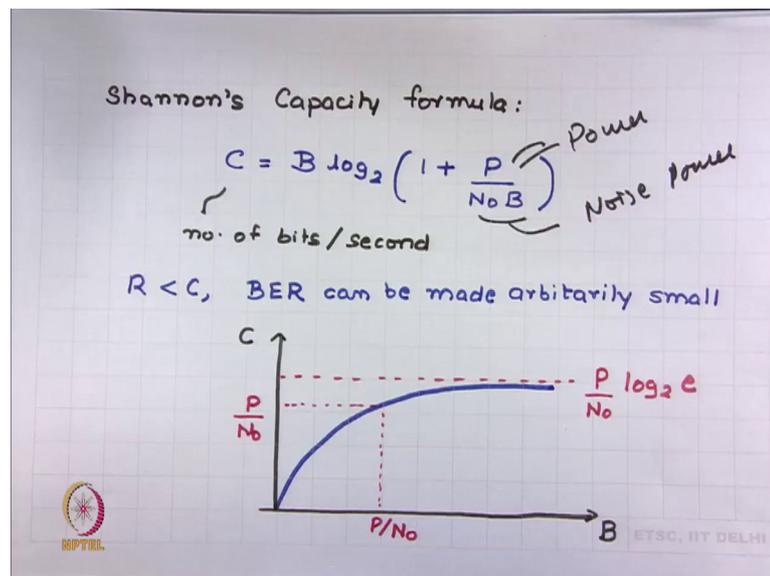


**Principles of Digital Communications**  
**Prof. Abhishek Dixit**  
**Department of Electrical Engineering**  
**Indian Institute of Technology Delhi**

**Lecture – 28**  
**Modulation**  
**Orthogonal Modulation Schemes**

Good morning welcome to new lecture on Modulation and today we will talk about Orthogonal Modulation schemes, in the last lecture we have concluded with the Shannon's capacity formula where we have said that essentially if you increase bandwidth you can have more capacity.

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Of course, it saturates to a value of  $\frac{P}{N_0} \log_2 e$  that means you can increase bandwidth increase capacity. But it saturates to this value, then we have seen by doing simple arithmetic that for a given SNR you need to have  $E_b/N_0$  larger than this expression.

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$$C = \frac{R}{B} < \log_2(1 + \text{SNR})$$
$$\text{SNR} / \frac{E_b}{N_0} < \log_2(1 + \text{SNR})$$
$$\frac{E_b}{N_0} > \frac{\text{SNR}}{\log_2(1 + \text{SNR})}$$

So, to have bit error rates arbitrarily small  $E_b/N_0$  should be larger than SNR divided by  $\log_2(1 + \text{SNR})$  and this allows us to trade off bandwidth with  $E_b/N_0$ .

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$$\frac{E_b}{N_0} > \frac{\text{SNR}}{\log_2(1 + \text{SNR})} \quad \text{SNR} = \frac{P}{N_0 B}$$

a)  $B \downarrow$   $\text{SNR} \uparrow$   $\frac{E_b}{N_0} \uparrow$   $P \uparrow$   
{ ideal for Bandwidth constrained channels, M-QAM modulation schemes }

b)  $B \uparrow$   $\text{SNR} \downarrow$   $\frac{E_b}{N_0} \downarrow$   $P \downarrow$   
{ ideal for power-constrained channels, Orthogonal modulation schemes }

So, you can have large bandwidth which reduces the signal to noise ratio which reduces the  $E_b/N_0$  requirement, but it also reduces the spectral efficiency and these kind of modulation schemes or this idea is really useful for power constraint channels. Where the power is the primary resource or you can have small bandwidth which increases your signal to noise ratio which increases the  $E_b/N_0$  requirements and which gives you a

good spectral efficiency and this is really the case in modulation schemes like M.com you are trading off the E b No with a spectral efficiency.

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Orthogonal modulation

- Capacity-reaching modulation schemes
- $\frac{E_b}{N_0} > \frac{SNR}{\log_2(1+SNR)}$
- $B \rightarrow \infty, \frac{E_b}{N_0} \rightarrow -1.59 \text{ dB}$

And there we have said that for power constraint channels orthogonal modulation schemes are really useful these are capacity reaching modulation schemes and if you allow to have infinite bandwidth you can reduce the E b No to as small as minus 1.59 dB. So, we will start with orthogonal modulation in this lecture.

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Orthogonal modulation

A set of  $M$  passband (real) waveforms,  
 $s_{mp}(t); m = 0, \dots, M-1$

such that

$$\langle s_{mp}(t), s_{np}(t) \rangle = \int_{-\infty}^{\infty} s_{mp}(t) s_{np}^*(t) dt = 0 \quad m \neq n$$

Moreover,

$$\int_{-\infty}^{\infty} s_{mp}(t) s_{np}^*(t-lT) dt = \begin{cases} 0 & l \neq 0 \\ E_s & l = 0, \\ & m = n \\ 0 & l = 0, m \neq n \end{cases}$$

So, if we have a set of  $M$  passband real waveforms; pass band waveforms are usually real waveforms, baseband waveforms can be complex for analytical ease otherwise baseband waveforms are also real waveforms, but we can think about the I and Q component together and we make it a complex. But in general if we have a pass band waveform it has to be a real waveform and if we have  $M$  pass band waveforms  $s_m(t)$ ,  $p$  represents that this is a passband waveform we are  $m$  takes in value 0 up to  $M - 1$ .

Such that if I take the inner product of the waveforms from the set then their inner product is 0 then we know that these passband waveforms are orthogonal waveforms right. How do we define Orthogonality? We define Orthogonality by taking the inner product of 2 waveforms; if the inner product of the 2 waveforms is 0 then we say that these waveforms are orthogonal waveforms ok.

So, in orthogonal modulation schemes we take a set of  $M$  wave forms and these wave forms are orthogonal with respect to each other, moreover if you take these wave forms and if we take the shifted version of these wave forms, if you take their inner product it must also be 0, this we have seen wise that this should be 0 for avoiding inter symbol interference and if  $l$  is 0 that means they are not shifted then this should give me energy of the symbols if  $m$  is  $n$  and if  $m$  is not  $n$  then this should be 0. So, this is just saying that these waveforms should be orthogonal, so if I look at this I am just considering that these should be orthogonal if these should be orthogonal then if  $m$  is same as  $n$  then we should get the energy corresponding to each signal. But if the signals are not same signals if they are different signal then their inner product should be 0 ok.

So, this should also be 0 only when  $m$  is not same as  $n$ , if  $m$  is same as  $n$  then you are taking the inner product of the signal with itself and then it should be the energy of the sector this we have seen in great detail earlier. So, let me define orthogonal modulation schemes again, so for orthogonal modulation schemes I am using  $m$  waveforms and these waveforms are orthogonal to each other ok.

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'b bit blocks' to form 'M = 2<sup>b</sup>' symbols,  
M waveforms

Baseband transmitted signal,

$$C(t) = \sum_{l=-\infty}^{\infty} S_m(t - lT)$$

$S_m(t)$  is the complex baseband representation of the passband signal  $S_{mp}(t)$

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Again I can use the same ideas, I can consider a block of b bit and I can form 2 to the power b symbols where this 2 to the power b represents the number symbols for which we are using M. So, if we want to consider the baseband transmitted signal, so in every symbol durations you are sending one of the symbol right  $S_m(t)$  is the complex baseband representation of the pass band signal  $S_{mp}(t)$ .

So, if I have a pass band signal I can also derive it is complex baseband representation. So, in one of the symbol duration you are transmitting  $S_m(t)$  alright and the  $C(t)$  is the baseband transmitted signal. So, this is what happens in orthogonal modulation scheme and there are various examples of orthogonal modulation scheme, basically you can make this waveforms orthogonal either in frequency domain or in time domain or by doing orthogonal coding and this we will see. So, the first example of these orthogonal modulation schemes that we will see is frequency shift keying ok.

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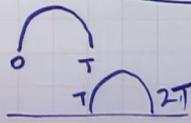
Example :

a) Frequency-shift keying (FSK)

$$s_i(t) = e^{j 2\pi f_i t} I_{[0, T]}(t)$$

$i = 0, \dots, M-1$

Obviously,

$$s_i(t) s_j(t - \Delta T) = 0$$


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So, here what we are talking about are these baseband signals we will focus on the baseband signals. So, let us consider the set of baseband signals and the set of baseband signals are obtained by changing the frequencies of these baseband signals and of course this baseband signals have the duration of capital T. So, this is an indicator function that means that you are truncating this signal in the duration of 0 to T that means this signal only spans between 0 to T outside this duration it is 0.

