is a exchange of heat which is negligible after let us say 5 or 6 time period.

So, we wait enough amount of time so, that this equilibrium is reached a the measurements. So, in equilibrium measurement in this technique we wait long enough for equilibrium to achieve. So, we have to wait for a long amount of time depending upon the RC time constant of thermal network. So, this is the disadvantage of equilibrium-based technique

we have to wait long amount of time to perform the measurement. However, the reading what we get in equilibrium technique will be more accurate.

So, this is the disadvantage and advantage is this is more accurate. So, in equilibrium based we have to wait for a longer amount of time if time constant is really high, but the accuracy of the reading will be much better. The other technique to overcome this problem of time is predictive based measurement. So, in predictive method the equilibrium is not reached. So, we do not wait enough for the equilibrium to reach we just make the contact of sensor with the body and depending upon the how fast the energy is being transferred from body to sensor the temperature rise is there.

So, looking at the temperature rise and doing some mathematical modeling we can predict what will be the steady state value of this graph because we know that the the charging of this the rising of this temperature is an exponential function and in how much time and how what will be the steady state value. So, we do some sort of a prediction to measure the temperature the however, the system is still not in equilibrium the temperature has not reached the sensor is still the reading is being updated, but we measure this graph for a small amount of for a small range and based on this range we predict what will be the steady state value. So, timing wise the predictive method is FAST which is an advantage of this technique. However, this prediction is not very good because we are taking data only for a small amount of range and because of that the extrapolation is not very accurate. So, this predictive method not very accurate, this is a disadvantage.

In equilibrium this is advantage that this technique is more accurate, but it takes long amount of time which is a disadvantage. So, these are the two techniques we apply in temperature measurement. So, now let us discuss how these techniques affects the accuracy of temperature measurement. So, for that we need to first make a model of our temperature sensor, the whole system we where we have body the sensor connected to it. So, we want to make a model of this whole system.

So, for example, we have a body which is at temperature T_B , this is our object. This is the object we want to measure the temperature. Now we connect a sensor to it which is generally small compared to the size of the body or the object. So, this is our sensor, this sensor temperature we can denote it by T_S , the object is T_B . And generally this sensor is always connected to some external electronic circuitry or external world.

So, we will have a cable or something which is connecting this sensor to the outside electronics or we call it an environment and this environment is at temperature T_0 . And this environment temperature we consider it as a boundary condition that this temperature does not change, remain fixed with whatever the system happens within the system. This T_0 temperature will always remain same. Now this is the complete system where we have just connected this sensor to the object and trying to measure the temperature this T_B and

the initial temperature of the sensor is T_S . So, now, we can make an equivalent model of this whole system.

So, we have this object T_B in between we have a sensor which is at temperature T_S . Now this temperature sensor has its own thermal capacity. So, what is the thermal capacity? Thermal capacity is the amount of heat to be supplied to an object to produce a unit change in the temperature. So, we have a object to raise a temperature by 1 degree centigrade of this object how much energy we need to give it to this object. So, that is the thermal capacitance of this object.

It is simply the product of mass into the specific heat of this of this sensor. So, we have this sensor which is here. Now this has some thermal capacity which we can represent using a capacitor which is connected to ground and this capacitor is analogous to electrical capacitor where in electrical capacitor we store charge here in this thermal capacitor we store thermal energy. So, this this is the thermal capacity of sensor. Now this sensor is connected to this body using a certain resistance.

Let us say this resistance is R_1 this is the thermal resistance. So, thermal resistance is analogous to electrical resistance where the electrical resistance is kind of stopping the current from flowing when we apply a voltage across it the higher the resistance the more it stops the current to flow. The same function applies here we have this thermal resistance R_1 which is the resistance between the body and the sensor temperature and it depends on multiple factor first of all is the surface of contact. So, when we are making this contact of this sensor with this body let us say we have this sensor and this is connected to this body. Now we can see only a very small portion of the sensor is connected to the body not all the surface of sensor is connected to body.

It means the thermal energy can flow only through that contact area which is which is in contact with both the object and the sensor. So, the cross section through which the energy can transfer is very less. So, accordingly the resistance is very high. So, this is analogous to electrical terminology. So, this is the thermal resistance which is between this object and the sensor.

This is our sensor and we have this node here. Now this sensor is also connected to a external environment. Let us say we call it thermal resistance R_2 and here this is connected to environment which is fixed. The temperature of environment we consider it the fixed at T_0 degree. So, we may be represent this like a rigid wall sort of thing. So, we have this environment which is fixed. This temperature sensor the temperature of this temperature sensor is rising when we are connecting it to body. Now this rising temperature is also different than the temperature of environment. The environment temperature is T_0 and sensor is T_s . So, there is a difference between these two temperature.

