Principles of Digital Communication
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Detection
Lecture – 38
Fading Channel

Good morning. Welcome to the last lecture of this course. And in this lecture, we will be talking about fading channels ok.

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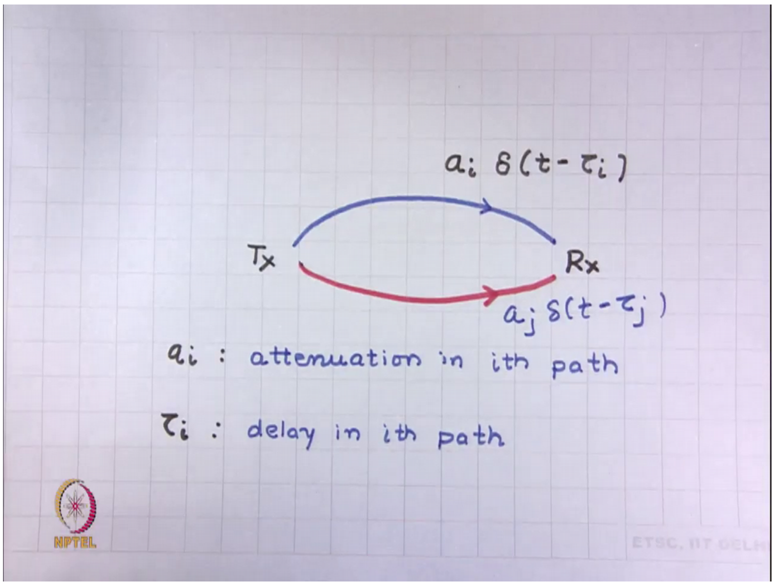

And today we will see what happens when we go to wireless domain. So, let us try to see what happens when you have a wireless transmission medium. Let us assume that we have a transmitter T x and a receiver R x. Transmitter transmits some signal, and this signal is received by the receiver. But because this transmission happens through electromagnetic waves, this signal may take different paths to reach a receiver ok. So, here I have shown two paths right, but there would be infinite paths between transmitter and receiver.

If I look at one of the path then what would happen because of this transmission media is that my signal will get attenuated with some constant a i, and the signal will be delayed with this time unit tau i. So, what I say is a i is attenuation in ith path and tau i is the delay in the ith path ok. And the channel in this path will create these two effects, namely it will attenuate the signal and it will delay the signal because of the propagation delay ok.

Now, similarly you can and research that this path will also create certain attenuation let us say a j and it will create certain propagation delay which I say as tau j ok. And now you can see as what is happening is that the signal is reaching the receiver at different time instances. For example, this signal will reach the receiver at tau i time instance, and this signal will reach the receiver as tau j time instance ok. So, how can I think about the channel?

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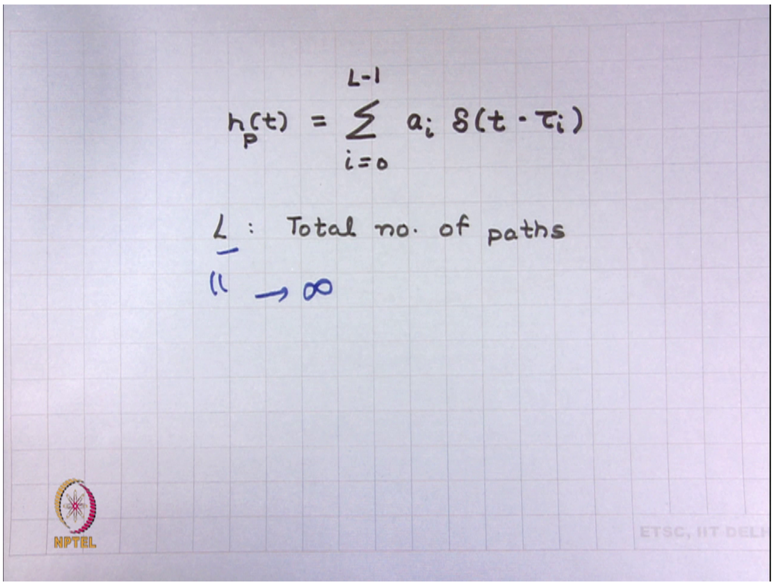

So, I can think about the impulse response of the channel as this, and where this would have contributions from L paths. Let us assume that there are L paths between a transmitter and receiver. This L might be pretty large number; it might tend towards infinity ok. So, in each path the attenuation is a i and the delay is tau i ok. And this is the impulse response of the passband channel. So, here the p represents that this is a passband channel, so that is what happens typically in a wireless communication ok.

So, the way to start modelling your channel is by assuming that there infinite path or large number of paths between a transmitter and a receiver. And you can think that in each path the effect would be that your signal will be attenuated by certain constant and it would be delayed by a certain propagation delay ok. And the net effect will be the sum of the individual effects.

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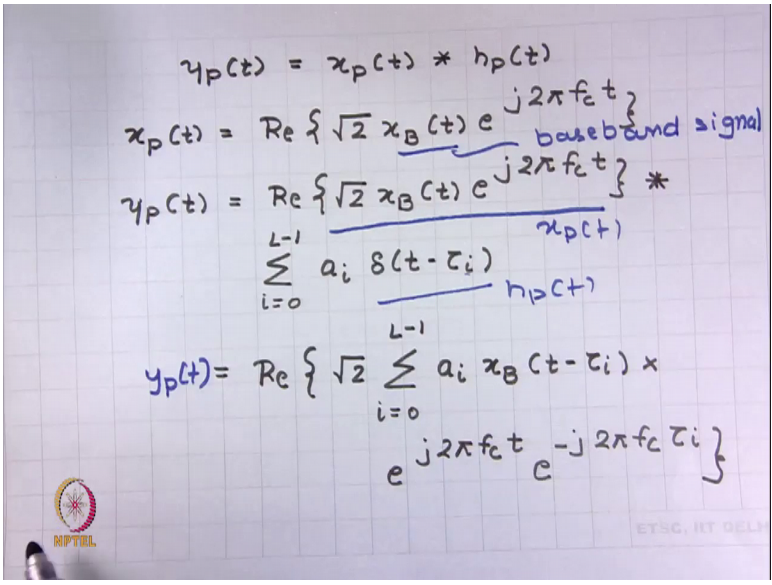

If I have to think about the received passband signal, the received passband signal can be obtained by convolving the input where the impulse response of the channel ok. This is clear we have been doing this all the while. Now, if I want to think about this passband signal, this passband signal can be thought in terms of the baseband signal. So, x B t is the baseband signal. Now, so this passband received signal can then be obtained by convolving this x B t with h p t. So, this is x p t represented in terms of the baseband signal, and this is h p t we have seen this.

Now, if you convolve this with this, what would happen is simply that this signal which was at t will shift to t minus tau i ok. We have seen the impact of convolving a signal with an impulse, the only effect that it has is it shifts the signal to the time where the impulse exists. So, this impulse exists at tau i. So, this signal will also shift to tau i ok. So, now, changing this t to t minus tau i, we get this; and then I have to change this t to t minus tau i, and I get two complex exponentials ok. So, this is actually your y p t, simple.

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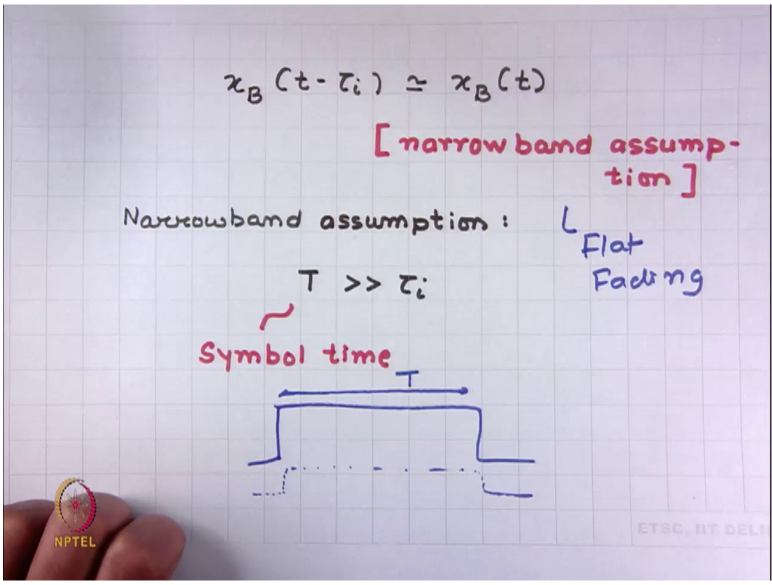

Now, to solve this further, I am making an important assumption here, I am making an assumption that x B t minus tau i is approximately same as x p t and this assumption is known as narrowband assumption. This assumption is true, in what is known as flat fading channels or a narrowband channels. So, in narrowband channels, you can approximate the signal x B t minus tau i as simply x p t. And what it simply means is that if the symbol time of the signal is much larger than this tau i, then its effect is negligible.

For example, if I have a signal whose symbol time is pretty large, and if I shift the signal with a very small delay tau i, so this signal shifts like this, then actually you can treat this signal same as the signal ok. This is the idea behind narrowband assumption. That means, if the data rate of the signal is pretty small, then this narrowband assumption holds. If the data rate of the signal increases, then this narrowband assumption might not be true. So, what we can say is if the symbol time is much much greater than tau i where tau i is the delay created by a path of the channel, then I can simply use this narrowband assumption ok.

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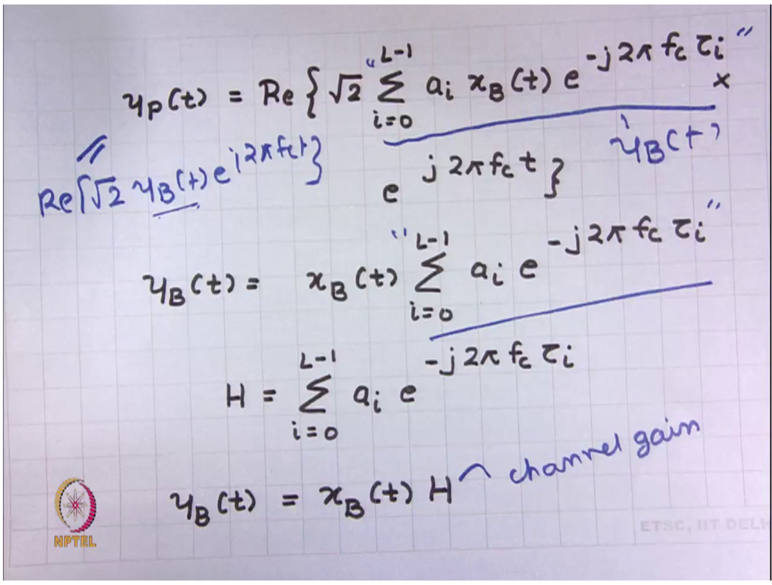

Now, using this narrowband assumption, I can write y p t as simply this. So, what I did is I simply have changed x p t minus tau i as x B t ok. Now, if you think carefully then this is actually y B t, because this is a passband signal. This passband signal can also be thought in terms of is baseband signal. So, I can think about this y p t as also real part of root 2 y B t times e to the power j 2 pi f c t ok. And y p t is this. So, I can simply say that y B t which is the baseband equivalent of this passband signal is simply this thing ok, which I write it like this.

