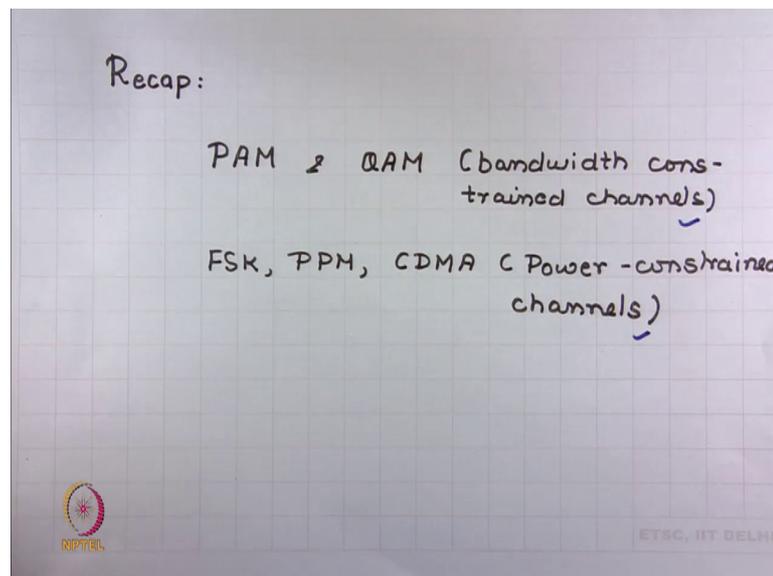


Principles of Digital Communication
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Lecture – 29
Modulation
Differential Modulation Schemes

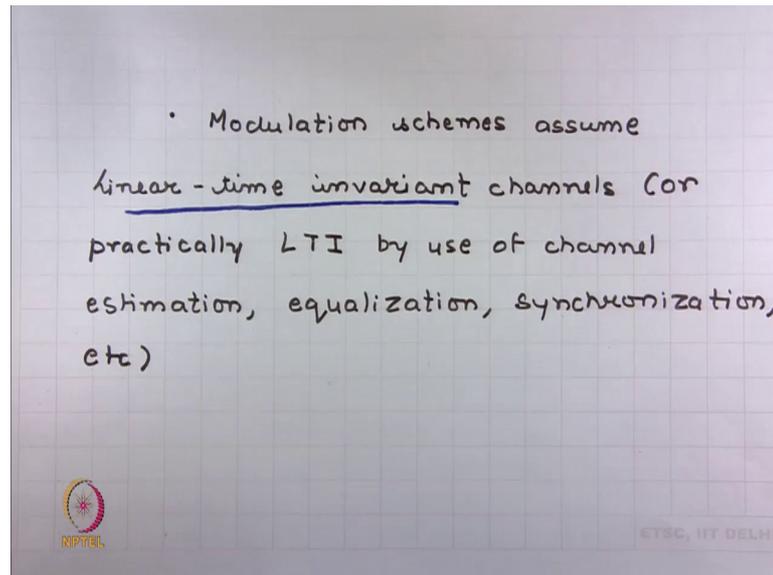
Good morning. Welcome to a new lecture on Modulation and in today's lecture we will be talking about Differential Modulation Schemes ok. So, in the previous lecture we have looked into the modulation schemes like pulse amplitude modulation and quadrature amplitude modulation.

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And we have said that these modulation schemes are ideal for bandwidth constraint channels ok. And we also looked into modulation schemes like FSK, PPM and CDMA and we have said that these modulation schemes are ideal for power constraint channels. And in today's lecture we will be talking about differential modulation schemes alright.

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So, the modulation schemes that we have discussed, these modulation schemes assumed Linear Time Invariant channels or LTI channels. And if the channel is not linear time invariant we can assume it to be practically linear time invariant by the use of channel estimation equalization and synchronization techniques.

So, with all of these other things channel estimation equalization and synchronization we can make our channel approximately as linear time invariant channel. And these modulation schemes that we discussed so far are good modulation schemes for linear time invariant channels. Let me revise something that we have done in pass band to baseband conversion.

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LTI

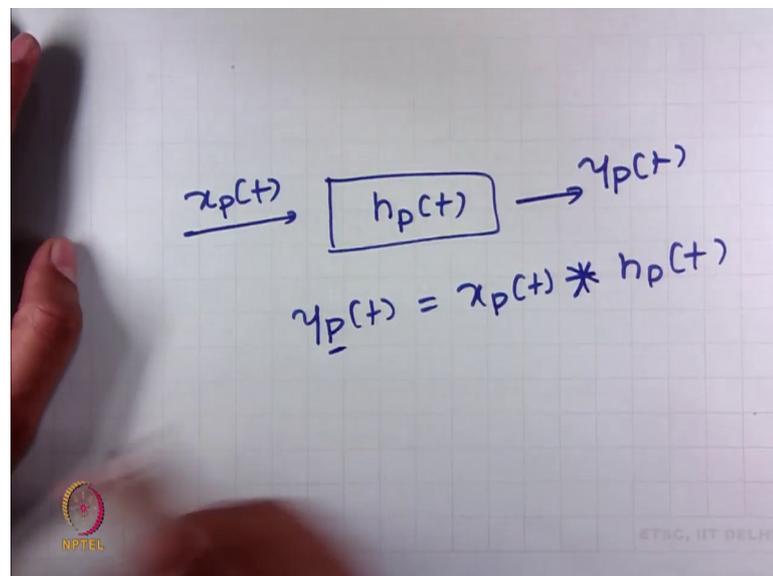
$$\left. \begin{aligned} y_p(t) &= x_p(t) * h_p(t) \\ y(t) &= \frac{x(t) * h(t)}{\sqrt{2}} \end{aligned} \right\} (19-21)$$
$$h(t) = h_s(t) \sqrt{2}$$

↳ channel gain (complex, as it is base band)

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We have seen that the output of a pass band system can be obtained by convolution of input with the impulse response of the system.

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For example, if we have a filter which is a pass band filter with an impulse response of $h_p(t)$, to this filter if I assume that the input is a pass band signal $x_p(t)$ then the output $y_p(t)$ of this filter can simply be obtained by the convolution of the input to this filter with the impulse response of this filter ok. So, these operations are carried out in pass band

domain. And how do we know that this is in pass band domain? Because of this subscript p.

So, if we want to think about this operation in baseband domain, we have said that $y(t)$ is also same as $x(t)$ convolution $h(t)$ divided by root 2. And $y(t)$ is the complex baseband equivalent of this signal $y_p(t)$, $x(t)$ is the complex baseband equivalent of the signal $x_p(t)$ and $h(t)$ is the complex baseband equivalent of the signal $h_p(t)$ ok. So, we have seen this between lecture 19 and 21; I would not be deriving it again we have derived it at that time.

And, why we have this a scaling factor of root 2? Because we want to conserve energy as we go from pass band to baseband domain. Now, let us assume that $h(t)$ that is the complex baseband equivalent of the pass band channel is some constant h times root 2 times delta t . It is an impulse response, $h(t)$ is an impulse response and we are assuming that the impulse response is impulse times some constant; h is a constant and this h is known as channel gain.

And in general this channel gain will be complex because, $h(t)$ is a complex baseband signal. And why we have this root 2? We have this root 2 hear to get rid of this scaling factor of root 2. So, this is what I am assuming; I am assuming the impulse response of the channel to be of this form. Let us try it now to figure out what is this $y(t)$ when we have an impulse response given by this expression.

