Principles of Digital Communication
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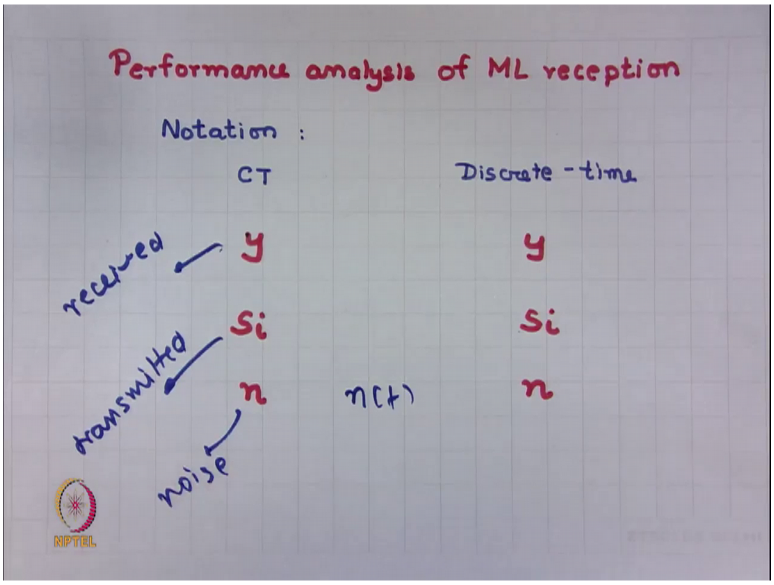
Lecture - 34
Detection
Performance of Binary Signaling Schemes

Good morning. Welcome to a new lecture and in this lecture we will start looking into the Performance of Various Signaling Schemes. So, till now we have covered the entire theory of detection, we have seen how to detect numbers, how to detect vectors, complex vectors, real vectors, wave form and sequence of symbols. Now, it is a time that we look into the performance of various signaling schemes and we start by looking into the performance only of ML signaling systems. Because, in ML the priors are equal and when the priors are equal we begin to have the neat expressions for probability of errors because, this annoying term elenita disappears from probability of error formulas ok. (Refer Slide Time: 01:11)



And we have also established the equivalence between continuous time and discrete time signals right. So, whether you look into a continuous time signal y of t or you look the vector corresponding to that continuous time signal that is y which belongs to a discrete time signal or you want to look into the noise as n t or you want to think about the noise as vector n, it does not matter. So, we will use the same symbols for denoting the continuous time signals and the ski time signals.

So, notations are bit sloppy here, but this is because there is complete equivalence between the two words right. So, we use the notation y to denote the received signal, it also denotes continuous time signal or discrete time signal Si we denotes we use for denoting transmitted signal. So, both it can mean a continuous time signal or a discrete time signal. So, y is the received signal and n is the noise and the channel. So right; so, get used to this fact right.

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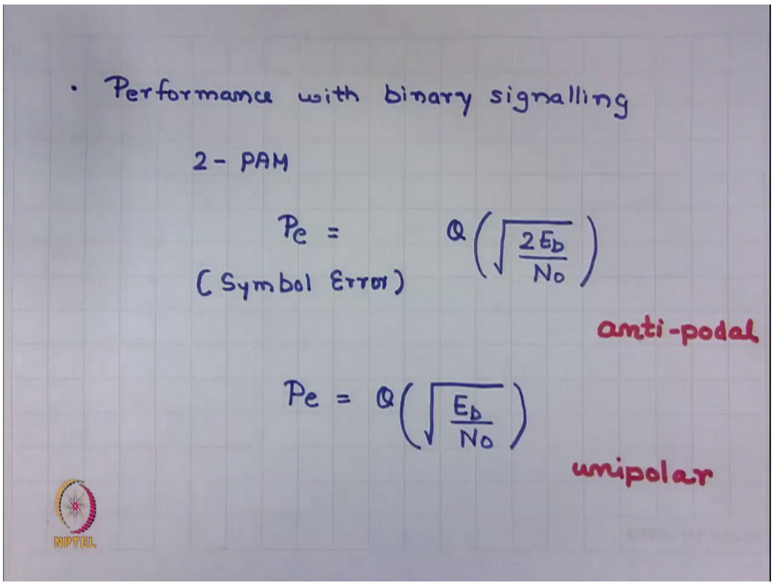

Now, we will start in this lecture we will concentrate on performance of binary signaling systems and we have already looked into this. So, for example, we have looked into the probability of errors in case of 2-PAM right a binary PAM. And, whenever we are saying probability of errors we means probability of symbol error, with what errors the symbols are decoded incorrectly ok. We will talk about probabilities of bit errors later on, for binary signaling there is no difference between bit errors or symbol errors because every symbol corresponds to a bit.

So, bit errors are same as symbol errors in case of binary signaling, but in case of a M-ary signaling this is not the case right. So, at this moment we are just concentrating on symbol errors and later we will see how can you relate the symbol errors to bit errors alright. So, probability of errors of probability of symbol errors in 2-PAM systems we have already obtained was Q of under root of 2 E b by N naught. In case of unipolar signaling the probability of error was Q of under root of E b by N naught, these formulas must be learnt by heart.

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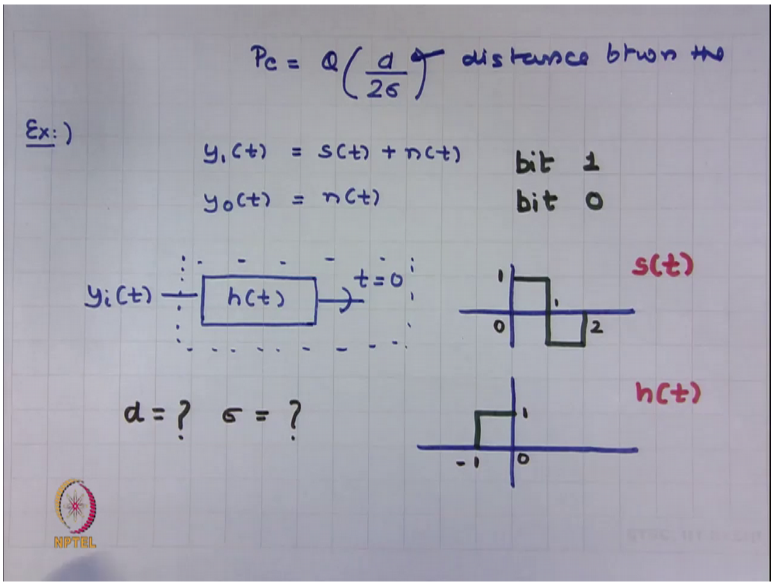

In general we can say that the probability of error is Q of d by 2 sigma where, d we said denotes the distance between the received signals and sigma was the standard deviation of the noise. Let us now try to understand this d and sigma in more depth and this is really useful and we will understand this through couple of examples. So, get used to the notations here.

So, y 1 t is the received signal at the input of a matched filter or at the input of a filter with an impulse response of h t followed by a sampler. So, this is the receiver that we are using and we have seen that this is the most optimal receiver. So, y i t is s t plus n t if bit 1 is transmitted and if bit 0 is transmitted then we just have noise waveform ok.

So, we just have noise waveform and 0 bit is transmitted and when 1 bit is transmitted we have signal plus noise; that means, it is an example of unipolar signaling. There is no signal when you are transmitting 0 bit, there is a signal when you transmit bit 1. And, what we are assuming is that the signal looks like this and the impulse response of this filter looks like this and we want to investigate what is d and sigma in this case ok. So, that is the question. So, we are trying to understand clearly what is this d and sigma and we are not trying to learn it by heart ok. What is d?

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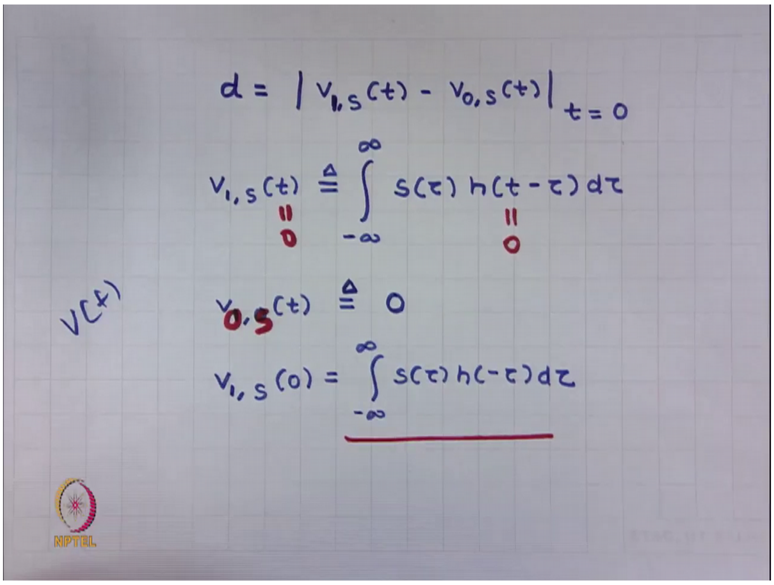

So, what is this V t, first let us clear the notation. V t is the output of this filter. so this is V of t and we say this is V 1 s of t; that means, this is the output when 1 bit is transmitted and this has only the signal component. We are not considering noise because, this is the distance between the two output signals, you do not have to consider noise in this. So, this simply denotes what is the signal content at the output of a filter, given that the bit 1 is transmitted minus the signal content and the output of a filter given 0 bit is transmitted.