Now, because x B t is not a function of i, I can also pull this out of the summation and I can simply obtain that y B t is simply x B t times this quantity. And this I call as H which denotes the channel gain ok. So, I take this quantity and I simply call as H. Now, this H is not a function of time right. So, it is a constant with respect to time. So, I can write this y B t is simply x B t times H ok. So, H remember is the channel gain and this channel gain is a still time invariant ok.

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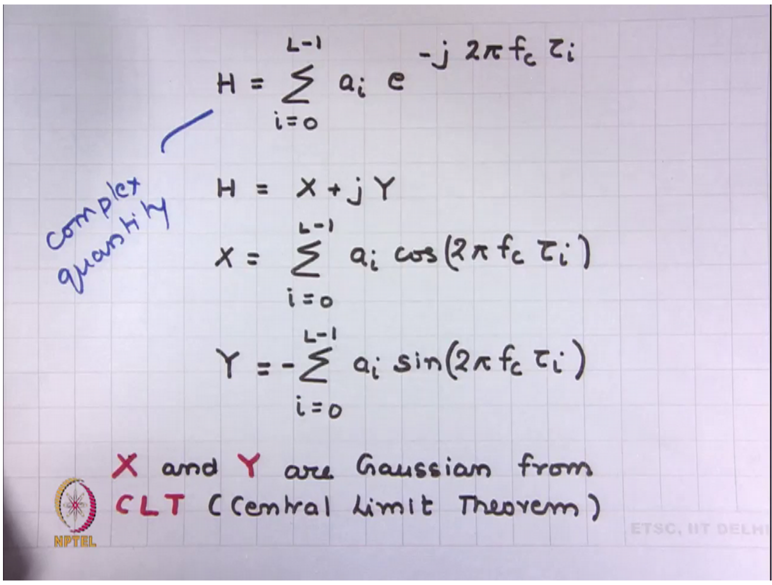

Now, so what we have seen that edge is simply this quantity. So, it is sum of the channel gains of each path, and there are L paths. Now, in general this H is going to be a complex quantity, and it can be a complex quantity because we are thinking about the baseband signals ok. Baseband signals can be complex signals right. So, H in general is a complex quantity because of this term sitting in over here. So, I can write this H as simply X plus j Y ok, where X is simply this thing. So, I have broken down this complex exponential using cos and sin. So, I get X as simply this thing and Y as simply this thing ok.

Now, what is this quantity? So, you have several random variables here and this is a sum of large number of independent random variables. And we have seen in one of the previous lectures that if you have sum of large number of independent random variables by central limit theorem, the resultant random variable will be a Gaussian random variable, and hence this X is a Gaussian random variable.

Similarly, Y is also a sum of large number of independent random variables, and hence y is also a Gaussian random variable. Why these random variables are independent, because we are assuming that the attenuation and the delay of each path is independent of the another path. And hence each term will be independent of the another term. And hence X and Y are sum of large number of independent random variables; and by central limit theorem they will tend to be Gaussian.

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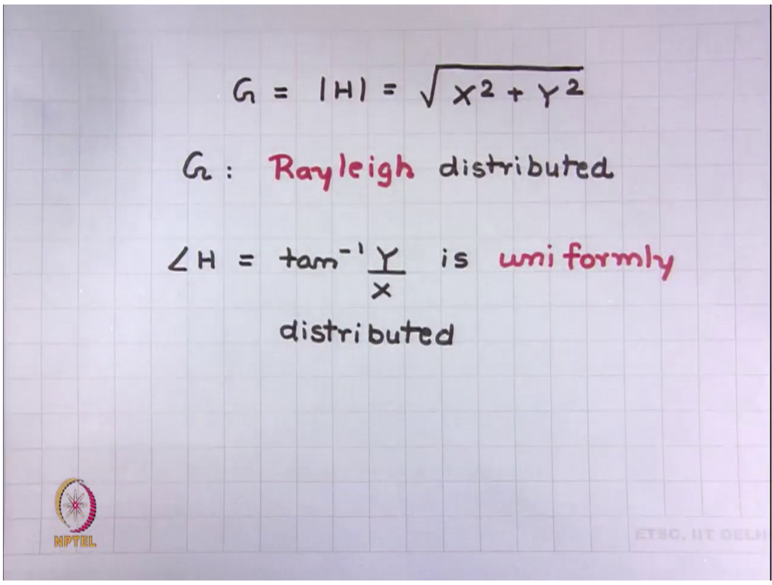

Now, we can define G which is mod of H, and thus the random variable G is simply square root of X square plus Y square and this is what we have seen before. G is a Rayleigh distributed random variable. And if I want to think about the phase of this H, the phase of the H is simply tan inverse Y by X. And this we have seen that this phase of H is a random variable which is a uniformly distributed random variable. So, this is what we have seen in the last lecture as well. Let us revise this Rayleigh distribution.

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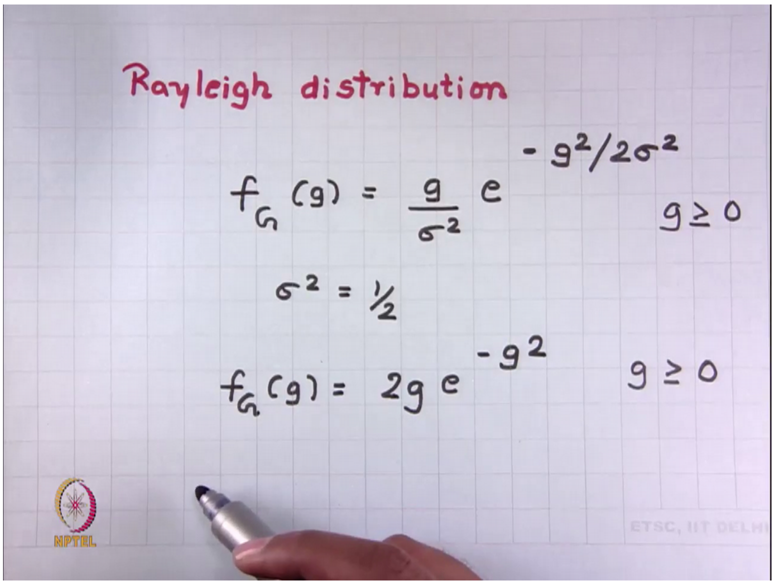

So, we have seen that the probability density function of Rayleigh distribution looks like this ok. And if you assume the variance to be half, then the probability density function of the Rayleigh distribution looks like this ok. So, for the derivation in this lecture, we will assume that the variance is simply half ok.

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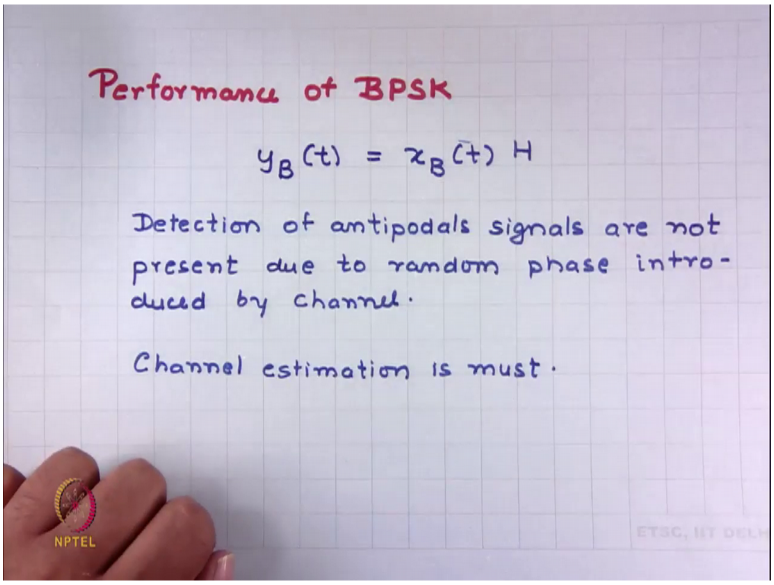

Now, with these assumptions, let us see if we can think about the performance evaluation of binary phase shift key. So, let us look at what happens in the baseband domain. So, in the baseband, you are receiving a signal y B t which is simply the transmitted baseband signal x B t multiplied by this H or complex channel gain.

Now, we have seen that this H induces a random phase, because its phase is uniformly distributed. So, they can be any phase angle which this H can assume. And thus this y B t also have can have any phase. Now, thus you know that if you want to detect BPSK, the information lies in phase. So, so this signal might have a phase of zero degree or this signal might have a phase of 180 degree.

Now, if this H induces a random phase, then this y B t also has a random phase, and thus the detection of x B t is not possible ok. And thus when you want to use these modulation schemes which embed information in phase, the only thing you can use them is by learning what this H is. If you know what this H is, then you know what the phase this H creates and then you can detect x B t. So, one of the assumption that we have to make is when we are detecting these modulation schemes, where information in valid phase is that we have to know what the channel is ok. So, the gist channel estimation is must.

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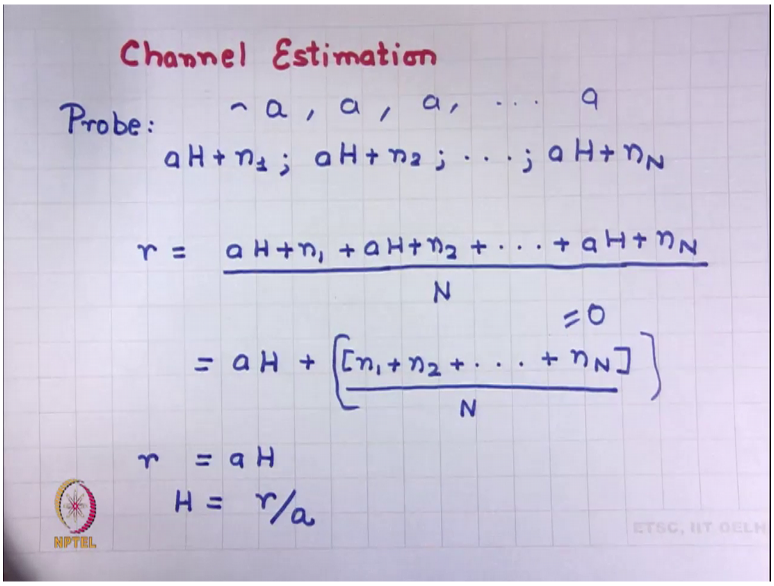

How can we estimate channel? Ok, this is a very simple idea. They are more sophisticated ideas of estimating channel which we will not look into, but simply what you can do is you can send a probing signal N times ok. So, I can think about this, I am transmitting a signal a, then I transmit a signal a, then I transmit a signal a, then I transmit the signal a, and I transmit a particular signal N times, and receiver also knows that I should be receiving this signal a ok. So, you decide a time frame in which you transmit are known sequence to the receiver. Receiver knows what it should be expecting.

And let us assume that we are transmitting that probe signal which has the signal value of a. What receiver receives is a H plus n 1. So, this a multiplies by this H some noise adds we call the sample of that noise as n 1, and similarly it receives n such values. What the receiver can do is simply it can take the sum of these n values and it can divide by N ok. So, what it would get is a H plus this thing. And what is this? This is the mean value of noise, and the mean value of noise is 0. So, the value of r that receiver would see than is simply a times H; and from this receiver can estimate H by dividing this r by a.

