Principles of Digital Communication
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Lecture – 31
Detection
Vector Detection

Good morning, welcome to a new lecture on detection in the previous lecture on detection we have discussed the idea of MAP detector and ML detector.

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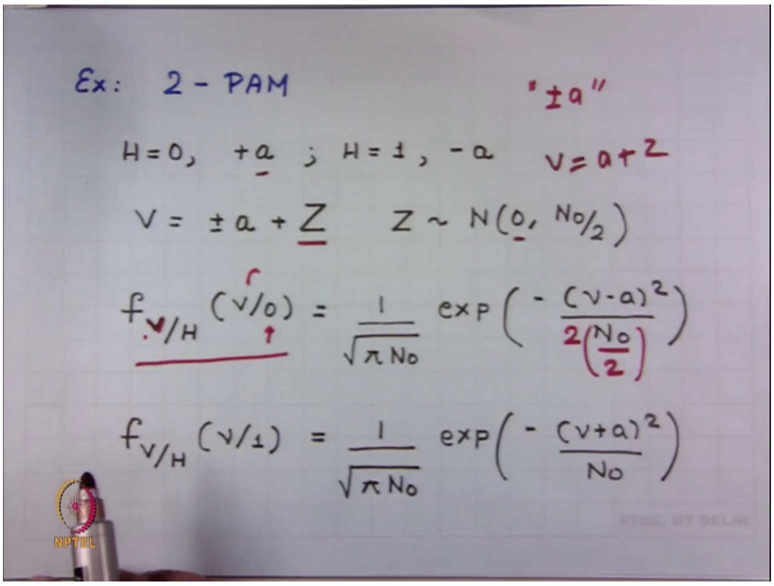

So, in this lecture we will start by looking into the example of detection for 2 PAM and we will derive the bit error rate performance of binary PAM. We assume that to send hypothesis 0, we are sending a symbol plus a and to send hypothesis 1 or bit 1 we are sending a symbol minus a. So, its an example of antipodal signaling right. So, if you send plus or minus a this falls into the category of antipodal signaling.

So, after demodulator we are going to receive a random variable V which is plus or minus a plus noise Z denotes that it is a Gaussian random variable with mean 0 and variance of N 0 by 2. So, if you transmit a symbol plus a this random variable will be plus a plus Z and if you transmit a symbol minus a this random variable will be minus a plus Z. So, first step is to identify the likelihood and first we are finding what is the likelihood given that hypothesis 0 has been sent. So, this is the conditional probability density function of random variable we give an H. So, what we are identifying is that if we have received the numerical value of random variable as a small v what is the probability density function given the hypothesis 0 has been sent.

So, in that case its very simple we will be Gaussian random variable. The only difference here is that now the mean of we would be shifted around plus a because if the hypothesis is 0 we are transmitting plus a. So, Z was a random variable with mean at 0. And so V a plus Z will have a mean at a right. The variance does not change if you add a deterministic signal to a random variable. So, the variance will remain same N 0 by 2. So, you can also write this as this 2 N 0 by 2 so. In fact, we are just having n 0 variance is intact N 0 by 2 only mean has been shifted to a and this is the likelihood when hypothesis 0 has been sent.

Similarly, we can find the likelihood when hypothesis 1 has been sent and it would have the same distribution, as in this case except now the mean will be shifted to minus a ok. So, instead of V minus a square, now we will have V plus a whole square that is easy.

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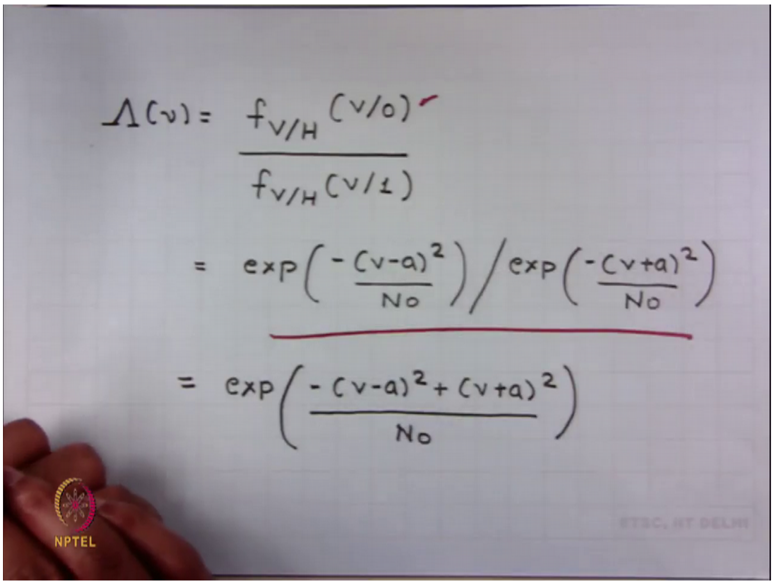

Now, what we have to find is the likelihood ratio to find the likelihood ratio we have to divide the likelihood, when hypothesis 0 has been sent by the likelihood when hypothesis 1 has been sent. And dividing is really straightforward what we will get is this. Now you can also arrange this in this form. So, simplifying this further we just have to span this up.

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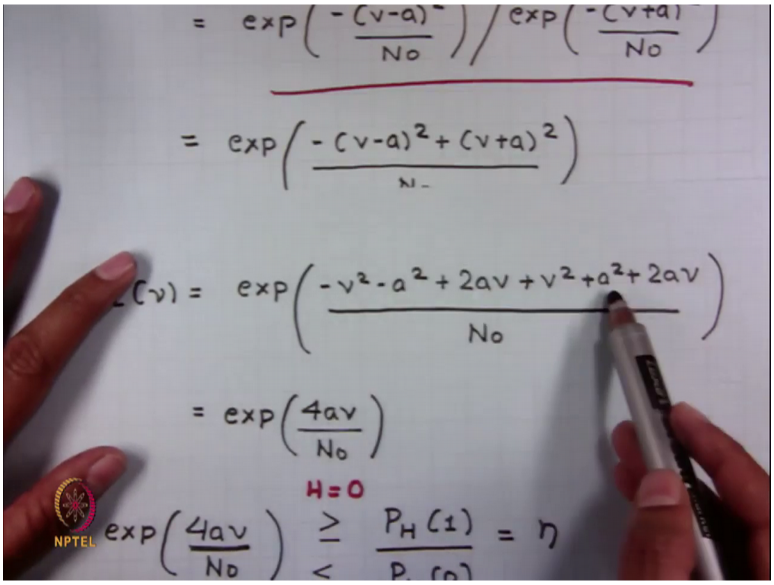

So, we will get minus v square minus a square plus 2 a b we have to expand this up we will get v square plus a square plus 2 a v divided by N 0.

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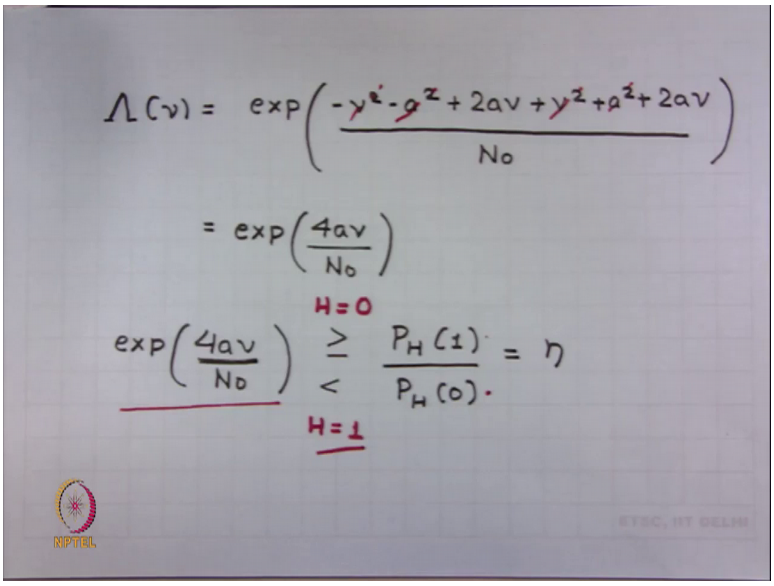

So, now you can see that this term cancels with this term, this cancels with this and what we have is 4 a v by N 0. So, likelihood ratio is pretty straightforward for binary PAM. Now what does our detector do detector compares this likelihood ratio with eta. Where eta is the ratio of a priori probabilities of hypothesis 1 and hypothesis 0 and when it compares this likelihood ratio with eta if it finds that likelihood ratio is greater than or equals to eta. Then it choose the hypothesis as 0 and if identifies that this likelihood ratio is less than eta then it chooses the hypothesis as 1 we can also think this in terms of log likelihood ratio.