So, this is distance and this distance we have to evaluate for t equals to 0 because this sampler is that t equals to 0. So, at the output of the sampler we would have this as d and this d corresponds to the distance between the signals the case 1 bit is transmitted and 0 bit is transmitted alright. So, what is this V 1 s of t? So, what is the signal content when 1 bit is transmitted? Signal content is s t isn’t it, when bit 1 is transmitted. So, we need to have the input as s of tau at the output of a filter we would have the output given by the convolution of input with an impulse response of the filter. So, you know from the coast and signals and systems the output of a filter is simply the convolution of input with impulse response. So, we are carrying out that. So, this is bit sloppy.

So, this is V 0 s so, this is the signal content when bit 0 is transmitted alright and when bit 0 is transmitted there is no signal content. Hence, V 0 s of t is 0. V 1 s at 0 so, we want to find out this thing at t equals to 0. So, you have to put t equals to 0, if you want to put t equals to 0 in here you simply get V 1 s of 0 as the thing. So, you look at the output, output is the convolution of input with impulse response and you look at that output at t equals to 0 ok so, you get this. Let us see how we can work this out.

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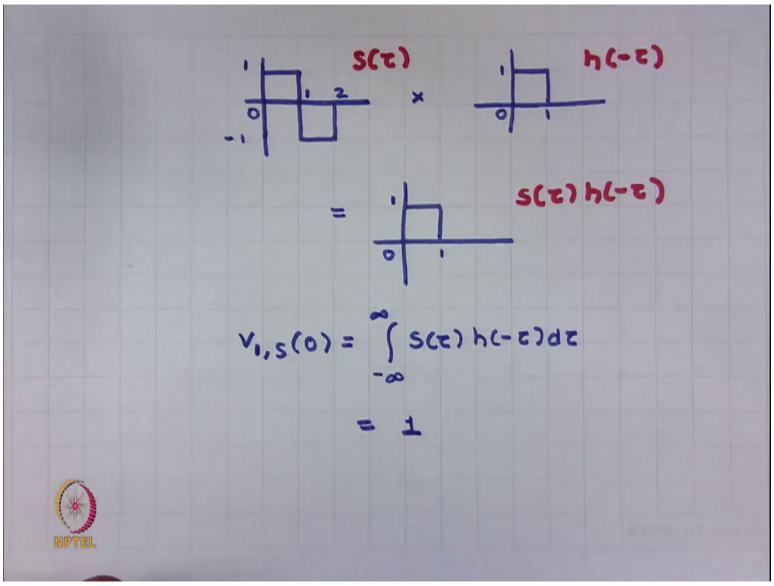

So, we know s of tau is given to us, h of minus tau is simply the flipped version of the impulse response. So, this is this, we need to multiply this with this and we get the signal. You integrate this thing from minus infinity to plus infinity you get 1. So, we have got the output corresponding to the bit 1 when you are sampling the output at t equals to 0 alright.

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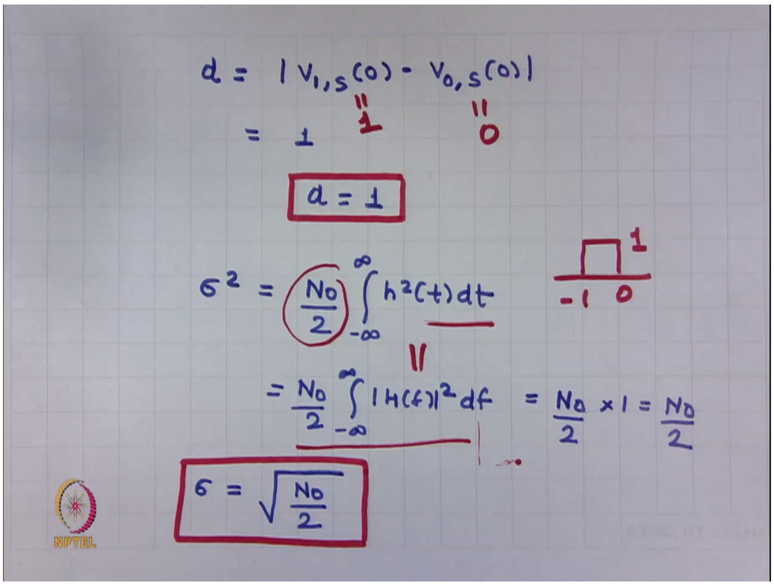

Now, what is the distance? Distance is this minus this and this we have seen is 1 and this we have seen is 0 so, the distance is 1 easy. How to find out the sigma? Variance of the noise at the output of a filter is given by the variance of the noise at the input and because input is a white Gaussian noise we know that is variance is N naught by 2. And, then the variance at the output of a filter is simply this thing, this we have looked into lecture 17. So, to find out the power spectral density of noise and the output of a filter you have to multiply the input power spectral density with this quantity.

And, from Parseval's theorem these two things are same whether you want to find out h square t d t from minus infinity to plus infinity or you want to integrate mod square H f from minus infinity to plus infinity from; Parseval's theorem both these integrations will lead to the same result. Integrating this thing is easier because, this has been given to us and what is this. So, this is 1 between 0 and minus 1 so, this integration is simply 1 the area is 1. And, hence the output power spectral density or output variance is same as N naught by 2. So, sigma is square root of N naught by 2 because, the standard deviation.

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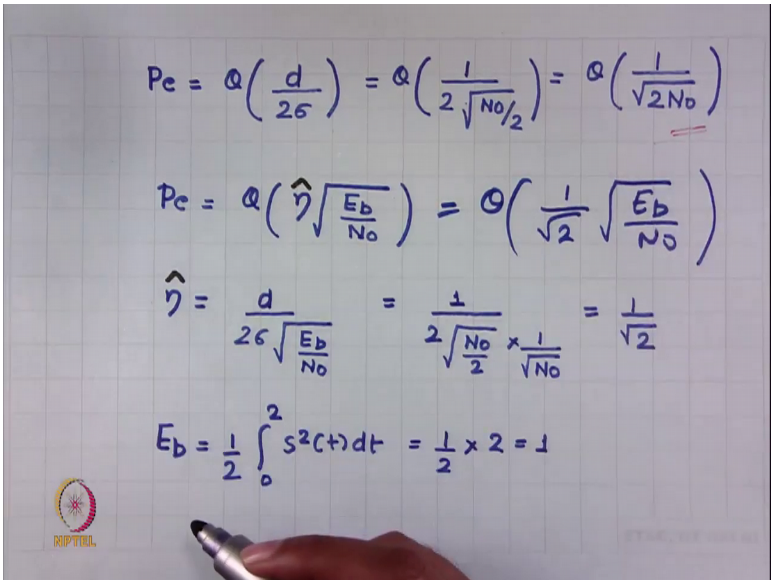

So, from this we can find the probability of error which is Q of d by 2 sigma d we have found as 1, sigma we have found as square root of N naught by 2 and this can be reworked to this easy. But, mostly we want to write the probability of error in terms of square root of E b by N naught. We have seen in the formula's before that probability of error is Q function half square root of E b by N naught and there might be some constants here and there. But, what we know for sure that probability of error should be containing this terms; Q function and it is mostly Q of under root of E b by N naught.

And, there might be some missing constant, that missing constant we say as eta cap and we want to investigate that eta cap; eta cap if you see will be simply d by 2 sigma because, we have seen that the argument inside Q function is d by 2 sigma. So, eta cap is simply d by 2 sigma divided by root of E b by N naught, d we have found as 1, sigma we found as square root of N naught by 2. What is E b? What is bit energy? So, in this case the bit energy is half integration of s square t dt. So, this is because let us remind this.

