Principles of Digital Communication
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Lecture - 26
Modulation
Pulse Amplitude Modulation & Quadrature Amplitude Modulation (Part- 1)

Good morning. Welcome to new lecture on Modulation and in this lecture we will be talking about Pulse Amplitude Modulation and Quadrature Amplitude Modulation. So, let us start with examples of modulation and we have already seen the basic equation for modulation is this.

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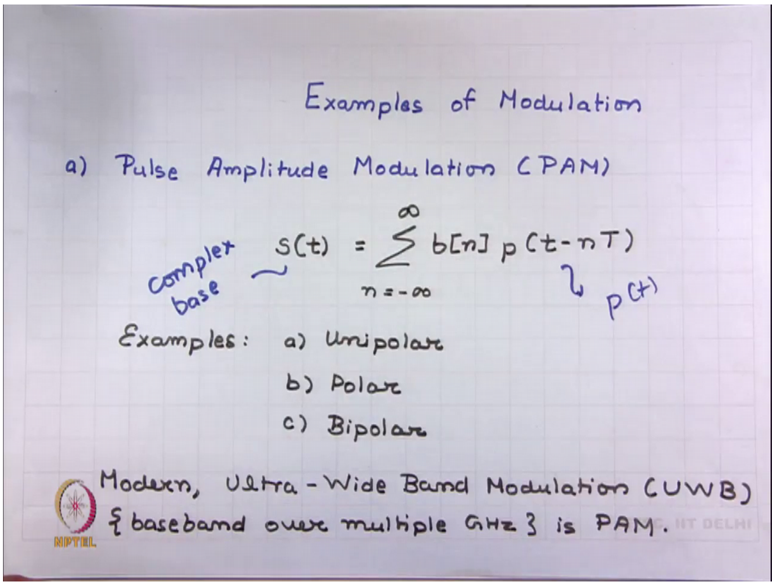

Where S t is complex baseband signal, b n are coefficients of this complex baseband signal. In general they can be complex and this p t minus n t is the basic pulse that we use, right. So, in the last lecture we have discussed what should be p t to avoid inter symbol interference. So, this equation describes basic linear modulation.

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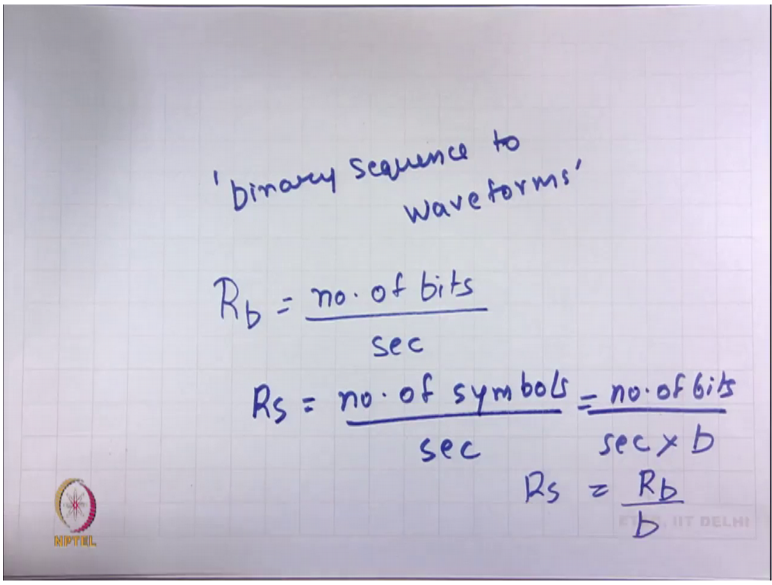

And we have seen that modulation in general is the way, how can you go from binary sequence to waveforms. You should always remember the basic underlying theme that we are discussing. This is what the modulation is, right. So, we have seen various examples of modulation before. For example, we have seen unipolar modulation, polar modulation, bipolar modulation. In all these modulation schemes what happens to based on the binary sequence, you choose this b n and b n has modulating the pulse.