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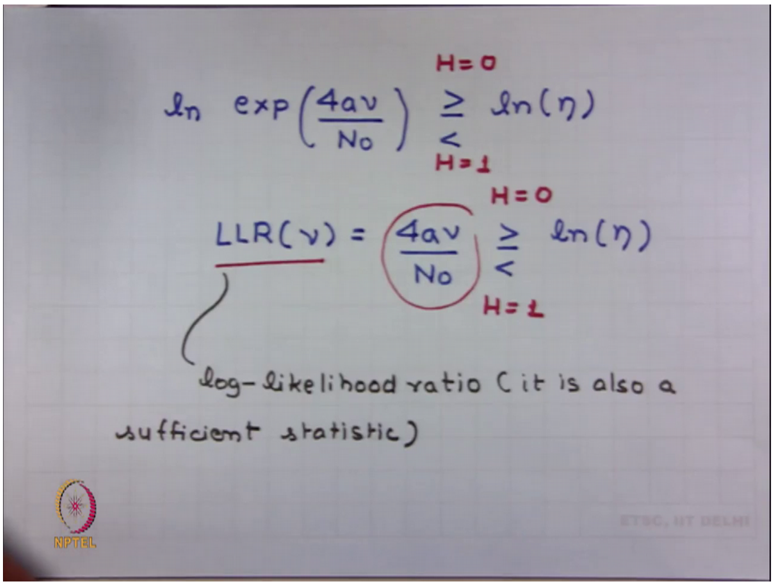

So, we take log on both side log to the base e or natural log on both side. So, we get the log likelihood ratio as simply 4 a v by N 0 and this becomes l n eta. So, it compares this log likelihood ratio with l n eta and if it finds that this log likelihood ratio is greater than this then it chooses hypothesis of 0. And if this log likelihood ratio is less than l n eta it chooses the hypothesis as 1. And as we have said this likelihood ratio is a sufficient statistic and so is log likelihood ratio, you can go back and forth between likelihood ratio and log likelihood ratio.

So, if likelihood ratio is the sufficient statistic log likelihood ratio is also a sufficient statistic. And what a detector needs to know is just this log likelihood ratio once it knows this it does not need to know what where the original likelihoods.

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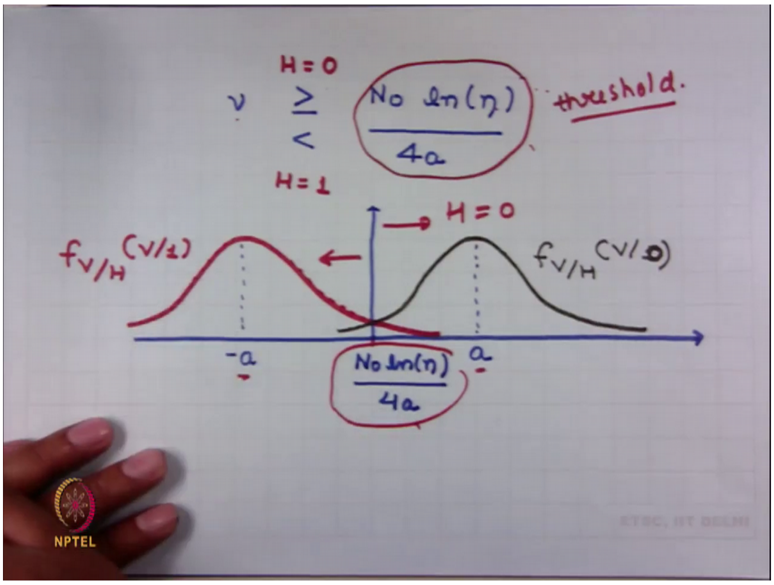

So, now we can shift all this thing to the side what the map rule that you can get is that the detector compares this v which is the received value or the numerical value of the random variable that a detector sees with this thing. And it chooses the hypothesis 0 if numerical value of random variable v is greater than this thing and it chooses the hypothesis this 1 if numerical value of random variable v is less than this thing. So, this is the threshold all right.

Now the next job once we have identified these likelihood ratios and log likelihood ratios and the threshold is to estimate the probability of error. And it will be helpful to plot the likelihood for these two hypotheses. So, we know that these likelihood follows a Gaussian distribution. And when a hypothesis 0 has been sent the mean of this Gaussian distribution is at a and when this hypothesis 1 has been sent the mean would be at minus a. And the detector compares the received value with this threshold and if the received value becomes larger than this N 0 l n eta by 4 a then it chooses hypothesis to be 0. If the received value becomes larger than this point then the hypothesis 0 is chosen and if the received value is less than this threshold then hypothesis 1 has chosen.

And you see that even if hypothesis 1 has been transmitted, there is a possibility that the received numerical value might exceed this threshold. And that will create errors and what would be that error that error would be given by the probability that the random variable numerical value is larger than this when 1 has been transmitted and you can easily calculate that.

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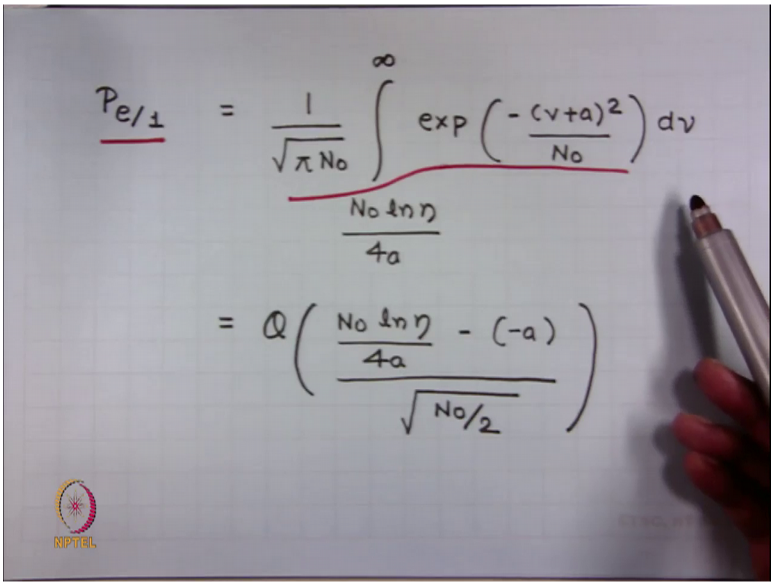

So, what we are saying we can easily calculate probability of error given that 1 has been transmitted when 1 is transmitted this is the probability density function. And we have to find the probability for which the received numerical value of random variable v exceeds this threshold. So, we have to calculate this integration between N 0 l N eta by 4 a to infinity. And we have learned how to solve these integrations we know that this would be calculated easily by using ideas of Q function.

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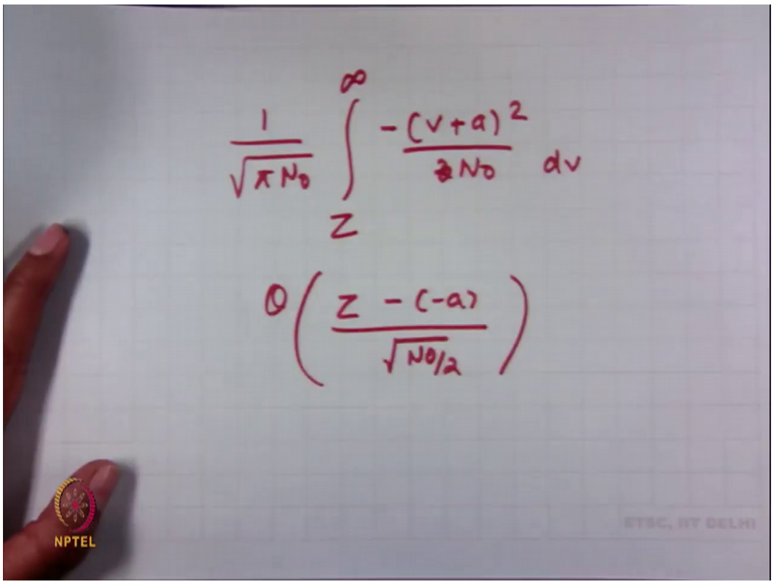

So, let me remind so if I have to calculate this integration, we have seen that this is Q of Z minus mean in this case it will be minus a divided by standard deviation, which is root N 0 by 2 this we have seen in the previous lecture. So, using this idea this integration simply boils down to this thing. So, this is z minus mean is minus a divided by standard deviation.

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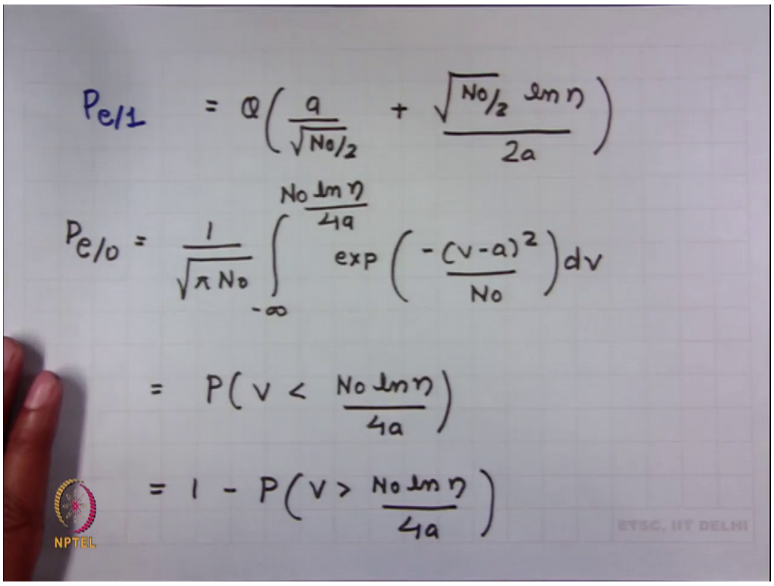

And we can rearrange this into this form. So, this is probability of error given 1. And similarly we can find the probability of error given 0 and what would happen in this case is, let us assume that hypothesis 0 is transmitted since hypothesis 0 is transmitted the detector should decide for hypothesis 0.

But even if the hypothesis 0 is transmitted there is a possibility and a probability for the numerical value of the random variable v to be less than this threshold. And this will create error because if the numerical value of v becomes less than this then detector will choose hypothesis to be 1. And we can evaluate this by integrating the p d f when 0 is transmitted, in the limits minus infinity to threshold and you can solve this by thinking that this is the probability of random variable v to be less than threshold.

And this probability as we have seen before is nothing, but 1 minus probability of random variable v to be greater than threshold right. And using this simple ideas we can express this probability of error given 0 in this way.