So, r is known because these samples are known. So, it can sum up these values of these samples. So, r is known to the receiver; a is also known to the receiver, because receiver knows what transmitter would be transmitting. And from this another receiver can estimate the value of H. Of course, this problem is really simple and trivial because what we have assumed is that this H remains constant across symbols that means, channel does not vary across symbols all right. And in reality this will not be the case and thus this problem is little bit more complicated than what we have presented ok.

So, what we are saying in this case is let us assume that the channel remains constant for certain time ok. And in that time duration, I am transmitting n symbols by receiving the samples values corresponding to n symbols and by summing them up and by dividing by N receiver will simply receive a times H, where a is the numerical value of the probe signal, and H is the channel gain. And thus from r receiver can simply calculate H. So, the point is when we are thinking about these modulation schemes H must be known all right.

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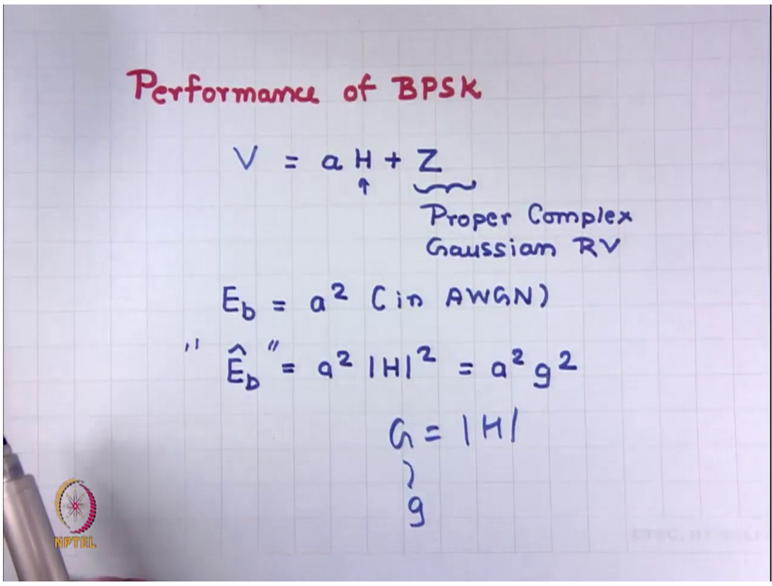

Let us look at one small point. Now, what receiver is receiving let us say it is receiving a random variable V which is not important how you call this random variable because we will not be using this random variable anymore in this lecture. So, I could have called this anything. I have called this as V, which is simply a times H plus a Z. And what is this Z; Z is a proper complex Gaussian random variable. H is complex right, that means, we are thinking in the baseband domain. So, z should also be a proper complex Gaussian random variable. H is also a proper complex Gaussian random variable. These complex

Gaussian random variables are always this proper complex Gaussian random variables, and thus I simply also call them sometimes complex Gaussian random variable ok, because whenever you see that is a complex Gaussian random variable, it has to be a proper complex Gaussian random variable, because i and q channels has to be independent, and it is also a good assumption to assume that noise in these channels is also identically distributed ok..

So, anyways, so what I am receiving is a times H plus Z. Now, we have seen in AWGN channel, we like to think about this energy per bit. And in case of BPSK, this is simply a square ok. Now, in these fading channels, what is this energy per bit, it is a square multiplied by mod of H square. And if I call G as mod of H, the numerical value of G, I call as a small g. And thus energy per bit in fading channels is simply a square times g square ok, where g is the numerical value of random variable capital G, and where this random variable capital G is simply mod of H. So, what happens in fading channel is your energy per bit changes by this channel gain ok.

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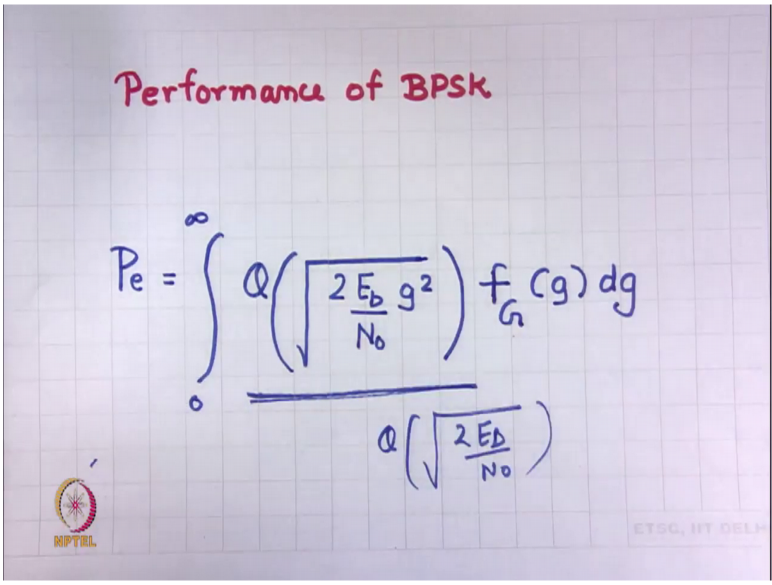

Now, if you have to think about the performance of binary phase shift key, we have seen in one of the previous lectures that probability of error in case of BPSK is Q of under root of 2 E b N o. And in case of fading channel this E b becomes E b times g square because of the reasoning that we have seen. So, this is the probability of error given a value of g, but g can take any value. And thus to find out the average probability of error, you have to multiply this quantity when the probability density function of random variable g taking in a numerical value small g, and then you have to integrate this thing over all possible values of g, g lies between 0 to infinity because it is a mod of H ok.

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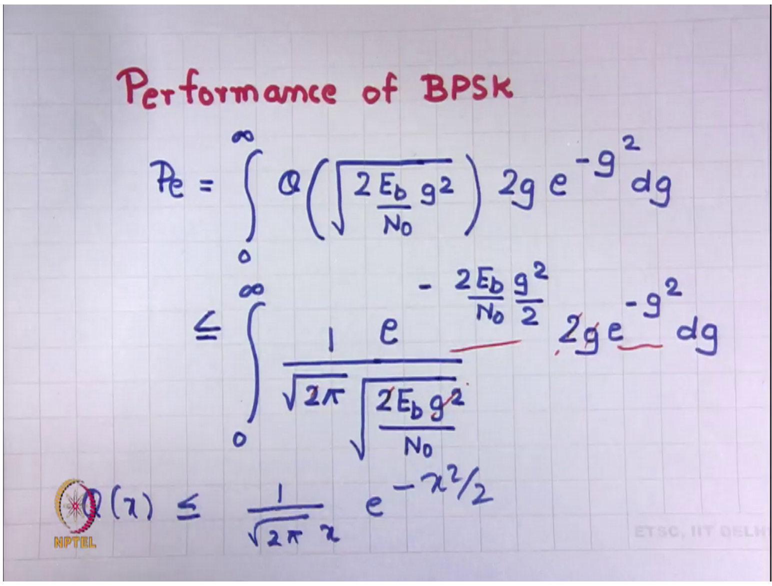

Now, when we have to solve this, let us look at the bound of this expression. We know that the probability density function of g is this. And we can use one identity that Q of x is always less than 1 by root 2 pi x e to the power minus x square by 2. So, this will give me substitution of this thing in place of Q of x will give me an upper bound on the probability of error. So, substituting Q of x as this thing, and multiplying by this thing we can get an upper bound on the probability of error ok. Now, let us try to see how can I solve this up. So, this cancels with this, this 2 cancels with these two 2s ok. And I can combine these two terms.

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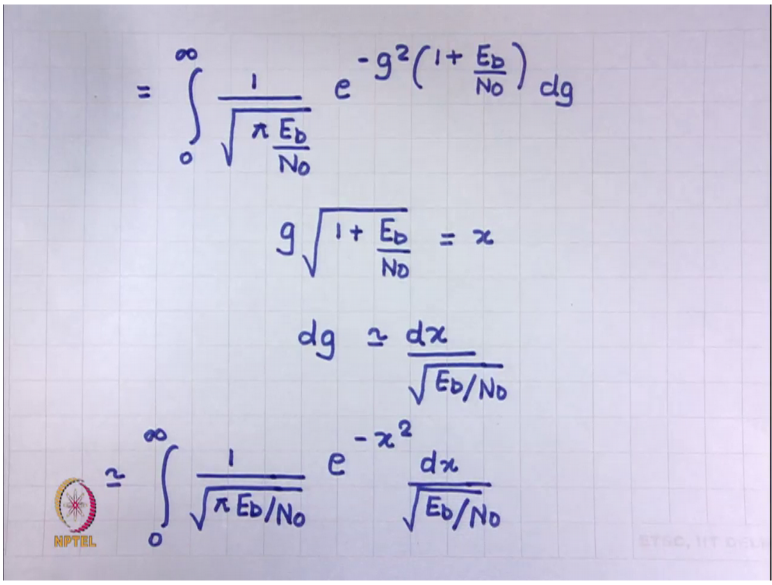

What I get is this expression. Then I can substitute g times under root of 1 plus E b N o as simply x. If E b N o is pretty large, I can approximate d g as simply d x by root of E b N o. I can replace this thing by e to the power minus x square and I can replace this d g by d x divided by root of E b N o. And I can solve this up I can combine this E b N o with this E b N o.

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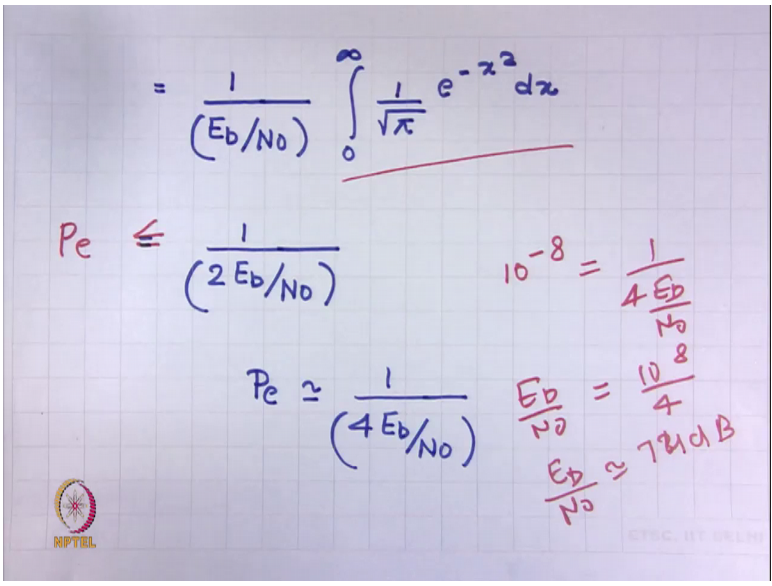

And I can solve this up to this expression. And I can solve this integration also by thinking this as Gaussian PDF. And by knowing that integration of Gaussian PDF from minus infinity to plus infinity is 1 ok, using this ideas try to think about what is the value of this integration I have given you a hint. So, you know that this looks like a Gaussian PDF. And you also know that if you integrate the Gaussian PDF from minus infinity to plus infinity, you should get a 1, because it is a PDF So, using this solve this, and what you get is probability of error should be less than 1 by 2 E b N o.