All these examples of the modulation schemes that we discussed unipolar, polar and bipolar falls into the category of pulse amplitude modulation right. What is pulse amplitude modulation? You have a basic pulse and the amplitude of this basic pulse is modulated by b n and there are various ways in which you can choose b n based on the binary sequence. All these examples are examples of pulse amplitude modulation.

In modern times there is very popular ultra wideband modulation scheme which is also an example of pulse amplitude modulation. The only thing that is in there, that this complex baseband signal that we use runs in over multiple gigahertz. So, this is a very wide band width signal, but that is also the example of pulse amplitude modulation and it is a good idea to start thinking about pulse amplitude modulation and from here we can go to other modulation schemes, ok.

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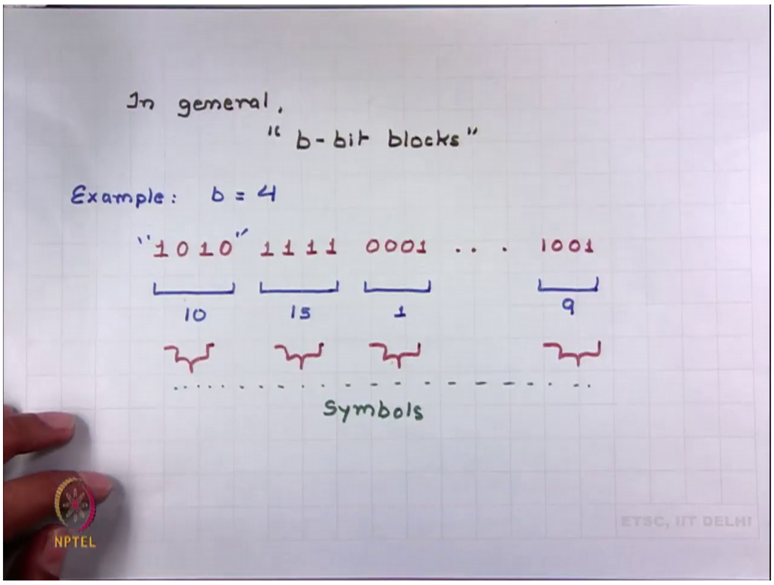

So, let us see what happens in modulation technique. We can consider the blocks of b bits. For example, suppose this is the binary sequence that we have and this binary sequence can be converted into blocks and here we have considered the blocks of 4 bits. So, this is one block, 2nd block, 3rd and 4th block and now these blocks of bit can be converted to a symbol and what is the symbol? Symbol is nothing, but it is a quantized real number ok.

Here what we have done is, we have just considered the symbol which is equivalent to the binary value of this block. For example, we can map this to a number 10, this to number 15, this to number 1 and this to number 9. So, first step in modulation is you convert these bits of blocks of bits into the symbols. B n are the symbols. So, we have converted this binary sequence into 4 numbers of 4 symbols, alright.

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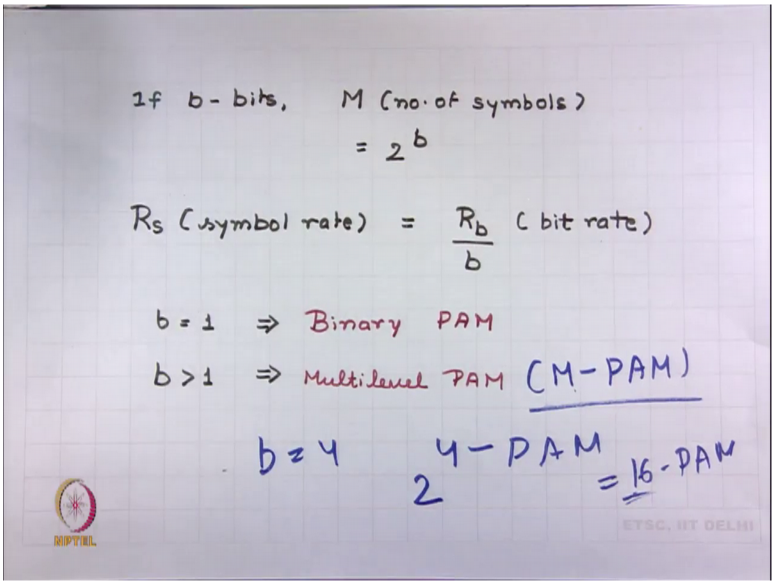

So, if we have blocks of b bits how many symbols you can have? You can have 2 to the power b symbols this also we have seen when we discuss quantization right. So, for example, if we have b binary bits then the number of symbols that you can have is 2 to the power b ok.