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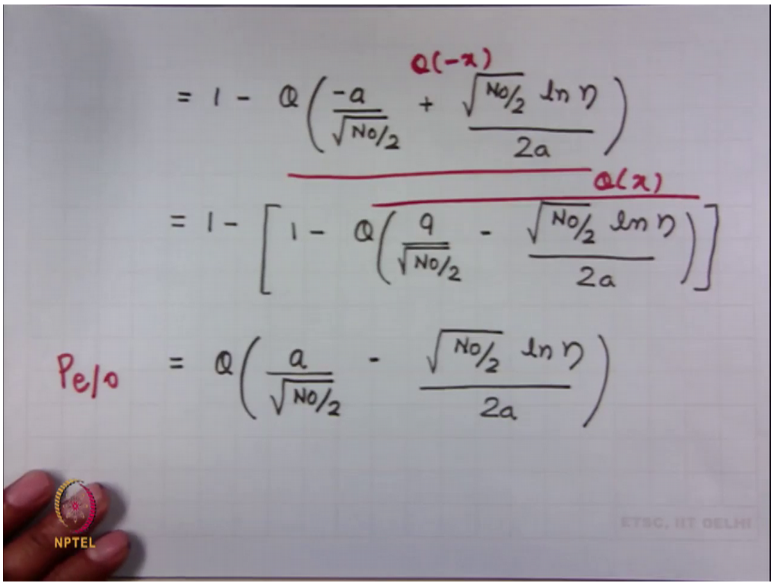

Then rearranging this and this let us say is Q of minus x this is Q of x and Q of minus x is 1 minus Q of x. And from this we can get to this form and we have calculated the probability of error when 0 has been transmitted.

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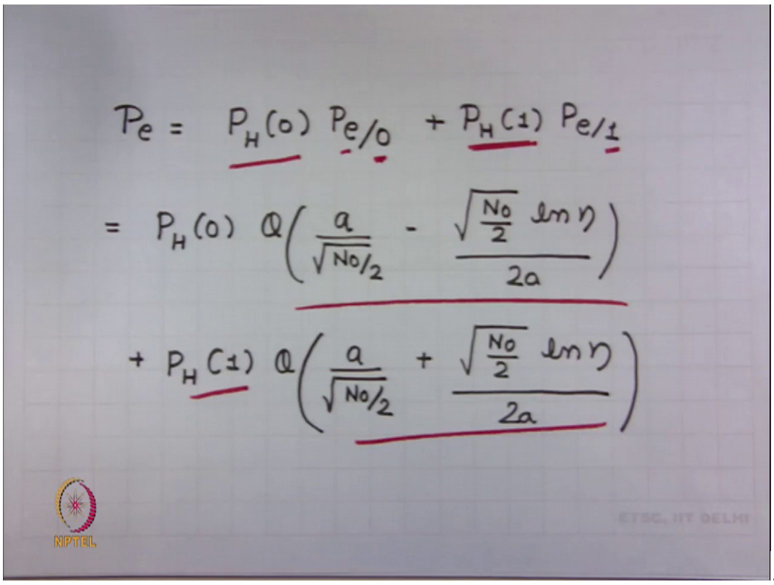

So, what is the total probability of error? The total probability of error is the probability of error when 0 has been transmitted multiplied by the probability that 0 is transmitted plus probability of error when 1 has been transmitted multiplied by probability that 1 is transmitted. So, you can work this out this we have already calculated and then you have to multiply this with the probability of hypothesis 0 and probability of hypothesis 1 and its very inconvenient this formula to remember.

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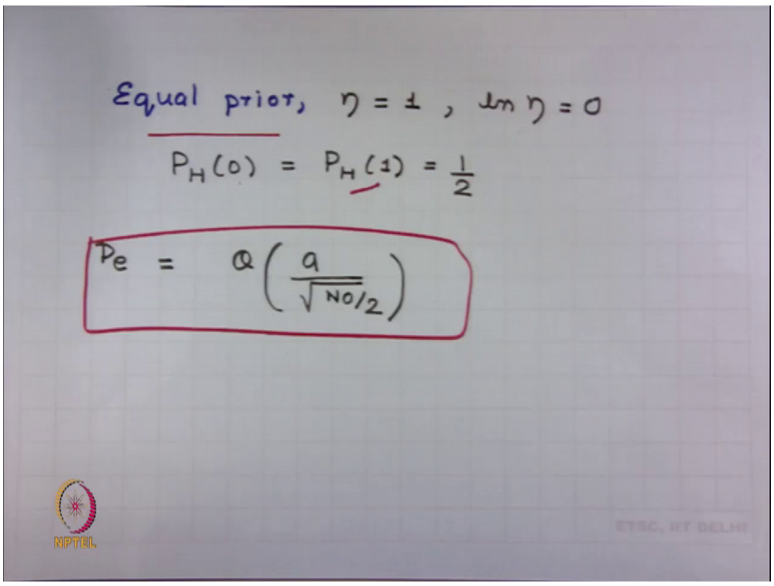

So, we will try to simplify this for the case when we have a situation of equal priors equal prior means that a priori probabilities are equal and same. So, probability of hypothesis 0 to be transmitted is same as the probability of hypothesis 1 to be transmitted and this should be then 1 by 2 because both of these probabilities are same eta is 1 l n eta is 0 and in that case the probability of error reduces to a very simple formula which of course, we can remember the probability of error is given by Q of a divided by root of N 0 by 2. Now, we have got a simple and a neater expression, how can we think about this even more.

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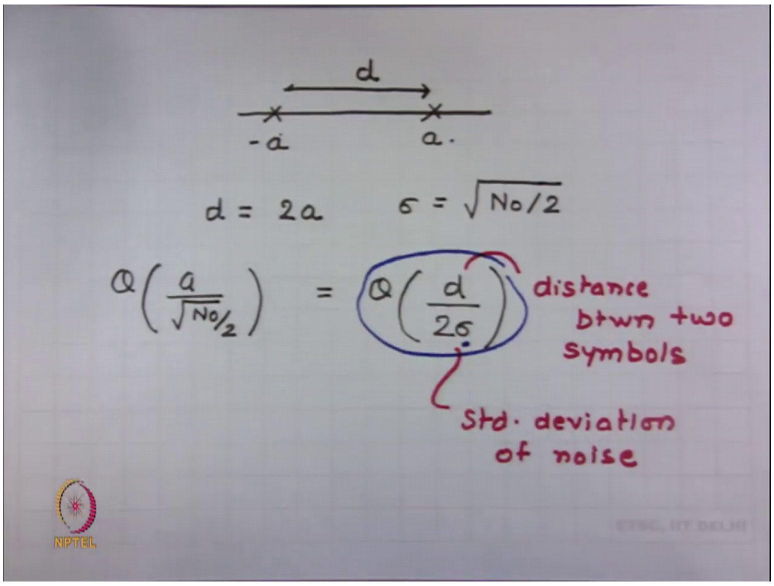

So, for binary PAM we have 2 symbols; a symbol a and a symbol minus a the distance between these two symbols is 2 a and we know that the standard deviation is root of N 0 by 2. So, a divided by root of N 0 by 2 is simply d by 2 because a is d by 2 and standard deviation of noise for the case of binary PAM and for the case of binary antipodal signaling the probability of error is Q of this function and this is simply the distance between the 2 symbols divided by 2 times the standard deviation of noise. And this is a formula that you must remember.

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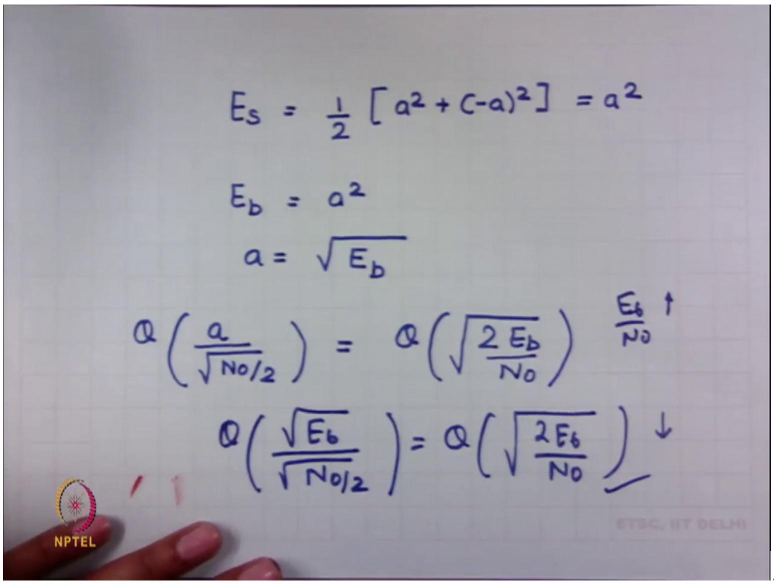

We also have 1 more form in which we can think about this we can think this in terms of energy per symbol. So, energy per symbol in this binary antipodal PAM is simply a square. Now, the bit energy is same as symbol energy because per symbol there is only 1 bit in binary systems. So, bit energy is also a square, so, a is root of bit energy. So, I can also write this, so a is root of E b divided by root of N 0 by 2 and this is simply Q of square root of 2 E b by N 0 this is also very useful formula it gives us the bit error rate performance of binary antipodal signaling as a function of N 0.

As I have no increases this function decreases and hence bit error rate decreases and this is expected. If we are spending more energy per symbol or per bit what would happen is the distance between the symbols will increase and as the distance between symbol increases we hope that the probability of error will decrease and that is what happens.