If you do it exactly what you can see is probability of error will be approximately 1 divided by 4 times E b N o. And let us see what is the E b N o requirement if you want to achieve a bit error rate of 10 to the power minus 8 ok. So, if I want to think it, so E b N o will be 10 to the power 8 divided by 4. And this is a pretty large E b N o that we need to have. So, what we see now is that in fading channel the probability of error varies proportionally to 1 by E b N o, and for low error rates the E b N o requirement is pretty large ok.

So, if we want to think about this increase in dB, the E b N o requirement would be around 74 dB ok. In AWGN channel, this E b N o requirement was 12 dB; in fading channel the E b N o requirement increases up to 74 dB ok. So, in fading channel that is the problem. The E b N o requirement increases to pretty large values and thus it prohibits the use of these simple modulation schemes in case of fading channels. We need to do something else to improve the probability of error.

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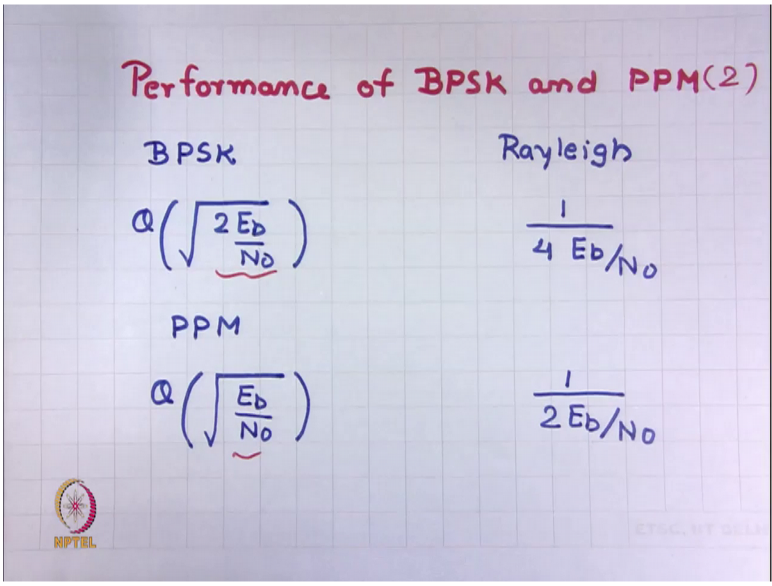

So, let us try to summarize this. The performance of BPSK in AWGN channel was this in Rayleigh channel this becomes 1 by 4 times E b N o. For PPM, the performance in AWGN channel was this, this we have seen when we have looked at the performance of PPM in AWGN channel and this is rather two PPM ok, binary PPM right. And in case of Rayleigh, this will simply be 1 by 2 E b N o. Using the same ideas that we have used for calculation of probability of error of BPSK in Rayleigh channel, you can simply use those ideas solve that integration and things like that, and you will get that the probability of error of PPM in Rayleigh channel is simply 1 by 2 E b N o ok. You can also relate this to these coefficients.

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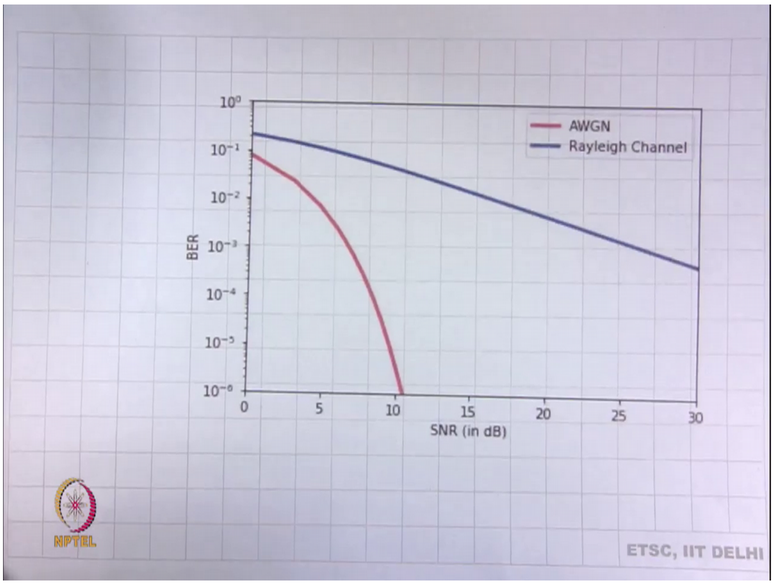

So, let us plot these curves. So, in AWGN channel we have seen that the bit error rate decreases following a waterfall curve with SNR, but here in Rayleigh channel this is a linear decrease if you plot the SNR in dBs, and bit error rate in log scale ok. So, this decrease in Rayleigh channel is much smaller than the decrease that we could have in AWGN channel. Let us move on.

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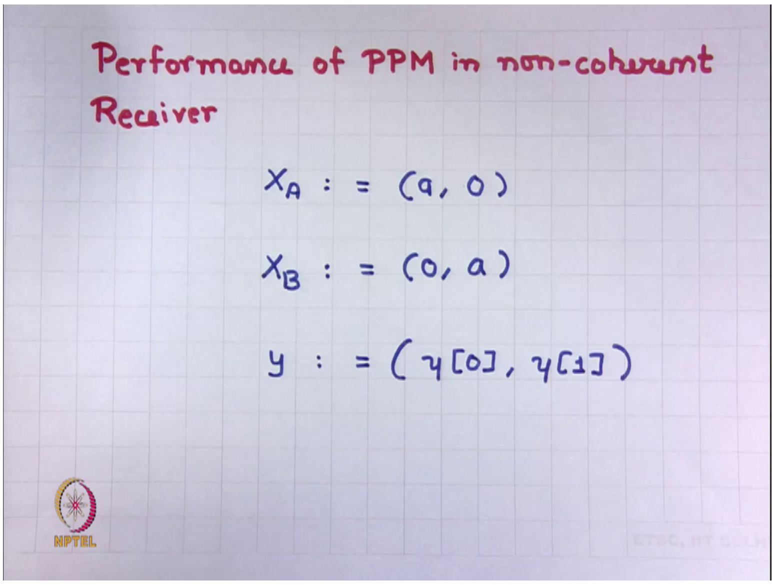

And before trying to think what we can do let us also estimate the performance of PPM in non-coherent systems. So, we have seen just now the performance of BPSK in fading channel. And what we assume there is that the channel information is known to us right. Now, we are going to look down into the performance of modulation schemes when we do not really require the channel information. It would be a good idea because learning channel would be creating some overheads and some issues.

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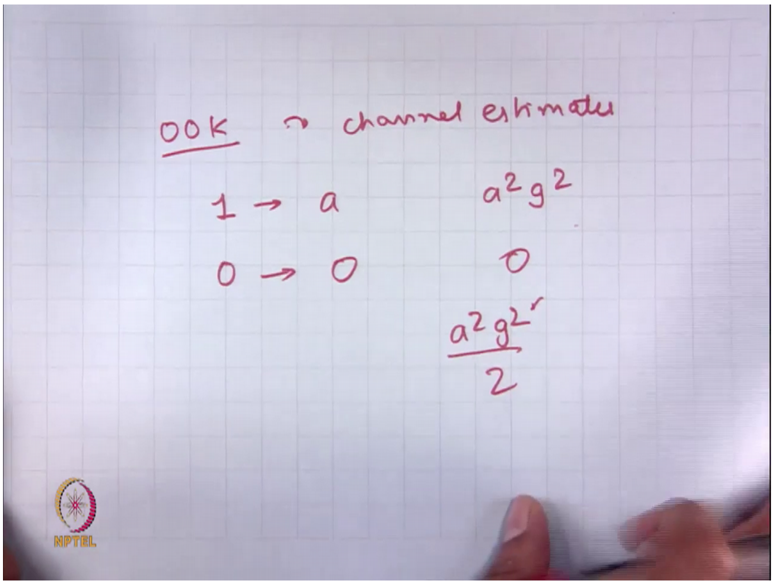

So, before going to discuss the performance of PPM which we will focus now in non-coherent channels, let us first ask the question can we detect OOK modulation scheme without having the channel estimates? An answer is no, because when you transmit 1 let us say we transmit a signal a, and the energy that you would receive will be proportional to a square times g square.

When you transmit 0, you transmit nothing and then the received energy will be 0. So, where will be set threshold assuming that this channel is hit with additive white Gaussian noise, and these symbols are equiprobable symbols. The threshold will sit at a square g square by 2. And thus to take a decision receiver must know what this g is not it. And thus in these OOK systems learning channel is necessary right.

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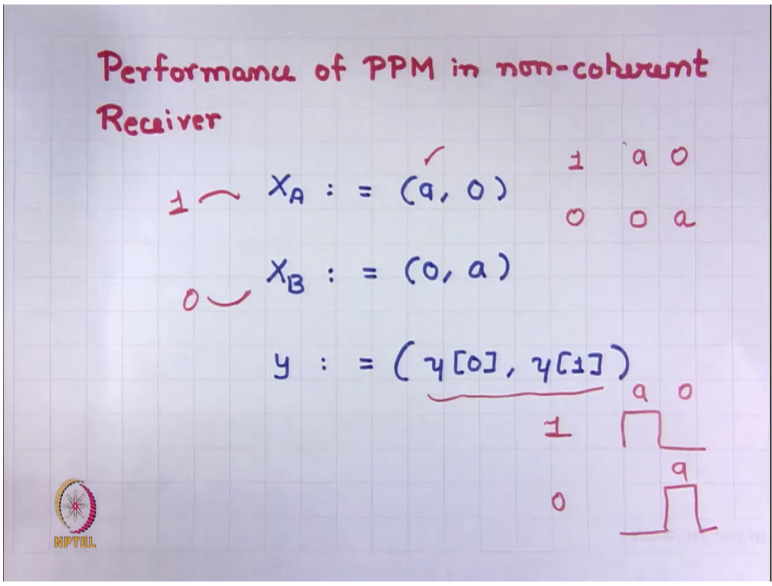

But now we are going to see the system or a scheme like PPM, where you do not have to know anything about the channel ok. So, in PPM essentially what we have seen is that lets say again we are focusing on binary PPM. So, let us say if you want to transmit 1, we transmit a and 0; and when we want to send 0, we will send 0 and a right. So, in this is in terms of numbers in terms of pulse position, you can think that if you want to transmit 1, we will send a pulse and nothing. So, this corresponds to a and zero and when we want to send zero we will send 0 and a ok, so that is what happens in pulse position modulation.

So, here we are saying this if you want to transmit one or let us say X A, X A is defined as a combination of a and 0, that means, in first symbol time we will be transmitting a, in x symbol time we will be transmitting 0. Now, to transmit 0, we would be sending X B, where X B is combination of 0 and a, that means, in first symbol time we will be transmitting 0 in next symbol time we will be transmitting a. Now, we can define an observation y which is built using two observations y 0 and y 1. And looking on this y 0 and y 1, we have to take the decision about these xs and x ps ok, so that is the goal.