So, this is the set of baseband waveforms that we are using in Frequency shift keying, these baseband waveforms only differ in the frequency right and hence the name frequency shift keying. Of course, I will taken a value 0 to M minus 1 and of course because these signals are time limited signals that means this span only from 0 to T. If you take a signal and shift a signal with T then the product of this signal will be 0, because these signals will be non overlapping signals in time.

So, what we are saying is if I have a signal which is limited to duration of T, if I shift this by T this will span from T to 2 T, then if I multiply this I will get a 0 because there is no overlap in time domain. So, every time limited signal would satisfy this property. So, we do not have to worry about this condition, so when l is not 0 this will always be satisfied if I have a time limited signal.

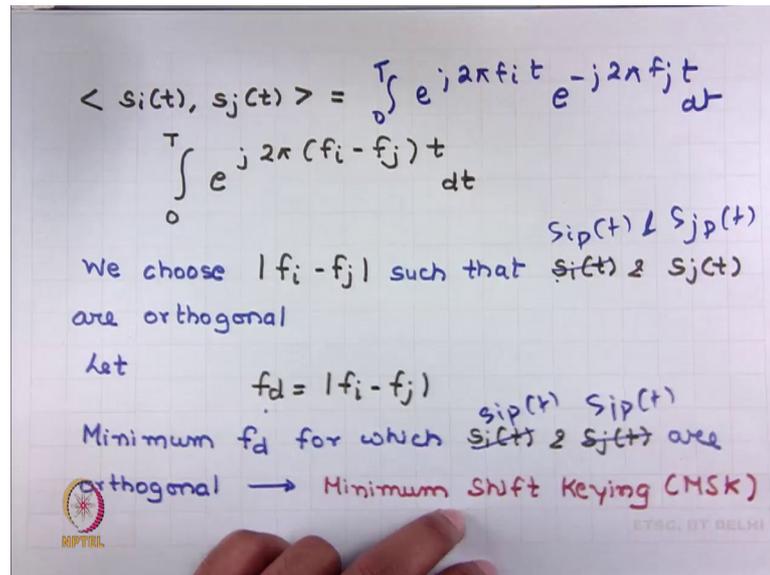
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$$\langle s_i(t), s_j(t) \rangle = \int_0^T e^{j2\pi f_i t} e^{-j2\pi f_j t} dt$$
$$\int_0^T e^{j2\pi (f_i - f_j) t} dt$$

We choose  $|f_i - f_j|$  such that  $s_{ip}(t)$  &  $s_{jp}(t)$  are orthogonal

Let  $f_d = |f_i - f_j|$

Minimum  $f_d$  for which  $s_{ip}(t)$  &  $s_{jp}(t)$  are orthogonal  $\rightarrow$  Minimum Shift Keying (MSK)



Then the inner product of  $S_i t$  and  $S_j t$  is simply obtained by multiplying  $S_i t$  which is this with conjugate of  $S_j t$  which is this and this indicator function has been absorbed in the limits ok. So, basically I get this expression, so this is how we calculate the inner product of these 2 signals. And we want to choose the difference between the 2 frequencies of these baseband waveforms in such a way that  $S_i p t$  and  $S_j p t$  are orthogonal.

So,  $S_i p t$  is the pass band equivalent of this baseband signal  $S_j p t$  is the pass band equivalent of this signal. We want to choose the difference between these frequencies in such a way that their pass band equivalent signals are orthogonal, this is what we want. We want the signals to be orthogonal in pass band domain and why we have a mod in here, because it simply does not matter whether you have a positive or negative this integration will give you the same value. So, let me define  $f_d$  as mod of  $f_i$  minus  $f_j$ .

So, what we are saying is usually it is important to select this  $f_d$  which is the minimum frequency which makes  $S_i p t$  and  $S_j p t$  orthogonal and when you do that you get to a modulation scheme which is known as Minimum Frequency Shift Keying or simply Minimum Shift Keying which is also abbreviated as MSK.

So, what is MSK? MSK is the frequency shift keying in which you are using the frequency of the baseband waveform in such a way that the differences in these frequencies make their passive and equivalent signals orthogonal and at the same time

the difference in their frequencies is minimum ok, so you use minimum possible value for  $f t$ . So, to understand this issue let us recall what we have learned in lecture 19 to 21.

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Recall : { lectures 19 - 21 }

$$u(t) = u_c(t) + j u_s(t) ; v(t) = v_c(t) + j v_s(t)$$

$$u_p(t) = \text{Re} [\sqrt{2} u(t) e^{j 2\pi f_c t}]$$

$$v_p(t) = \text{Re} [\sqrt{2} v(t) e^{j 2\pi f_c t}]$$

$$\langle u_p(t), v_p(t) \rangle = \text{Re} \langle u(t), v(t) \rangle$$

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In those lectures we have seen that if we have a complex baseband signal,  $u(t)$  is a complex baseband signal, this complex baseband signal can be understood in terms of cosine part and sine part of a signal. So,  $u(t)$  is a complex baseband signal similarly  $v(t)$  is a complex baseband signal, once you have these complex baseband signal you can also derive their equivalent passband signals, so  $u_p(t)$  and  $v_p(t)$  are the passband signals.

So, in those lectures we have also seen that if you want to calculate the inner product of the pass band signals, this inner product is same as the real part of inner product of the baseband signals. This is an important idea that we developed in those lectures and we will use it. So, inner product of 2 pass band signals is same as real part of inner product of complex baseband signals.

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Condition for orthogonality for Coherent FSK

$$\langle s_{ip}(t), s_{jp}(t) \rangle = 0$$
$$\text{Re} \left[ \langle s_i(t), s_j(t) \rangle \right] = 0$$
$$\text{Re} \left[ \int_0^T e^{j2\pi(f_i - f_j)t} dt \right] = 0$$
$$\int_0^T \text{Re} \left[ e^{j2\pi(f_i - f_j)t} \right] dt = 0$$


Using this idea let us develop the condition for orthogonality of Coherent FSK systems. So, we have said in coherent FSK systems we want the inner product of the 2 passband signals to be 0, from this we can think about this equation which says that because inner product of two pass band signals is same as real part of inner product of the 2 passband signals. So, we can also identify the condition for orthogonality that is the real part of inner product of 2 baseband signals should be 0 and we have already seen what is the inner product of these two baseband signals namely  $s_i(t)$  and  $s_j(t)$ .

So, inner product of these 2 signals is this, so what we want is real part of this function should be 0. Now real part of integration is same as integration of real part so from this equation we can easily derive this equation ok. Now, what is the real part of complex exponential? It is a cos function so from here we can arrive at this equation which says that this function should be 0.

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$$\int_0^T \cos 2\pi(f_i - f_j)t dt = 0$$
$$\sin[2\pi(f_i - f_j)T] = 0 \quad \text{sin } m\pi = 0$$
$$2\pi(f_i - f_j)T = m\pi$$
$$|f_i - f_j| = \frac{m}{2T}$$
$$\therefore |f_i - f_j|_{\min} = \frac{1}{2T} \quad (\text{at } m=1)$$


Now if we integrate cos. we get sign putting in this limit of integration we get to the condition that this function should be 0 and we know that sine  $m\pi$  is 0. So, from this we get that this argument of sine namely  $2\pi f_i$  minus  $f_j$  times  $T$  should be  $m\pi$ , from here we can say that mod of this difference should be same as  $m$  by  $2T$ , so we can cancel  $\pi$  by  $\pi$  and we can shift  $2T$  to this side. Now for minimum difference in this frequency separation I can choose  $m$  as 1 and I can get that minimum difference in the frequency separations mod should be same as  $1$  by  $2T$ .

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$$|f_i - f_j| = \frac{1}{2T}$$

This corresponds to minimum BW  
and also condition for Minimum  
frequency Shift keying (MSK)



So, this is the condition that we have caught for establishing the minimum frequency separation that is required to have orthogonality in coherent FSK systems ok. So, this condition corresponds also to minimum bandwidth requirement and it also corresponds to a special kind of frequency shift keying which is known as minimum frequency shift keying or it is often abbreviated as MSK. So, MSK is a kind of a frequency shift keying which uses this frequency separation between baseband signals. Now, let us see what happens when we have something different when we have non coherent frequency shift keying.