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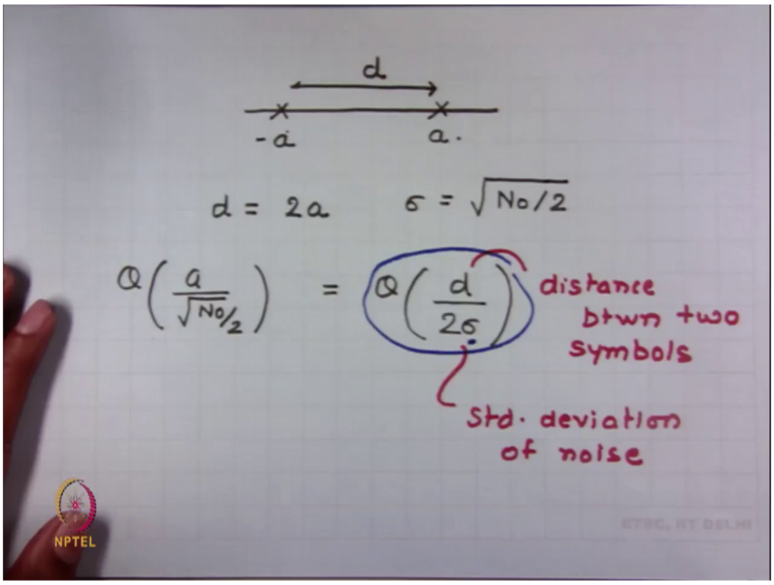

And when we discuss modulation we talked a lot about constructing constellation ensuring that the distance between 2 symbols is appropriately chosen. And this is the reason the bit error rate performance of a modulation scheme is a very strong function of the distance between the symbols. Now let us try to look into binary PAM system.

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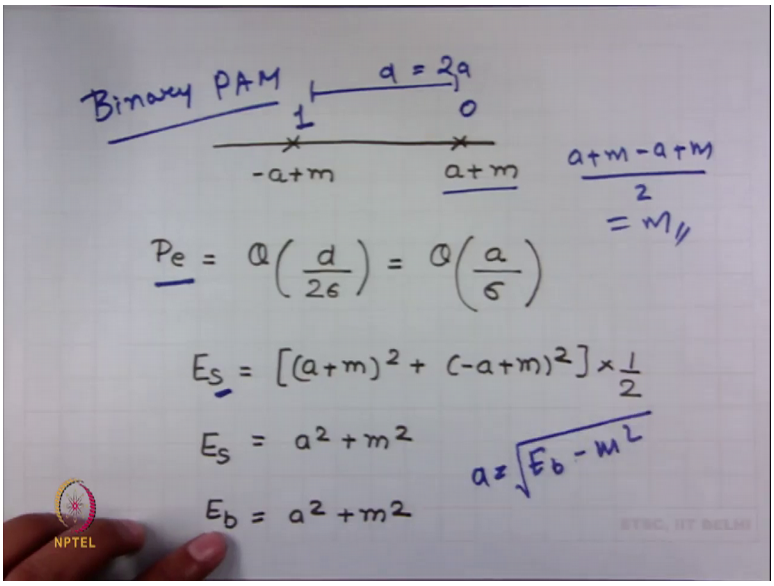

Which is not antipodal so for example you should send this hypothesis 0, by sending a symbol with amplitude a plus m and you send hypothesis 1 by sending a symbol with amplitude minus a plus m what is the mean in this case? So, if you find mean is m. So, this signaling system is with a certain mean whereas, in antipodal signaling we have mean equals to 0. So, we are generalizing this modulation scheme. Bit error rate performance is again a function of distance between the 2 symbols and the standard deviation of noise that is it nothing changes.

So, this signaling scheme will have the same b r performance as the binary antipodean signaling scheme that we discussed. This d is again 2 times a so d by 2 sigma is simply a by sigma and energy per symbol in this case changes. So, energy of this is a plus m whole square energy of this is minus a plus m whole square because there are 2 symbols we have to divide by 1 by 2 I am assuming that these symbols are equal and so mean can be simply obtained by adding the energy of the symbols and dividing by the number of symbols. And you can work out this arithmetic and find that the energy of symbol is simply a square plus m square energy per bit is also a square plus m square because in 1 symbol there is only 1 bit.

So, if I want to find out better performance in terms of E b. So, a is simply E b minus m square root. So, because beta rate is Q a by sigma, bit error rate is also Q of square root of E b minus m square by sigma.

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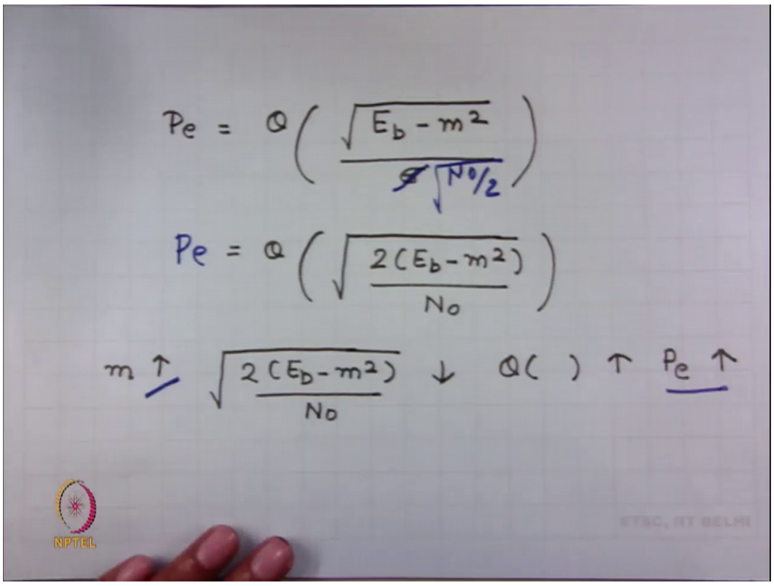

And you can also substitute sigma as N 0 by 2 square root and you get this expression. So, this is the probability of error in terms of bit energy.

So, what you see is when m increases the argument of Q function decreases when argument of Q function decreases Q functions value increases and thus probability of error increases and thus the modulation schemes which have a certain mean, have a poorer better rate performance for the same bit energy spent; that means, if I have a constellation scheme which has got a 0 mean.

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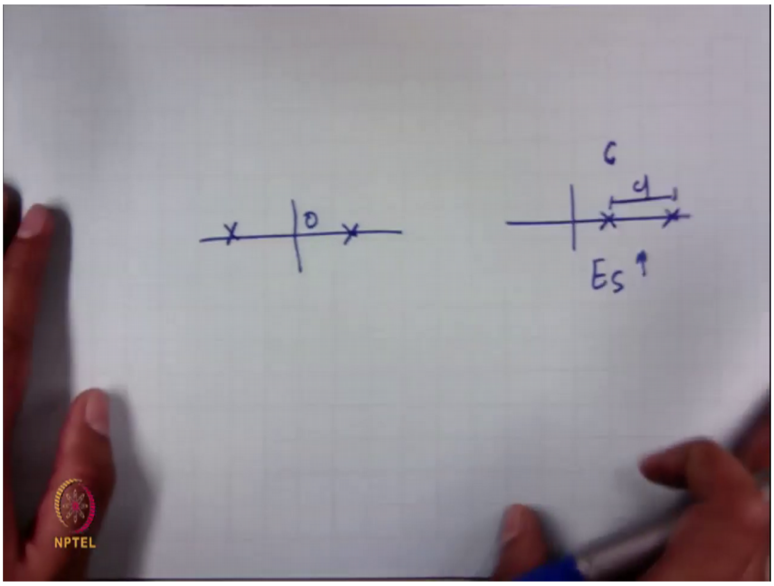

If I simply translate this to this constellation scheme, both these constellation schemes will have the same distance between the 2 symbols. So, both these constellation schemes achieve the same b r performance, but this constellation scheme has to spend a higher energy per symbol. And thus if I want to estimate the performance for the same energy spent per bit, this scheme will have a poorer performance for the same energy spent per bit and that is what this expression is saying. If you want to have the best performance the value of mean should be 0.

And that is the idea behind simplex signaling that we have seen before what was simplex signaling doing? Simplex signaling was just making the mean of the constellation to be 0 and thus it was a long standing conjecture, that simplex signals achieve the best beer performance for a given f no it was a standing conjecture meaning that it was neither being proved nor was it approved, but people tend to believe that this is correct.

So, people used to believe that simplex signals achieve the lowest b r for the given f norm; however, it was disapproved in 1994 by paper published in transactions on communication. And the authors of that paper proved that at ridiculously low levels of signal to noise ratio there is a constellation system which can achieve a better b r for a given f norm.

So, simplex signals are not optimal at ridiculously low levels of SNR, but otherwise they are believed to have the best b r performance for a given f norm and this is because they have 0 mean. So, any modulation scheme be our performance for a given f norm can be improved by just subtracting the mean from that constellation structure. Let us see the b r performance for the unipolar signalling scheme.