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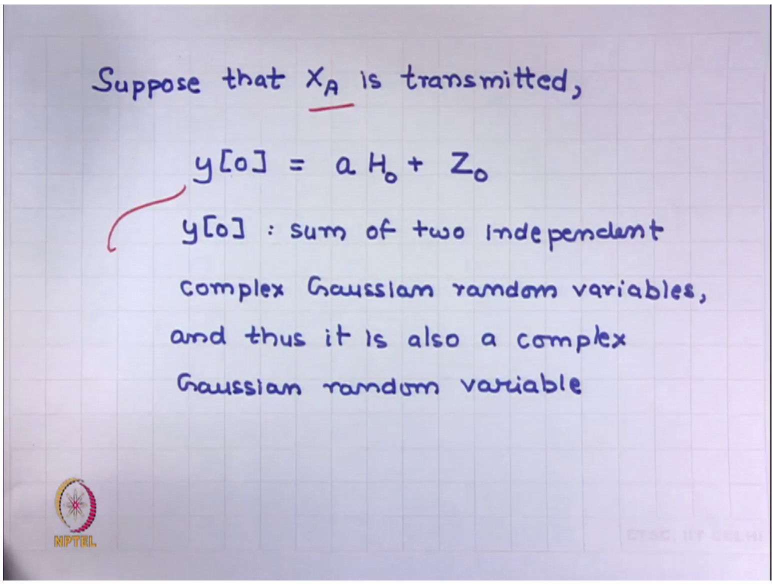

Now, let us assume that we are transmitting this X A ok, this is what we assume. Now, what would be y 0, y 0 would be a times H of 0 plus Z of 0 ok, where Z of 0 is the complex number created by the noise in the first symbol time ok, and H of 0 is a complex number created by the channel in the first symbol time. This y 0 it is a sum of two independent complex Gaussian random variables complex or proper complex does not matter.

So, because complex Gaussian random variables are always proper complex Gaussian random variables, and thus I have simply stated them to be complex Gaussian random variables. But you know that when I am saying complex Gaussian random variables, it simply mean that these random variables are also proper complex Gaussian random variables ok; y 0 a sum of two independent proper complex Gaussian random variables and thus y 0 is also a proper complex Gaussian random variable ok.

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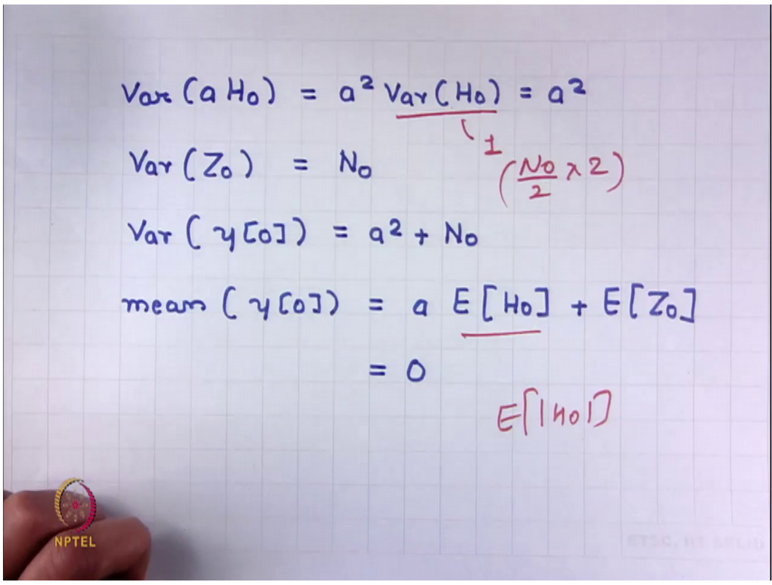

So, if we want to find what is this variance of a times H naught, this is simply a square times variance of H naught. And we have already assumed that this channel has a variance of half in one real dimension, thus this will have a variance of one in complex dimension ok. So, variance of H naught is simply 1, and hence variance of a times H naught is a square. What is variance of Z naught? We know that noise has a variance of N naught by 2 in one real dimension, and because this is a complex Gaussian random variable, it will have a variance of N naught in one complex dimension. So, N naught by 2 times 2, because here we are thinking in terms of complex dimensions.

So, what is variance of y 0? Variance of y 0 is simply a square plus n naught right because these are independent random variables. And there is the variance of y 0 is simply obtained by adding the variance of a H naught and the variance of Z naught. What is mean of y 0, mean of y 0 is a times mean of H naught plus mean of Z naught.

And this mean of Z naught is 0, because it corresponds to a noise; and mean of H naught is also 0, and hence mean of y naught is 0. Remember here we are taking mean of H naught and not mean of mod H naught ok. Mean of mod H naught will not be 0, but mean of H naught will be 0, because it is a proper complex Gaussian random variable.

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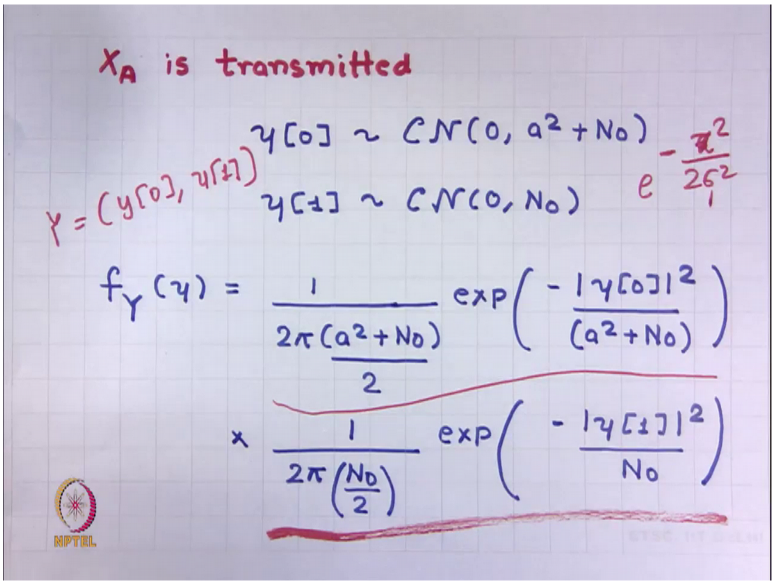

Now, if X A is transmitted this is what we are saying y 0 will be simply a proper complex Gaussian random variable with mean 0 and variance of a square plus N naught. Similarly, y 1 will also be a proper complex Gaussian random variable with mean 0 and variance of N naught right.

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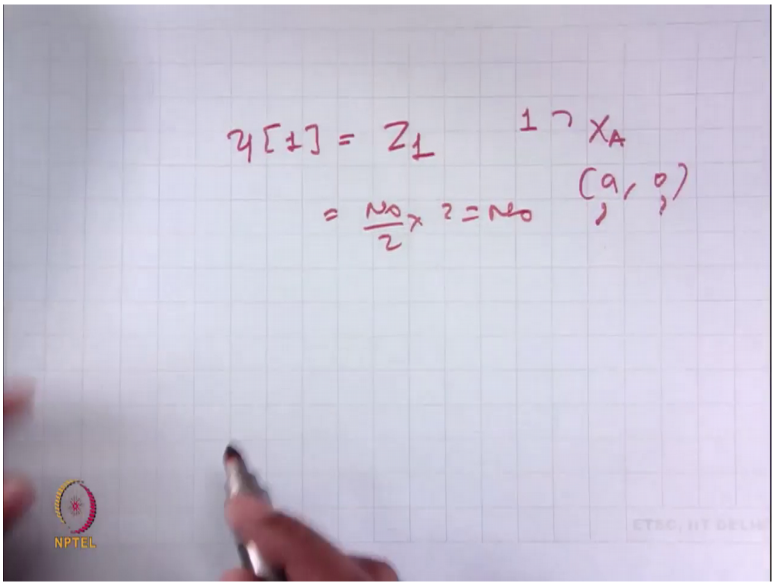

In y 1 what would happen in y 1, y 1 is simply Z 1 is it not, it there is no signal present in this symbol 1 if hypothesis 1 is transmitted which corresponds to X A being transmitted. And X A is simply understood to be a combination of a and 0. So, in symbol 0, you transmit A; and in symbol 1, you transmit nothing. And hence y 1 is simply Z 1 and hence the variance of y 1 is simply N naught by 2 times 2 which is N naught ok. And hence y 1 is a proper complex Gaussian random variable with mean 0 and variance of N naught.

So, from this we can derive the probability density function of y which is defined to be composed of y 0 and y 1. And these two received numbers are also independent, and thus the PDF of y can simply be obtained by multiplying the marginal PDF of y 0 and y 1. Now, this is the marginal PDF of y 0. It has got a mean of 0 and it has got a variance of a square plus N naught. This variance is in complex dimension, and hence you do not have a term two in here ok.

So, we have sometimes a term like 2 sigma square, remember then the sigma square must be in real dimension. If we are writing this variance in complex dimension, then we do not have to have this 2 in here and hence this does not have a two in here. Similarly, you have 2 pi multiplied by sigma square in one real dimension. So, sigma square in one real dimension is simply a square plus N naught by 2. And again this factor two will cancel out. Similarly, you can find the marginal PDF of y 1 just the variance in this case is N naught instead of a square plus N naught ok; and multiplying these two terms we can get the PDF of y.

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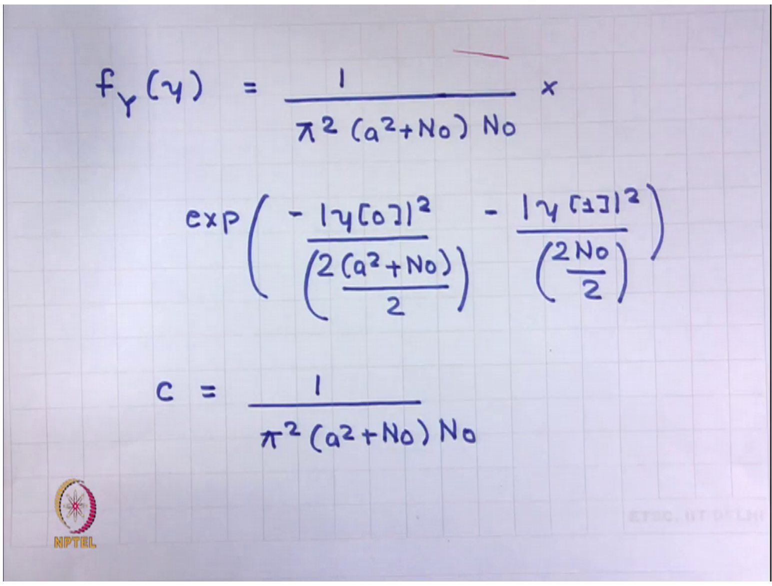

So, I can write this in this form. I have combined these two marginal PDFs and I have written it like this. Moreover I define a constant c which is just this thing just to make my equations more compact, we will see that this does not influence my threshold ok. So, I have reduced this to a constant c.