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Orthogonality for non-coherent FSK

$$u_p(t) = \text{Re} \{ \sqrt{2} u(t) e^{j 2\pi f_c t} \}$$

$$\hat{v}_p(t) = \text{Re} \{ \sqrt{2} v(t) e^{j 2\pi f_c t} e^{j\theta} \}$$

$$\langle u_p(t), \hat{v}_p(t) \rangle = \text{Re} \{ \langle u(t), v(t) e^{j\theta} \rangle \}$$

Condition for orthogonality:

$$\text{Re} \{ \langle u(t), v(t) \rangle e^{-j\theta} \} = 0$$


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Once we want to talk about the non coherent FSK the condition of orthogonality is slightly different ok. So, what is the difference between coherent and non coherent this will become clear now. So, when we are talking about the non coherent systems you cannot assume that the two passband signals have the complex exponential at the same frequency. Because, if you assume that the two pass band signals have the complex exponential at the same frequency inherently what we assume is that the receiver has the same frequency generation as used at the transmitter and this requires synchronization and other things.

In non coherent systems we assume that this rotating complex exponential is offset by theta with respect to the rotating complex exponential that is used at the transmitter. And we have also seen in one of the lecture a possibly lecture 21 how to get rid of this theta

ok. Anyways at this point what we are assuming is that we are having 2 passband signals, but the phase of one pass band signal is offset with the phase of the another pass band signal. If I want to take the inner product of these two signals, the inner product can be simply obtained by taking the real part of inner product of the baseband signal. But now the baseband signal is not  $v(t)$ , the baseband signal is  $v(t)$  times  $e^{j\theta t}$ .

So, this inner product would be same as real part of inner product of these 2 functions. Now if you want the orthogonality of these pass band signals you want this to be 0, so you want this to be 0. So, when this  $e^{j\theta t}$  pulls out of the inner product it will come with a conjugation, because inner product operation satisfies Hermitian by linearity, this we have seen. Now for orthogonality we want this to be 0.

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$$\text{Re} \{ \langle u(t), v(t) \rangle e^{-j\theta t} \} = 0 \text{ for all } \theta, \text{ only if } \langle u(t), v(t) \rangle = 0$$

For non-coherent FSK :

$$\int_0^T e^{j2\pi(f_i - f_j)t} dt = 0$$

$$e^{j2\pi(f_i - f_j)T} - 1 = 0$$

And this will be 0 for all values of theta only if inner product of  $u(t)$  with  $v(t)$  is 0, only if this is 0 then only this quantity the real part of this thing will be 0 for all values of theta and thus the condition of orthogonality for non coherent FSK, in which we assumed that receiver does not have the complete information about the frequency of the transmitter. Where we assume that there is a some frequency and phase offset at the carrier at user the receiver with respect to the carrier used at the transmitter, the condition of orthogonality is that the inner product of 2 baseband signals should be 0.

The condition of orthogonality for coherent FSK on the other hand was that the real part of the inner product of the baseband signals should be 0, so these 2 conditions are

different and thus you would get a different answer. So, once you want that inner product of baseband signals to be 0 what we want is this should be 0 and this integration evaluates to this. So, it simply means that from this condition we can get that the mod of  $f_i$  minus  $f_j$  for minimum value of this quantity is nothing but  $1/T$ .

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The image shows handwritten notes on a grid background. At the top, the equation  $|f_i - f_j| = \frac{1}{T}$  is written. Below it, the word "Summary:" is written in red. Then, two equations are listed:  $|f_i - f_j| = \frac{1}{2T}$  (Coherent FSK) and  $|f_i - f_j| = \frac{1}{T}$  (non-coherent FSK). A hand is visible at the bottom left corner, and there is a small NPTEL logo in the bottom left and "ETRC, IIT DELHI" in the bottom right.

$$|f_i - f_j| = \frac{1}{T}$$

Summary:

$$|f_i - f_j| = \frac{1}{2T} \text{ (Coherent FSK)}$$
$$|f_i - f_j| = \frac{1}{T} \text{ (non-coherent FSK)}$$

So, in summary we have seen that mod of the difference between the two frequencies in case of coherent FSK should be  $1/2T$ , whereas mod of difference between the two frequencies in case of non coherent FSK is  $1/T$ , because the orthogonality conditions are different ok. So now, as we have done for QAM we would be calculating the bandwidth requirement and we would be estimating the spectral efficiency in these modulation schemes.

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Coherent FSK

$$B, \text{ Bandwidth required} \approx M \times \frac{1}{2T}$$
$$\text{Bit Rate, } R = \frac{\log_2 M}{T}$$
$$P = \frac{R}{B} \approx \frac{\log_2 M}{T} \times \frac{2T}{M} = \frac{2 \log_2 M}{M} \text{ (b/s/Hz)}$$


So, we are using  $m$  symbols and each symbol is separated by a frequency of  $1$  by  $2T$ . So, bandwidth requirement in case of coherent FSK is  $m$  times  $1$  by  $2T$  bit rate on the other hand is  $\log_2$  to  $M$ . So, this is the number of bits times the symbol rate and symbol rate is  $1$  by  $T$ . So, bit rate is locked to  $m$  times  $1$  by  $T$  from this spectral efficiency can simply be obtained as bit rate divided by the occupied bandwidth. So, you have to divide this thing with  $m$  by  $2T$  which is the bandwidth required and for coherent FSK the spectral efficiency that we get is  $2 \log_2 M$  by  $M$  and again the units are in bits per second per Hertz, also let us do it alternatively in a different way the same thing.

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Or,

$$B \approx \frac{M}{2T}$$
$$"BT" \approx \frac{M}{2}$$

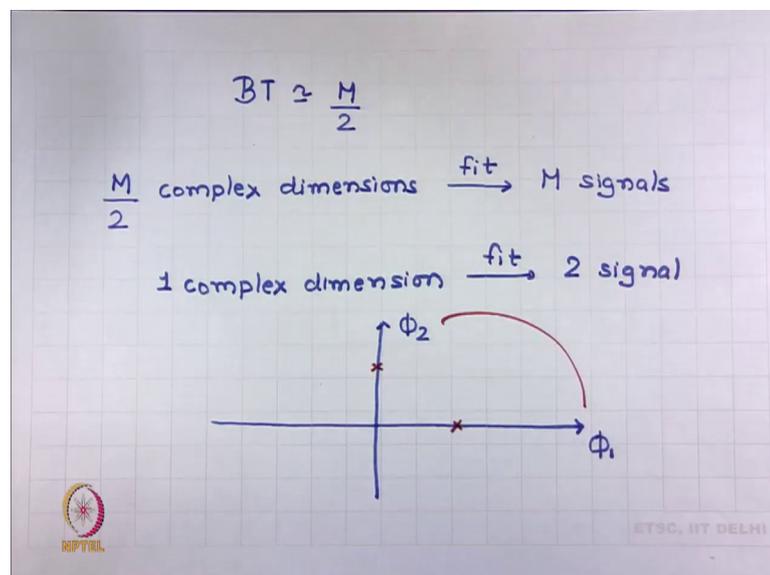
no. of bits =  $\log_2 M$

$$P = \frac{\text{no. of bits}}{\text{Complex dimensions}} = \frac{\log_2 M}{(M/2)}$$
$$= \frac{2 \log_2 M}{M} \left[ \frac{\text{bits}}{\text{Complex dimension}} \right]$$


So, we know that bandwidth in coherent FSK is  $M$  by  $2 T$  from this we get bandwidth time product or time bandwidth product is approximately  $M$  by  $2$ , number of bits that we have per symbol is  $\log_2 M$ . So, spectral efficiency as we have seen it is number of bits divided by number of complex dimensions, number of complex dimensions is same as time bandwidth product which is  $M$  by  $2$ . So, spectral efficiency is  $2 \log_2 M$  by  $M$ , the units is bits per complex dimension or bits per  $2$  real dimension alright.

Why do we have number of bits as  $\log_2 M$ ? Because, we want to consider how many bits you can transmit in a given time bandwidth product and the time that we are using is corresponding to one symbol that means this time bandwidth product corresponds to one symbol. So, in this time you can transmit only one symbol and thus in this time bandwidth product the number of bits that you can transmit is  $\log_2 M$  alright. So, number of bits that you can transmit in a given time bandwidth product is  $\log_2 M$  and the time bandwidth product or number of complex dimensions is  $M$  by  $2$ . So, we get the same answer, but we are using a different philosophy so that you also get trained in using these ideas.

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Little bit more, so we have said that time bandwidth product is approximately  $M$  by  $2$ , that means the number of complex dimensions is  $M$  by  $2$  and in these many complex dimensions we can fit  $M$  signals right. We are using this FSK scheme to support  $M$  signals. So from this I can get in one complex dimension I can fit 2 signal and this is

very clear if I have 1 complex dimension that means 2 real dimensions I can support 2 FSK signals, let us see what happens in case of non coherent FSK.