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The image shows a handwritten derivation on a grid background. It starts with the passband signal $x(t) = \sum_n x[n] p(t-nT)$. The baseband signal $y(t)$ is derived as $y(t) = \sum_n x[n] p(t-nT) * h(t)/\sqrt{2}$. This is then simplified to $y(t) = \sum_n (h x[n] p(t-nT))$. At symbol time $t = iT$, the output is $y(iT) = \underbrace{h x[i] p(0)}_{\text{channel gain}} \{p(0)=1\} + \sum_{n \neq i} h x[n] p(i-nT)$. The ISI term is noted as $ISI = 0$. The final result is $y[n] = h x[n]$, where $h(t) = h \delta(t) \sqrt{2}$.

$x(t)$ is a complex baseband signal that was used at the transmitter and we have seen several times that this complex baseband signal is nothing, but it is given by the weighted pulse train where, this $p(t)$ is the pulse used at the modulator, x_n are the symbols used to encode binary sequence. And in general you have a complex baseband signal given like this, $y(t)$ does would be obtained by the convolution of $x(t)$. So, this is $x(t)$ convolving with $h(t)$ divided by root 2 so, this is $y(t)$. So, $x(t)$ is this and $h(t)$ divided by root 2 is this. So, $y(t)$ can simply be obtained by just this thing h is a constant, it is not a function of time x_n is not a function of time. So, what we would have is $p(t - nT)$ would convolve it $\delta(t)$.

And we have seen a several time if you have a signal getting convolved with an impulse you get the same signal. So, this is basic stuff of how we carry out convolution. So, this is the complex baseband output; meaning that this is the complex baseband signal which is equivalent to the pass band signal by $p(t)$. If I sample this $y(t)$ and I am sampling it at integer multiples of T , where T is the symbol time. So, we are carrying out the same notation from previous lecture and i belongs to set of integer. If I sample this $y(t)$ at integer multiples of T what I would have? So, instead of considering this whole quantity for all values of n what I am doing is, I am subtracting the case when n equals to i . So, let me work this out.

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$$y(t) = \sum_n h x[n] p(t - nT)$$

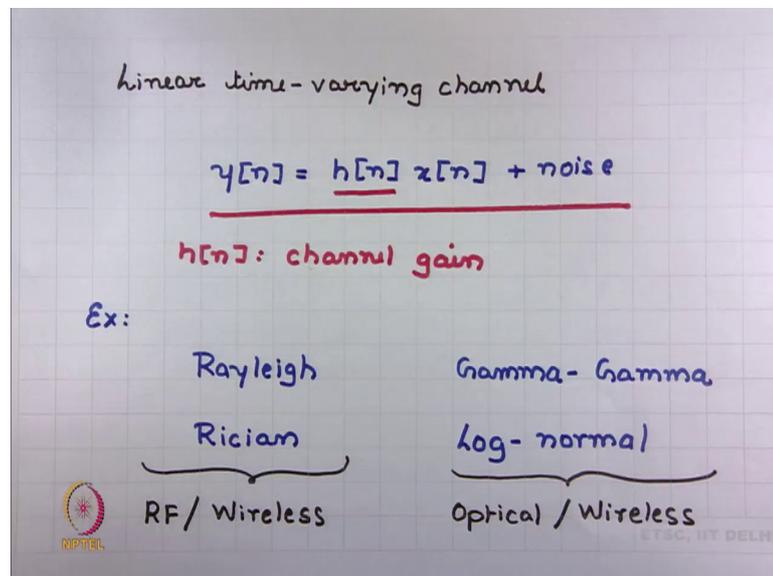
$$y(iT) = \sum_n h x[n] p(iT - nT)$$

$$= \underbrace{h x[i] p(0)}_{p(0)=1} + \underbrace{\sum_{n \neq i} h x[n] p((i-n)T)}_{\text{ISI}}$$

So, if $y[n]$ is $h[n] x[n]$ plus noise, first thing that I am doing is I am sampling this at integer multiples of T . So, I will get $h[n] x[n]$ plus noise. Now, I can assume n to be same as i , then I get $h[n] x[n]$ this will become p_0 then I have other terms when n is not same as i ok. And we have seen in one of the previous lecture that this corresponds to inter symbol interference and normally we assume that p_0 is 1 alright. So, this is what I am having here $y[n]$ is this thing plus inter symbol interference and if I assume that my inter symbol interference is 0 what I have is $y[n]$ is $h[n] x[n]$ ok.

So, this is actually a discrete time signal and I now choose i to be n and this i also becomes n . So, in short I can write that $y[n]$ is $h[n] x[n]$. And what is h ? h is channel gain, remember it is not the impulse response is the channel gain alright. So, this is when my channel has an impulse response which is time invariant because, what I have assumed is my $h[n]$ is $h[n]$ time root 2. So, this channel will have this impulse response irrespective of when you want to measure this impulse response. In that case what I have caught is this $y[n]$ is some constant times input, this h is some constant this h is not a function of time.

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Now, if I have a linear time varying channel, this channel gain will become a function of time and this I denote with $h[n]$. So, if I have an output $y[n]$ which is $h[n] x[n]$ plus noise and because, this channel gain is no longer a constant, but now this channel gain has become a function of time. This output represents the output of a linear time varying

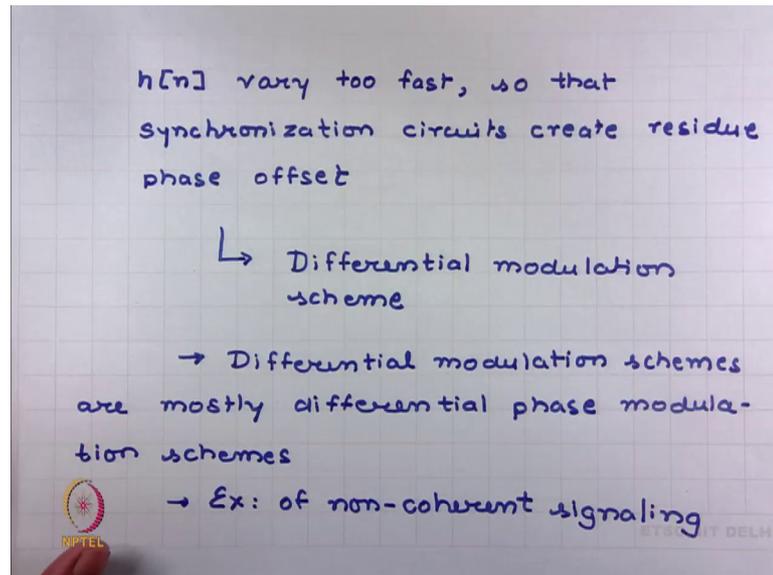
channel. So, what we are doing is we have identify the output for a linear time invariant channel. And we have said that for a linear time invariant channel this channel gain will not be a function of time, it will be some constant and from this I can easily think about the output for a linear time varying channel.

Where, have what happens is only that this channel gain becomes a function of time or n and of course, we also have some noise addition. So, in practical situation what we have is a channel which is a time varying channel. For example, in mobile communication, the channel that a mobile phone sees depends upon various factor. The time of the day, the atmospheric conditions, the pollution in the air, the distance between the mobile phone and the base station and several other factors.

And these factors are time varying factors, they change with time and thus the impulse response of the channel that a mobile phone sees is also time varying. And this equation can easily model the output of a linear time varying channel and there are various models that have been used in literature to model RF wireless channel.

For example: you can assume these amplitudes to be Rayleigh distributed, Rician distributed or in optical wireless you can assume that these amplitudes are gamma gamma distributed or log normal distributed. We will not discuss about them, but what I am trying to point out is when you have a wireless communication system; the channel that you have is a linear time varying channel. And what happens in those linear time varying channel is the output of the channel depends upon the channel gain times input and this channel gain is a function of time. And of course, there is always some noise bothering us alright. So, this we have understood, this is the basic equation from which now we would start.