Whenever there is a difference in the temperature the heat energy will flow. The heat will flow from higher temperature to lower temperature. So, assuming that sensor temperature is high and it is being rising this sensor is going to lose some energy from the sensor to the environment. And how fast that energy will be transferred from sensor to environment that depends on the contact resistance of this sensor and the environment which is R_2 . So, now we have two thermal resistances one between the sensor and the object which depends on the contact area how well connected this sensor is with the object R_1 .

And we have second thermal resistance R_2 how isolated this sensor is from the environment whether it is losing some energy to the environment or it is not losing. So, we have these two thermal resistances after the movement of coupling when we connect this sensor physically to the object that is the movement of coupling. So, after this movement of coupling or after the object temperature actually changes the sensor temperature is T_S and the sensor is losing energy from T_S to T_0 because these are different temperature. Almost never these temperatures are equal because the amount of heat energy which is raising the temperature of sensor this also depends the difference between the temperature. So, the higher the difference the more energy will flow and raise the temperature, but when both the bodies come very close into the same temperature the amount of heat transfer will be lesser because it depends on T_1 minus T_2 .

So, lesser energy flow means the rate of change of temperature will be less. So, this follows some sort of exponential behavior where we have this is time and this is the temperature of sensor. Initially because the T_B is very higher than T_S the rate of change will be very fast, but as the time passes and T_S is now very close to T_B then the rising temperature of the sensor will be very less and if we see here there will be certain difference because this is this is an exponential graph and it will take infinite amount of time in reaching equilibrium means when both the temperature becomes same. So, this is not possible to achieve this equilibrium in theoretical ah time limits we practically assume after certain point after long after waiting for so long the system again becomes into equilibrium where there is a heat flow between T_B minus T_S which is charging this thermal capacitance. So, there is a heat flow from T_B to T_S there is a heat flowing from here to here and the same amount of heat is actually transferred from sensor to the environment when the both of these rates are equal it means the system is in equilibrium and sensor reading is stable.

So, that is that condition we call it the equilibrium case when this whole system again reaches into a thermal equilibrium. Now, when both of these rates are equal then we can write the equation for them or maybe we can write here itself when both of these rates are equal, we can write T_B minus T_S divide by R_1 the difference between the temperature of object and sensor divide by the thermal resistance R_1 this rate is equal to T_B minus T_0 divide by R_1 plus R_2 . So, we are saying here this rate T_B minus T_S divide by R_1 this rate is equal to the complete rate from object to the environment it means the sensor is not storing further energy in itself and sensor has come into the equilibrium. So, the rate is

$T_B - T_{\text{naught}}$ divide by $R_1 + R_2$ the total resistance in the path. So, this is the equation we can write if both of these rates are equal and we can now rearrange this equation to get T_S equal to $T_B - \frac{T_{\text{naught}} R_1}{R_2}$.

So, this is the equation or we can further solve this and this comes out T_S equal to $T_B - \frac{\Delta T R_1}{R_2}$. So, this is the equation we get for our system where the sensor temperature T_S is equal to T_B the object temperature minus certain term which is $\Delta T \frac{R_1}{R_2}$. So, this ΔT will always be there we cannot control it much which is the difference between the body and the environment which we do not have any control environment temperature is fixed and the body temperature is what we are going to measure. So, this is what it is. So, this ΔT we do not have much of a control.

For accurate measurement what we need T_S equal to T_B when the sensor temperature is equal to the object temperature that time the sensor will give the accurate reading. Our desired condition is T_S equal to T_B and what we have this equation is T_S equal to $T_B - \frac{\Delta T R_1}{R_2}$. So, now we can note two points here this R_1 which is the contact resistance the thermal contact resistance between the object and the sensor. So, this temperature this thermal resistance need to be as small as possible.

So, we have R_1 should be as small as possible. What do we mean by this statement R_1 should be as small as possible. That resistance between the object and the sensor should be as small as possible means there should be an efficient heat flow the thermal energy flow from the object to the sensor. It should not be limited by the contact area of between these two objects and other factors as well. So, the sensor should have a very good contact with this body the whole of the surface of the object should be in very close contact with the with the object then the resistance R_1 will be very less. Even we can put some artificially epoxy or some other materials to increase this contact area to decrease this R_1 the contact this thermal resistance.

So, we can decrease this R_1 to make this term smaller $\Delta T \frac{R_1}{R_2}$. We either decrease this R_1 or second thing we can do is R_2 should be as high as possible. R_2 should be as high as possible. What do we mean the resistance the thermal resistance between the sensor and the environment it should be very high. It means there should not be any heat flow from the sensor to the environment.