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Non-coherent FSK

$$B \approx M \times \frac{1}{T}$$
$$R = \frac{\log_2 M}{T}$$
$$P = \frac{R}{B} \approx \frac{\log_2 M}{T} \times \frac{T}{M} = \frac{\log_2 M}{M} \text{ (b/s/Hz)}$$

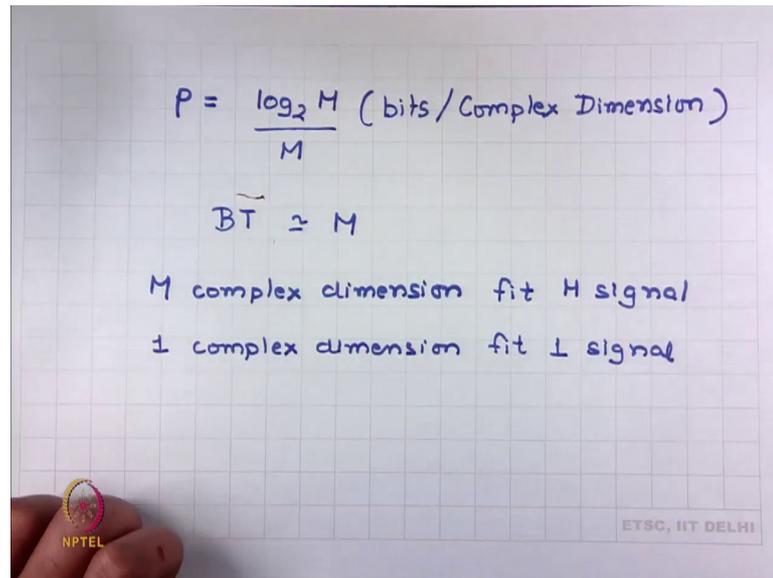
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Non coherent FSK the bandwidth is M times the frequency separation between 2 symbols which is  $1/T$ , rate is same as before  $\log_2 M / T$ . So, spectral efficiency is also same as before just that there is a factor of 2 missing and because here you have a higher bandwidth requirement, because the frequency separation has to be larger for non coherent FSK as compared to coherent FSK alright. So, spectral efficiency in case of non coherent FSK is  $\log_2 M / M$ .

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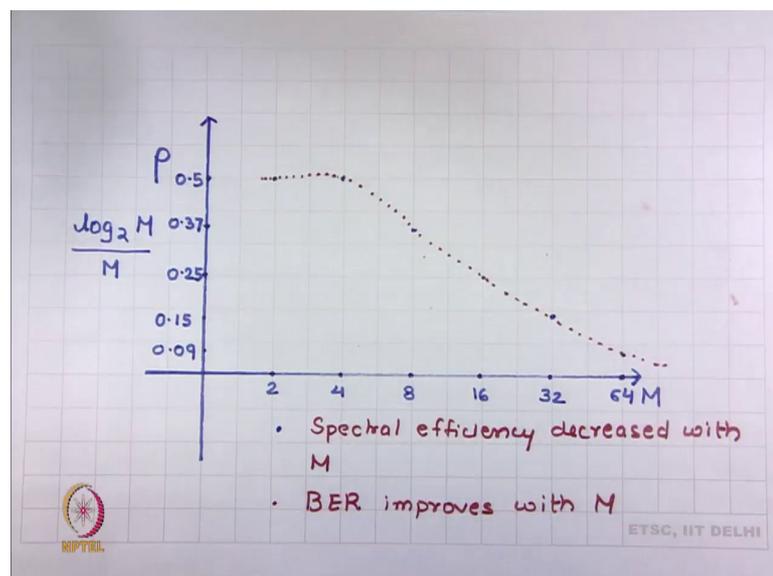
$$P = \frac{\log_2 M \text{ (bits/Complex Dimension)}}{M}$$
$$\widetilde{BT} \approx M$$

M complex dimension fit M signal  
1 complex dimension fit 1 signal



In bits per second per hertz or in bits per complex dimension does not matter and because time bandwidth product is M, you can simply get this if you take TB it is approximately M. That means, in m complex dimension we were fitting M signal, in 1 complex dimension, we will be able to fit only 1 signal and there is the spectral efficiency of non coherent FSK reduces compared to coherent FSK right. So, that is the price that you have to pay by having a receiver which does not synchronize well with the transmitter ok.

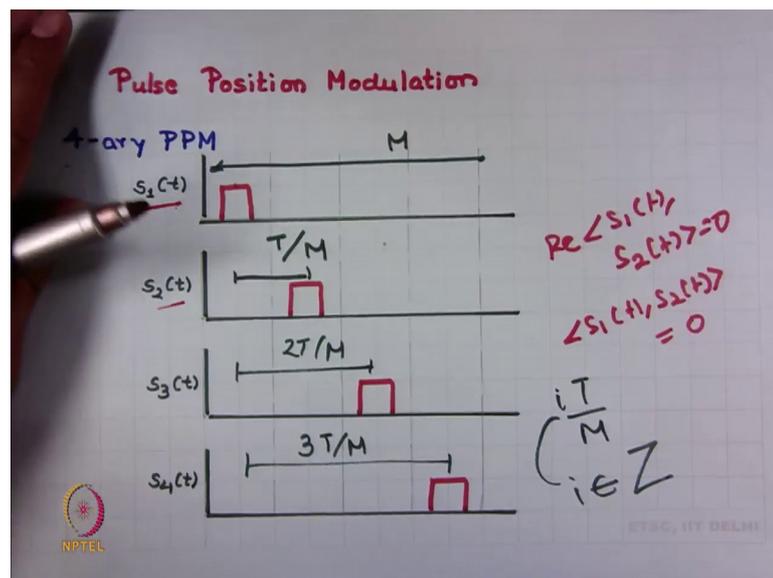
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If you look at this spectral efficiency verses M something strange we see; we see that spectral efficiency in this case reduces with respect to M and that is strange because earlier we have seen in case of QAM the spectral efficiency improves with M. Whereas in this FSK modulation schemes spectral efficiency reduces with M and as M increases we get to very poor values of this spectral efficiency, because these modulation schemes are not optimized to harness higher spectral efficiency as QAM was.

These modulation schemes are optimized to improve bit error rate for the same signal energy and this we will see when we see detection that these modulation schemes have very low levels of signal energy requirement. But at this point in modulation we are just focusing on this spectral efficiency and clearly FSK based modulation schemes runs out on the basis of spectral efficiency.

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So, we have looked into one kind of orthogonal modulation technique that is frequency shift keying, then the question comes are the other ways in which we can construct a set of orthogonal waveforms and answer is yes. So, to understand that let us look at this example of Pulse Position Modulation.

So, in pulse position modulation we use pulses and these pulses are non overlapping pulses and if these pulses are non overlapping we know that these signals  $s_1(t)$  and  $s_2(t)$  will be orthogonal signals. So, if you want to multiply these two things you will get a flat 0, if you get a flat 0 then real part of inner product of these 2 signals will be 0. Also the

inner product of these 2 signals will also be 0, because the pulses of these signals are non overlapping and this is the main idea behind pulse position modulation. In this case we are looking at 4-ary PPM that means we have 4 signals.

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**M-PPM**

$$s_i(t) = \text{sinc}\left(t - i \frac{T}{M}\right)$$

$$i \in [0, \dots, M-1]$$

$\frac{T}{M}$  spaced sinc pulses are orthogonal.

Each  $s_i(t)$  is transmitted at  $\frac{T}{M}$ .

Now, mathematically we can understand M-PPM as a modulation scheme which has signals set like this, so what we mean by this to understand that let us first look back at this waveforms. So, here we can see that this waveform, this pulse is shifted by  $T$  by  $M$  compared to this pulse if there are  $M$  slots. So, if the number of slots is  $M$  and if in each slot I have a pulse then I can have M-PPM system and thus the difference between any two pulse is  $T$  by  $M$ . This pulse is shifted by  $2 T$  by  $M$  compared to this pulse and this pulse is shifted by  $3 T$  by  $M$  compared to this pulse.

That means, in PPM system each pulse is used in a slot, there are  $M$  slots to have M PPM and the separation between these pulses is  $i T$  by  $M$  where  $i$  belong to a set of integers. So, that is what we are saying in that mathematical equation, that means we have different signals in the signal set and these signals are arrived by just shifting one pulse by  $T$  by  $M$  units. So, you can derive different signals by shifting the pulses by different integer multiples of  $T$  by  $M$ , here we are saying that the basic pulses a sinc pulse.

Whereas, in the picture we have used the rectangular pulses of course, because it is easier to draw rectangular pulses. Whereas, sinc pulses are more practical right because they have minimum bandwidth requirement, of course we could have also used root raised

cosine pulses. But for this case we are sticking with sinc pulses and at this moment we are ignoring any issues of inter symbol interference ok. So, this  $i$  should belong to this set it can take a value from 0 up to  $M$  minus 1.