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Now, what might happen is this h of n that is the channel gain it might vary too fast. So, that whatever synchronization circuits that we have would not be able to offset phase differences. And there might be some residue phase offset and then we have a problem of course, if this channel gain also varies too fast then you have to learn the channel too often and it also creates overheads in our system. So, if we have a channel which is vary too fast the modulation schemes, that we have seen before might not work well.

First of all because, this synchronization in channel estimation techniques that we would require either may begin to work poorly Or we need to learn channel too often creating overheads in the system and for this scenario differential modulation schemes are better. Though we say differential modulation schemes but, when we say differential modulation schemes we invariably mean that we are talking about differential phase modulation schemes right. And we will see this why that is the case and we will also see that these differential modulation schemes are the examples of non coherent signaling systems.

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$$y[n] = h[n] x[n]$$

$$y[n-1] = h[n-1] x[n-1]$$

$$y[n] y^*[n-1] = \underline{h[n]} \underline{h^*[n-1]} \underline{x[n]} \underline{x^*[n-1]}$$

$h[n]$ is stable over two symbols,

$$h[n] h^*[n-1] = \frac{|h[n]| e^{j\angle h[n]}}{|h[n]| e^{-j\angle h[n]}}$$

So, let us start and think about the problem. So now, what we are assuming is that there is no noise; let us first deal with the case when there is no noise and then when we will do detection we will think about how to handle this noise too. So, when there is no noise the output $y[n]$ is $h[n]$ times $x[n]$ and the output delayed by a 1 time unit of course, depends upon the channel gain at $n-1$ time instants or a sample number times the input at $n-1$ sample. Let us try to find out what is this $y[n]$ times $y^*[n-1]$. What we are doing in this?

Actually, we are trying to calculate something like in a product and for the calculation of this inner product we are considering the sample or the received symbol at n th time instants in it $n-1$ th time instants. And if I see this what is this? So, this $y[n]$ is $h[n]$ times $x[n]$ and $y^*[n-1]$ is $h^*[n-1]$ times $x^*[n-1]$. Let us now assume that this $h[n]$ is a stable at least over 2 symbols $h[n]$ might be varying too fast, but for 2 symbols.

Let us assume that this is constant, this is stable it does not change between 2 symbols. So, what we mean with this is $h[n-1]$ is same as $h[n]$, if $h[n]$ is stable over 2 symbols. And now what I am doing is I am interested in writing this $h[n]$ which in general is a complex number in terms of polar coordinates.

So, I write this $h[n]$ as magnitude of $h[n]$ times the phase of $h[n]$ ok, you must have seen this representation of a complex number. So, I have this is the phase of $h[n]$ or angle of h

n and this is magnitude of $h[n]$. And any complex number can be represented in terms of the magnitude of $h[n]$ and the phase of $h[n]$ because, $h[n] e^{j\theta}$ minus 1 is same as $h[n] e^{-j\theta}$ conjugate of $h[n] e^{j\theta}$ minus 1 is same thing as $h[n] e^{-j\theta}$ conjugate. So, when I want to conjugate $h[n] e^{j\theta}$ the magnitude of $h[n]$ would not change in conjugation, the phase of $h[n] e^{j\theta}$ will change. So, instead of phase this now we have a phase with negative sign that is it. So, if $h[n]$ is a stable over 2 symbols $h[n] e^{j\theta}$ in into $h[n] e^{-j\theta}$ minus 1 is this thing and you can clearly see that this thing will cancel out with this thing.

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$$h[n] h^*[n-1] = |h[n]|^2$$

$$y[n] y^*[n-1] = |h[n]|^2 x[n] x^*[n-1]$$

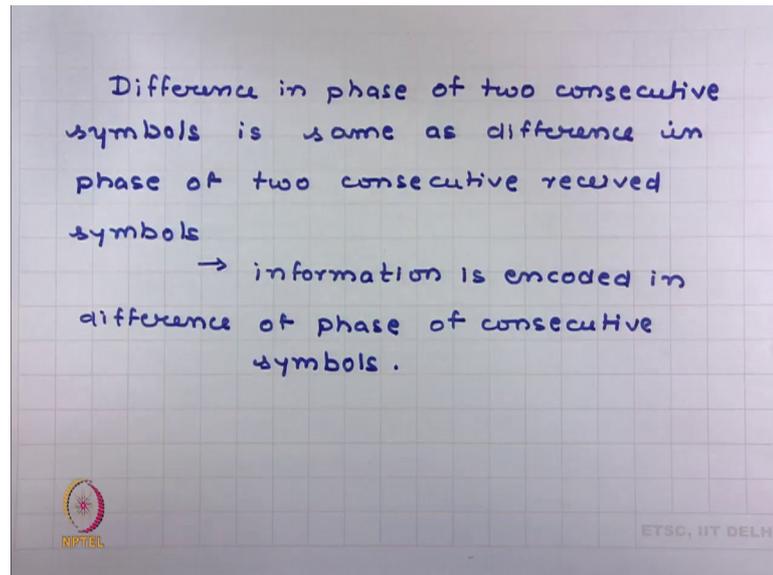
$y[n] y^*[n-1]$ has the same phase as $x[n] x^*[n-1]$

$y[n] y^*[n-1]$ has different amplitude than $x[n] x^*[n-1]$

And what you would have is $h[n]$ times h conjugate of n minus 1 is simply mod square of $h[n]$. Now, let us see what happens to this thing. So, we now see that this becomes mod $h[n]$ square. So, this $y[n]$ times y conjugate of n minus 1 is nothing, but mod square of $h[n]$ times $x[n]$ times x conjugate of n minus 1. Now, this is some number a real number.

So, the phase of this is same as a phase of this, $y[n]$ times y conjugate of n minus 1 has the same phase as $x[n]$ times x conjugate of n minus 1. But, the amplitude of $y[n]$ times y conjugate of n minus 1 is not same as amplitude of this thing. And that is why it is harder to use differential amplitude modulation schemes, it is relatively easier to use differential phase modulation schemes.

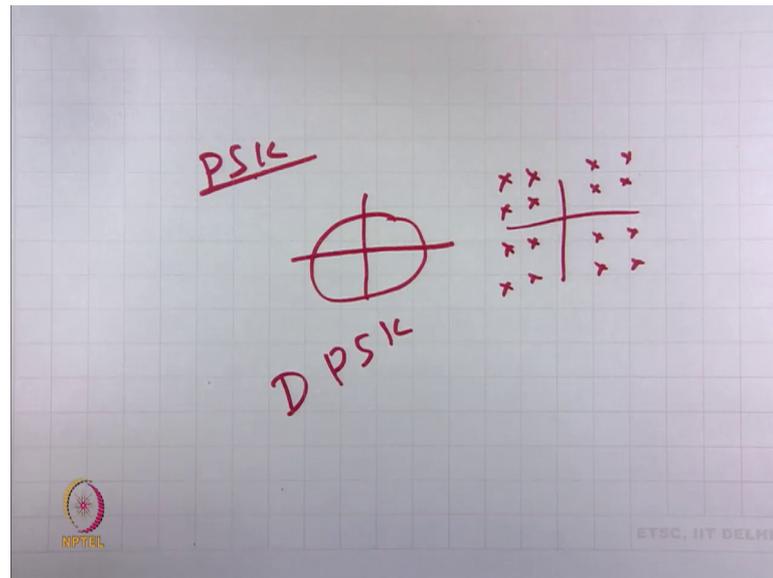
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Because, we have seen that difference in phase of two consecutive symbols is same as difference in phase of two conjugative received symbols and thus, you can easily encode the information in difference of phase of consecutive symbols. So, if you encode the information in difference of phase of two consecutive symbols; this information is easily carried out at the receiver.