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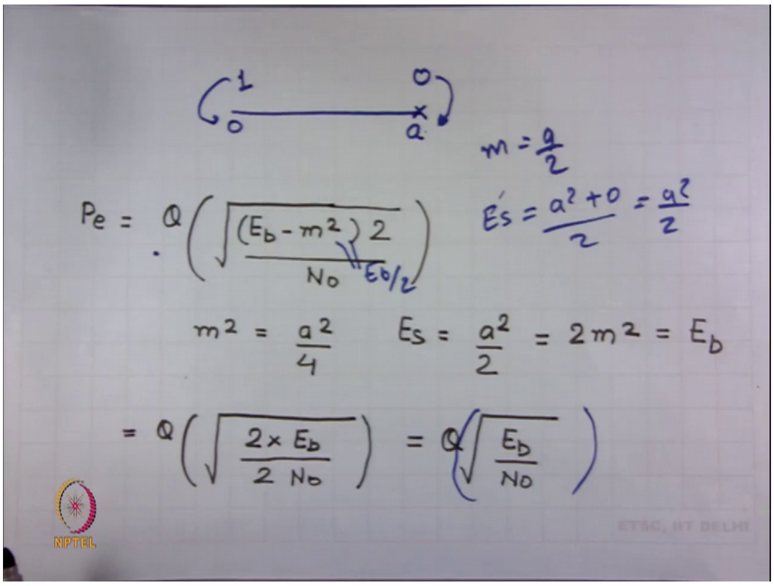

So, in unipolar let us say we are transmitting a symbol a for hypothesis 0 and symbol 0 for hypothesis 1 and we know that we have to use this formula, this formula should be used when this constellation has a mean and what is the mean in this case. So, mean is a by 2. So, m square is a square by 4 similar energy is a square plus 0 by 2, so, a square by 2. So, this is simply 2 m square and similar energy is also same as E b. So, we get E b minus m square is simply E b by 2. So, we get E b by 2 and 2 cancels with this 2, so, we get simply Q of root E b by n 0. And because this scheme has certain mean this will achieve a lower beer performance for the same ab no all right.

So, we have understood it quite well how does the detector work in case of binary antipodal signals. And so far what we have been assuming is when you have to transmit a hypothesis you are transmitting one numerical value, plus a or minus a let us have more fun by generalizing this. And let us now say that when you have to transmit a hypothesis instead of transmitting 1 numerical value you transmit a vector and why this is important because actually when we have to transmit a hypothesis we will be transmitting a waveform and waveform is nothing, but it is a vector.

So, let us see what happens when for a hypothesis we transmit a vector instead of a numerical value and to generalize these issues as much as we can we assume these vectors to be complex vectors. So, that we cover things intality. Once we have finished with this idea of sending complex vectors for hypothesis we will see later on why this is important and how is this related to other things and other blocks like how is this related to basement to passman conversion how is this related to d modulator and so on and so forth.

But at this point without worrying about what is the physical significance of transmitting hypotheses with complex vectors. let us assume that we are doing this and let us see what is the log likelihood ratio in this case and how can we think about be a performance ok.

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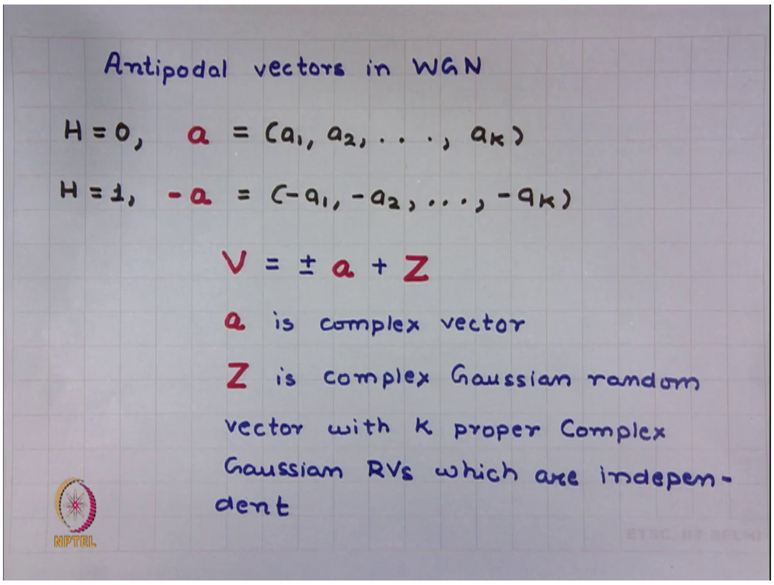

So, let us get us started with antipodal vector detection in white Gaussian noise and we will see that detection of vectors is not different from detection of a single numerical value. So, let us assume that we have 2 hypotheses so we are considering antipodal case. So, we have 2 hypotheses and when the hypothesis is 0; that means, when we have to transmit bit 0 we send a vector with coefficients a 1 a 2 and a k.

And when we have to transmit hypothesis 1 we transmit a vector minus a and does the coefficients of this vector minus a 1 minus a 2 and minus a k. And we assume that these vectors are complex vectors because we want to generalize issues we will see what happens, when these vectors are complex and then from complex going to real vector detection is trivial ok. So, we want to start thinking about detection of complex vectors all right.

So, at the output of a demodulator we are going to have a random vector V this random vector V is this vector a plus a vector Z and what is Z? Z is a complex Gaussian random vector because what we are transmitting is a complex vector. So, the noise vector will also be a complex Gaussian random vector and this complex question random vector will contain k proper complex Gaussian random variables. And these complex question random variables will be independent; we will see shortly what are these proper complex Gaussian random variables.

But let us just assume at this moment that this Z contains k proper complex Gaussian random variables and these proper complex Gaussian random variables are independent. And these are independent because the underlying noise process is quite Gaussian noise. So, any samples of a white Gaussian noise are independent samples right. And the samples of a white Gaussian noise are random variables right. So, these k proper complex Gaussian random variables will be independent. So, the first thing that we will see is what are these proper complex Gaussian random variables?

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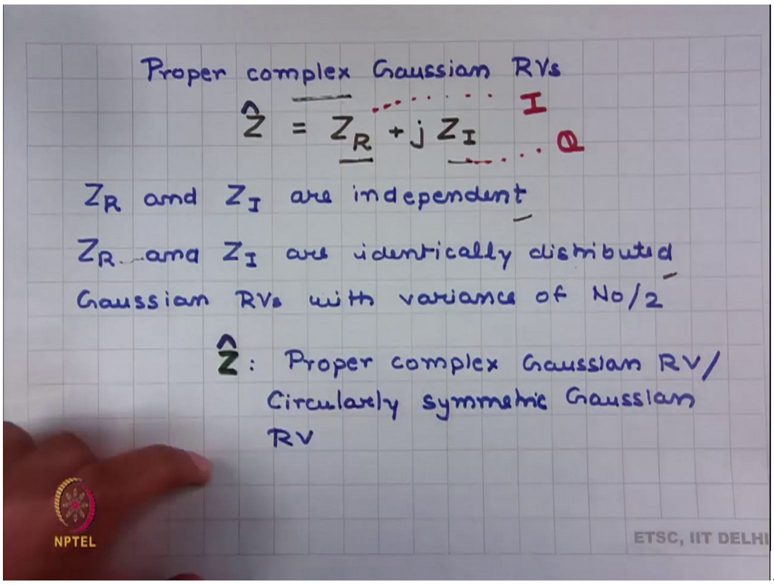

So, let us assume that I have a random variable Z cap and this random variable is composed of 2 random variables Z R is real random variable and Z I they notes imaginary random variable. That means, that this is the imaginary part of this proper complex Gaussian random variable Z cap. So, what we assume is that these Z R and Z I are independent. And secondly, what we assume is that these Z R and Z I are identically distributed and we assume that they are Gaussian random variables with a variance of N 0 by 2. So, if Z I are and Z I are independent and they are identically distributed Gaussian random variables then this Z cap is a proper complex Gaussian random variable.

So, what is the proper complex Gaussian random variable? Is first of all is a complex Gaussian random variable; that means, its composed of 2 real Gaussian random variables and these random variables are independent and these random variables are also identically distributed ok. So, if Z cap has 2 real independent random variables, which are identically distributed Gaussian random variables then we say that random variable as a proper complex Gaussian random variable.

And in detection we usually end up with these proper complex Gaussian random variables because first of all this Z R what it would denote it would denote the sample of noise in I channel and Z I would denote the sample of noise in Q channel right. We already know from where we are getting these complex signals we get this complex signals because we want to think about 2 real signals in 2 channels.

So, we have a real signal in I channel and we have a real signal in Q channel noise we assume has the same distribution in I and Q channel. And that is why first of all these random variables will have identical distribution and what also we assume is this I channel and Q channel is completely independent. So, noise in these channels will also be independent and that is why this complex Gaussian random variable is usually proper complex Gaussian random variable. Now this proper complex Gaussian random variables are also known as circularly symmetric Gaussian random variables ok. So, sometimes we call this as circularly symmetric Gaussian random variable sometimes we call this as proper complex question random variable.

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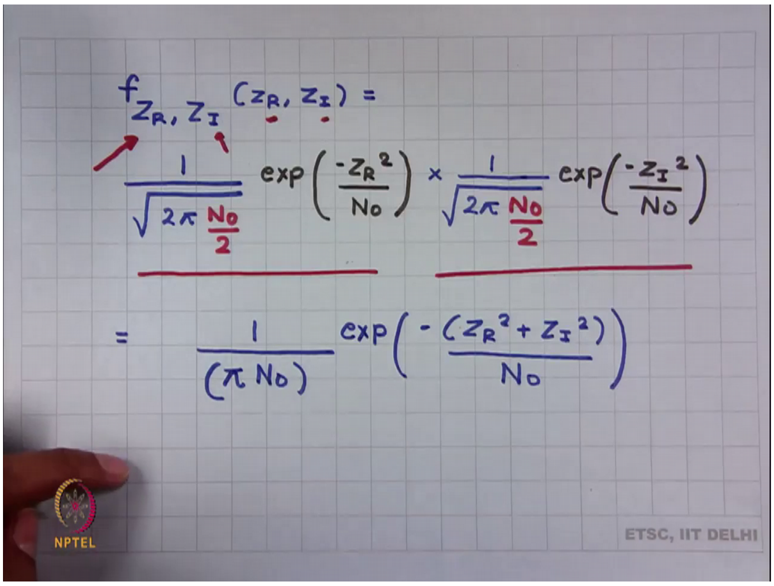

So, let us look at the joint p d f of Z R and Z I. So, Z R is the random variable in the real part of proper complex Gaussian random variable and Z I is the random variable in the imaginary part of proper complex Gaussian random variable. And what we are interested in is trying to find out what is the joint p d f of Z R and Z I. Now because these Z R and Z I are independent random variables the joint p d f is simply the product of marginal p d f and what is the p d f of Z R? Z r is a Gaussian random variable. So, its p d f can be obtained as we have done it several times its variance is n 0 by 2 its mean is 0 and it has a Gaussian distribution why is mean 0? We always assume that the random variable is resulting from noise has 0 mean.