Now in one of the previous lectures we have derived that  $t$  spacing pulses are orthogonal right, they are not perfectly non overlapping what we do not want is strictly non overlapping pulses, what we simply want are the pulses which are orthogonal and these sinc pulses are orthogonal pulses if they are spaced by  $t$  units or  $T$  by  $M$  units it does not matter. Also remember that though these pulses typically lie in a slot of a duration  $T$  by  $M$ . But each signal is only transmitted at time instances which are integer multiples of this  $T$ . Let us now investigate what is the bit error rate in this  $m$  PPM.

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Bit Rate,  $R = \frac{\log_2 M}{T}$

Bandwidth,  $B \approx \frac{1}{2(T/M)} \approx \frac{M}{2T}$

$\rho = \frac{R}{B} \approx \frac{2 \log_2 M}{M}$

$BT \approx \frac{M}{2} \Rightarrow \frac{M}{2} \text{ D fit } M \text{ signals}$   
 $1 \text{ D fit } 2 \text{ signals}$

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So, if there are  $M$  waveforms the number of bits would be  $\log_2 M$  and each waveform is transmitted at a time interval of  $T$ . So, bit rate which is number of bits, but time is simply  $\log_2 M$  by  $T$ . This was also the case in frequency shift keying the idea is essentially similar.

Now we have seen that bandwidth requirement is typically  $1$  by  $2T$ , where  $T$  is the pulse duration and here each pulse duration is  $T$  by  $M$ , because each pulse essentially is confined to this  $T$  by  $M$  slot. So, the bandwidth requirement in case of  $m$  PPM is approximately  $M$  by  $2T$ , so from this we can also derive the spectral efficiency which is

bit rate divided by bandwidth and this will be simply  $2 \log_2 M$  by  $M$ , all these things are exactly same as fitted in frequency shift keying.

So, there are no surprises involved here, if I am interested in time bandwidth product I can shift this  $T$  to this side, so I can get  $BT$  is approximately  $M$  by  $2$  and we know that this time bandwidth product is same as number of complex dimensions. So, in  $M$  by  $2$  complex dimensions I can fit  $M$  signals that means in  $1$  complex dimension I can fit  $2$  signals this is also same as before. So, what we have assumed so far is this is baseband PPM. Now you can also modulate this PPM at higher frequencies, so you can convert this baseband PPM into passband PPM.

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**Passband PPM**

a) DSB

$$B \approx 2 \times \frac{M}{2T} = \frac{M}{T}$$

$$R = \frac{\log_2 M}{T}$$

$$P = \frac{R}{B} \approx \frac{\log_2 M}{M} \quad (\text{similar to non-coherent FSK})$$

$\langle S_i(t), S_j(t) \rangle = 0$

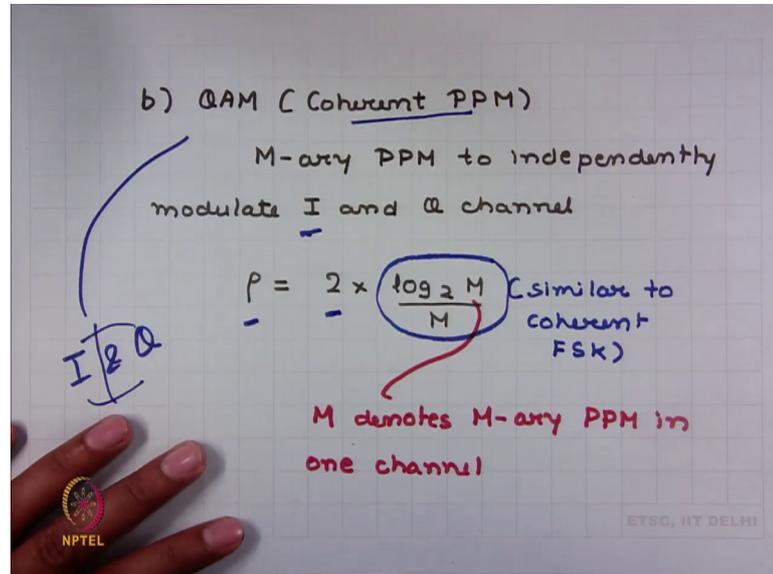
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And we know how to go from baseband to pass band domain, we can use DSB double side band modulation scheme and let us now see what is the spectral efficiency. If we use this double side band modulation scheme when you go from baseband to pass band the bandwidth requirement doubles.

So, now bandwidth is  $2$  times  $M$  by  $2T$  which is  $M$  by  $T$  bit rate remains same, but this spectral efficiency reduces by a factor of  $2$  and this is similar to non coherent FSK systems. PPM essentially can be used in non coherent systems because, the inner product of the two baseband signals is  $0$  this is what we require in non coherent systems. And if we use DSB modulation scheme to go from baseband to passband then the spectral

efficiency that you derive is exactly same as that we derived in the case of non coherent FSK systems.

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If for this baseband to passband conversion we use Quadrature Amplitude Modulation scheme and this case you can think as coherent PPM, because if you are using this quadrature amplitude modulation essentially we have I and Q components and to isolate these I and Q components you want a receiver which is synchronous to transmitter. Otherwise these channels will bleed into each other and the performance will deteriorate.

So, what we are assuming is a synchronous receiver and we assume that this receiver can maintain isolation between I and Q channel and if I use an M-ary PPM to independently modulate I and Q channel my bit rate will simply double, because these channels are independent channel and then the spectral efficiency would be 2 times the spectral efficiency that we derived in the previous case where we are just using 1 channel and that case will be corresponding to the case of non coherent PPM. If you want to call it that way because if we are using just 1 channel we do not want synchronization ok, but if we want to have QAM and if you want to maintain isolation between 2 channels essentially we are talking about synchronous receivers.

So, remember this M denotes the M-ary PPM that is used to modulate 1 channel. So, essentially we have considered two kinds of orthogonal modulation scheme; we have

seen this frequency shift keying which uses baseband waveforms like this and we have seen PPM which uses baseband waveforms like this.

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Handwritten notes on a grid background:

- FSK:  $S_i(t) = e^{j2\pi f_i t}$
- PPM:  $S_i(t) = p(t - iT/M)$
- Walsh-codes:  $S_i(t) = \sum b[n] p(t - iT)$

Additional handwritten annotations:

- A bracket on the right side groups the FSK and PPM equations with the label "non-linear".
- Next to the PPM equation, it says "p(t) = sinc(t)".
- The term  $b[n]$  in the Walsh-codes equation is circled.
- The term  $p(t - iT)$  in the Walsh-codes equation is boxed.

Logos for NPTEL and IIT DELHI are visible at the bottom of the slide.

Of course, for the example that I have shown  $p(t)$  as  $\text{sinc}(t)$ , but it does not matter and if you look carefully then you see that these modulation schemes use different pulse shapes to transmit a symbol and these can be taught as examples of non-linear modulation scheme. Because to transmit one symbol sometimes I am using a waveform  $e^{j2\pi f_1 t}$  and to transmit another symbol I may be using a different complex exponential.

Whereas, in linear modulation the basic pulse remains same and the amplitude of these pulses are altered by this  $b[n]$  and you can also obtain the orthogonal waveforms by just manipulating this  $b[n]$  and keeping these  $p(t)$  same and this is the topic of next discussion.

So, now we will be focusing on orthogonal modulation scheme which manipulates the symbols to obtain orthogonality between waveforms. The third example of generating these orthogonal waveforms follows from the idea of Walsh Hadamard codes.

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3. Orthogonal Walsh Hadamard Codes

$$s_1(t) = \sum_n b[n] p(t-nT)$$
$$s_2(t) = \sum_n g[n] p(t-nT)$$
$$\langle s_1(t), s_2(t) \rangle$$
$$= \int_{-\infty}^{\infty} \sum_n b[n] p(t-nT) \sum_m g^*[m] p^*(t-mT) dt$$

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These codes are used in the systems like CDMA systems Code Division Multiple Access systems, CDMA systems have been very popular and used in mobile communication systems as well. So, third generation mobile telecommunication systems used the CDMA codes ok.

So before carrying into this Walsh Hadamard codes let us revise one important idea of this orthogonality, for example let us take these two baseband waveforms  $S_1(t)$  and  $S_2(t)$  and we know this equation we have used to several times denoting the linear modulation systems. If I want to take the inner product of this  $S_1(t)$  and  $S_2(t)$  I can take this by having this expression here taking the conjugate of this thing and integrating it.