Because, this difference in phase of two consecutive symbols is same as difference in phase of two consecutive received symbols. And having a linear time varying channel and having a channel which varies too fast does not break out this transformation of information. And this is the idea why we use differential phase modulation schemes and thus we start with phase shift keying base modulation schemes.

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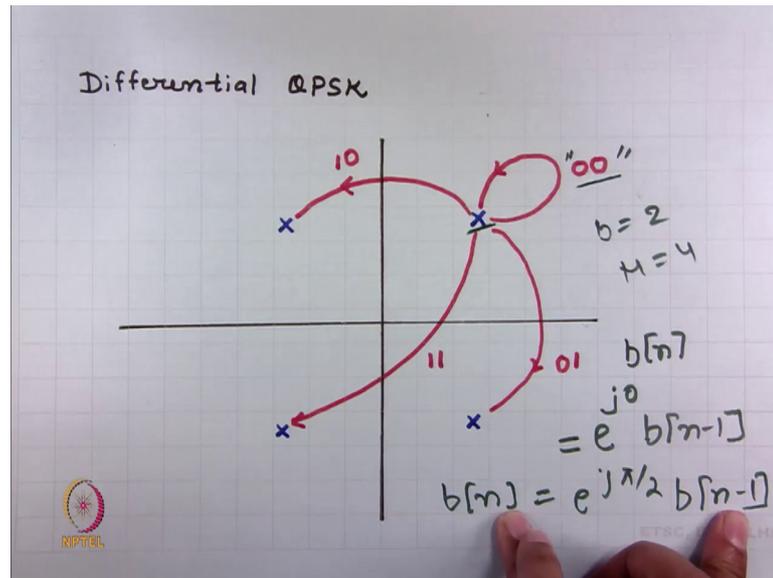


Because remember in PSK systems, we have the information in the phase of the symbols. We said that PSK systems are an example of quadrature amplitude modulation systems where, the symbols can only lie on a circle. If the symbols only lie on a circle; that means, that the information is only in the phase of the symbols, amplitude of the symbol is the same. And because in these differential modulation schemes we are only encoding the information in the phase these differential modulation schemes are also examples invariably of differential phase shift keying modulation schemes ok. Can we use this differential modulation schemes with modulation schemes like QAM in general?

For example if I have this constellation of QAM, where I allow the symbols to lie on a square grid. Now, these symbols have information both in amplitude and phase. Now, to use differential amplitude modulation schemes this thing will annoy us because, now the impulse response of the channel will come into picture. And that is why it is not really straightforward to use differential amplitude modulation schemes ok; it is really straightforward to use differential phase modulation schemes. In fact, this is an area of connective research at this moment how can we use these differential amplitude modulation schemes in an optimal way, but this has not still been sorted out ok.

So, when we say differential modulation schemes in this course we invariably we will mean that this is differential phase modulation scheme.

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Let us see 2 examples of these differential modulation schemes. The first example is of this differential quadrature phase shift keying. So, the idea is let us say we are at this symbol, remember in QPSK we consider 2 bits at the same time in QPSK, b is 2 and that is why M is 4. So, we consider the group of 2 bits and if these 2 bits happen to be 0 0 the phase of the next symbol, let us say b_n is the next symbol and b_{n-1} is the current symbol. Then the phase of this b_n is chosen same as the phase of this b_{n-1} ok.

There is no phase difference between the phase of b_n and b_{n-1} , if the transmitter sees the incoming bits as 0 0. If transmitter sees the incoming bits as 1 0 the phase of the next symbol b_n is chosen as 90 degrees more than the phase of the current symbol. If the transmitter sees the incoming bits is 1 1 the phase of the next symbol is chosen as 180 degree more than the phase of the current symbol. If the transmitter sees the incoming bits as 0 1, the phase of the next symbol is chosen as 90 degrees smaller than the phase of the current symbol.

So, what do you do? You encode the information about the bits in terms of the phase difference of the two conjugative symbols. Let me take one example.

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	00	10	00
0°	0°	90°	90°

Rx	00	10	00
	0°	90°	0°

So, if you have 0 0 let us say we are starting with a phase difference of 0 degree. So, this is where we have to start with transmitter sees the incoming bits is 0 0. It chooses the same phase as this phase then transmitter sees incoming bits is 1 0, it chooses a phase of 90 degree. Transmitter sees next incoming bits let us say again 0 0, it chooses this phase same as this phase. So now, the information is encoded in the difference of the phase and not in the absolute values of phase because, the same 0 0 has 2 different phases.

The receiver on the other hand it calculates the difference in the phase. So, it calculates the phase, it calculates the difference in the phase. And now when it says that the difference in the phase is 0 degree it knows that 0 0 must have been transmitted. If it sees that the difference in the phase is 90 degree, it knows 1 0 has been transmitted. If it sees that the reference in the phase is 0 degree it knows that 0 0 must have been transmitted ok. And this is the simple idea behind this differential quadrature phase shift keying.

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$$\begin{aligned} 00, & \quad b[n] = b[n-1] \\ 10, & \quad b[n] = e^{j\pi/2} b[n-1] \\ 01, & \quad b[n] = e^{-j\pi/2} b[n-1] \\ 11, & \quad b[n] = e^{-j\pi} b[n-1] \end{aligned}$$

initial phase is ambiguous (unknown)
and not necessary

So, we have seen this mapping before, if the transmitter sees 0 0 the phase of the current symbol is chosen same as the phase of the previous symbol. If the transmitter sees 1 0 the phase of the current symbol is chosen as 90 degrees more than the phase of the previous symbol and so on so forth. You can see that initial phase is ambiguous. For example, this is what we have assumed to start with, you could have chosen any initial phase and is still this scheme will work. So, initial phase in this system is ambiguous, it is unknown and it is not necessary for the demodulation.

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Differential PSK (DPSK)

- Binary differential PSK

$$\begin{aligned} 0, & \quad b[n] = b[n-1] \\ 1, & \quad b[n] = -b[n-1] \end{aligned}$$
$$s[n] = (b[n-1], b[n])$$

signal defined over 2 symbols

We can go to this differential phase shift keying as well when we say differential phase shift keying, it means that this is binary differential phase shift keying; though you have no clue from DPSK that this is binary but, it is by custom. So, and now we also see one more example of lousy notation adopted. So, when you hear this term differential phase shift keying it should ring the bell in your head that we are talking about binary differential phase shift keying. And what is the rule here? If the transmitter sees a bit 0 incoming bit as 0 now, because this binary we only care about 1 bit at a time and we do not anymore look to 2 bits at a time.

If the transmitter sees 1 bit and that bit happens to be 0, then a phase of the present symbol is chosen same as the phase of the previous symbol. If the transmitter sees the incoming bit as 1 the phase of the current symbol is chosen opposite to the phase of the previous symbol. So, this is symbol differential phase shift keying. Let us try to explore this beauty of differential phase shift keying by construing a signal S_n which is defined over 2 symbols and why we are doing this will become clear in a minute. So, what we are saying is we are construing a signal and this signal is defined over 2 symbols b_{n-1} and b_n .

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Handwritten mathematical equations on a grid background:

$$s[n] = (b[n-1], b[n])$$

$$s_0 = (+1, +1) \quad 0$$

$$s_1 = (+1, -1) \quad 1$$

Below these, there are two lines of equations:

$$0, \quad s[n] = \pm s_0$$

$$1, \quad s[n] = \pm s_1$$

To the right of these equations, there is a handwritten equation:

$$b[n] = -b[n-1]$$

In the bottom left corner, there is a logo for NPTEL. In the bottom right corner, there is text that reads "ETWC, IIT DELHI".