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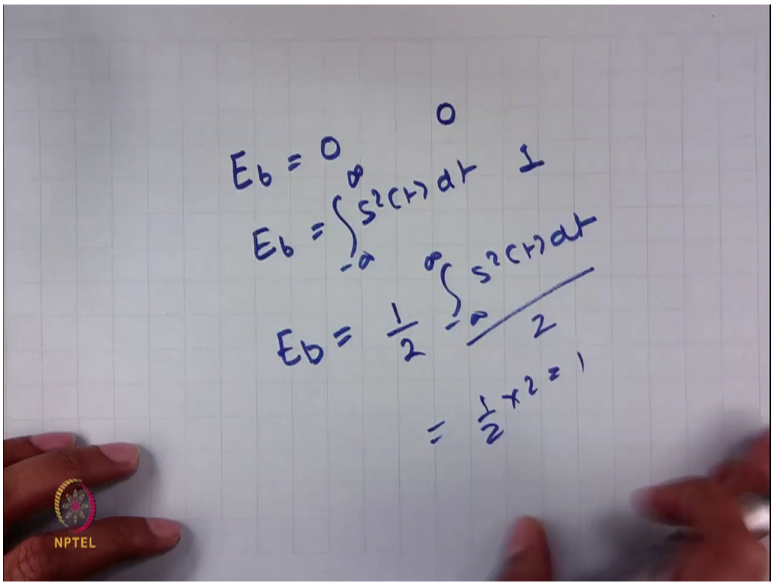

So, you have bit energy 0 when 0 is transmitted, you have bit energy integration of s square t d t when 1 is transmitted. So, on an average if they are equiprobable and they are equiprobable because, we are in the ML regime. This will be half this thing alright and this we have evaluated to be 2. So, this is half times 2 which is 1 alright. So, we have got E b as also 1. So, eta cap is 1 by root 2 alright. Now, if you want to write this probability of error in terms of eta cap this would be Q of 1 by root 2 times root of E b by N naught.

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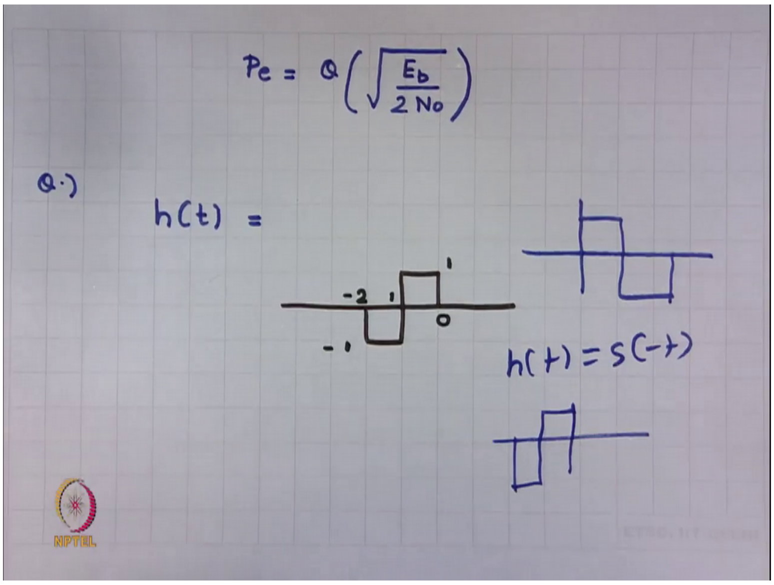

And you can write this as this meet; so, for this system which was not fed with a mash filter, but it was just some filter; we can also evaluate probability of error. And, why we are doing this because we want you to become an expert in evaluating probability of error for whatever kind of a filter is present and this will make things more generic and interesting. Now, let us take an impulse response so, every setting remains same. So, we are doing this question for the same setting, we will be just changing few things here and there; other than that everything remains same as was in this question.

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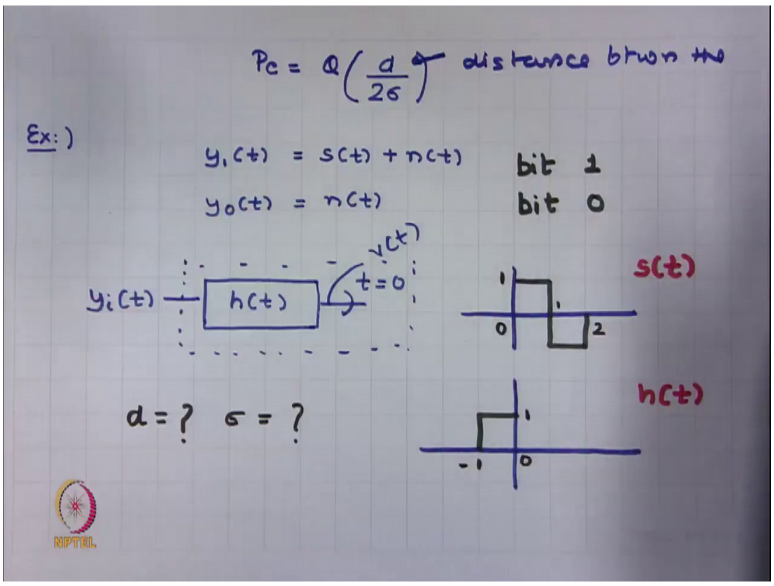

So, we are having y of y i of t feeding to a filter and a sampler and when there is a bit 1 we are sending this, when there is bit 0 we are sending this, s t remains this; whatever we will change we will point out that to you. So, here we are changing as the impulse response of the filter. So, actually we are using matched filter. How do you know that this is matched filter?

So, we know that the signal is like this and match filter impulse response should be s of minus t. So, you just have to flip this around and when you flip this around you will get an impulse response like this. So now, we have changed the filter to a matched filter and we want to investigate the same things.

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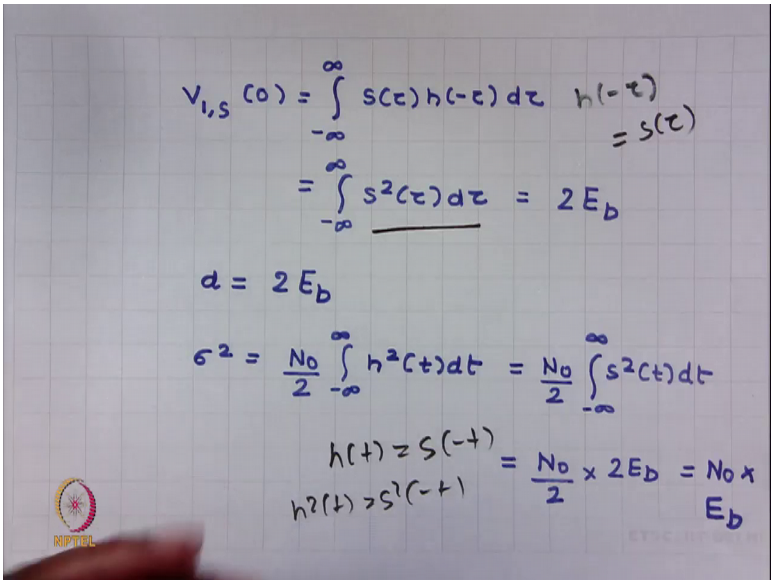

So, V 1 s of 0 is same as before, but now h of minus tau is simply s of tau because it is matched filter. So, this is simply integration of s square tau from minus infinity to plus infinity and this we have seen assembly 2 times E b. Distance in this case is thus 2 times E b. What is sigma square? Sigma square if you evaluate everything remain same h t is s t. So, h of t is s of minus t so, h square t is simply s square minus t.

And you know that if you want to integrate this or you integrate this you should get the same answer because, we are integrating from minus infinity to plus infinity. So, if you take a signal or you take the time reversed version of the signal, if you integrate both signals you get the same answer. And, we know that what is this quantity, this is 2 times E b. So, sigma square is N naught times E b.