So, what is in the product? Multiplied two signals and integrate right multiplied with conjugation if things are complex ok. So, this is how we can get to the inner product of  $S_1(t)$  and  $S_2(t)$  and rest is really arithmetic.

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$$\begin{aligned}
 & \langle s_1(t), s_2(t) \rangle \\
 &= \int_{-\infty}^{\infty} \sum_n \sum_m b[n] \hat{g}[m] p(t-nT) p^*(t-mT) dt \\
 & \quad p(t-nT) \text{ is orthogonal to } p(t-mT) \\
 &= \int_{-\infty}^{\infty} \sum_{n, m=n} b[n] \hat{g}[m] |p(t-nT)|^2 dt \\
 &= \sum_{n, m=n} b[n] \hat{g}[m] \int_{-\infty}^{\infty} |p(t-nT)|^2 dt
 \end{aligned}$$

So, then we know from a Fubini's theorem that I can always arrange these summations, I have arranged these summations like this and we know that this  $p(t-nT)$  is orthogonal to  $p(t-mT)$  and this is important to avoid inter symbol interference we use the idea.

So, this pulse should be orthogonal to the pulse which is carrying the  $m$ th symbol, this pulse is carrying the  $n$ th symbol and to avoid this inter symbol interference we know that this should be 0 if  $m$  is not same as  $n$  right. And thus what we want is  $m$  should be same as  $n$  otherwise this product would be 0 and thus instead of running out this summation for  $n$  and  $m$  we can run it only for  $n$ , because when  $m$  is not same as  $n$  this is going to be 0 and these terms will not contribute to the summation.

So, we replace this double summation by single summation running over  $n$ ,  $m$  should be chosen as  $n$  and then we have  $b[n] \hat{g}[m]$  conjugate. So,  $m$  is same as  $n$  and when  $n$  is same as  $m$  we just have  $p(t-nT)$  multiplied by conjugate of  $p(t-nT)$  and that boils down to the square of mod of  $p(t-nT)$  ok. Then I pull out this summation outside this integration and I can get to this expression, if I assume further that this  $p(t-nT)$  are orthonormal waveforms then I know that this would be 1. That means, the energy of these pulses is assumed to be one that is what happens in orthonormal case.

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$$= \sum_n b[n] \hat{g}[n]$$
$$\langle s_1(t), s_2(t) \rangle = \langle b[n], g[n] \rangle$$

Thus, to make wave forms orthogonal, we can make their respective vectors orthogonal

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So, in that case the inner product of  $s_1(t)$  and  $s_2(t)$  simply boils down to inner product of  $b_n$  with  $g_n$ . So, what I am saying is if you want to consider the orthogonality and for that orthogonality if you want to force in the inner product of the complex baseband signals to take in some value, you can also do this on the basis of the symbols that you are forming. Because inner product of the wave form is the same as the inner product of the symbols that these waveforms have; so we can work also on this  $b_n$  and  $g_n$  to make these waveforms orthogonal and that is the idea behind orthogonal codes.

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Generation of Orthogonal Codes

$$W_1 = \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \quad \begin{matrix} 1 \times 1 + 1 \times (-1) \\ = 0 \end{matrix}$$

Rows and columns are orthogonal

$$\begin{cases} s_1(t) = 1 p(t) + (1) p(t - T/2) \\ s_2(t) = 1 p(t) + (-1) p(t - T/2) \end{cases}$$
$$\langle s_1(t), s_2(t) \rangle = 0$$

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So, we start with this Hadamard matrix as simple 2 by 2 Hadamard matrix has these elements. So, you can see for this matrix that this is an orthogonal matrix, meaning that the rows are orthogonal and the columns are also orthogonal. How do we test for rows being orthogonal? We have  $1 \times 1$  plus  $1 \times \text{minus } 1$ , we already know how to compute the inner product of the rows and this is going to be 0 right.

So, the inner product of the rows is 0, inner product of the columns is also 0 and hence this is an orthogonal matrix. So now, if you know this orthogonal matrix you can construct this  $S_1(t)$  by multiplying the first pulse with 1, you can multiply the second pulse which is shifted to  $T/2$  there are 2 symbols in this waveform right. Or you use 2 pulses for a symbol, however you want to think of it it is and this pulse is modulated by this one. This pulse is modulated by this one, this pulse is modulated by this minus 1 and hence because the rows were orthogonal you can similarly say that these  $S_1(t)$  and  $S_2(t)$  are also orthogonal right, because the inner product of  $S_1(t)$  and  $S_2(t)$  is 0.

So, this is what we are saying that the inner product of  $S_1(t)$  and  $S_2(t)$  is 0 and hence they can also used in non coherent systems right. Because this is a more strict condition for orthogonality the stricter condition for orthogonality is that the inner product of the baseband signals is 0.

If inner product of the baseband signals is 0 of course the real part of inner product of the baseband signals will also be 0 which is more a relaxed condition for orthogonality which can be used in coherent systems. But because we are using the Walsh code, Walsh code will ensure that the inner product of the baseband signals is also 0 and thus they can be used in non coherent systems as well ok. So, this is for 2 by 2; can we have more terms in the column? Of course yes.

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$$W_n = \begin{pmatrix} W_{n-1} & W_{n-1} \\ W_{n-1} & -W_{n-1} \end{pmatrix}$$
$$W_2 = \begin{pmatrix} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \\ \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} & \begin{pmatrix} -1 & -1 \\ -1 & 1 \end{pmatrix} \end{pmatrix}$$

$W_1$

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You can construct  $n$ th order Hadamard matrix from  $n - 1$ th order Hadamard matrix by using this simple idea. So for example, if I have to find  $W_2$  from  $W_1$   $W_1$  is already obtained, so this is  $W_1$ ,  $W_1$ ,  $W_1$  and minus of  $W_1$ . So, from  $W_1$  you can easily obtain  $W_2$  and so on so forth you can obtain  $W_3$  and  $W_4$  and so on so forth ok.

So, in this way you can generate these Hadamard matrices of different orders and once you have this Hadamard matrix you take different pulses you generate different baseband signals by modulating the amplitude of these pulses with the elements of the rows of Hadamard matrix. And this we have already seen that because, here we are making sure that the inner product of  $S_1(t)$  and  $S_2(t)$  it can be used in non coherent systems.

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$\langle s_1(t), s_2(t) \rangle = 0$   
can be used in non-coherent systems.  
For coherent systems, use independent I and Q channels.

Summary:

a) FSK, PPM, CDMA (Orthogonal Walsh Hadamard Codes) are examples of orthogonal modulation schemes

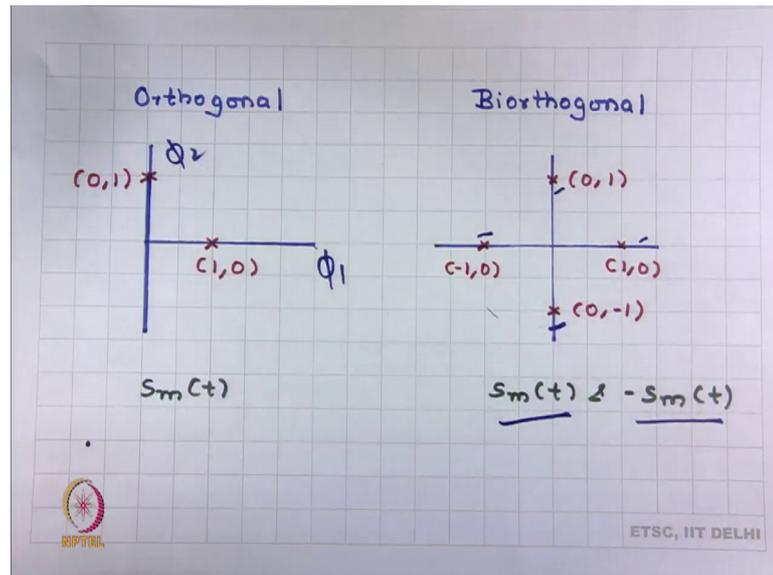
b) All these systems can be analyzed using the same framework

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For coherent systems what we can do? We can simply use independent I and Q channels ok. We will not be deriving the spectral efficiency and other things for these Walsh codes because, they follow the exact ideas as we developed in PPM and FSK all the answers that we obtain for FSK and PPM also applies here they have the same kinds of a spectral efficiency and bandwidth requirement. So, in summary we have discussed three kinds of orthogonal modulation schemes we have seen FSK PPM and CDMA, all these are examples of orthogonal modulation schemes and all these systems can be analyzed using the same framework.