What will be the symbol rate? For example, if I have a bit rate R b which is number of bits per second and in this example that we have considered for every 4 bits I have a symbol. So, symbol rate which is number of symbols per second is nothing, but it is the number of bits per second divided by the block length which is in this case b isn't it and number bits per second is nothing, but it is the bit rate. So, I get a very neat expression that is relating the symbol rate and bit rate. So, we can say that the symbol rate is nothing, but it is the bit rate divided b because for every b bits you are going to have one symbol, right. So, this is the relationship between symbol rate and bit rate.

If b is 1 that means for each binary bit I will be generating a symbol, then we are in the regime of binary pulse amplitude modulation. If b is greater than 1, then I am talking about multi-level pulse amplitude modulation or in short we can say it as M-PAM ok. M stands for multilevel pulse amplitude modulation. For example, if b equals to 4, then we are talking about 2 to the power 4 PAM which is 16 PAM. So, M represents the number of multi levels that you have. So, if b is 4 the total number of symbols that we have is 2 to the power 4, that means we are going to have 16 symbols.

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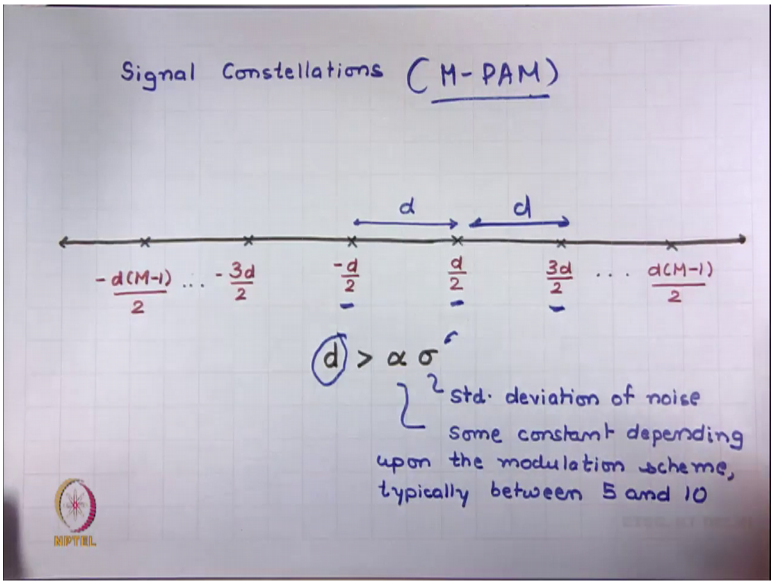

Let us now look at the signal constellation for M-PAM. So, this is the signal constellation of M-PAM and what we are showing is that we are putting the symbols on the real line. So, in pulse amplitude modulation the signals are confined to be real signals, ok. So, the symbols that we have can only take real values, ok so, this is the restriction. Once you allow the symbols to be complex symbols, then from PAM be go to Quadrature Amplitude Modulation which we will discuss after we have finished with PAM, but for a starting we are talking about PAM and the basic restriction in PAM is that the signals are considered only to be real signals or symbols are real symbols and we are plotting the symbols on this real line.

Symbols are quantize discrete time signals and in literature the symbols and signals are used synonymously and sometimes you also referred to symbols as signals. So, please do not get confused. So, for example one symbol can take a value of d by 2, another symbol can take a numerical value minus d by 2. All this is real next symbol can be assigned an amplitude of 3 d by 2 and so on and so forth.

Interesting thing that you can look in here is that the symbols are separated by constant distance d. For example, the difference between the two symbols is d. The difference between these two symbols is also d, right. So, symbols and at least separated by a distance of d and this we will see in a while and this will become more clear when we discuss a detection that the performance of any modulation scheme depends upon this minimum distance d, ok. So, to ensure that your modulation schemes achieves a certain error performance, you want to make sure that the symbols are separated by a certain distance d, and this has to be mostly uniform.