Now, what is the rule? If incoming bit happens to be 0 then b_n is same as b_{n-1} ; that means, they use the same phase and we get to a signal S_0 . If the incoming bit is 1 then the b_n is minus b_{n-1} and this brings us to a signal S_1 where, b_n has the

opposite polarity as b_{n-1} . When you have incoming bit as 0 b_n has the same polarity as b_{n-1} . So, when you have 0 bit coming in your signal is either plus or minus S_{naught} . What should happen is that this b_n and b_{n-1} should have the same polarity whether it is plus S_{naught} or minus S_{naught} it depends upon the history.

When the transmitter sees an incoming bit as 1, it transmit plus or minus S_1 . What is most important is that this b_n and b_{n-1} are opposite polarity ok. And again whether its plus S_1 or minus S_1 depends upon the history and you can work this out yourself. Assume a bit pattern, identify S_{naught} S_1 from there and you would see that sometimes it is plus S_{naught} and sometimes it is minus S_{naught} when the transmitter c is the bit 0 ok.

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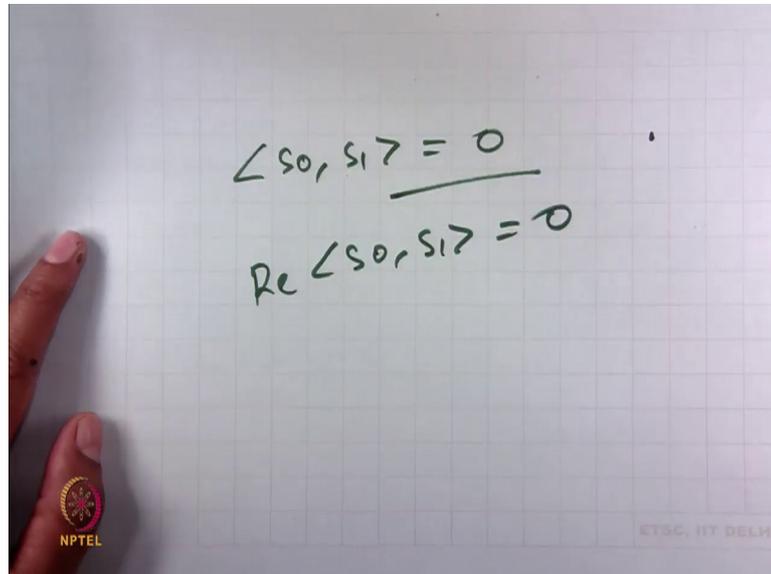
$\langle S_0, S_1 \rangle = 0$
 DPSK is an orthogonal signaling scheme { in fact, non-coherent orthogonal signaling }
 $S_0 = (1, 1)$
 $S_1 = (1, -1)$
 $\langle S_0, S_1 \rangle = 1 + (-1) = 0$

Interesting thing and why we are taking this approaches to see that these S_{naught} and S_1 are orthogonal because, the inner product is 0. So, S_{naught} is 1 and 1 and S_1 is 1 in minus 1 what is the inner product? Inner product is 1 times 1 which is 1 plus 1 times minus 1 which is minus 1 and this is going to be 0; that means this S_{naught} and S_1 are orthogonal.

And thus this DPSK scheme can be thought as orthogonal signaling scheme because, the signals that is using for transmission S_{naught} or S_1 are orthogonal signals. And because their inner product is 0 rather than the real part of their inner product being 0 this DPSK

scheme can be used for non-coherent orthogonal signaling. We have seen this, do not forget and this is an important idea.

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The image shows a hand pointing to two equations written on a grid background. The first equation is $\angle s_0, s_1 = 0$ with a horizontal line underneath it. The second equation is $\text{Re} \angle s_0, s_1 = 0$. In the bottom left corner, there is a small circular logo with the text 'NPTEL' below it. In the bottom right corner, there is the text 'ETSC, IIT DELHI'.

That if a modulation scheme satisfies this condition, it can be used for non-coherent orthogonal signaling scheme. If a modulation scheme satisfies this condition then this should be used for coherent orthogonal signaling scheme. So, because they are inner product of s_0 and s_1 the basic signals used in DPSK is 0. DPSK is an example of orthogonal signaling scheme, more precisely it is an example of non-coherent orthogonal signaling scheme alright.

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Bits 1 0 1 1 0 1 0

DPSK $(1 -1 -1 1 -1 -1 1 1) e^{j\theta}$

$P = \frac{n}{n+1}$ bits/Complex Dimension

$= 1$ bit/Complex Dimension

Conventional binary FSK (non-coherent),

$P = \frac{\log_2 2}{2}$ bits/D = 1 bit/2 CD

So, let us now look at the spectral efficiency of this DPSK scheme and for this we are assuming this as a bit sequence. Now, let us assume that we are starting with some initial phase θ . When the transmitter sees 1, it transmits minus e to the power $j\theta$. If it sees 0, it transmits the same symbol as the previous symbol, if it sees 1 it transmits the symbol with opposite polarity than the previous symbol and so on so forth. To look at the spectral efficiency we need to see that how many complex dimensions we require to transmit 1 bit, that is the spectral deficiency bits per number of complex dimension.

For example, here let us assume that we have n bits and how many complex dimensions we need to use to transmit these n bits, the number of complex dimensions will be n plus 1; 1 more than this n bits to have some initial phase. So, the spectral efficiency of DPSK is n divided by n plus 1 bits per complex dimension. And if n is pretty large this will boil down to 1 bit per complex dimension and this is an improvement over conventional non-coherent FSK based orthogonal signaling schemes. Because, we have seen that in conventional binary FSK non-coherent kind this spectral efficiency is $\log_2 m$ divided by m for binaries 2.

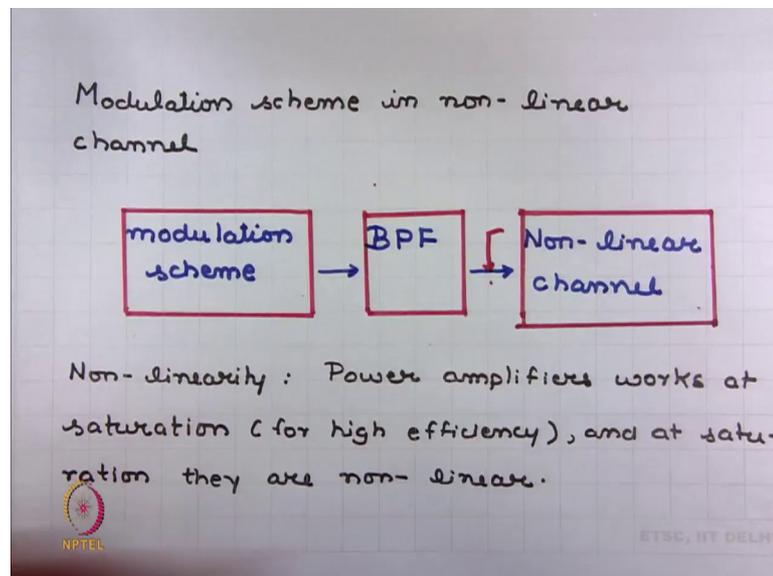
So, this spectral efficiency in conventional binary FSK is 1 bit per 2 complex dimensions and here in case of DPSK which is also an example of non-coherent orthogonal signaling scheme; we have improved the spectral efficiency from 1 bit per 2 complex dimensions to 1 bit per complex dimension ok. So, there is a factor of two improvement when you go

to DPSK and that is why this DPSK is also useful because, it also increases the spectral efficiency ok.

So, why we are using DPSK? The one reason is that these differential modulation schemes are ideal for channel where, the channel gain is varying too fast. And so, we use this differential modulation schemes, invariably they fall into the categories of differential phase modulation schemes.