So, you do not have to use different frameworks from for PPM and CDMA, essentially they have the same structure same properties and the same ideas and once we do detection we will see again the array performance of all these modulation schemes follows the same trend and they are also similar; these modulation schemes differ in the implementation methods ok. So, that is conclude our discussion on orthogonal modulation schemes, but we will like to stretch this little bit further and let us try to see can we construct other interesting signal sets from this orthogonal signal sets ok.

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So, let us see the simple case for orthogonal modulation schemes, let us say that we are using binary frequency shift keying. So, we have 2 orthogonal basis and we are transmitting a signal on this orthogonal basis and one signal on this orthogonal basis these signals are orthogonal to each other.

So, from this orthogonal signal set we can construct what is known as Biorthogonal signal set, biorthogonal signal set is simply the orthogonal signal set plus the negative of this orthogonal signal set. That means, if I am using a symbol I also use a symbol which has a negative amplitude than this symbol. So in orthogonal signal set only these two symbols will be present. In biorthogonal what I do is I produce the negative replicas of these signals.

So, I have this signal and this signal in addition with the 2 signals that I had before, so this signal set is known as biorthogonal signal set. What about the energy per symbol? Energy per symbol will be same as the energy per symbol here right, because these symbols are exactly at the same distance right, so average energy per symbol will not change.

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Orthogonal vs. Biorthogonal

- $E_s$  remains same
- Prob. of Error in Biorthogonal is approx. twice than in Orthogonal
- $D \rightarrow 2D$  signals (orthogonal)
- $D \rightarrow 4D$  signals (biorthogonal)

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We will see later that probability of error in biorthogonal signals set is approximately twice in orthogonal signals sets and that is why it is not used right, so this does not perform well with respect to probability of error. But what it does is it improves this spectral efficiency because here as you can see in one complex dimension we are packing 2 orthogonal signals, whereas in the case of biorthogonal in one complex dimension we can pack 4 signals. Let us now look at this spectral efficiency of biorthogonal codes.

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Spectral Efficiency of biorthogonal codes

Examples: Coherent Signaling System

$\frac{M}{2} \rightarrow M$   $P = \frac{\log_2 M \times 2}{M}$

$\frac{M}{2} \rightarrow 2M$   $P = \frac{\log_2 (2M) \times 2}{M}$

$P = \frac{2}{M} + \frac{2 \log_2 M}{M}$

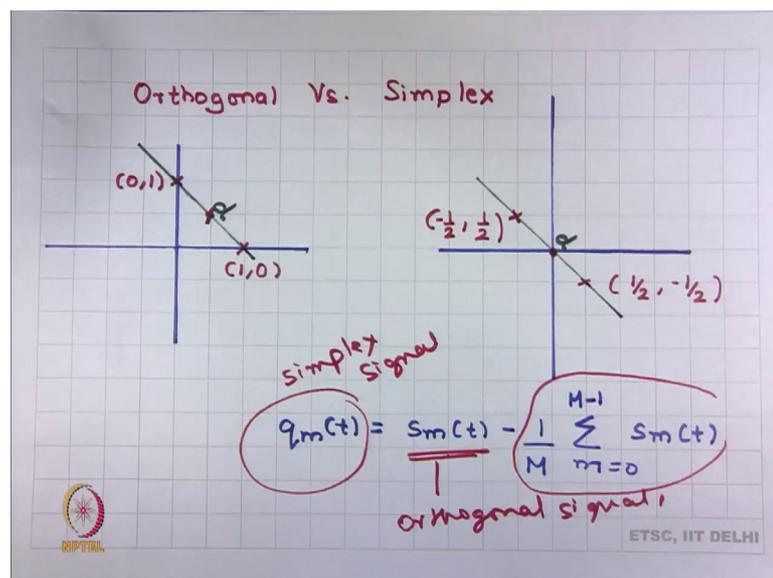
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So, first let us see that these biorthogonal codes are example of coherent signaling system, because if I am using a signal  $S_1(t)$  I am also using a signal  $-S_1(t)$  and thus I need a system which can detect the phase and thus I need a coherent system and we have seen earlier that in coherent system in  $M$  by  $2$  complex dimensions you can pack  $M$  signals. So, spectral efficiency is number of bits which is  $\log_2 M$  divided by number of complex dimensions number of complex dimensions is  $M$  by  $2$ .

So, spectral efficiency in case of coherent FSK systems is  $2$  by  $M$  times  $\log_2 M$ , in biorthogonal systems what changes is the number of symbols grows by a factor of  $2$ , thus now this spectral efficiency becomes  $\log_2 M$  because numbers symbols have grown from  $M$  to  $2M$ . But the numbers of complex dimension have remained same, so this is the spectral efficiency of biorthogonal codes. So, if you want to compare this with this you can also write this as.

So now, you see that the spectral efficiency improves by this factor  $2$  by  $M$  so if  $M$  is pretty large then there is not much saving. So, this spectral efficiency of biorthogonal signal set is large compared to orthogonal signal set, in orthogonal we know that in one complex dimension we can pack  $2$  signals. So, in  $D$  complex dimension we can pack  $2D$  signals in biorthogonal in  $D$  complex dimensions you can pack  $4D$  signals. Another signal set that can be derived from orthogonal signal set is a simplex signal set.

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So, we had these orthogonal signals set and the simplex signal set can be obtained from this by subtracting from this signal set the mean of signals. So,  $q_m$   $t$  represents the simplex signal  $S$   $m$   $t$  are orthogonal signals. So, if you subtract from these orthogonal signals the mean of this signal set you get 2 simplex signal set let us see this with one example, so in this case we had 2 orthogonal signals one orthogonal signal is 1 0 other orthogonal signal is present at 0 1. What is the mean?

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The image shows a handwritten derivation on a grid background. It starts with two orthogonal signals:  $[1, 0]$  and  $[0, 1]$ . These are added together to get  $[1, 1]$ , which is then divided by 2 to find the mean:  $[1, 1]/2 = [1/2, 1/2]$ . Next, the first orthogonal signal  $[1, 0]$  is subtracted from the mean to get a simplex signal:  $[1, 0] - [1/2, 1/2] = [1/2, -1/2]$ . Finally, the second orthogonal signal  $[0, 1]$  is subtracted from the mean to get another simplex signal:  $[0, 1] - [1/2, 1/2] = [-1/2, 1/2]$ . The NPTEL logo is visible in the bottom left corner, and 'ETSC, IIT DELHI' is written in the bottom right corner.

$$\begin{aligned}
 & [1, 0] \\
 + & [0, 1] \\
 = & [1, 1] / 2 = [1/2, 1/2] \\
 \underline{[1, 0]} - & [1/2, 1/2] = [1/2, -1/2] \\
 [0, 1] - & [1/2, 1/2] = [-1/2, 1/2]
 \end{aligned}$$

So, mean is you add these 2 things you add this 2 vectors you get to this vector and divide by the number of vectors which is 2. So, this is what the mean is now to get a simplex signal you taken an orthogonal signal and subtract the mean and you get a simplex signal half and minus half. So, 1 simplex signal here. Similarly from another orthogonal signal you subtract the mean and you get another simplex signal.

So, what you are doing in this process is this signals has a mean let us say somewhere here and you are subtracting the mean, so you get this signal which has got a mean of 0 and this is important, this is why you obtain the simplex signals. In simplex signals what you want to do is you want to have a mean of 0. And why is this important? What is the key motivation for a simplex signal set.

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Motivation:  $E[X^2] = \text{Var}(X) + E[X]^2$

Energy = Energy in fluctuations +  
Energy in mean

Mean = 0 in simplex

Energy is reduced but Error performance  
remains same.

Having mean eases out implementation  
(clock recovery, synchronization are  
easier)

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We know that energy in general is energy in fluctuations plus energy in mean right. So, we have done a similar expression in the case of random variables where we have seen this expression, so this is the energy in the mean, this is the energy in the fluctuations and the total energy is energy in fluctuations plus energy in mean or the total power is AC power plus DC power and because what we are doing in simplex signal set we are making the mean to be 0 we are reducing the signal energy.

When we are reducing the symbol energy the error performance remains same, why does it remain same because error performance just depends upon the distance between the 2 signals. We are not changing the distance between the 2 signals, these signals for here we are just shifting them down. So, we are changing the mean without changing the distance between the 2 symbols and thus the error performance does not change when you go from orthogonal signals there to simplex signal set and so we are having the best of two worlds. We have been able to reduce the average symbol energy and we have not sacrificed anything on error performance and that is why simplex signal sets are good.

However, the problem comes in the ease of implementation, so certain things becomes easier when you have a mean for example, clock recovery and synchronization in general becomes easier when you have certain mean and that is why from implementation point of view orthogonal signals were used. But now because implementation is no cost simplex signals are also becoming more and more important ok. Let us finish this lecture

by just looking at 2 properties of simplex signal, the first property is finding out the energy of simplex signal set if we know the energy of orthogonal signal set.