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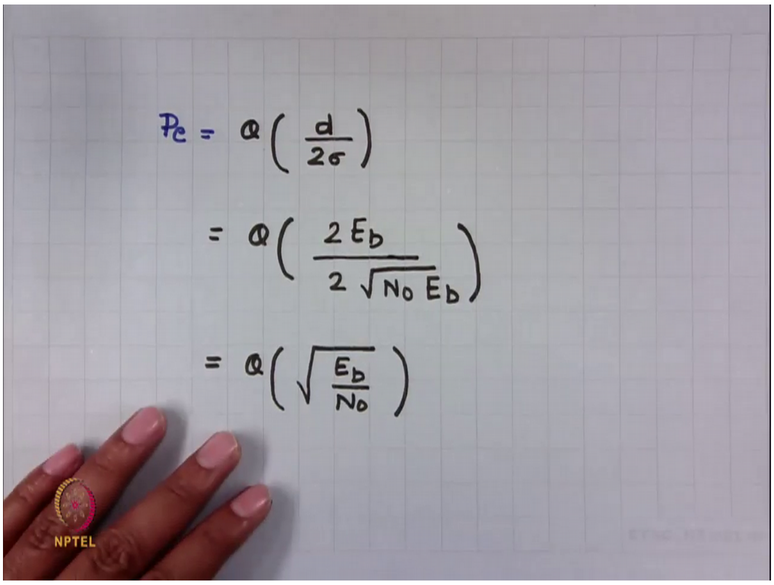

What is probability of error? Is Q of d by 2 sigma; d we have calculated as 2 times E b, sigma we have calculated as square root of N naught E b. So, this is the answer that we get is probability of error, if we use a mass filter is Q of root of E b by N naught alright. So, this is the answer that we have also got for unipolar signaling mechanism and this in fact, is a unipolar signaling mechanism.

And, when we have gone from waveform to vectors we said that the way that we want to do is you have to use correlators or mash filters. And, if you get the mash filters that time optimal way to convert waveforms to vectors. So, the probability of error formula that we derived was actually assuming that I have a matched filter and a sampler. And, we see that if you run this out from basics you get the same answer. Can we think about this in some other ways, as looking into again why is this matched filter the most optimum filter.

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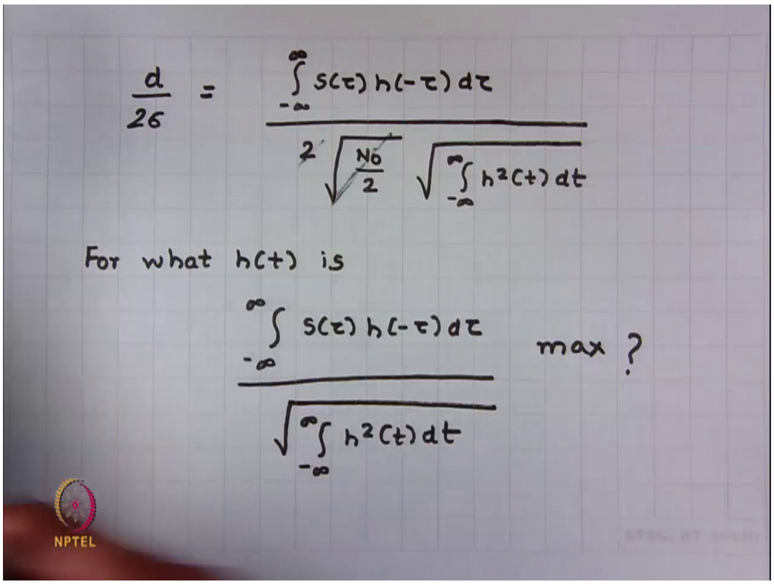

Because, what you want to do is you want to maximize this quantity d by 2 sigma. And what is d? In case of unipolar signaling is this thing isn't it. And what is sigma?

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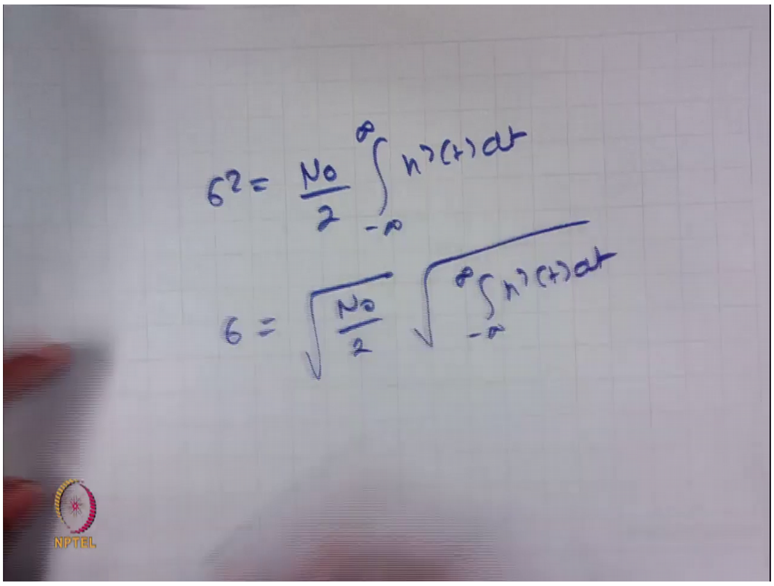

Sigma is if you see this, sigma square is N naught by 2 times h square t d t, we have written this formula several times. So, sigma is square root of N naught by 2 square root of this thing alright. So, substituting sigma in here and the question that we ask is for what h t is this quantity maximum. This is not a function of h t this rather some constant, we do not worry about constants.

We want to find out the impulse response for which this quantity is maximum and if you have not done this vector analysis trying to find out h t might worry you. But, because we have done this vector analysis things are pretty trivial. What is this? This is then inner product of signal s t with h of minus t; inner product is same as dot product.

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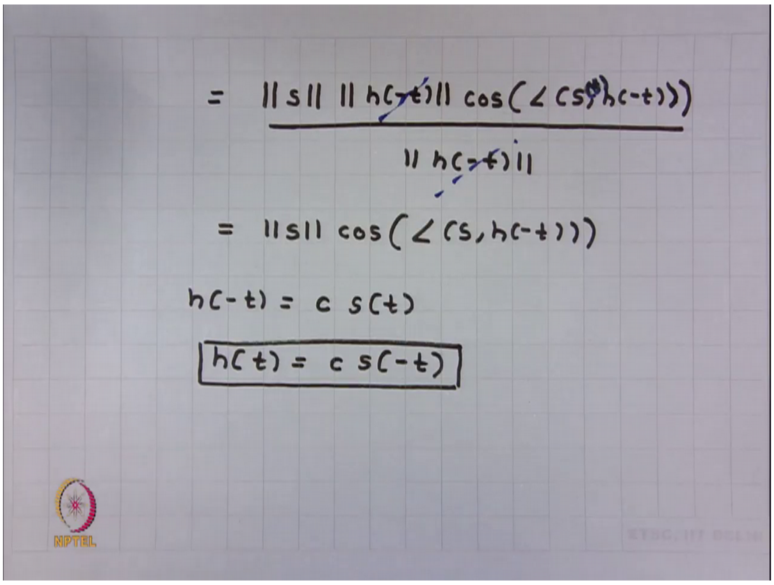

So, this is simply norm of s times norm of h of minus t cos of angle between s t and h of minus t. If you want you can have t in here, does not matter because s t is same as s in our notations divided by; what is this? This is simply the square root of energy and the square root of energy is simply the norm of the vector right. I am treating signals as vectors and I can cancel this with this and I get this thing is simply norm of s times cos of angle of s and h of minus t.

And when is this quantity maximum? When this angle is 0, then cos of 0 is maximum 1. And, when is this angle 0? When h of minus t is some constant times s of t; that means, these two vectors are co-linear, h of minus t is simply s of t times some constant ok; c is a constant. So, if this is there then h of t is simply some constant times as s of minus t. And, this simply proves that matched filter is an optimum receiver, is an optimum filter to use alright. And what is that maximum value?

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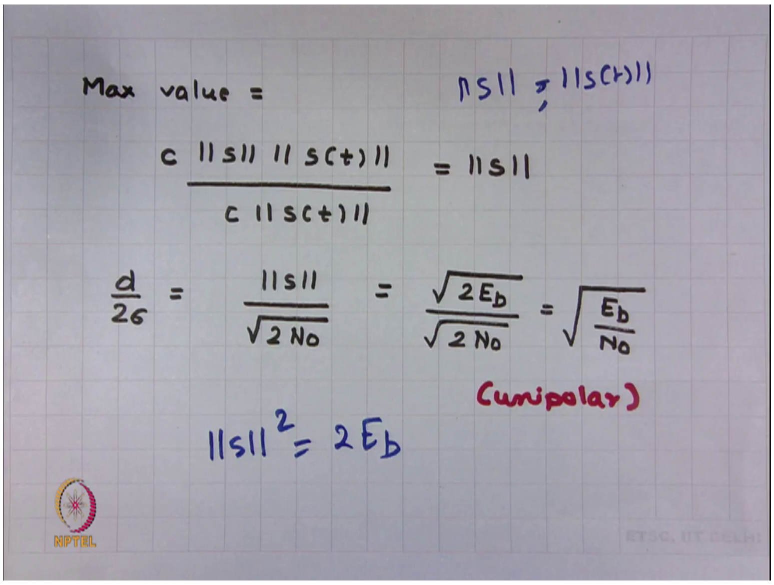

So, if you put h of minus t as c times, if you substitute this in here you get c and in you get norm of s instead of this you get c times norm of s t. And, this is also c times norm of s t, this cancels with this, c cancels with this you simply get norm of s. Do not worry about whether I should use norm of s or norm of s t these are one and the same thing. So, the maximum value of the expression, this expression is simply norm of s and we killed out this root of 2 N naught. So, I have it back here and norm of s is simply so, norm square of s is 2 E b this we have seen.