Because, we have seen that the difference in the phase of two consecutive received symbols is same as the difference in the phase of two consecutive transmitted symbols right. And the channel gain has no influence there and that is why it is easier to use this differential phase modulation scheme. We have seen that this DPSK is an example of non-coherent orthogonal signaling scheme and it improves the spectral efficiency by a factor of 2; it achieves spectral efficiency of 1 bit per complex dimension. So, let us now see the examples of modulation scheme that are useful for non-linear channels ok, let us understand this.

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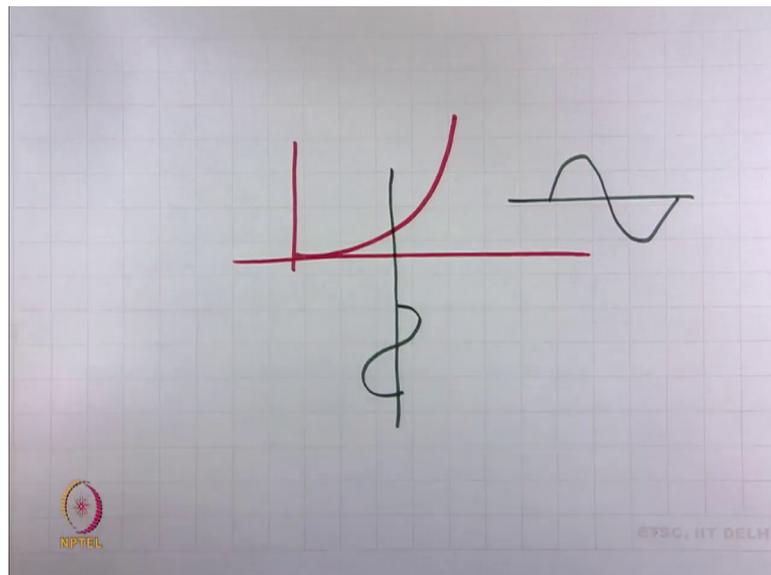
So, let us say we have a modulation scheme generator and we pass this modulation scheme through band pass filter. This is useful because you want to restrict the bandwidth of this modulation scheme within a certain band, you do not want to violate FCC regulations or TRI regulations. So, then you would have invariably a power

amplifier and this power amplifier is a non-linear device that makes a channel non-linear channel. Why power amplifier is a non-linear device?

Because, we mostly want to make it work at saturation, at saturation power amplifier is a non-linear device. We want to make it work at saturation because, this is there where it has the maximum power efficiency right. So, you want to maximize the power efficiency of your power amplifier that is why it is operated in saturation.

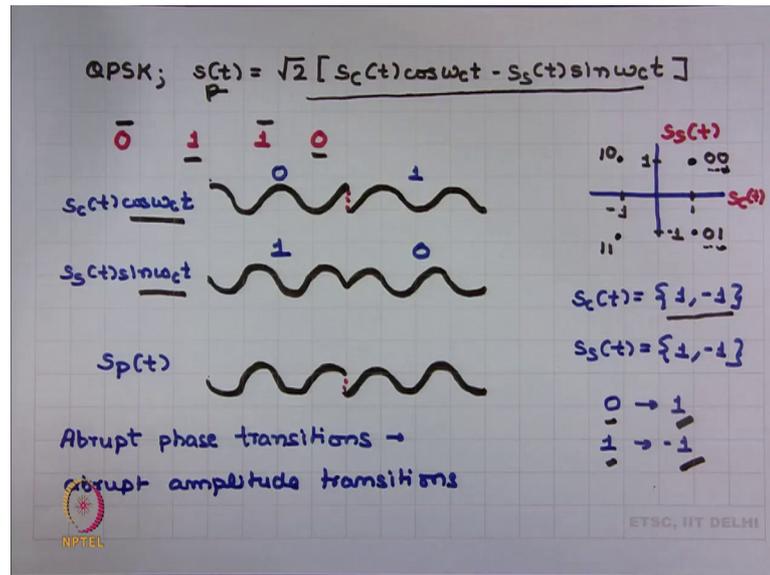
But once its operates in saturation it is a non-linear device ok. Now, if you have a non-linear device what do you want? You want that at the input of this non-linear device what we have is constant amplitude because, if you have a non-linear device; non-linear device would have different gain for different amplitudes. So, if at this input you have a modulation scheme which has lot of amplitude variations at the output of this non-linear device you would have distortion. For example, let us take a simple case.

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Suppose, I have a device which has non-linear characteristics and to this device at the input I have let us say a sinusoidal waveform. Then you can think yourself that at the output of this non-linear device you would have a waveform which would not be sinusoidal, it would be distorted. And that is why what you want to make sure is that the input of this non-linear device we have constant amplitudes. Let us see whether the modulation schemes that we use have constant amplitudes.

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For example, let us say QPSK so, we know that this QPSK signal equation is given by this. So, this is the pass band signal and this pass band signal can be thought in terms of the baseband signals $S_c(t)$ and $S_s(t)$ are the i and q parts of the complex baseband signal. Now, QPSK is an example of QAM systems where the symbols are constrained to lie on a circle; that means, the amplitude of the signals is same, it is constant and this is the constellation diagram of QPSK. So, you can see that QPSK has four symbols and that is why the name quadrature and we also show the binary representation of these symbols.

So, each symbol is assigned some binary bits and if you look at this carefully you can see that these bits are assigned to the symbols using what is known as gray coding; we have already looked at this gray coding in lecture 18. So, what gray coding does is it ensures that the difference between the binary representation of 2 neighboring symbols does not differ by more than 1 bit. So, if you see the binary representation of this symbol and compare it to the binary representation of this symbol, you see that these binary representations only differ by 1 bit so, this bit in here.

So, on this we have $S_c(t)$ and on this axis we have $S_s(t)$. So, for this case we are assuming that $S_c(t)$ draws in the amplitude levels from this set; that means, either it can take an amplitude of 1 or minus 1. $S_s(t)$ similarly takes the amplitude levels from this set and what we assume is for bit 0 we are using an amplitude level of 1 and for bit 1 I am using

an amplitude level of minus 1 and this is also clear and evident from this constellation diagram ok.

So, let us assume that we have a binary sequence coming in and the first thing that the transmitter does is it separates this binary sequence into 2 binary sequence; one binary sequence consisting of odd bits. So, these are the odd bits and the second binary sequence consisting of even bits and these are the event bits. So, let us assume that these odd bits modulate this $\cos \omega_c t$ and let us assume that even bits modulate $\sin \omega_c t$. So, for example, when we have 0 $S_c t$ takes in a value 1. So, the resultant signal is $\cos \omega_c t$ which is this waveform and when we have 1 $S_c t$ takes in a value minus 1. So, the resultant signal is $-\cos \omega_c t$ which is this and similarly you can think about $S_s t \sin \omega_c t$.

So, $S_s t \sin \omega_c t$ would be modulated by a bit 1 and 0, when you have 1 $S_s t$ takes in a value minus 1. So, we have $-\sin \omega_c t$, a waveform something like this and when we have 0 you have $\sin \omega_c t$ which is a waveform something like this. And then you subtract these two waveforms to get S_t or $S_p t$, we use p to denote that this is a pass band signal so, this is my $S_p t$.

So, what you should notice from this waveform is that there are abrupt phase transitions. These abrupt phase transitions create abrupt amplitude transitions and our non-linear power amplifier does not like this. So, we have to modify this QPSK to another signal set where we can avoid this problem. And that is the idea of going from QPSK to offset QPSK and let us see how can we do that.