How do we choose d? So, we can choose d as some constant alpha time sigma where sigma is the standard deviation of noise, right and this constant depends upon the modulation scheme. Typically you can choose the value of alpha as something between 5 and 10. So, what we are saying is depending upon the noise level and depending upon the some constant which varies from modulation scheme to modulation scheme, you decide on a certain distance d, and once you have decided on a certain distance d, then what you want to make sure as these symbols are separated by distance d or the minimum distance between any two symbols is at least d.

And once you have decided for that you can easily draw the signal constellation, signal constellation is just drawing the signal alphabets. So, we are just drawing the amplitude levels of various symbols that we have. And what is this idea behind choosing a certain constant d? For example, if you want to look at the error performance of this modulation scheme what would happen is the possibility of error is coming only from the neighbors.

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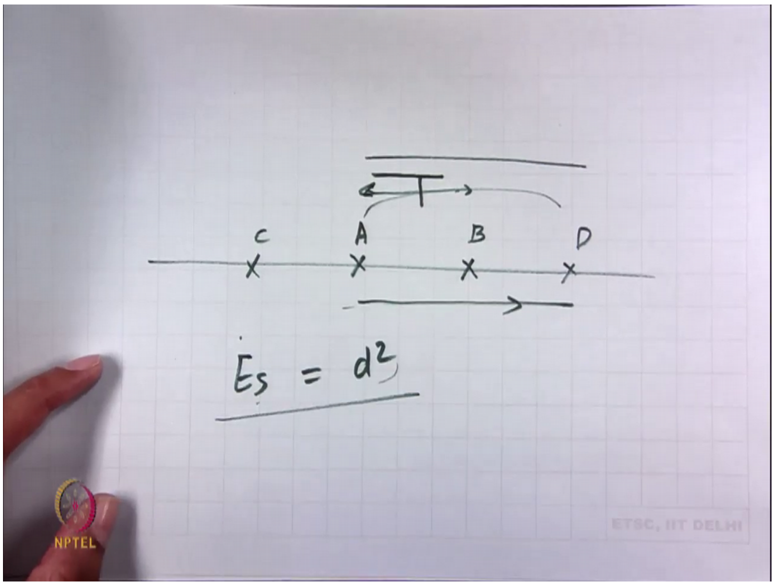

For example, suppose you are transmitting a symbol. The error may happen when you receive a confused, the symbol with one of the neighboring symbol we have transmitting let us say symbol A. The receiver might confuse it with A, symbol B or symbol C. The symbol B and symbol C are the neighbors of the symbol A. The possibility of receiver confusing it with symbol D is very remote. Why is this? So, because for receiver to confuse A with D, the noise amplitude levels must be larger than the noise amplitude levels, that would create confusion for the receivers confusing A and B, ok.

For example if this is the distance between A and B and let say if noise amplitude levels are larger than the half of this distance, then the receiver would confuse A to B and if the receiver has to confuse A to D, then the noise amplitude levels has to be larger than half of the distance between A and D, ok. These things will become more clear later on, but the point that we are emphasizing is that receiver is most likely getting confused from the neighbors and thus for the error performance it is only important that we maintain a certain minimum distance between any two symbols, ok. So, we want to make sure that the distance between any two symbols is greater than certain threshold.

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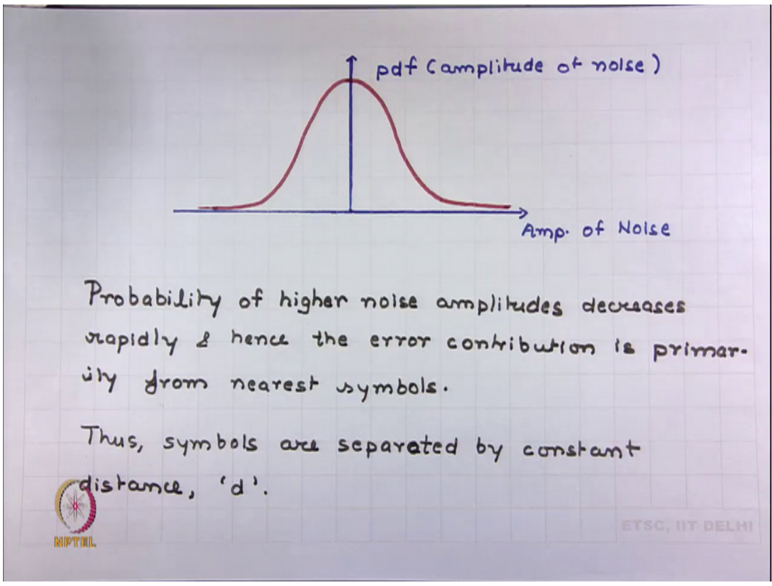