The sensor should not lose any amount of thermal energy to the environment. It means it should be very much isolated the sensor should be very much isolated from the environment then we have R_2 should be very high. So, when we make these R_1 should be very less and R_2 should be very high this second term is going to decrease and we will have T_S as close to T_B as possible by applying this R_1 and R_2 conditions. But these R_1 and R_2 are finite values these we cannot make this 0 R_1 and R_2 no matter how small R_1 is or how high R_2 is this term $\Delta T \frac{R_1}{R_2}$ will always be a finite number and

that will give us some error in the measurement and T_S will never be equal to T_B . So, based on these two analysis what can we do during our measurement to make this error as small as possible.

So, one way of doing R_1 as small as possible is you embed this the sensor into the object itself. So, we try to have more and more contact area between common between these object and the sensor. So, one way is let us say we have this object what we considered earlier which is a temperature T_B this is object to measure. And we somehow dig a cavity inside this. So, we have this object now we dig a cavity in this and we put the sensor deep inside this object.

So, this is our sensor and we put some epoxy as well here and this is connected to the outside world which is at T_0 . So, this is our sensor now which is buried deep inside the object. So, that we can see now this sensor is covered almost from every side by this object it means the heat can be exchanged from all the sides decreasing its thermal resistance R_1 . So, this is one way of one way of making R_1 as small as possible. So, this is this is one technique of making R_1 as small as possible.

We can do in multiple techniques this is engineering task actually now to reduce this contact resistance. We can put some epoxy next to this next to this sensor. So, that because of the surface roughness if the contact is not proper. So, we put epoxy in between. So, that there is a close contact between these object and sensor.

Second we can do is we kind of put a spiral over here for example, this sensor is connected to a external circuitry using this wire. This wire actually we put multiple turns here inside this object only. So, that because this sensor is losing thermal energy through this cable only because this is the connection between sensor and outside world. We are increasing the length of this cable here inside this. So, that the resistance R_2 becomes a little bit higher and sensor does not lose much energy to the outside world.

So, this is one way of making R_1 smaller. Next technique we can apply where we can make R_2 as large as possible. So, we are making R_2 large. So, how we can do that? Let us say we have this is the body or the object at temperature T_B and we connect our sensor to this. Now we do not want this sensor to lose any energy to the environment, but the problem is environment temperature is fixed which is at T_0 we cannot do much about it and the sensor temperature is rising. So, there is a difference always between the sensor temperature and the environment whenever there is a temperature difference heat is going to flow.

Whatever we put in in between these objects everything has certain amount of thermal conductivity higher or lower that depends on material, but it is always there it is finite. So, the sensor is always going to lose some energy to environment because of this temperature difference. So, now we can apply a technique which can be used to reduce this error. The problem here is the temperature difference the sensor temperature is different than

environment temperature. So, if we can somehow give a artificial environment to this sensor where this local environment the temperature of this local environment is same as sensor temperature then we do not have any temperature difference between sensor and local environment and then there will be no heat loss or the heat energy will not transfer from sensor to an that local environment.

So, this we can do like this for example, we have this sensor connected through this cable. Now, we put an artificial cavity around this let us say this is artificial cavity this cup sort of thing which is inside this there is some space and sensor is inside that cavity. So, let us say we have this cavity now this cable comes out from this cavity and this cavity temperature is also being tracked through other cable which goes to some instrumentation amplifier also. Let us say we have amplifier coming in which is plus and here it is minus and then it is some sort of driver circuit which is controlling this. So, we have this cavity which is containing this sensor the temperature of this cavity also can be controlled using this driver circuit.

This driver is controlling the temperature of this cavity and here we have this comparator plus and minus plus comes from the sensor and minus comes from the cavity. Now this comparator is tracking the sensor temperature and bringing the cavity temperature this is cavity. So, this comparator tracking the sensor temperature and adjusting the cavity temperature so that they are matched whatever is the sensor temperature at that point the cavity will be heated up or cooled down so that there is no temperature difference. So, when there is no temperature difference between this sensor and this local cavity so there is no heat exchange between these two objects so this heat exchange is not there. So, in that way we can reduce this heat exchange between sensor and the environment and we bring down that error by the error factor $\Delta T \frac{R_2}{R_1}$ upon R_2 to the minimum possible.

So, this is one way of making R_2 large. This thermal shield which is here which we have made artificially this is usually made by metals or some aluminium which the metal should have very good conductivity so that we can adjust the temperature of cavity as soon as the temperature of sensor is changed. So, these are the techniques where we can minimize this error R_1 upon R_2 into ΔT by applying certain techniques. Now let us consider the dynamic case. The dynamic case is when you connect the sensor to the object and the temperature of sensor start rising and it has not reached equilibrium yet. So, let us consider the dynamic case where the temperature is still changing with time.