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QPSK to Offset QPSK

$$s(t) = \sum_n \frac{b_c[n] p(t-nT) + j \sum_n b_s[n] p(t-nT)}{s_s(t) s_c(t)}$$

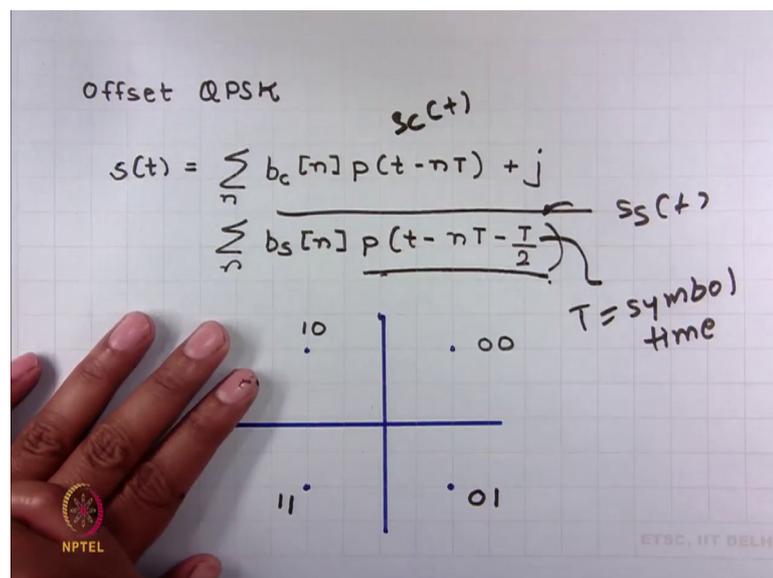
{ QPSK }

$$= \sum_n \underline{b[n]} p(t-nT)$$

$$s(t) = \underline{s_c(t) + j s_s(t)}$$

So, to do that you have to go from QPSK to offset QPSK, to understand that first let us revise the baseband waveform for QPSK signal. So, we have seen several times that $S(t)$ is $S_c(t) + j S_s(t)$, this is $S_c(t)$, this is $S_s(t)$ and I can write this also in this form, all these forms are equivalent right. So, here what I have done is I have separated this $b[n]$ which is a complex number into 2 real numbers and here I have represented it in a more compact notation by just stating that this $b[n]$ is a complex number. So, this is how the baseband signal of QPSK looks like.

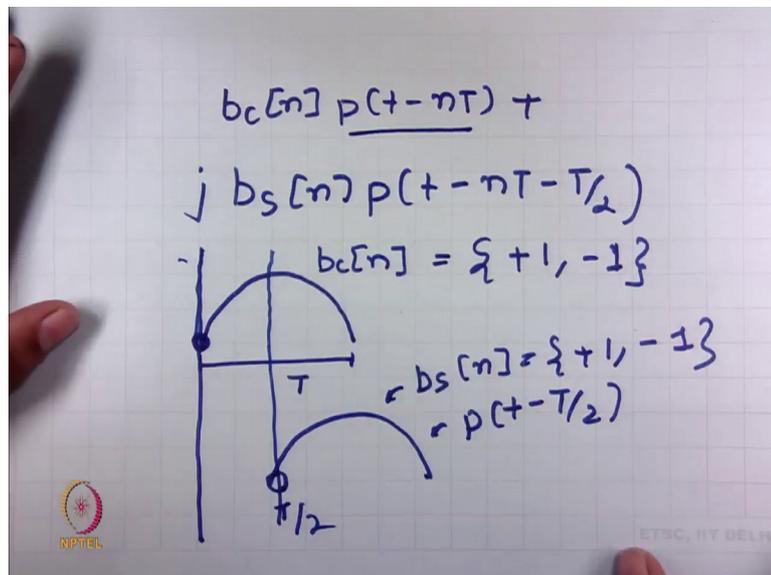
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Now, to go from QPSK to offset QPSK what we do is we shift the pulse along the q direction by $T/2$ that is it, that is the main difference between QPSK and offset QPSK. In offset QPSK the pulse that you use for S s t; so, this is S s t is shifted by $T/2$ compared to the pulse that you used at S c t. And what is T? T is nothing, but as usual it is the symbol time.

Now, what happens when you use this approach? You make sure that at every $T/2$ time interval only one of the channel changes. So, either i channel changes or either q channel changes, but not both of the channels will change at the same time. For example, let me draw one waveform.

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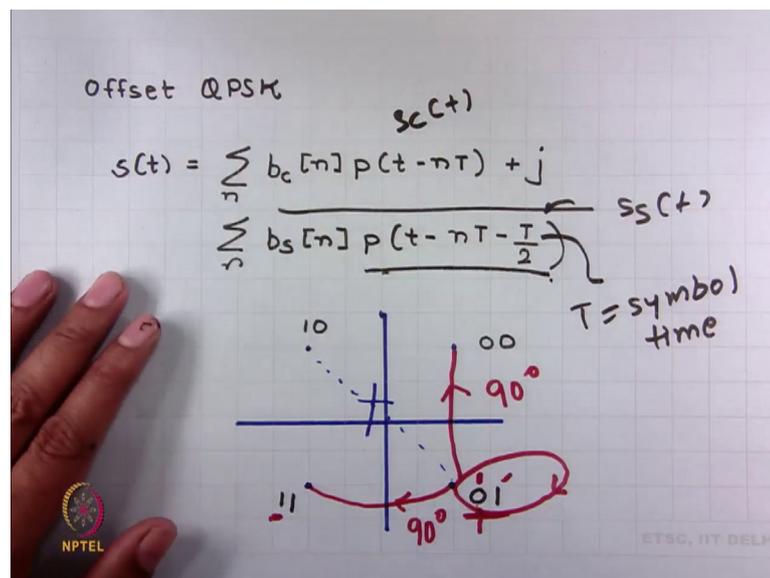


So, what I am saying is this $b_c[n] p(t - nT)$ and then there is a j times $b_s[n] p(t - nT - T/2)$. So, if I am using let us say one pulse with a duration of T and this pulse is modulated by $b_c[n]$; for QPSK this $b_c[n]$ would take either a value plus 1 or minus 1. Now, for this part what would happen is this would be shifted by $T/2$. So, this is $p(t - T/2)$ let us just consider for n equals to 0 and $b_s[n]$ also would draw values from this set. Now, let us consider this in totality.

So, this value of $b_c[n]$ will change at this time instants and the value of $b_s[n]$ would change at this time instants. So, when I am looking at the waveform at this time instants i component would change and when I will look at the waveform at this time instants the q component would change. But these would not change at the same time ok, if they were

not shifted by T by 2 these pulses will overlap. And the coefficients corresponding to these pulses might change at the same time. But once you have shifted one of the pulse by T by 2 you have made sure that the coefficient only along one channel will change at a time.

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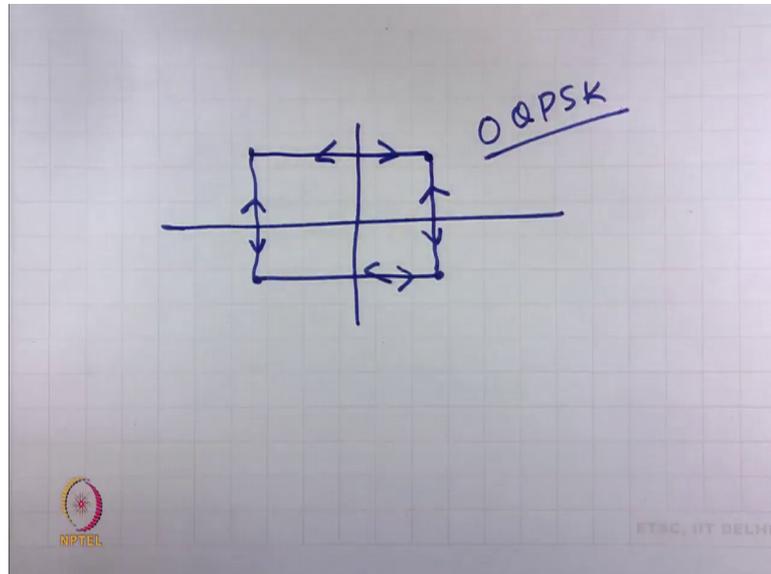
So, what does will lead to is suppose you are at this place then what would happen is, if I am looking at integer multiples of T only i component can change; that means, you can go from this symbol to this symbol if 0 does not change. So, we are focusing only on this first bit so, this bit can only change. So, if I receive 0 along the i component I remain to the same symbol or from this symbol I can go to this symbol because, 0 might change to 1.