So, norm of s is the square root of 2 E b and we get d by 2 sigma is a square root of E b by N naught. We are getting the same answer again and again, I am just trying to say to you that whatever methods you choose as generic as trying to find out what is the distance between the two receive signal at the output of a sampler. And, you start to evaluate d by 2 sigma from that or you choose any other method does not matter, you get the same answer. What also we have proven in this course is we have proven that matched filter is actually an optimum receiver and this is quite clear right; if you look at this ratio of d by 2 sigma. Let us have more fun and now, let us use the impulse response.

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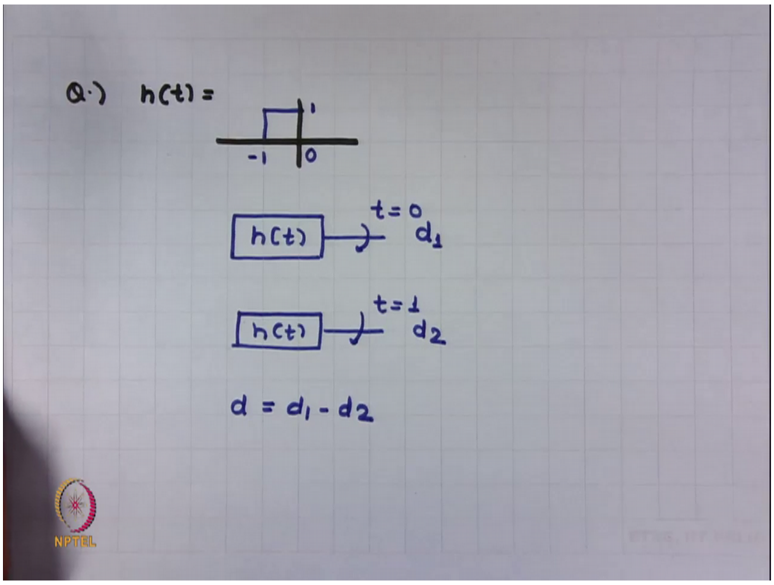

Again which is not same as matched filter, it is an impulse response that we have used in the first example. But now, what I do is I have a sampler at t equals to 0, I collect a number d 1. I use a sampler at t equals to 1, I collect a number d 2 and I form my decision statistics by subtracting this d 2 from d 1 and I define this as number d and see what happens in this case alright.

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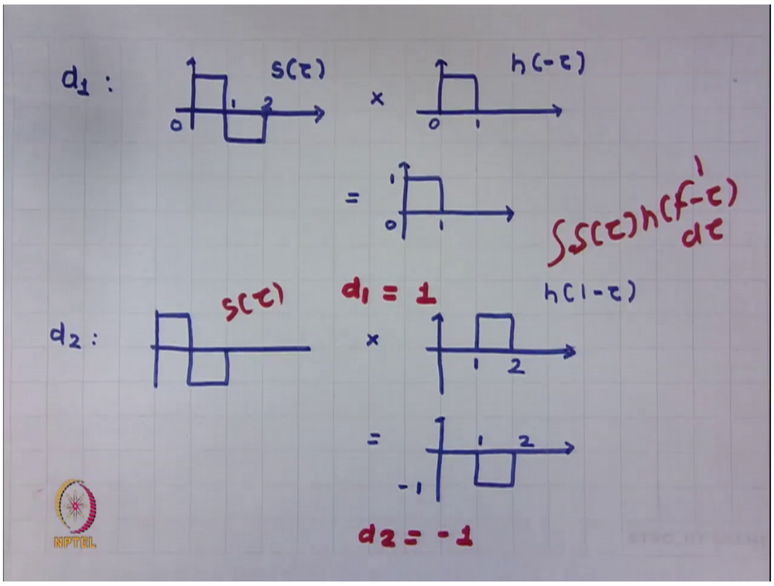

So, if you look at d 1, d 1 would be obtained by multiplying s of tau with h of minus tau you get this thing and you integrate this and you get d 1 is 1. I am doing it too fast because, we have already covered two examples right. You work this out first and then look at the answer, this is kind of answer to the question that I have asked. When thinking about d 2 I have to take s of tau.

So, this is s of tau and this is h of 1 minus tau because, I am looking down the output at t equals to 1. So, remember in convolution we had s of tau h of t minus tau d tau. So, if t is 1 then I have to multiply s of tau with h of 1 minus tau which is the signal. When I multiply these two things up I get this, if I integrate this I get minus 1. And, thus the decision is statistic which is formed by d 1 minus d 2 is simply 2.

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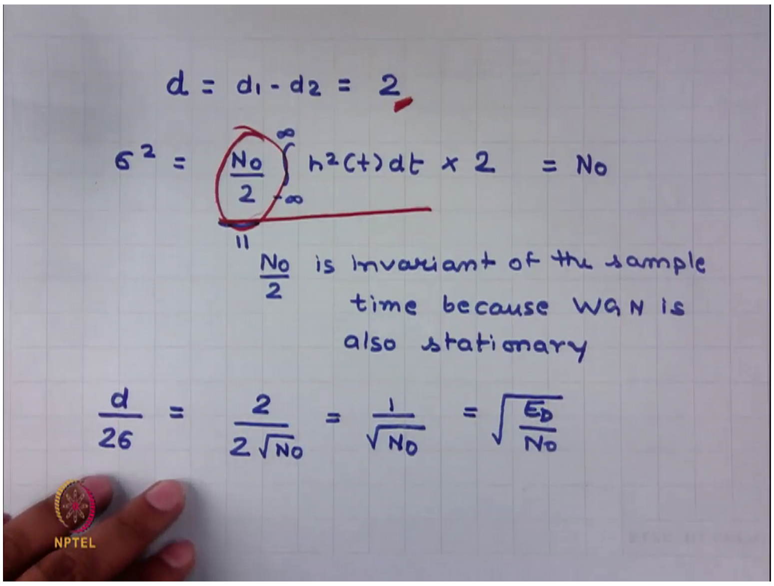

What is sigma square? Sigma square is this thing we have been using this couple of times, times 2 because whenever I am sampling I am getting noise in 1 degree of freedom. Noise power in 1 real degree of freedom to be sighs and that noise power in 1 real degree of freedom is N naught by 2. I am sampling twice so, I will have noise power in 2 real degrees of freedom. And, because the noise is independent, its power will simply add and I have a factor 2 in here which by substituting in the value of h square t that we have sigma square is simply N naught. And the question is also an interesting (Refer Time: 26:15).

So, whenever I am sampling the output at t equals to 0 or at t equals to 1 does the input noise power spectral density remain same or does it change. It remains same because we assume white Gaussian noise and one of the properties of white Gaussian noise is also a stationary random process. If it is the stationary random process at whatever time you want to look at it you are going to have the same noise power available per degree of freedom. And, thus N naught by 2 is invariant of this sample time. So, noise power does not change with the sampling time instances.

The signal power to change right because, at certain times the signal power may peak or it may be 0, the sampling instances determine what signal power you have at the output, but it does not influence the noise power alright. So, d by 2 sigma d we have said is 2 and sigma is square root of N naught. So, we get 1 by root of N naught and this is same as root of E b by N naught. That is what we have seen is in this case we are getting the same d by 2 sigma as we got in the case of a matched filter, this is interesting.

The filter that we have used is not a matched filter, it is not the optimum filter to start with, but the answer that we are getting the performance, that we are getting is similar to the performance that we got when we used a matched filter. And why is this so? The reason is that we are sampling it at 2 time instances instead of 1 time instance.