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**Properties of Simplex signals**

$$1) E_q = \int_{-\infty}^{\infty} |q_m(t)|^2 dt = \left(1 - \frac{1}{M}\right) E_s$$

Proof:

$$E_q = \int_{-\infty}^{\infty} \left| s_m(t) - \frac{1}{M} \sum_{m=0}^{M-1} s_m(t) \right|^2 dt$$

$$= \int_{-\infty}^{\infty} \left( s_m(t) - \frac{1}{M} \sum_{m=0}^{M-1} s_m(t) \right) \times \left( s_m^*(t) - \frac{1}{M} \sum_{m=0}^{M-1} s_m^*(t) \right) dt$$

$|X|^2 = X \bar{X}$

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Whenever we are saying energy of power we always mean that we are talking about the average energy or average power. So,  $E_q$  energy in a simplex signal set can be obtained by carrying out this integration  $q_m(t)$  we have seen is  $s_m(t)$  minus the mean of the orthogonal signal set and from this we can obtain this mod square of this quantity by having this quantity multiplied with the conjugate of this quantity this you know that mod square is nothing but  $X$  into  $X$  conjugate. So, from this we can get this and now we can multiply term by term. So, we can multiply this  $s_m(t)$  with  $s_m(t)$  conjugate and in the end we will have four terms.

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$$\begin{aligned}
 &= \int_{-\infty}^{\infty} |s_m(t)|^2 dt - \frac{1}{M} \int_{-\infty}^{\infty} |s_m(t)|^2 dt \\
 &\quad - \frac{1}{M} \int_{-\infty}^{\infty} |s_m(t)|^2 dt + \frac{M}{M^2} \int_{-\infty}^{\infty} |s_m(t)|^2 dt \\
 &= E_s - \left( \frac{1}{M} E_s \times 2 \right) + \frac{1}{M} E_s \\
 &= E_s \left( 1 - \frac{1}{M} \right)
 \end{aligned}$$

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So, this with this, this with this, this with this and this with this so we multiply term by term you have 4 terms and finally by working out the arithmetic the no concepts involved it is just some simple arithmetic operations that you have to do, you can easily obtain the average energy of the simplex signal given by this. So, as  $m$  increases this  $1$  by  $M$  reduces and thus there is not much gain when you are using simplex signals. If  $M$  is a small of course, there is a little bit more gain compared to the orthogonal signals and thus for large values of  $M$  it does not matter in terms of energy requirement whether you are using simplex signals or orthogonal signals.

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2) Simplex signals are not orthogonal

$$\int_{-\infty}^{\infty} q_m(t) q_n^*(t) dt = \frac{-1}{M} E_s \quad (\text{if } m \neq n)$$

Proof:

$$\int_{-\infty}^{\infty} \left( s_m(t) - \frac{1}{M} \sum_{m=0}^{M-1} s_m(t) \right) \left( s_n^*(t) - \frac{1}{M} \sum_{n=0}^{M-1} s_n^*(t) \right) dt$$

$$= \int_{-\infty}^{\infty} s_m(t) s_n^*(t) dt - \frac{1}{M} \int_{-\infty}^{\infty} \sum_{m=0}^{M-1} s_m(t) s_n^*(t) dt = \frac{-E_s}{M}$$

$$- \frac{1}{M} \int_{-\infty}^{\infty} \sum_{n=0}^{M-1} s_n^*(t) s_m(t) dt + \frac{1}{M^2} \int_{-\infty}^{\infty} \sum_{n=0}^{M-1} s_n^*(t) \sum_{m=0}^{M-1} s_m(t) dt$$

$$= \frac{-E_s}{M} - \frac{2E_s}{M} + \frac{1}{M^2} M E_s = \frac{-E_s}{M}$$

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The next property that we want to prove for simplex signals is that these simplex signals are not orthogonal. In fact, if you take the inner product of 2 simplex signals you can easily show that this inner product corresponds to minus  $E_s$  by  $M$  and the prove is trivial it is simple it is on the same lines as we did for the first property. So, you can simply expand this  $q_m t$ . So, this is  $q_m t$  and you can write in for this  $q_n t$  conjugate which is this. So now, you multiply this term with these two terms and this term with these two terms you end up with these four terms.

So, I have  $S_m t$  multiplied by  $S_n t$  conjugate and then you have to carry out this integration then you have this multiplied by this you get this. Similarly you have to multiply this with this and you get this term and finally you have to multiply this with this and you get this term. You can easily see that this integration will boil down to 0 because  $S_m t$  and  $S_n t$  are orthogonal signals, so this will be 0 now when you multiply this  $S_n t$  with this summation.

So, the only non trivial term which will have some effect is when  $S_n t$  conjugate multiplies with  $S_n t$  because all other products will contribute to 0, because  $S_n t$  will be orthogonal to all other signals and thus finally you can show that if you multiply this  $S_m t$  with  $S_n t$  conjugate.

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$$\int \sum_{m=0}^{M-1} S_m(t) S_n^*(t) dt$$

$$\int S_m(t) S_n^*(t) dt = E_s$$

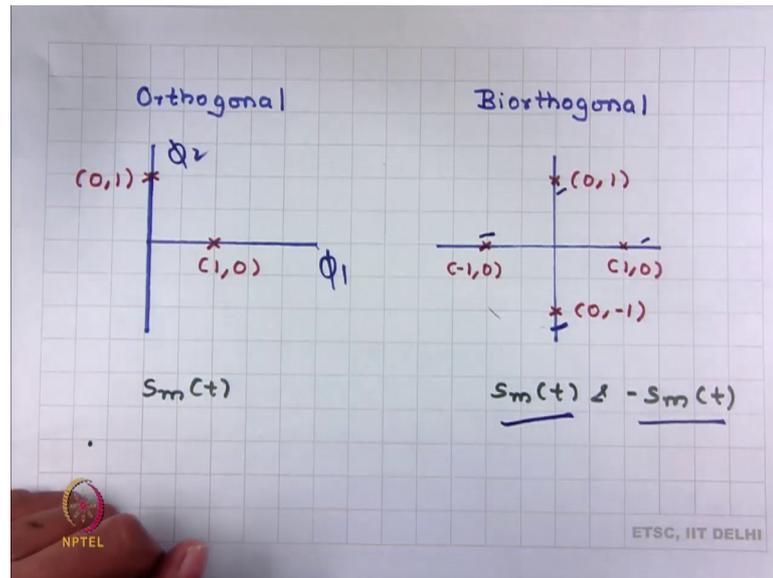
So, what we will have is just  $S_m t S_n t$  with  $S_n t$  conjugate  $dt$  because, when  $S_n t$  conjugate multiplies with other signals as those signals are orthogonal they will have no

contribution. So, finally we will get this and this is nothing but energy of the signal, so when I multiply this with this I get  $E_s$  by  $M$  with minus. So, this will lead to minus  $E_s$  by  $M$ , similarly using the same logic I have minus  $E_s$  by  $M$  from here. So, I have minus  $2 E_s$  by  $M$  and using the same logic you can prove that this is nothing but this is also  $E_s$  by  $M$ . So finally, I get the answer is minus  $E_s$  by  $M$  and thus it completes the proof that simplex signals are not orthogonal signals, so this brings us to the conclusion of this lecture.

So in this lecture we have started with orthogonal codes which are really important for power constraint channels like satellite channels and we have seen 3 examples of orthogonal codes FSK pulse position modulation then we have seen Walsh Hadamard codes based modulation schemes. For example, which are used in CDMA systems, we have seen that all these modulation schemes ends up with having the same spectral efficiency and bandwidth requirement. And thus the only difference in all these varieties of orthogonal modulation schemes is their implementation rest the framework remains same.

And we have seen interesting thing that the spectral efficiency of this modulation scheme decreases as  $m$  increases this is really in contrast with what we have seen for  $M$  QAM case and finally towards the end the lecture we have seen that you can also derive some other interesting signal sets from orthogonal signal sets we can derive biorthogonal signal sets.

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In which you just have the orthogonal and negative of those signals that is biorthogonal, in simplex signals sets you just remove the mean of the orthogonal signal set. We have seen an interesting property of simplex signals that simplex signals does not degrade performance still they reduce the average symbol energy and hence they are important, of course them they might be some implementation issues with simplex signals.

So, in the next lecture we will look at differential modulation schemes, these differential modulation schemes are also important modulation schemes particularly in the context of the channels where the phase response of the channel where is randomly and too fast.

Thank you.