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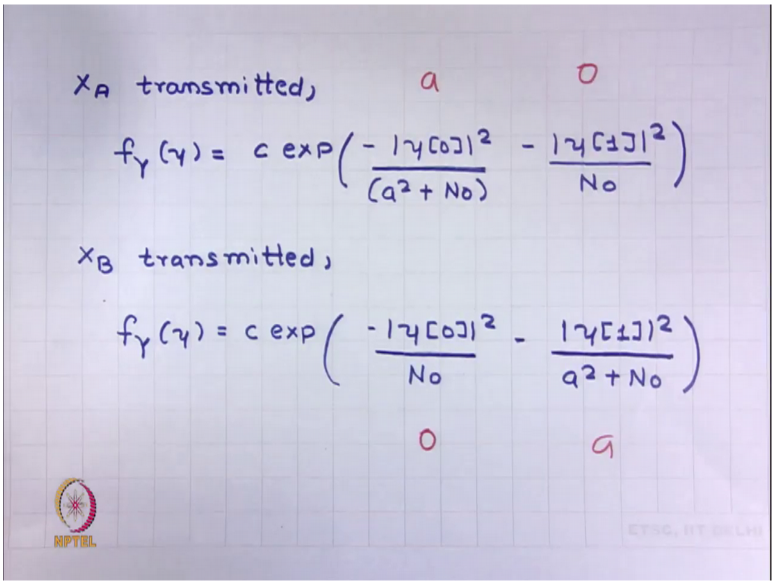

Now, when x is transmitted PDF of y is simply this we have derived this here just compacting the equation. And if X B is transmitted my PDF of y will be this. What changes, in case of X B; we have nothing for symbol 0 and we have a for symbol 1. And hence the variance in this case is a square plus N naught; the variance in this case is N naught. The things that were different here is that the variance in this case is a square plus N naught; and the variance here is N naught, because in X A we are transmitting a and 0 ok, easy.

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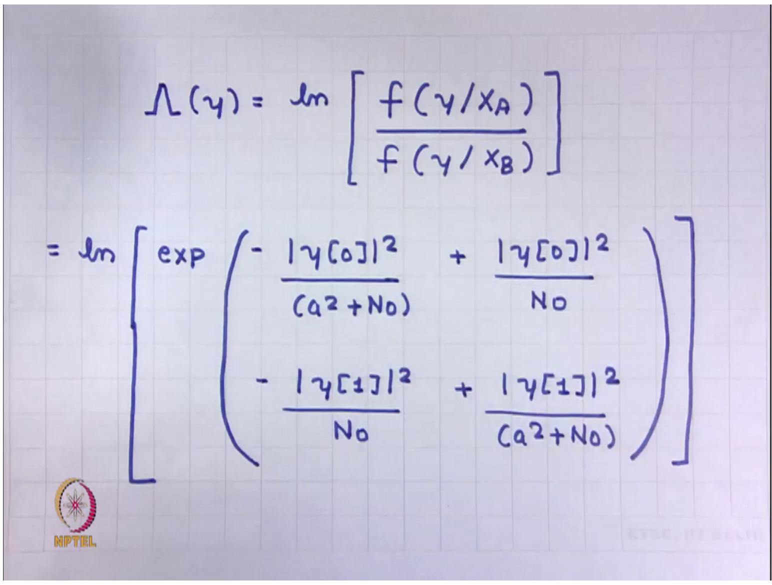

Now the job is to find the log likelihood ratio. The log likelihood ratio is obtained by taking L n of the ratios of the PDF when X A is transmitted and when X B is transmitted ok, then you can divide the two PDFs that we have obtained. So, we have obtained these two PDFs, we divide these two PDFs we take the L n of it and we get the log likelihood ratio. So, when you divide this PDF by this PDF, you get this thing ok. Then we just have to take the ln of it. It is trivial arithmetic, thus I will not do this. I think you can do all of this stuff.

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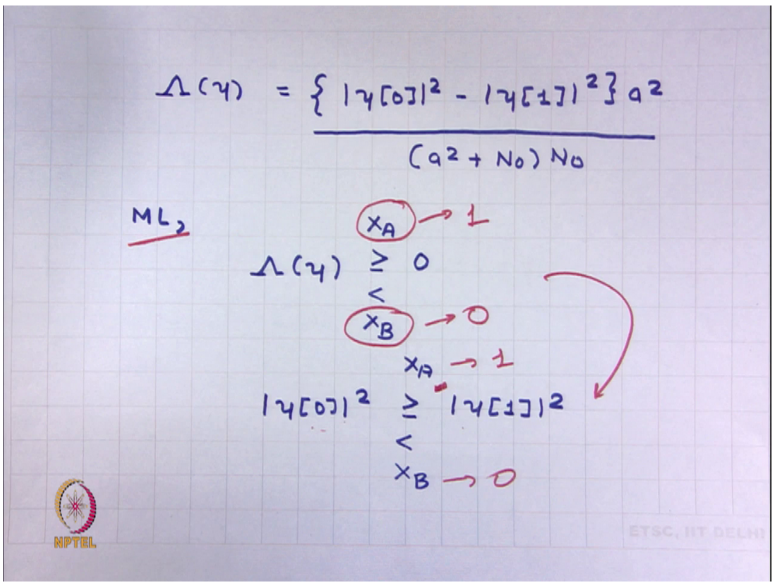

From this we can get the log likelihood ratio as simply this ok. The interesting thing here is if I am assuming M L rule then log likelihood ratio if it is greater than or equals to 0, I can decide that x is transmitted that in this case would correspond to hypothesis one being transmitted. And if this log likelihood ratio is less than 0, then I would assume that x b is transmitted, in this case it would correspond to hypothesis 0 being transmitted ok. Remember in maximum likelihood detection L n eta is simply 0 or threshold is 0.

So, what we have to see is when is this log likelihood ratio is less than or equals to 0. And this corresponds to the situation when mod of y 0 square is greater than mod of y 1 square the log likelihood ratio will be a positive quantity. And hence this condition translates to this condition that if mod of y 0 square is greater than or equals to mod of y 1 square, then you consider that the symbol X A is transmitted which corresponds to hypothesis 1 being transmitted. And if this is not the case then you assume that symbol X B is transmitted which corresponds to hypothesis 0 being transmitted ok.

Now, you notice something very funny in here. Here the decision does not depend at all on the channel knowledge. So, you see y 0, you take the mod of it, you square this up you receive y 1, you take mod of it, you square this up. And based on these two quantities, you can simply decide whether the hypothesis 1 is transmitted or hypothesis 0 is transmitted. Here there is no dependence on g ok.

That is what we have shown is even if there is no channel information present in the case of PPM, we assume that we do not know the magnitude of the channel you also do not know the phase of the channel. And what we have seen in that case that threshold does not at all depend upon this channel knowledge ok. And we have seen in one of the previous lectures that log likelihood ratio is the sufficient statistic ok. Based on the log likelihood ratios, you can always take the decision. This log likelihood ratio does not depend at all on the knowledge of H ok.

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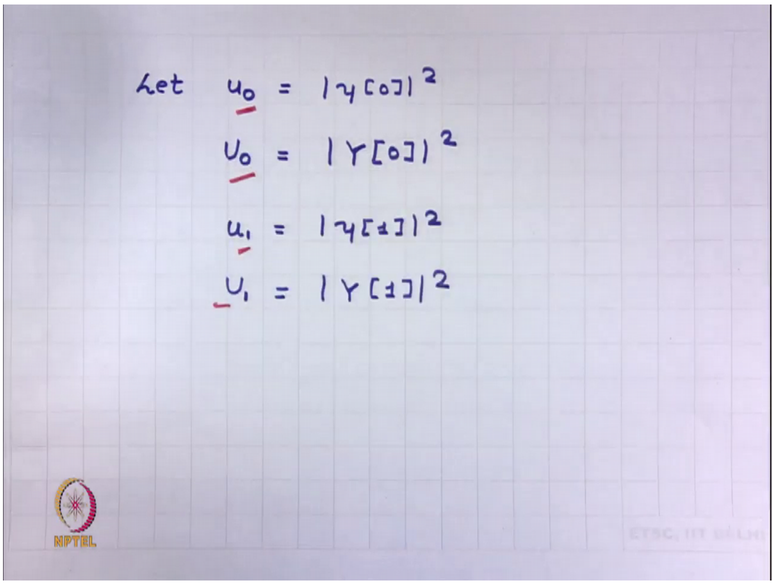

Now from here we can find out the probability of errors. And to simplify arithmetic a bit, we make these substitutions. So, we say that u naught is simply mod of y 0 square, and this corresponds to a random variable capital U naught being mod square of random variable Y 0. Similarly, we can define u 1 to be mod square of y 1 and this corresponds to a random variable U 1 which is equal to mod square of random variable Y 1 ok. The first thing that we would like to do is, we want to compute the probability density function of this random variable u naught in terms of probability density function of the random variable y naught ok.

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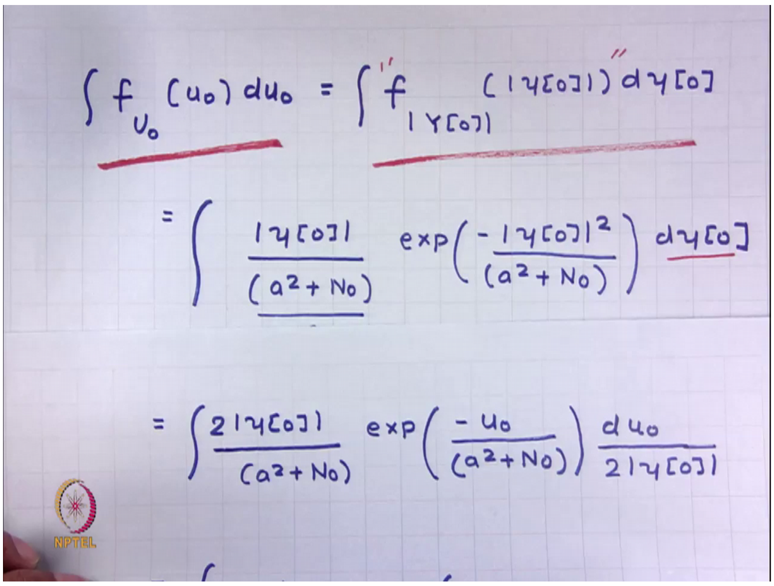

And we know that probability remains conserved when you want to go from one space to another space. So, this is the probability when I want to think in terms of random variable u naught, and this will give me probability when I want to think in terms of random variable mod of y naught ok. And I know what is this quantity ok. This quantity is simply mod of y naught divided by variance. And we know that the variance of mod of y naught is simply a square plus N naught by 2 times exponential minus of mod square of y 0 divided by a square plus N naught ok. And I have to integrate this thing over this numerical values y 0.

Now, if u naught is mod square of y 0, then d u naught is simply 2 times mod of y 0 times d y 0. Now, what I can do is instead of d y 0, I can write this as d u 0 or d u naught divided by 2 mod of y 0 ok. And instead of mod square of y 0, I simply write u 0 because that is what u 0 is.