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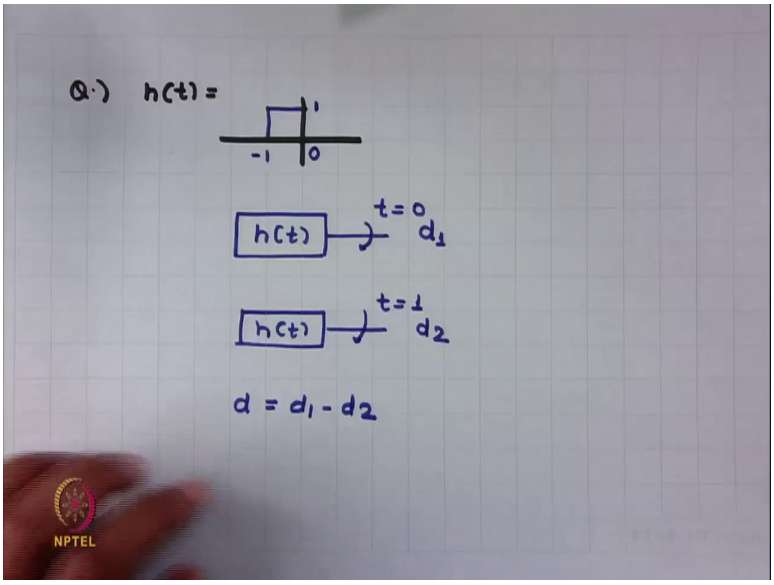

So, when we look at this picture, when we sample it at t equals to 0 the filter behaved like this and when t sample this at t equals to 1 and I have taken the negative of this statistic, they actually filter behaved like this ok. So, at t equals to 0 filter behaved like this, at t equals to 1 and with a negative sign in there the filter behaved like this. And, thus I am constructing a matched filter from a simple filter by sampling the output process at multiple time instances, in this case 2 time instances ok.

So, the idea is that you can construct something like matched filter by having more samples, you can also understand this from a different perspective. So, when you pass a process through a filter you get more information about the signals, if you have chosen the sample points appropriately to be precise. And, in this case by just having two sample points I have got a performance which is same like that of a mesh filter. Can I get a better performance by sampling it even more than 2 times, let us say if I have sampled it 10 times. The answer is no, think the reason why is that ok.

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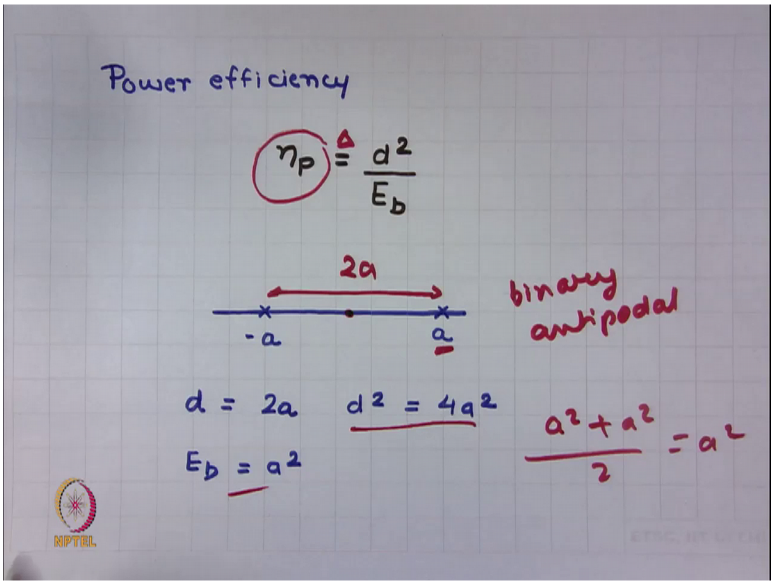

To have more fun in this performance analysis, let us introduce a new parameter which we call as power efficiency. This power efficiency will help us in two things; first it will help us in arriving at this bit error rate or symbol error rate per formula rather quickly and then it will give us some quick insight into the performance of various signaling schemes alright. Let us get it started and then see whether it serves the two purposes which I have just pointed out.

So, power efficiency we define as d square by E b, d is the distance between the two signals or symbols and E b is the bit energy. Let us start understanding this from a simple antipodal binary PAM system, binary antipodal PAM system and d in this case is 2 a right. So, d square is 4 a square. What is a bit energy? So, bit energy is this energy a square plus this energy is square divided by 2 so, a square.

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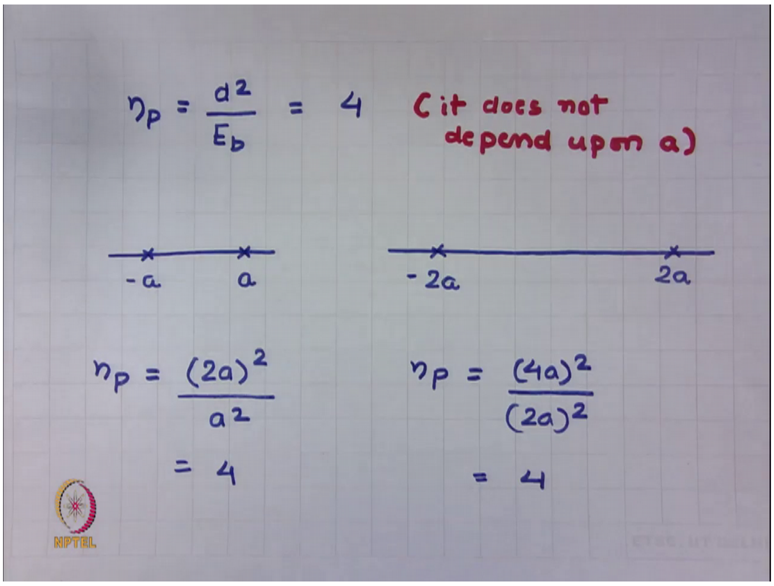

So, we have got d square, we have got E b and we have got power efficiency which is d square by E b which is 4. Interesting thing is it does not depend upon a, where you; what a you have chosen for this constellation. That means whether you calculate the power efficiency of this constellation or power efficiency of this constellation you get the same power efficiency. Hence, we call that power efficiency is a scale invariant parameter. What does it mean? If you take in this constellation, if you zoom this up on zooming you are changing the scale, but zooming a constellation will not influence its power efficiency. And hence, power efficiency is scale invariant parameter and that is why we love this.

More so ever any constellation scheme can have a better error performance if you have a lot of distance between the symbols, but then they have to pay the penalty in terms of bit energy. So, having this factor d square by E b tries to really quantify how power efficient a constellation scheme is. So, these are two important things that eta P serves; first thing is that this is an invariant parameter. So, it really characterizes a constellation structure and secondly, it gives us a good idea about how power efficient a modulation scheme is because, it is a ratio of the distance squared between the symbols and it also depends upon how much energy you have to spend per bit.

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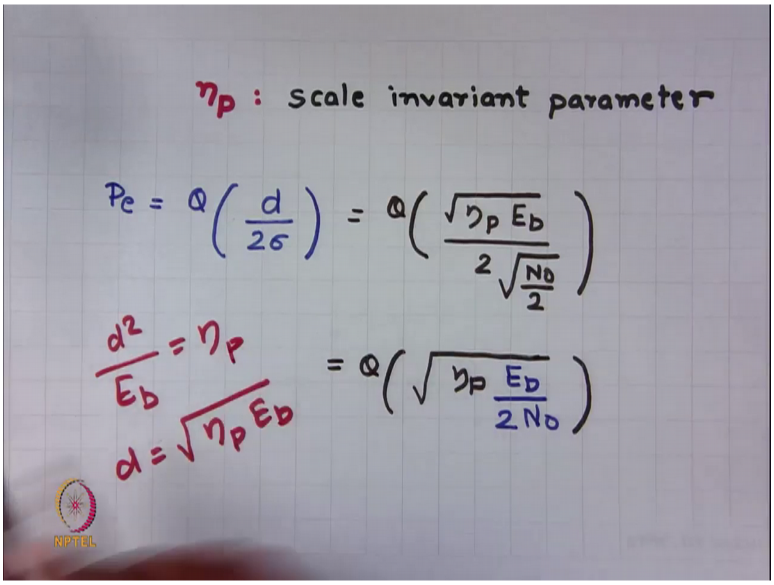

So, we have already looked into this probability of error and we have said that this is Q of d by 2 sigma. And what is d? So, we know that d square by E b is eta. So, d is actually square root of eta P E b, this is eta P. So, d is square root of eta P E b and sigma is a square root of N naught by 2 ok.