And you can also see this from the noise characteristics we have already seen that the noise is caution and this is the probability density function of the noise on the times axis we have the amplitude of noise. So, it is very clear that the probability of higher noise amplitude decreases rapidly and hence, the error contribution is primarily from nearest symbols because it is very unlikely that the noise amplitudes are so large that it gets confused with d, right because the probability of larger noise amplitudes is smaller and this most likely receiver gets confused only from the nearest symbols or nearest neighbors and does its only important to separate the symbols at least by certain distance d.

So, what happens in signal constellation design, let me go back to that signal constellation design is you choose a value of d which you can choose depending upon the standard deviation of noise and some constant. Once you have decided upon this value of d, you can place all these symbols separated by this distance d, and by this you can get to the signal constellations of M-PAM.

So, let me repeat what is PAM. PAM is the linear modulation scheme in which the symbols are modulating the amplitudes of the pulse and these symbols are confined to be real symbols, Ok. Once they has real symbols we can just draw them on the real line and M denotes the number of symbols that you have. For example, in this case we have M symbols and thus we call this as M-PAM.

Let us look at the average symbol energy. So, how can we find the symbol energy? Symbol energy we already have seen in one of the lectures that energy is nothing, but distance square. So, if we consider the signals as vectors. The energy of the signals is nothing, but it is the distance of the signals from the origin and square of it. Isn't it. So, this is how we interpret the energy of this signals or symbols.

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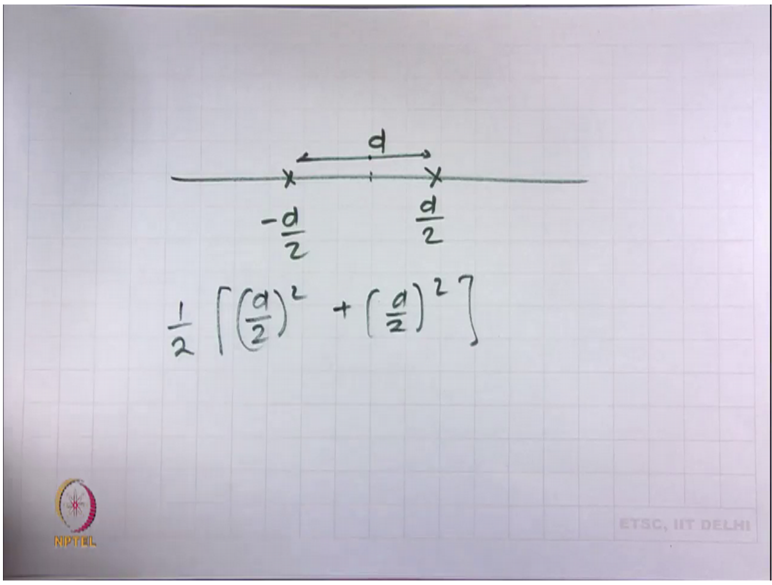

Suppose I have two symbols d by 2 and located at minus d by 2 with a distance of d. What is the energy of this symbol? It is d by 2 square. What is the energy of this symbol? It is again d by 2 square and what is the average energy. So, there are two symbols thus we divide by 2. So, this will give me the average symbol energy of this constellation right.