So, that is why it has got a mean 0. How do you think about the p d f of Z I is exactly same as the p d f of Z r. So, what changes is the numerical value is Z I and the numerical value in this case is Z R because we are trying to identify what is the probability distribution of Z R at numerical value small z R what is the probability distribution of random variable Z I at numerical value a small z I. So, we can simply obtain the joint p d f by multiplying these 2 p d f and we get an equation like this. So, now if you seen that equation if we assume that this Z R square plus Z I square is r square.

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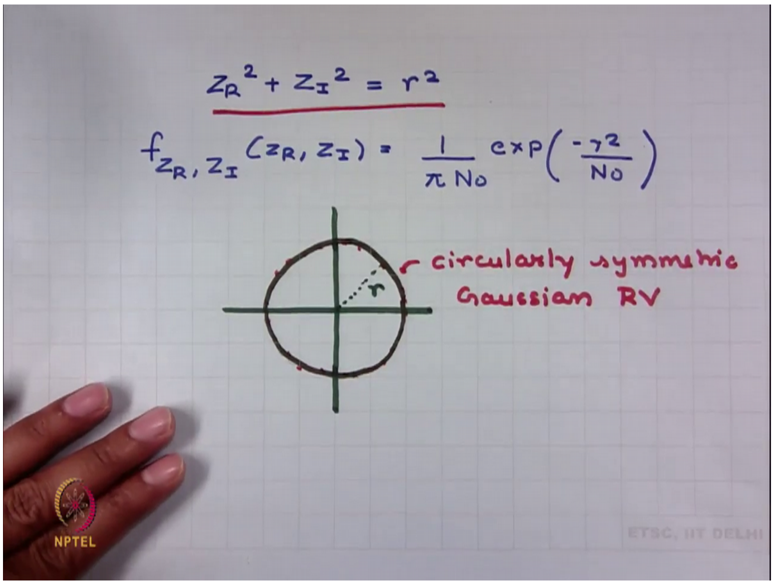

Then this is the equation of a circle. So, we can assume a circle of radius R where R is given by this equation. Then what you see is this joint p d f is simply 1 by pi N 0 exponential of minus R square by N 0. That means, for the same r the joint p d f would be same; that means, all these points on a circle will have the same p d f. And hence you have a circular symmetricity and that is why it is known as circularly symmetric Gaussian random variable we can also write this in terms of proper complex Gaussian random variable.

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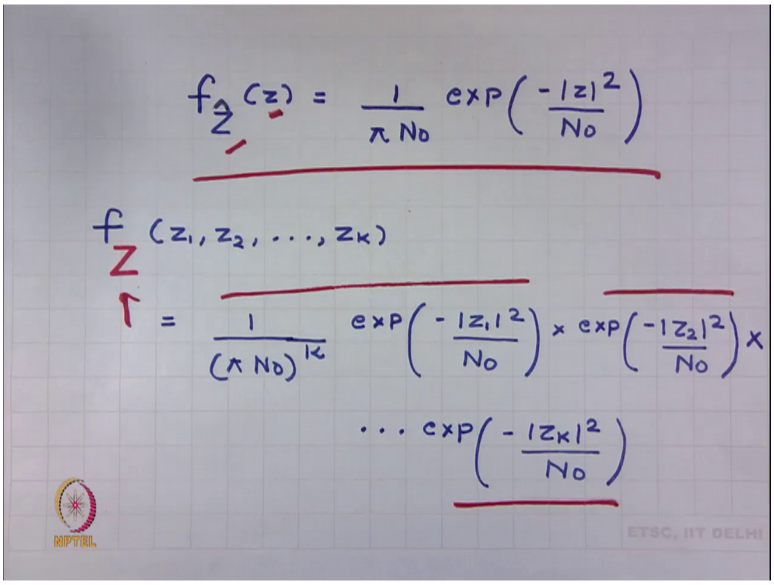

So, what we are asking is what is the p d f of this random variable taking a numerical value small z and this p d f can be easily followed from this expression. So, what we have to substitute in here is mod of Z instead of r because mod of Z is simply r right.

So, from this we can easily get the joint p d f of a proper complex Gaussian random variable. And if we have investigated and obtained the p d f of 1 proper complex Gaussian random variable we can easily obtain the p d f of Gaussian random vector z; this Gaussian random vectors is composed of k proper complex Gaussian random variables which are independent and hence its p d f can be simply obtained by multiplying the marginal p d f of these k proper complex Gaussian random variables. And we know that the p d f of 1 proper complex Gaussian random variable is given by this expression. Similarly the p d f of the second proper complex Gaussian random variable will be given by this expression and so on so forth.

So, multiplying k such marginal p d f we can obtain the p d f of the Gaussian random vector Z and when you multiply this you get a very simple expression which is this.

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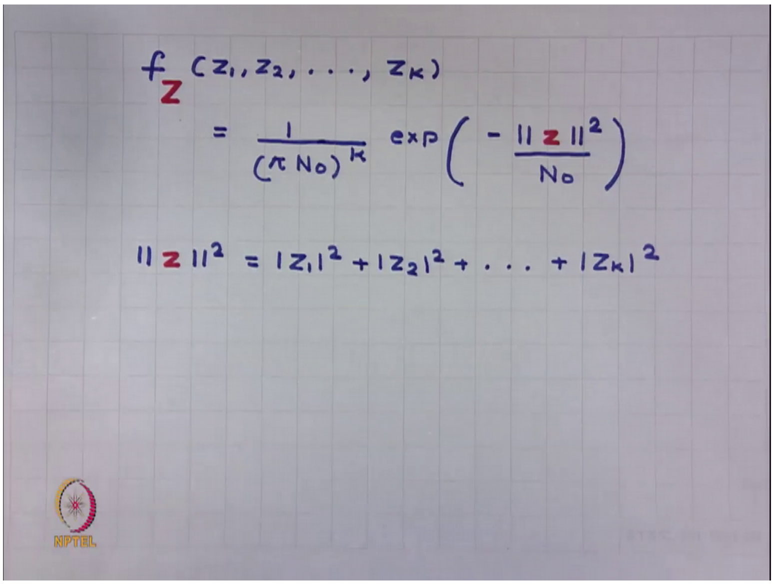

So, you get an expression in terms of norm of Z and we already know that norm square of Z is simply mod square of Z 1 plus mod square of Z 2 up to mod square of Z k. So, we have obtained the p d f of this Gaussian random vector Z.

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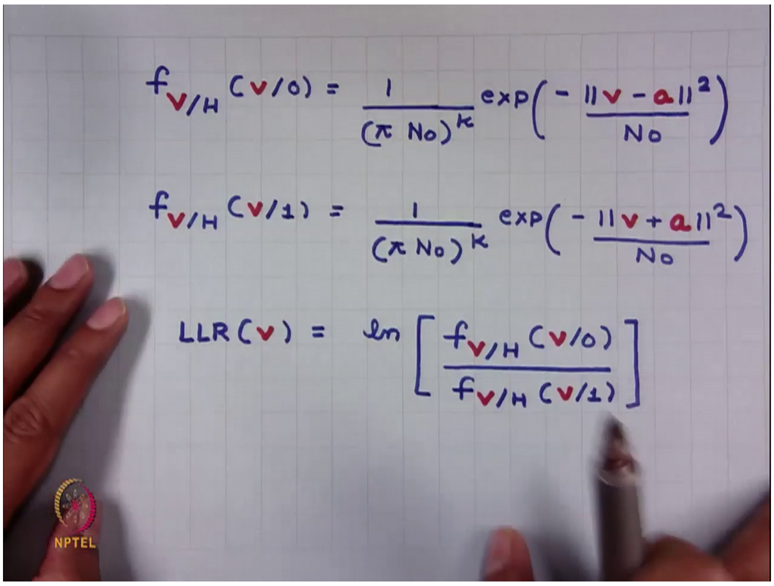

Now, what we have to find is the likelihood given that hypothesis H is 0 when you have to find that you can easily derive this from this expression. This is the p d f, when the Gaussian random vector Z is transmitted. And when the hypothesis is 0 the mean of this Gaussian random vector will shift around a. So, we have done this in the case of detection of single numerical values as well. We have seen that when you transmitted hypothesis 0, we were transmitting a numerical value a corresponding to that hypothesis and what happened was the mean of the Gaussian random variable shifted at around a ok.