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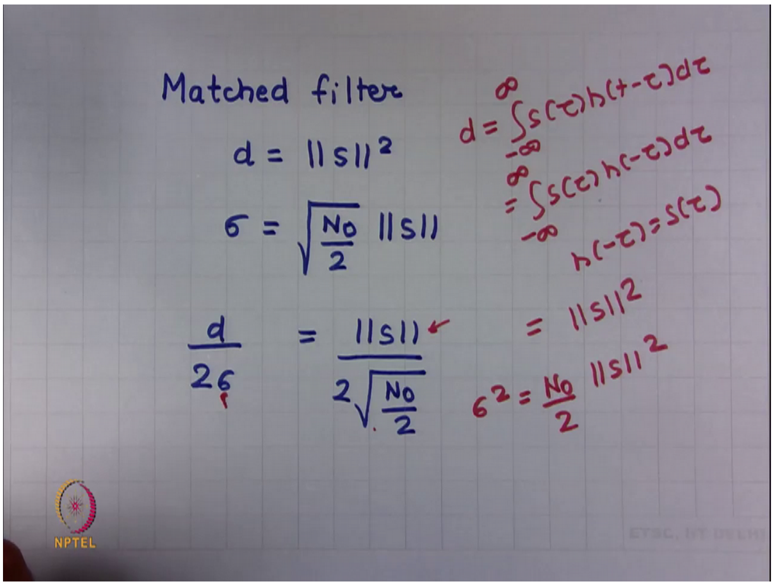

So, let us first revise what we have learned in the matched filter. So, we know that in case of matched filter this d is simply norm square of s and why is this because, we have seen that d is s of tau h of t minus tau d tau integration from minus infinity to plus infinity. And, when we want to sample this at t equals to 0 this simply is s of tau h of minus tau integration from minus infinity to plus infinity. And when it is a matched filter, this h of minus tau is simply s of tau and so, this corresponds to energy of s right. So, when we are having a matched filter we know that d is simply energy of s. What is the standard deviation of noise at the output of the sampler?

It is simply square root of N naught by 2 multiplied by norm of s right and why is this because, we know that variances N naught by 2 times norm of s square. So, the standard deviation is a square root of N naught by 2 multiplied by norm of s. So, what is this d by 2 sigma? d by 2 sigma is simply norm of s divided by 2 times square root of N naught by 2.

So, I can also interpret d as the distance of the input signal and I can also interpret the sigma as the standard deviation of noise at the input to this matched filter. So, when I am having a matched filter and this is normally the case, I can assume sigma to be the standard deviation of noise at the input of a filter and, I can interpret d as the distance between the input signals. So, what we have got is that, this could be rearranged to this thing; that means, probability of error is Q of root of eta P this is deciding factor times E b by 2 N naught.

So, if for a modulation scheme or for a constellation system you know eta P you can easily obtain the probability of error. This is also one formula that you must know right or you must learn one of the formula and should have the ability to go back and forth between various formulas. I personally try to remember this one ok. So, the probability of error is Q of a square root of eta P times E b by 2 ok. Let us see if we can use this formula and can derive the probability of error for various constellation schemes.

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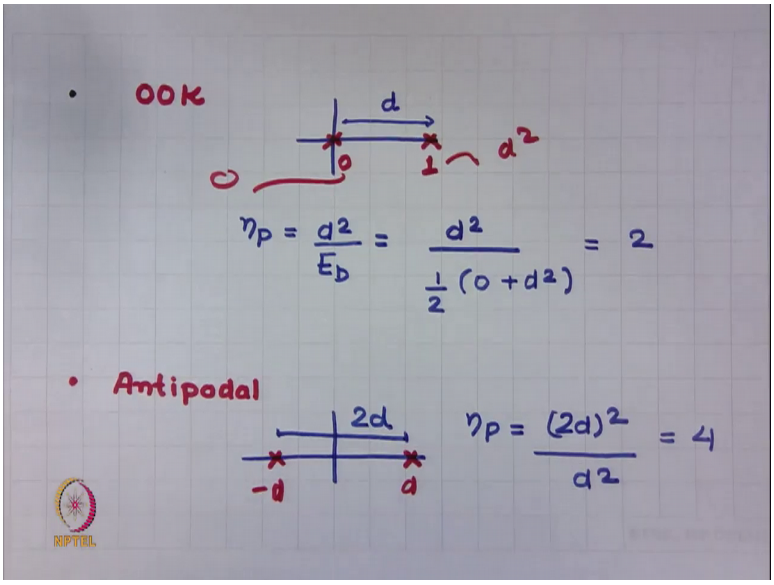

Let us start with on off keying which is also unipolar; that means, you transmit let us say 1 this symbol and for 0 you transmit this symbol. It does not matter whether you assign this a bit 1 or this a bit 1 right except for implementation issues where, you want to match the soaring operation to multiplication operation and this we have seen before ok. But, that is basically implementation point of view otherwise theoretically there is no difference.

So, power efficiency is d square by E b, d, in this case is d E b is so, for this you are spending d square, for this you are spending 0. So, E b is d square by 2. So, power efficiency is 2 ok, for antipodal let us assume that the distance between these two symbols is 2 d. So, this is at d this is at minus d. So, eta P is 2 d square divided by bit energy which is t square and thus, power efficiency in this case is 4.

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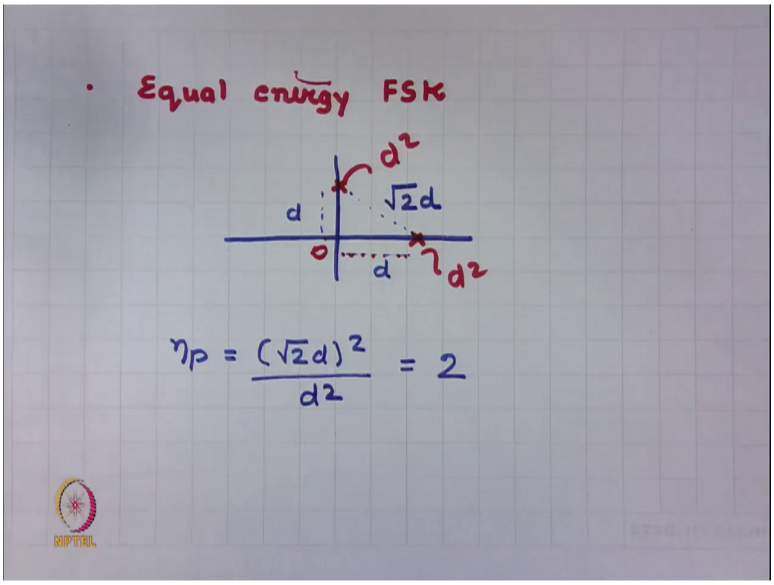

For equal energy frequency shift keying binary frequency shift keying, equal energy means that both these symbols have the same energy. If I assume that this symbol is at d distance from origin, this symbol is at d distance from origin. In frequency shift keying we are using to two orthonormal basis functions right.

So, the distance between these two symbols is root of 2 d power efficiency is root of 2 d square divided by d square, bit energy is simply d square. For this you need d square energy, for this we need d square energy. So, power efficiency is 2.

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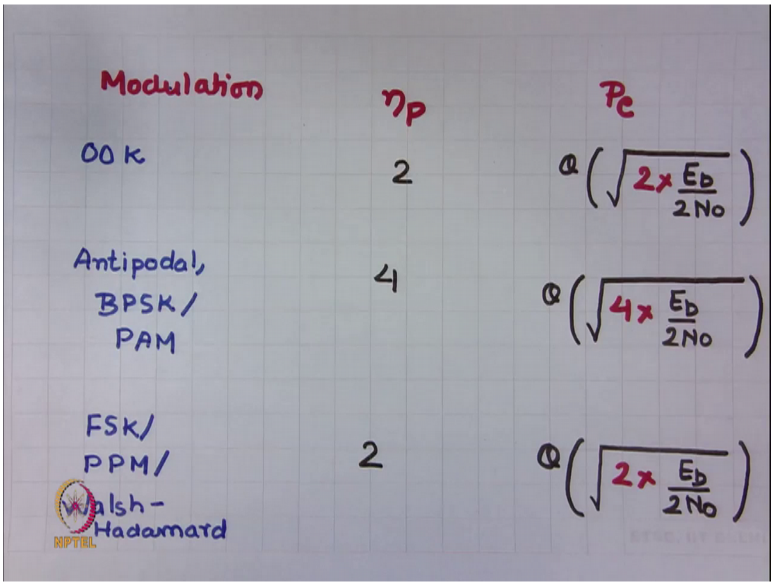

We can look at the different modulation schemes OO K power efficiency is 2, antipodal for which we can consider binary phase shift keying. So, binary phase shift keying is also same as antipodal PAM, the power efficiency we got is 4. For FSK, PPM or the orthogonal systems or orthogonal modulation using Walsh-Hadamard codes; they have the same constellation because, they work on the same underlying principle.

Only implementation methods of these modulation schemes are different, performance wise they are same at least in AWGN channel ok. So, the power efficiency for this modulation schemes is 2. And how do we get probability of error? A simple, you just have to substitute in that formula the power efficiency alright. Why this is known as power efficiency that is also important, why is this term power efficiency used for this ratio; let us try to understand this.