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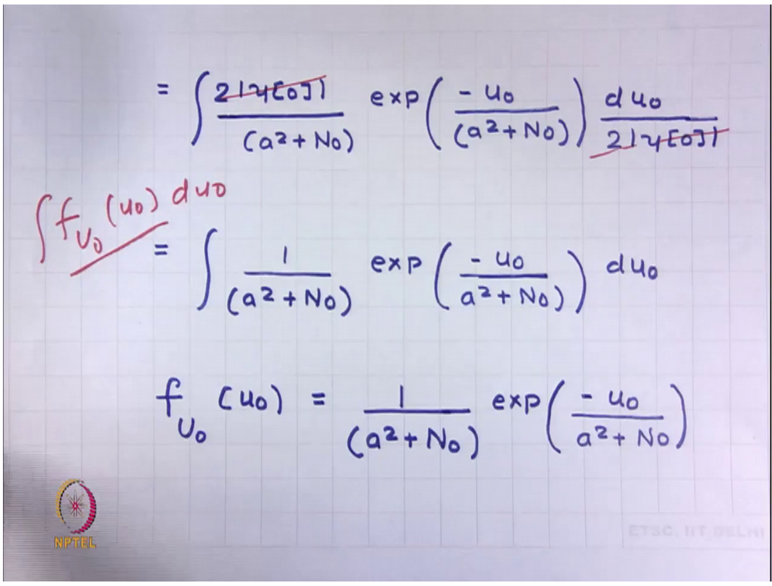

Now, this is simple. I can cancel this with this and I get this times d u 0. And this is what this is right. And from here I get that probability density function of random variable u 0 is simply this, all right.

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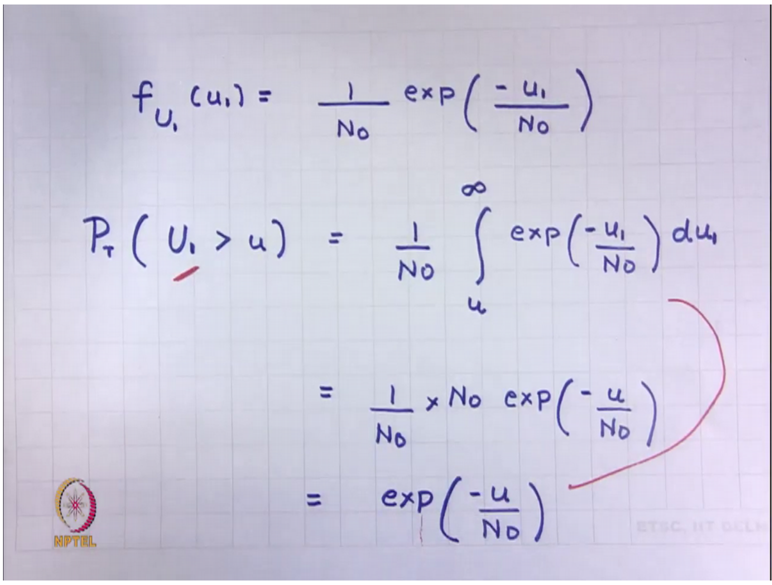

Now, what is the probability density function of random variable u 1? Following the same steps you can go to this, what changes in here is instead of variance of a square plus n naught you just have a variance of n naught, because we have seen that when x is transmitted for symbol 1, the variance is simply N naught ok.

Now, the first thing that we have to see before we can arrive at the probability of error formula is what is the probability that, this random variable U 1 has a numerical value larger than u. And we know how to solve this up you simply have to integrate this PDF from u to infinity. This will give me the probability that random variable U 1 is larger than u. And solving this is pretty trivial. If you solve this up, you get that this quantity is simply exponential of minus u by N naught ok, this is what we will use. So, probability that U 1 is larger than u is simply exponential of minus u by N naught.

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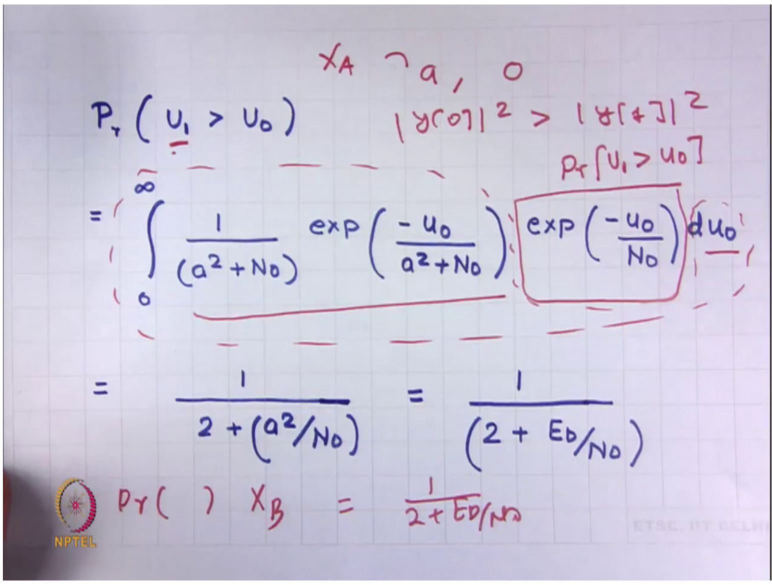

Now, we can find out the probability of error, when X A is transmitted. When X A is transmitted, there will be an error when U 1 is larger than U naught, because when X A is transmitted we are transmitting a and 0. And we hope that y is 0 mod square is larger than mod square of y 1, because then the detector will assume that hypothesis x is transmitted. But if this is not the case, then you get an error. And when will this situation happen when U 1 becomes larger than U 0 ok.

So, this is the probability of random variable U 0 taking in a numerical value small u 0 ok. And when this happens there will be an error when random variable U 1 has a value larger than U naught. And this probability is given by this. So, this corresponds to probability of random variable U 1 taking a value larger than U naught this is what we have derived in here. And this is the probability of random variable U 0 taking in a numerical value small u 0 ok.

Now, we can integrate this thing up, and we get probability of error when X A is transmitted is simply 1 divided by 2 plus a square by N naught, and a square by n naught is also E b N o. Similarly, what we can do is we can find that the probability of error when X B is transmitted is also the same right. There is no reason why the probability of error will be different when X B is transmitted. And hence the average probability of error will also be this ok. So, what we have done now is we have calculated the probability of error in case we have PPM scheme and we are using non-coherent detection.

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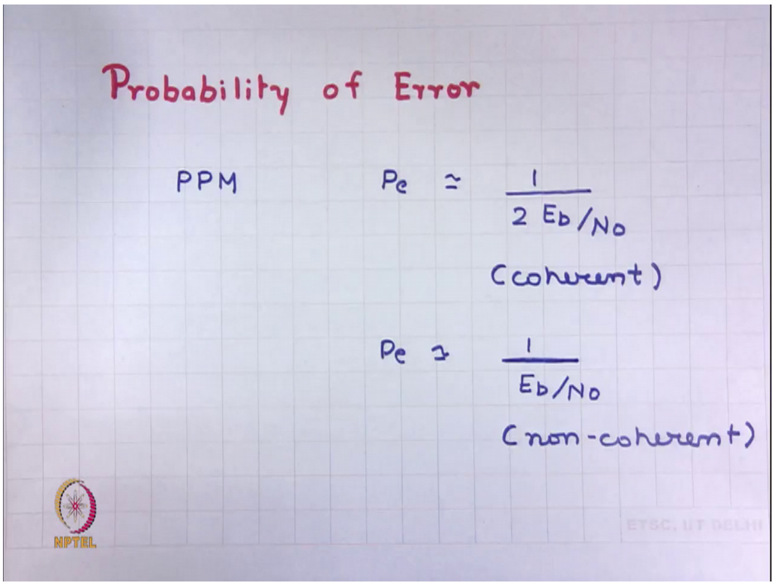

Now, let us see in totality the probability of error if we are using non-coherent scheme if you are using PPM is simply 1 by 2 E b N o if we are using coherent systems; and this probability of error is approximately 1 by E b N o if we are using non-coherent systems ok. Thus you lose out 3 dBs when you are going from coherent systems to non-coherent systems. What an advantage in non-coherent systems is that you do not have to learn the channel anymore, and thus the overheads corresponds to learning channel and errors in learning channel are not there ok.

What we also see here is when you go from coherent systems to non-coherent systems or you go from non-coherent systems to coherent systems, you do not improve your E b N o requirements considerably. I mean we have already seen that when we had the coherent systems the E b N o requirement was around 74 dB for BPSK streams right. And by using non-coherent systems, you also have an additional 3 dB (Refer Time: 46:38) right. Thus the problem in fading channels is not in learning channel ok. You might completely have the channel estimate, but it is not going to do you any good ok. So, now what we want to see is how we can reduce this probability of error. And to understand that we have to understand what is this deep fade ok.

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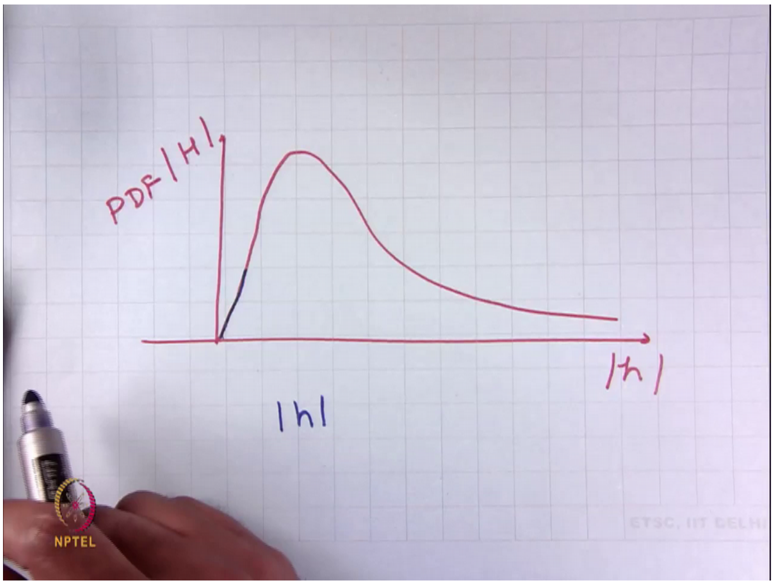

And before understanding this deep fade let me draw Rayleigh PDF which looked something like this. So, we have on this axis probability density function of let us say mod of H; on this axis, we have numerical value of mod of H. Now, what you can notice here is that there is a very significant probability of mod of h taking in very small values ok. And thus if your channel is in what is known as the deep fade, and we will see what does this deep fade mean, but let us just assume at this point that deep fade is the condition when channel is very badly behaved.

So, when the channel is deep fade it simply means that you have a very small value of this mod of h. Now, because of this very small value of mod of h whatever E b N o you may have the resultant E b N o at the receiver will be pretty small ok. And thus by having a larger E b N o does not help you anymore ok. So, let us quantify what we mean by deep fade.