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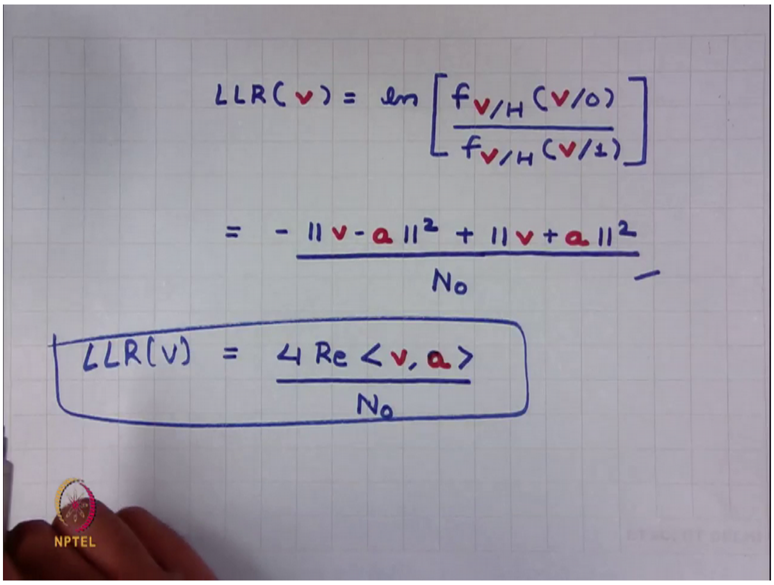

And the way we have thought about it there is if V is a plus Z and the mean of v will be simply a plus mean of Z and if mean of Z is 0 the mean of v is simply a and mean of the random variables arising because if the noise is always 0. So, mean of this random variable is simply shifted at around the value of the deterministic signal.

So, here we had a p d f of this Gaussian random vector Z this p d f was like this. So, when a hypothesis 0 writes over this what happens is that mean shifts around that is it. Similarly you can find the likelihood when hypothesis 1 is transmitted and what changes in this case is that the mean shifts now at around minus a, these things are exactly same as was in the case of detection of single numerical value. And again we can find the log likelihood ratio which is l n of the ratios of these 2 likelihoods.

And once you do that what we end up with is a simple expression like this again this is exactly same as what we obtained in the case of detection of single numerical value there it was v minus a whole square plus v plus a whole square here we are just having norms we are thinking in terms of norm square because these are vectors that was numerical value.

Now, if you work out this identity you can show easily that this thing is nothing, but it is 4 times real part of inner product of v with a. And hence the log likelihood ratio that we obtain is this before talking more about this let us first see and work out this identity will work out this time.

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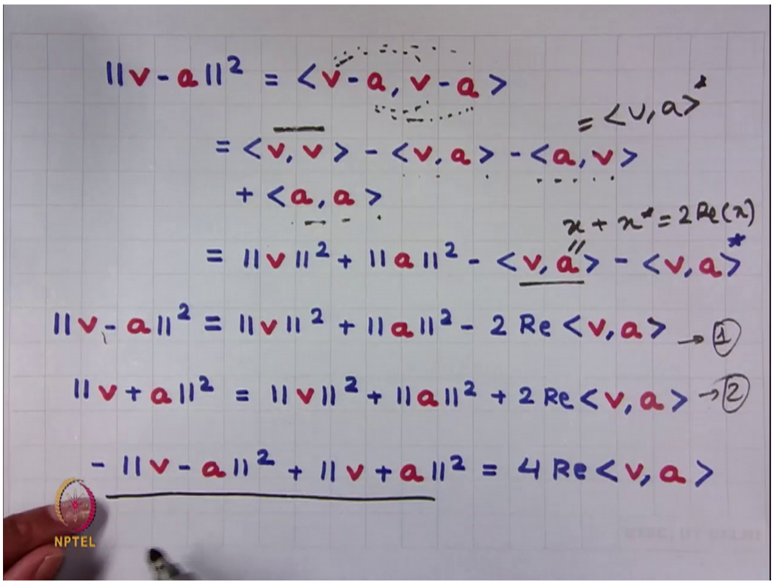

And I will hope that you will remember it the next time that we will be using it. So, how can I think about norm square of v minus a? Norm square of v minus a, is simply inner product of v minus a with v minus a. We know and we have seen that this is the way you should interpret norm square. Take the inner product of the vector with itself and now this inner product follows Hermitian by linearity. So, what you can do is you can multiply term by term.

So, you can multiply this with this you get this then you can multiply v with minus a and we get to this term then we can multiply this with this we get this term then we can multiply this with this we get this term. So, we get 4 terms and inner product of v with v is nothing, but norm square of v inner product of a with a is nothing, but norm square of a and we have this term in here and what is inner product of a with v? We know because inner product satisfies Hermitian by linearity. If you want to think about this you can think this in terms of inner product of v with a. So, this thing will be same as inner product of v with a, but with a conjugate.

So, what we are saying is if you have to calculate the inner product of these two vectors you can simply flip the vectors, Compute the inner product now this inner product will be simply the conjugate of this inner product. So, what we get is inner product of a with v is simply inner product of v with a, but with a conjugate. So, finally, we can say that norm square of v minus a is simply norm square of v plus norm square of a minus 2 times real part of inner product of v with a; you know that if you add x to x conjugate and let us assume that this is x this is x conjugate.

Then we simply get 2 times real part of x and this is what is happening in here. So, this is an important identity which you should remember that norm square of v minus a is simply norm square of v plus norm square of a minus 2 times real part of inner product of v with a. Similarly, norm square of v plus a is this thing. So, what changes in here is instead of minus now we have a positive sign. So, its like a square plus v square plus 2 a b kind of equality.

So, if I know this and this I can easily solve for this and we get that this is simply 4 times real part of inner product of v with a. So, we have got the log likelihood ratio.

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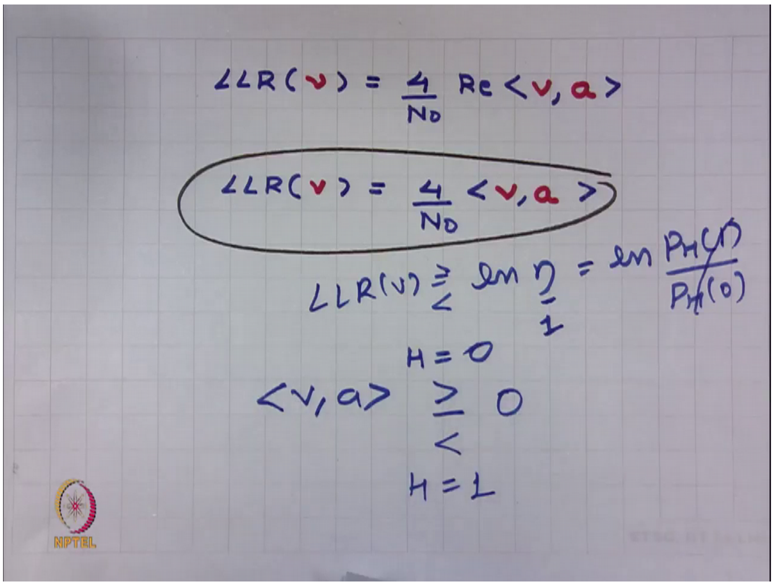

Log likelihood ratio is simply 4 times real part of inner product of v with a divided by N 0; if these vectors are real if you do not have this real part anymore because this anyway already gives you a real product.

So, in case that the vectors involved are real vectors log likelihood ratio simply reduces to this. Now its a very important thing because what is happening is that this vector we consisted a bunch of numbers, complex numbers real numbers this vector a consisted of bunch of real numbers of complex numbers, but the log likelihood ratio is just a single number. So, we are converting a bunch of numbers into a single number and by looking at that single number you can decide for the hypothesis and that is why it does lot of savings.

So, previously we said that log likelihood ratio is a sufficient statistic, because just by looking at log likelihood ratio your detector can decide for the hypothesis. And this is a great saving in this case, because you might have a bunch of numbers and you can convert this bunch of numbers into a single number by carrying out this operation and by looking at that single number you can decide for the hypothesis ok. Let us move on and let us look what is the geometry in case of ML detection.

So, we are first starting with m l detection and then we will see what happens when we go to map detection. And we are thinking in terms of real vectors only, geometry for complex vectors is little bit harder, but conceptually similar the drawing becomes harder and that is why we confine ourselves to just real vectors geometry all right.

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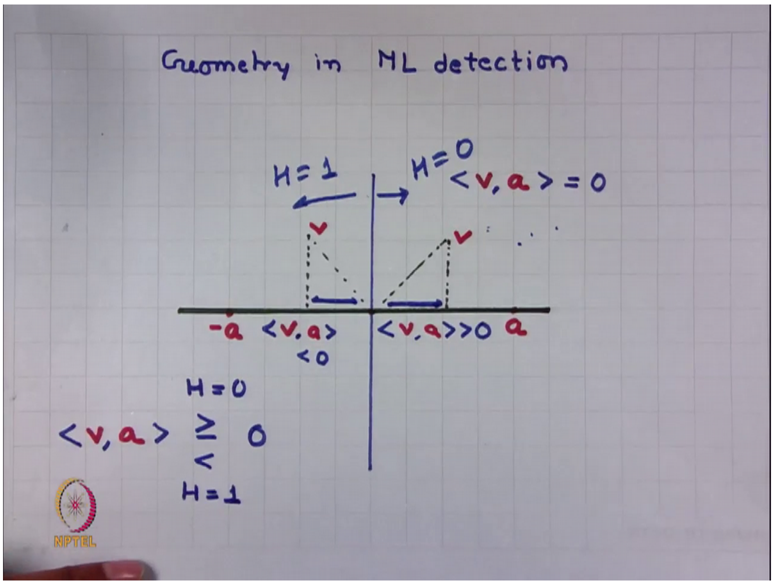

So, let us look where is the threshold? So, if we are talking about M L detection what we assume is that we have equal priors; once you have equal priors, you know that we have to compare this log likelihood ratio with l n of eta and this l n eta is simply l n of ratios of prior probabilities. And if these priors are equal eta is 1 l n eta is 0. So, basically when priors are equal you have to compare this log likelihood ratio with 0. So, in case of real vectors you simply have to ask the question whether the inner product of v with a is greater than 0 or is less than 0. If it is greater than 0 you decide for the hypothesis as 0 and if it is less than 0 you decide for the hypothesis as 1. So, in case of M L your threshold is the 0 alright.