So, at integer multiples of T either I will remain at the same phase or I can go to this symbol and here the phase changes by 90 degree at odd integer multiples of T by 2. The q component can change so, either I can stay in the same symbol or I can go to this symbol where also the phase change of 90 degree happen.

But, what will never happen is you can never go from here to here, this transition has become redundant now because, to go from here to here both bit should change at the same time. And this transition is no longer valid because, by shifting one of the pulse by T by 2 we are making sure that only 1 bit will change at a time. And thus, at max in offset QPSK I can have phase transitions of 90 degrees. In QPSK I can have phase

transitions of 180 degree because, both i and q channel changes at the same time and this is the main idea behind offset QPSK.

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So, if I want to look at the transition diagram for offset QPSK. This transition diagram will look something like this. So, you can go from here to here, you can go from here to here both directions. So, this is the transition diagram of offset QPSK.

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Offset QPSK

$$p(t) = \sin \frac{\pi t}{T} I_{[0, T]}(t)$$

$$p(t - T/2) = \cos \frac{\pi t}{T} I_{[T/2, 3T/2]}(t)$$

half-cosine

$$s(t) = \sum_n b_c[n] p(t - nT) + j b_s[n] p(t - nT - T/2)$$

$$s(t) = \pm \sin \frac{\pi t}{T} \pm j \cos \frac{\pi t}{T} \quad \left(\frac{n+1}{2} T < t < \left(\frac{n+3}{2} \right) T \right)$$

$$|s(t)| = 1 \quad (\text{Constant amplitude schemes})$$

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Let us now consider in spatial case of offset QPSK where, this p t that we choose is sin pi t by T multiplied by this indicator function. And these pulses are also known as half

cosine pulses because, anyway these are sin pulses or cosine pulses. But they are not ordinary sin and cosine pulse which runs from minus infinity to plus infinity, but these sin cosine pulse runs for half time period and that is the name half cosine. So, if you consider $\sin \pi t \text{ by } T$ it would be $\sin \pi t \text{ by } T$ I 0 T t would be a pulse like this. So, it is a sin pulse which is restricted to half a period and that is the name half sin pulse or half cosine pulse ok. They belong to this family of half cosine pulses. So, if $p t$ is this $p t$ minus $T \text{ by } 2$ would be pulse like this.

So, sin changes to cos and if I look to this $S t$ which is this complex baseband signal and if I restrict myself to a time interval like this, what you can easily see is $S t$ will be a pulse of this form. So, what would be arbitrary is the sin of this $\sin \pi t \text{ by } T$ whether it will be plus 1 or minus 1 depends upon this $b c n$. Similarly, the sin of this cosine pulse will be arbitrary depends upon $b s n$.

So, what change in time interval to time interval is these plus or minus signs, but basic complex baseband signal would be like this. And if you see the magnitude of this $S t$ you can easily see that the magnitude of $S t$ is 1 is constant. And thus this baseband waveform is the constant amplitude baseband waveform.

And this is pretty interesting because, we have converted offset QPSK which also had a phase transitions of 90 degrees. Those phase transitions would also make amplitude transitions and this would not be good considering a non-linear power amplifier. But as soon as you also decide on the pulse shape and choose it to be half sin pulse, what you get immediately is a baseband pulse with constant amplitude of 1. And this is very good modulation scheme for non-linear channels. Why are we discussing it so late? Why we have not talked about this before?

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$$s(t) = \pm e^{\pm j \frac{\pi}{T} t}$$

$$T = 2T_b$$

$$s(t) = \pm e^{\pm j \frac{2\pi}{2T_b \times 2} t}$$

$$= \pm e^{\pm j 2\pi \left(\frac{1}{4T_b}\right) t}$$

$$f_0 = \frac{1}{4T_b} \quad f_1 = -\frac{1}{4T_b}$$

$$f_0 - f_1 = \frac{1}{2T_b} \quad (\text{MSK})$$

Handwritten notes: $f_0 = \frac{1}{4T_b}$, $e^{\pm j 2\pi f_0 t}$, $e^{j 2\pi f_1 t}$

Because, this looks interesting; if you see this $S(t)$ you can also convert this $S(t)$ by using Euler's theorem into this form. The plus and minus signs are arbitrary that depends from time period to time period, but the basic form is of this kind. Now, T is the symbol time and the symbol time is 2 times the bit time because, in one symbol there are 2 bits. So, I can also write this $S(t)$ in terms of T_b which is the bit duration by just changing this T to $2T_b$. And then just carrying out some arithmetic multiplying with 2 here and 2 here, we can express this into this form. And if I look at this what would happen is, I have a waveform like this where f_0 I choose as $\frac{1}{4T_b}$; this is plus and minus.

Sometimes I transmit a waveform where, the frequency of $\frac{1}{4T_b}$ and sometimes I transmit a waveform with frequency of minus $\frac{1}{4T_b}$ and if you look at the difference between f_0 and f_1 . We get the difference as $\frac{1}{2T_b}$ and which is nothing, but minimum shift keying that is very interesting. So, this offset QPSK that we have chosen where, we have chosen this pulse shape to be half cosine pulse or you could have chosen it to be half sin pulse whichever; you get a waveform with constant amplitude.

And if we carry out certain arithmetic then we see that this is nothing, but this is a waveform corresponding to MSK signal. Remember in MSK we also used such a waveform right and where, we have seen that the minimum frequency differences between 2 orthogonal wave forms must be $\frac{1}{2T_b}$. And this is exactly what we have caught in here.

So, with this we have come to the conclusion of this lecture and in general the unit on modulation. In this unit on modulation we have covered some very useful ideas. We have seen that how can we think about this conversion between a pass band signal to a baseband signal. And how can we estimate the power spectrum of a random waveform, what are the some good ways to do modulation.

We have seen that these modulation schemes offers different flavors and they are very different. So, if we want to maximize the bandwidth efficiency of our communication channel, we should go to this MQAM based modulation schemes. If we want to talk about reducing EBNO's or to minimize the power consumption, we have to go to these orthogonal modulation schemes.

These orthogonal modulation schemes can get you an EBNO of as small as minus 1.5 nano T b, if you are willing to sacrifice the bandwidth ok. The third thing that we have seen is if your channel is varying too fast either you pay some penalty in learning the channel too often or you adopt these differential modulation schemes. Extending this ideas of differential modulation to differential amplitude modulation seems like a promising idea and this is actively explored nowadays. If this is achieved then you can have benefits of both the worlds, you can tolerate channel which varies too fast. At the same time you can accrue lot of spectral efficiency, but at this time we have to live a differential phase shift keying base modulation schemes.

Lastly, we have seen that this power amplifier that we use at the modulator also restricts the modulation scheme to have constant amplitude. And if we want to ensure that the effects of nonlinearities are minimized then you can go to this offset QPSK basis schemes; particularly the offset QPSK where the pulse shape is chosen to be half sin or half cosine pulse. But, this is nothing different from the minimum shift keying modulation scheme that we have seen before.

So, from the next lectures we will start thinking about detection, where we will estimate the probability of errors of these various modulation schemes.

Thank you.