So, now first we need to make some assumptions to simplify our calculations and these assumptions we will relax later on as we go along. So, the first assumption I am making here is the thermal resistance between the sensor and the environment is very large means the R_2 the amount of R_2 is very high the sensor is not losing any energy to the environment.

So, this is the first assumption I am making. The thermal resistance between sensor and environment is infinite.

R_2 tends to infinity. So, this is the first assumption I am making to solve my system that the R_2 is infinite the sensor is not losing energy to environment. The second assumption I am making here is the object temperature does not change when I connect the sensor to this. It means the size or the thermal capacity of The object is very very high compared to the sensor. The sensor thermal capacity is very small or the sensor is very small in size compared to the object. So, this is the second assumption object temperature does not change after sensor is attached.

So, these are the two assumption I am making before proceeding to solve the dynamic case. Now, I can write the heat transfer. This I can write ΔQ equal to $\alpha (T_B - T_S)$ into ΔT . So, this is the heat transfer equation and the same thing I can write in terms of the thermal capacity of the object as $M C$ into ΔT . So, these are the two equations for heat transfer where this C is the specific heat and M is the effective mass.

So, when I put both of these equations equal to each other then we have $\alpha (T_B - T_S)$ into ΔT equal to $M C \Delta T$ or we can define a time constant which is τ_T equal to $M C$ divided by α or M into C into R_1 . So, these are all the constants this $M C$ and α these are all constant we can put all of these constants together and call it with a different name which is the thermal time constant of the system and this is very much analogous to the RC time constant we have in electrical system. So, this thermal time constant exactly the calculations are same just like the RC network within for example, in one thermal time constant temperature reaches almost 63 percent of the initial gradient. So, this time constant is exactly same as electrical time constant we have. So, now coming back to this equation we have $\alpha (T_B - T_S)$ into ΔT equal to $M C \Delta T$ we rearrange the terms and putting this time constant we have ΔT divided by $(T_B - T_S)$ equal to ΔT divided by τ_T which is the time constant or you can write the final equation which is T_S equal to $T_B - \Delta T \exp(-t/\tau_T)$.

So, this is the complete equation of the system assuming we have made two assumptions after those assumptions we have this is the equation for the system. Now we can plot this equation the x axis is time and this is very very much similar to the RC charging equation, but on the x axis we have time and the y axis we have temperature here instead of the potential and the final the temperature of the body is T_B which the sensor is going to achieve in infinite time which is T_B and the actual rise in the sensor temperature is something like this exponential. This will achieve in infinite time, but here now we can define the thermal constants which is τ_T . So, this graph says that sensor temperature achieves almost 63 percent or 0.63 of T_B in one time period. In one time period sensor temperature will rise to 63 percent of the initial temperature difference and of course, in second in two time period it will reach certain amount and in third in fourth in a almost 5

or 6 time constant it will practically achieve the temperature of object this is the temperature change of the sensor. Now we study a more realistic case when we start removing those assumptions what we made earlier we made two assumptions. So, first now we are going to remove let us remove the first assumption which is that the resistance between the sensor and the environment is not infinite it is has certain finite value it means instead of achieving a final value of T_B the sensor is never going to achieve that final value T_B because there will always be a heat exchange between the sensor and the environment which will keep this temperature always lesser than T_B . So, after removing first assumption carbon tends to infinite this is the condition we are removing it means the sensor steady state value will not be the object temperature T_B , but somehow to a lower value than T_B . So, let us say this is T_{B1} which is the body temperature now because sensor is losing energy to environment it will reach to a steady state temperature to a different temperature which is not exactly T_B , but let us we call it T_{B1} where T_B is not equal to T_{B1} both the temperatures are unequal.

So, there is there is a difference between the final value of the sensor. So, this happens because of the finite value of R_1 . Now let us we remove the second assumptions as well which we made earlier. So, after removing second assumption which is the object is not very big and the sensor is not very small. So, if they are comparable or at least not negligible then as soon as we connect the sensor to the object, object has certain thermal energy stored within as soon as this thermal energy transfers to sensor that there will be a certain amount of dip in the temperature of object because certain amount of energy is transferred to some to the sensor.

So, if we plot the response after removing this assumption. So, this is T_B and because of the first assumption we have different value T_{B1} the final value of sensor. Now as soon as we connect the sensor there will be a certain dip in the characteristic of sensor voltage change and this is because of the finite mass of object. So, instead of having a smooth curve now we have a dip in the sensor temperature because of the finite mass of object or the sensor size is comparable to the object. So, here we considered the dynamic case where when we connect the sensor to the object what will be the time response of the system.

So, this is all for today.

Thank you.