So, we can draw this inner product of v with a and this will be simply perpendicular bisector to this factor connecting minus a and a. In simple terms you just draw a line between minus a and a you draw the perpendicular bisector and this perpendicular bisector will denote the points for which inner product of v with a 0 and why is this? Let us draw it again.

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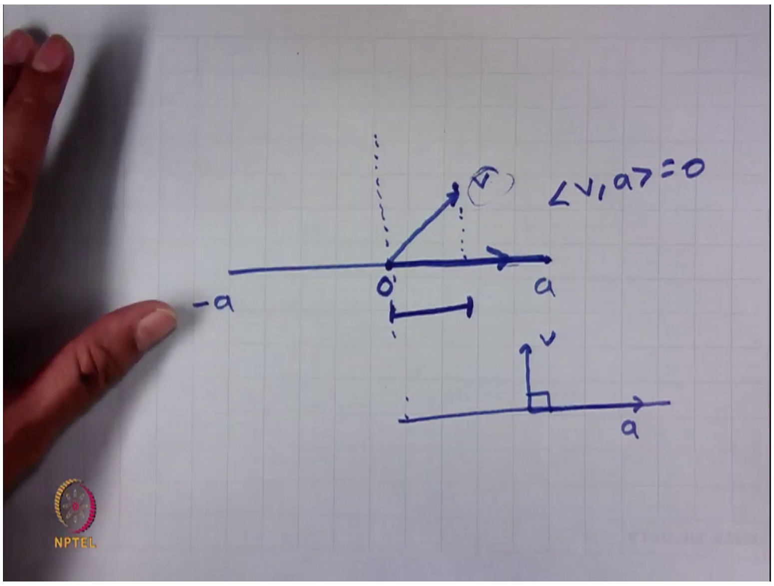

So, suppose you have 2 points and let us say vector a is this. So, vector a is from 0 to this point. And if let us say I am considering a v vector here. So, what is the inner product of v with a the inner product of v with a is simply the projection of v on a multiplied by norm of a projection of v on a will give you this length and this inner product will not be 0, until and unless this point v lies on the perpendicular bisector. When this point v lies on the perpendicular bisector then you have such a situation. So, v is here a is here and then there is no projection of v on a because v and a are orthogonal or perpendicular.

So, when the inner product of v with a is 0 you know that this points lies on a perpendicular bisector, for a line which is connecting minus a and a. So, this threshold is simply perpendicular bisector of the line connecting minus a and a and all points lying to the right of this perpendicular bisector denotes the situation when inner product of v with a is greater than 0.

And hence hypothesis 0 should be chosen, all points lying to the left of this perpendicular bisector denotes the situation when inner product of v with a is less than 0 and hence for these points hypothesis 1 should be chosen. In simple terms if the points are closer to this a then hypothesis 0 should be chosen if the points are closer to this minus a then hypothesis 1 should be chosen. So, this is what this threshold test tells us you can also appreciate this with the simple picture.

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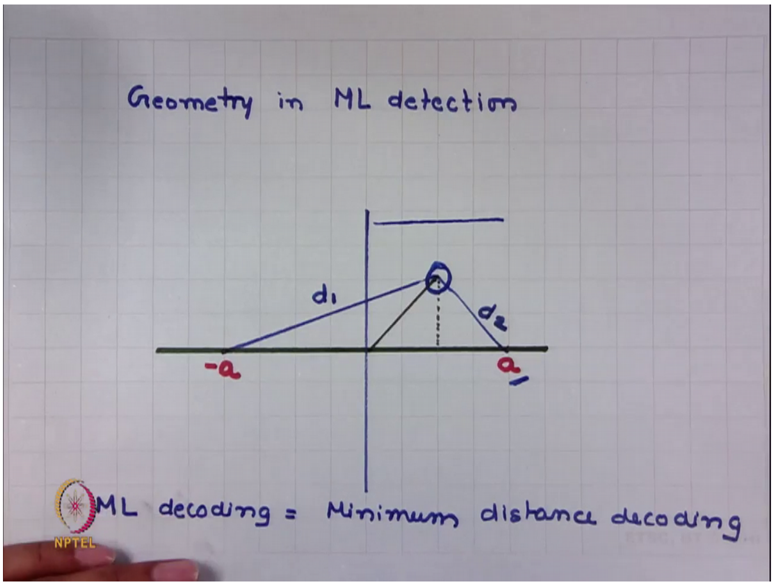

So, if this point lies here you know that hypothesis a should be chosen and this also tells me that if the points lie to this side, then the distance of this point from this hypothesis is closure. So, receiver does some kind of minimum distance decoding in which it calculates the distance of the received point from the hypothesis and it selects the hypothesis which has minimum distance from the received point or from the received vector. And hence M L decoding is also known as minimum distance decoding what happens in map detector?

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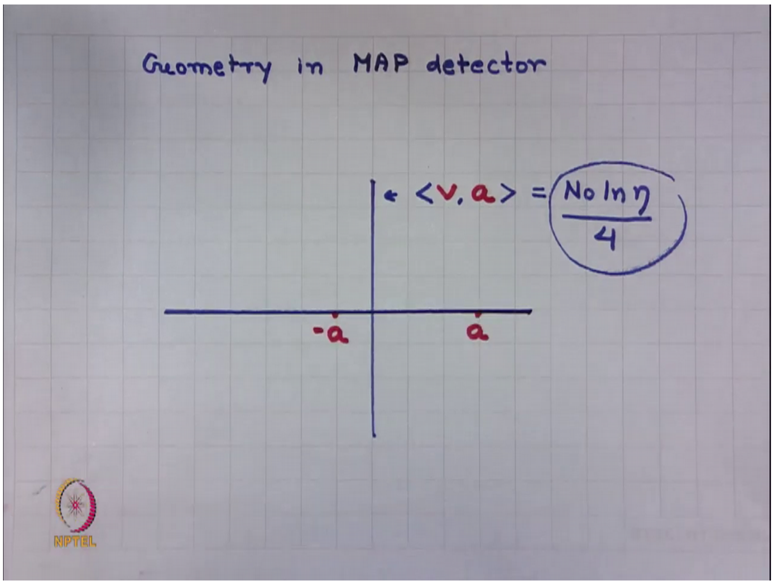

Nothing difficult happens just your threshold changes. So, threshold instead of being 0 now is given by this thing and now the detection regions are not decided by the perpendicular bisector of the line connecting minus a and a.

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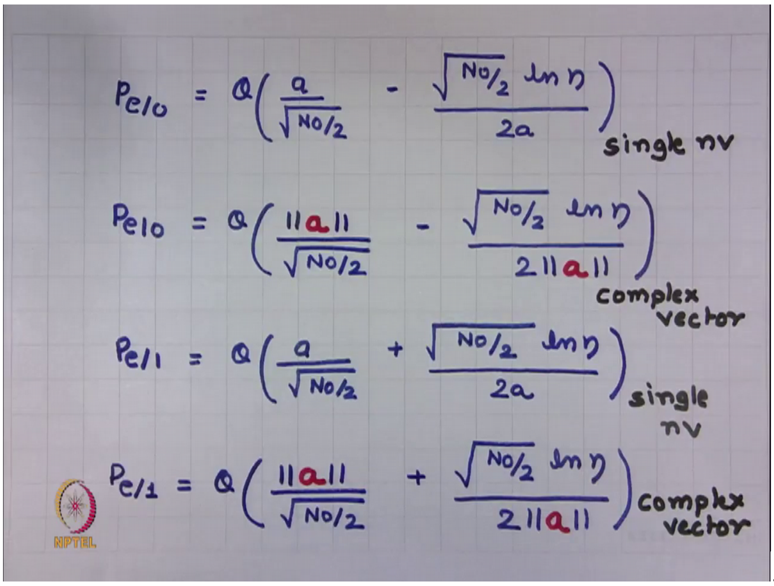

So, what about the error performance? So, we already have calculated the error performance in case of reception of single numerical values there we have calculated probability of error given 0 probability of error given 1. In case of complex vectors we get the same probability of errors the only thing that changes in here is instead of a now we have to talk in terms of norm of a. Because in case of single numerical values a decides the distance in case of vector norm of a decides the distance that is it and the probability of errors and all these formulas remain same.

So, with this we have come to the conclusion of this lecture in this lecture we have done few important things, first of all we have looked into what is the probability of error for binary PAM, we have looked this probability of error for 2 cases when we have antipodal signaling and when we have unipolar signaling. Then we looked into the aspects of vector detection particularly complex vector detection.

In case of complex vector detection what changes is that now you have the noise random variable, which is a proper complex Gaussian random variable; that means, its composed of 2 real Gaussian random variables which are independent and which are identically distributed. What we have formulated is the probability density function of a proper complex Gaussian random variable and then once we have understood the probability density function of a proper complex Gaussian random variable, we can easily extend this to the case of caution random vector which is consisting of these proper complex Gaussian random variables.

So, probability density function of a vector is simply obtained by multiplying the marginal p d fs of these proper complex Gaussian random variables because these proper complex Gaussian random variables are also independent. So, from that p d f we can easily obtain the likelihood ratios everything remains same other, than we have norms popping up in this case.

And we get the log likelihood ratio as 4 times real part of inner product of v with a where v is the received vector and a is the transmitted vector divided by N 0. And we have seen that in case of geometry is decided by perpendicular bisector which is bisecting the line connecting the 2 signal points ok. In the next lecture we will see how to do detection when multiple hypotheses are involved and then we will go to waveform detection.

Thank you.