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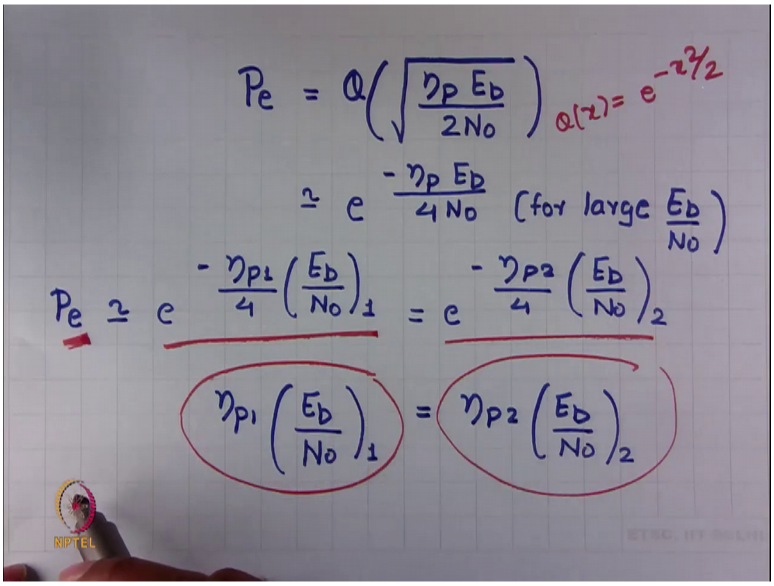

Let us see what is this P of e, P of e we know as a Q function of a square root of eta P times E b N 0 by 2 and we have seen when x is pretty large Q of x can be approximated as e to the power minus x square by 2. In that case P e is simply this, remember this is for large x; that means, is for large E b N 0's. And let us now assume that two modulation schemes have the same P e.

If the two modulation schemes have the same P e then for both modulation schemes this should be same. And, if this is same we can conveniently understand that this product for modulation scheme should be same; namely power efficiency multiplied by E b N 0 in one modulation scheme should be same as power efficiency multiplied by E b N 0 in another modulation scheme.

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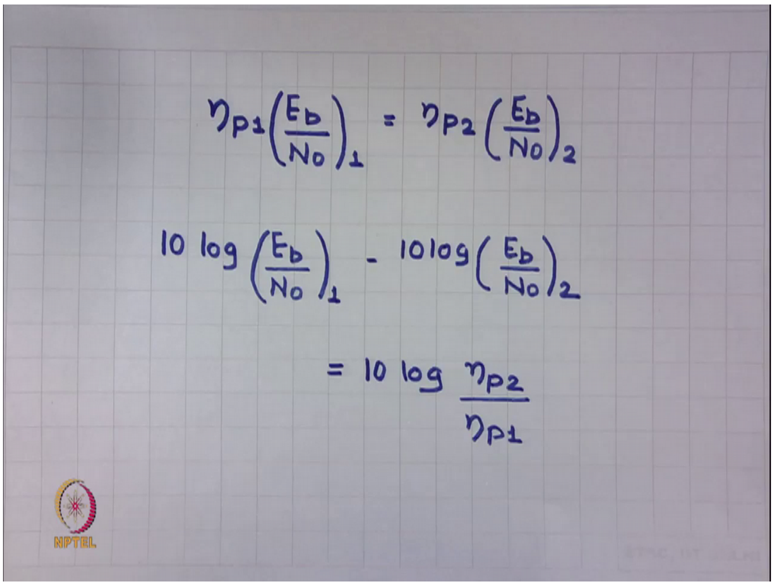

I can also write this in log scale. So, we can also say that E b N 0 requirement in first modulation scheme in dB scale minus E b N 0 requirement in second modulation scheme in dB scale should be this ratio of power efficiency in dB scale alright.

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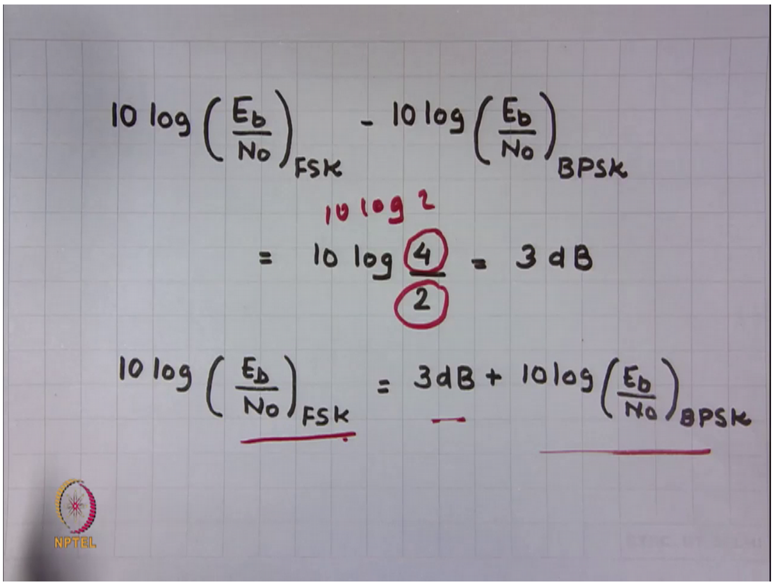

So, for example, if the underlying schemes are FSK and BPSK we can find out that the difference in the E b N 0 requirement in FSK and BPSK can be simply obtained by the ratios of the power efficiency in the two cases. So, BPSK has a power efficiency of 4, FSK has the power efficiency of 2. We divide these two power efficiencies we get 10 log 2, 10 log 2 is 3 dB; that means, the E b N 0 requirement in FSK systems is 3 dB more than the E b N 0 requirement in BPSK systems for the same error performance. And, we are making this equivalence remember at large E b N 0's.

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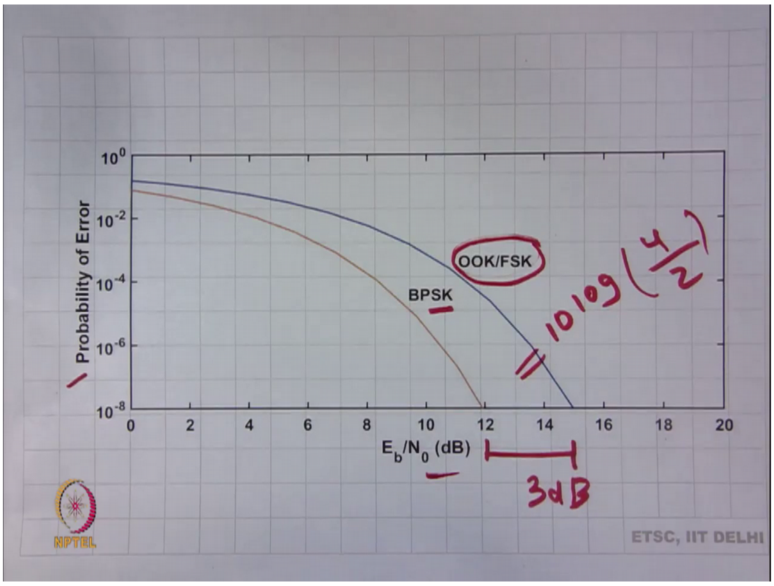

So, if I plot the probability of error versus E b N 0 for BPSK which is antipodal PAM and for OOK and FSK which has the same power efficiency. So, they will have same probability of error versus E b N 0; you should notice few things first probability of error with E b N 0 falls like a waterfall; if the probability of error and E b N 0's are using log scales. So, here also we are using a log scale and E b N 0 we are using a dB scale which is also a log scale. So, if you plot probability of error versus E b N 0 in dB scale or in log scale you get a probability of error versus E b N 0 which looks like a waterfall.

And, if you look at this distance you can easily see that this distance corresponds to 3 dB. So, this distance would correspond to 10 log of the ratios of power efficiencies of BPSK and FSK. So, if you know the power efficiencies of modulation schemes you can quickly estimate what this distance would be alright and hence, this power efficiency is a useful metric. So, with this we have come to the conclusion of this lecture. In this lecture we have looked into the performance of binary signaling schemes considering ML detection rules. So, whenever we are talking about ML remember that we are assuming that trials are equal.

And, we have looked into the BER performance for various modulation schemes, we have seen that these probability of errors can be easily derived in terms of power efficiency. Power efficiency is a scale invariant parameter and it simply depends upon d square by E b. Once you can calculate this power efficiency, you can easily derive from this the probability of errors for various modulation schemes. In the next lecture we will continue with m error detection, where things are little bit more interesting and little bit more harder.

Thank you.