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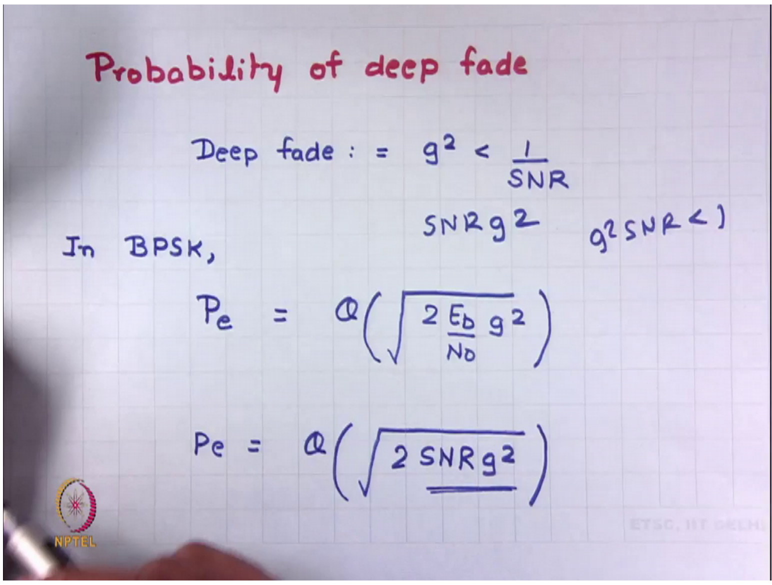

So, deep fade is the condition when g square is less than 1 by SNR. Remember we have seen that in BPSK systems for example, we had a probability of error for a given g like this right. Now, in BPSK E b N o is also SNR. So, I can also write this probability of error in terms of SNR like this. Now, when g square is less than 1 by SNR, this quantity will be less than 1; and because this is less than 1, you will have probability of errors right.

So, what we are saying if g square is pretty small number such that g square times SNR is less than 1, we say that the channel is a deep fade. And if channel is in deep fade whatever SNR you may have because of this very small values of g you are going to have errors. Let us investigate what is this probability of deep fade.

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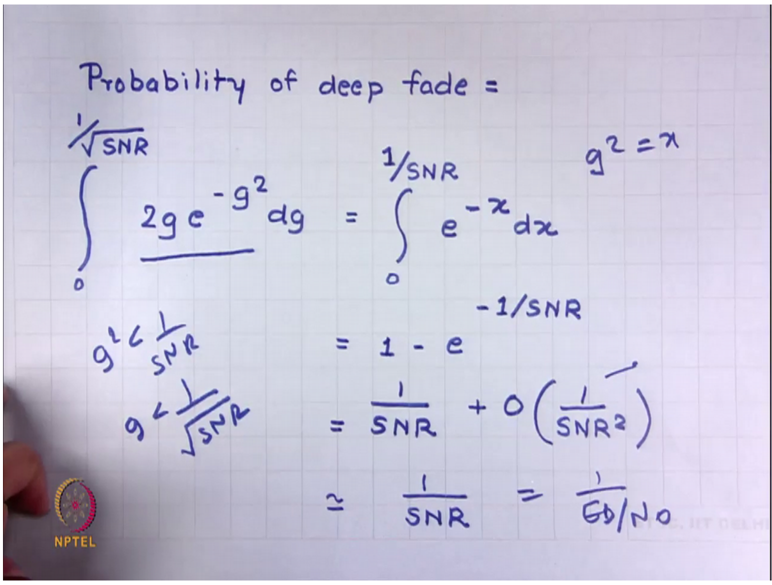

So, we know that PDF expression is this. And deep fade will happen when g square is less than 1 by SNR that means, g is less than 1 upon square root of SNR. So, I have to find out what is the probability of deep fade by integrating this PDF between 0 to 1 upon root of SNR. And I can convert this expression to this expression by simply substituting g square as x, and everyone knows how to integrate this thing up. This is simply 1 minus e to the power minus one by SNR.

And by using Taylor series, I know that this is 1 by SNR plus some terms which are proportional to 1 by SNR square. If SNR is large, then this is negligible and I get probability of deep fade as simply 1 upon SNR right or in this case this will also be 1 upon E b N o. Now, what we have noticed is that there is a striking similarity between this probability of deep fade which is 1 upon E b N o, and the probability of errors which was one upon 4 times E b N o.

And thus you can relate that most of these errors happen because of this deep fade right. And no matter what SNR you might have because of this deep fade the overall received symbol power or signal power is pretty feeble, and there will be errors happening because of that ok. So, you can only improve upon the performance by reducing the chances of the channel to be in deep fade ok and to do that what you can use are these diversity systems right.

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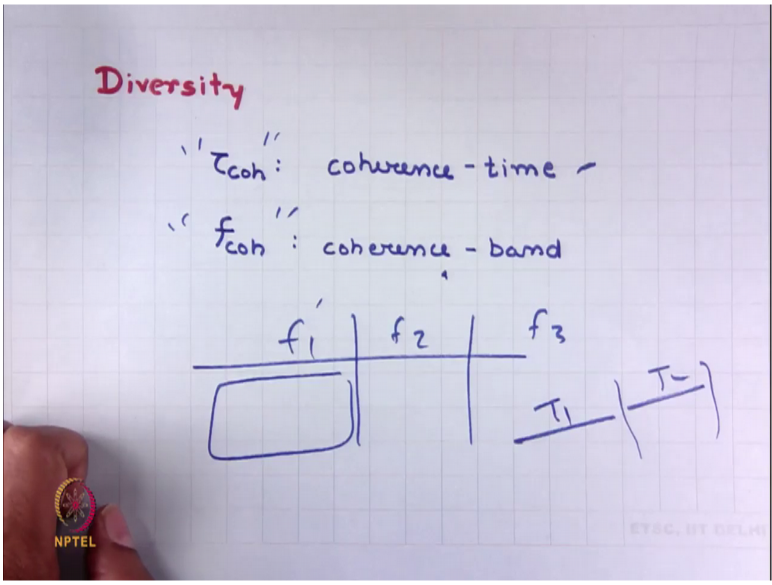

So, in channel you have these two important properties. Every channel has got some coherence time which simply means the time period over which the channel retains is statistics. And you also have a coherence band which simply means that the band of frequencies over which the channel has the same statistical properties. So, again the coherence time simply means the time period over which the channel has the same statistical properties. And coherence band means the band of frequencies over which the channel has the same statistical properties. Thus if a channel is in deep fade, you can transmit the signal in a different coherence band ok.

So, for example, if let us say we have three coherence bands f 1, f 2 and f 3, and if we know that the channel is deep fade in this band, I can use another band right. I can use this f 2 or f 4. And thus I can reduce the probability of deep fade by choosing a band in which the channel is in good condition.

The modulation scheme which does is for example, CDMA. What happens in CDMA, you have the signal and you spread the signal across the entire bandwidth. And thus if one segment of the bandwidth is bad or if the channel is behaving poorly in one coherence band, you have the another coherence band which will allow the receiver to have large values of signal transmitted ok.

What you can also use is you can repeat the signal in time, so that you have less error rates when you are transmitting a signal out of the coherence time ok. For example, if my channel is bad in this time duration T 1, I can transmit the signal or I can repeat the signal transmission in a different time duration, so that in this time duration the statistical properties of channel might be good, and thus I can have a small error rates ok.

So, idea is either you extend transmission time of your signals, so that you make sure that the signal is spreaded in time such that in certain time regions at least the it sees the channel in good condition or you can spread the signal in frequency domain, so that you can transmit the signal in a frequency band where the channel is good ok.

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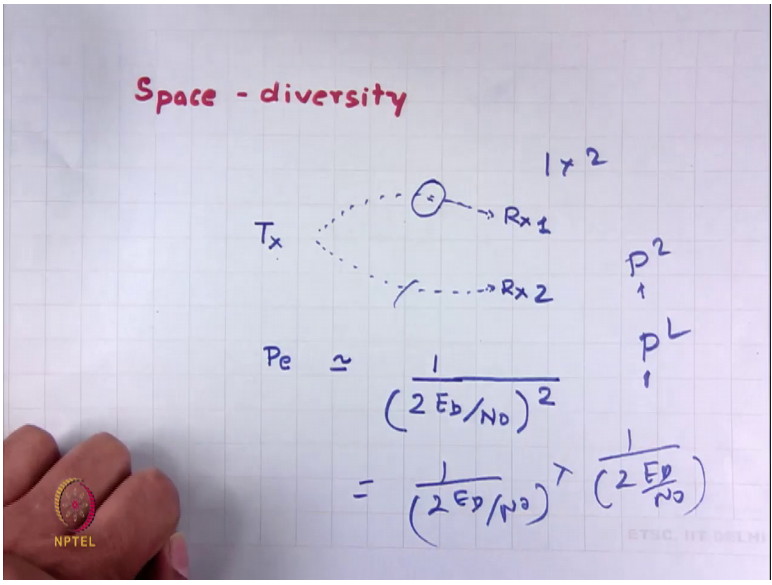

What you can also use much more simpler to understand is this idea of space diversity systems where you create multiple channels between transmitter and receiver. So, in this case, I have two channels between transmitter and receiver ok. So, I have two receivers and one transmitter. This is 1 by 2 space diversity system. If I assume that, this channel is independent of this channel, I can say to a large extent that when this channel is in error or is in deep fade, this channel might be good. The probability that both these channels will be in error will be simply P square, where P is the probability of one channel to be in error ok.

So, by having this multiple channels between transmitter and receiver, you reduce the probability of all channels to be in error simultaneously ok. So, for example, if you have L channels between transmitter and receiver, the probability of deep fade of all channels at the same time is simply P to the power L, where P is the probability of deep fade of one channel and thus you reduce the probability of error.

For example, if I have this 1 by 2 space diversity system, my probability of error reduces approximately to this value ok. So, probability of error when I just had one channel in case of PPM systems was this. If I have two channels my probability of error will simply be obtained by multiplying the probability of error when I just had one channel two times ok.

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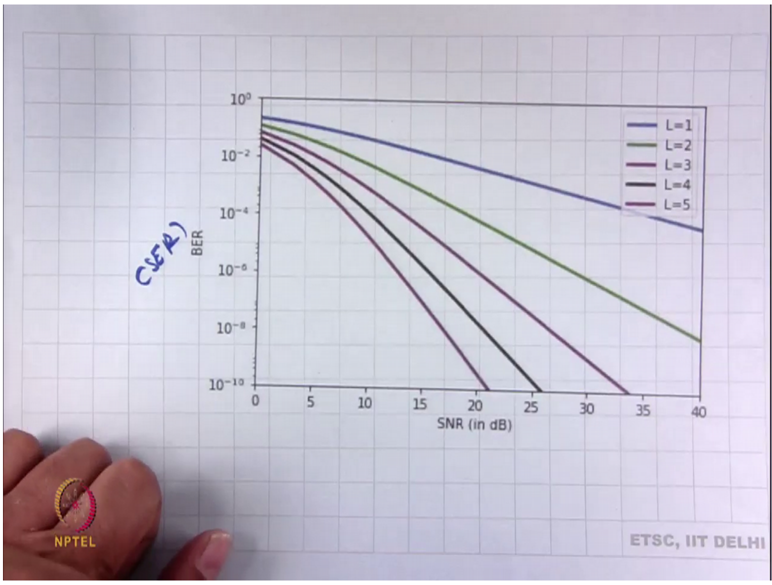

And here I have shown how does this probability of error reduces as we increase L ok. You can think this as bit error rate or symbol error rate does not matter, because bit error rate and symbol error rate are same when we are talking about binary schemes ok. So, as I increase L, my probability of error reduces ok. And thus by having large number of channels between transmitter and receiver, you can reduce the E b N o requirement to practical values.

So, with this we have come to the conclusion of this lecture and of this course in digital communication. I hope that you have enjoyed this course; and you have learned modulation, detection and now you can think about the noise.

Thank you.