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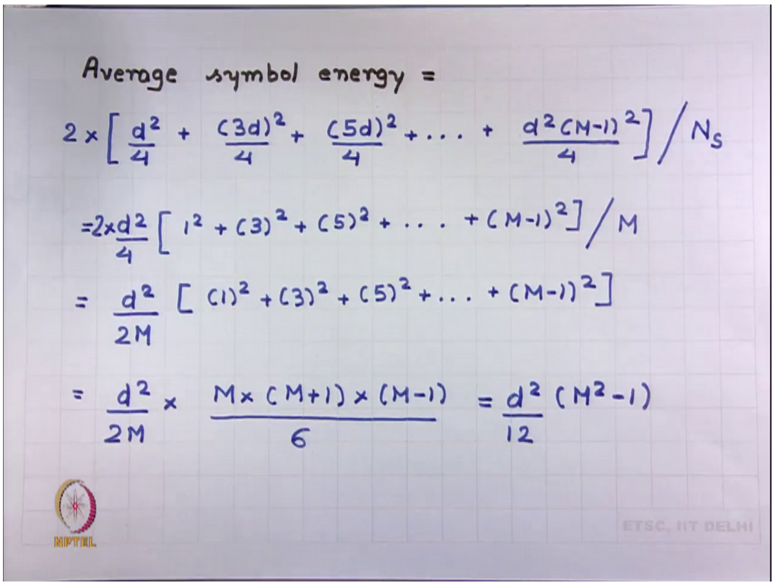

Similarly I can find the average symbol energy of this constellation here the difference is that we are having M symbols. So, we have d square by 4 which corresponds to the energy of this symbol plus 3 d by 2 square, 5 d by 2 square and so on so forth up to d square M minus 1 square by 4.

And then I have to account for these negative symbols and these symbols have the same energy as these symbols. So, I just have to multiply by a factor two and dividing by the total number symbols that I have and then, it is just arithmetic. You can reduce this to this the number of symbols is M and then we get in each series like 1 square plus 3 square and so on and so forth up to M minus 1 whole square and we know that some of the series is nothing, but M into M plus 1 into M minus 1 by 6.

Solving all this out what we get is average symbol energy is d square by 12 times square minus 1 and I request you to remember this formula because it is an important formula and it denotes how the average symbol energy varies in M-PAM ok. The average symbol energy in M-PAM is d square by 12 times M square minus 1. Let us work it out little bit more.

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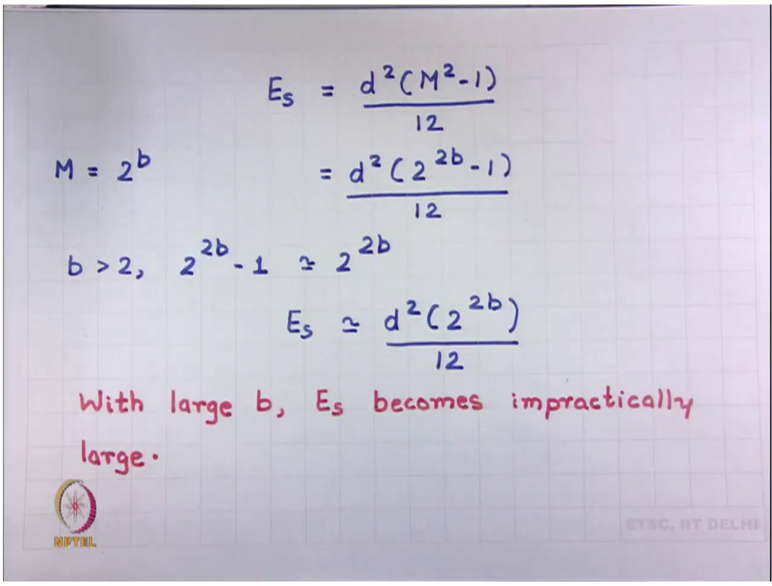

So, we have said average symbol energy is d square times M square minus 1 by 12 and M. We know its 2 to the power p b is the block length, right. So, remember that we are taking a group of b bits at a time and we are converting this group into a symbol. So, M is 2 to the power b substituting that in here we get average symbol energy by this expression and if b is greater than 2 2 to the power of 2 b minus 1 can be approximated as 2 to the power 2 b.

And, thus I can approximate the average symbol energy by this thing and one thing that you should note here is as the value of b increases, average symbol energy becomes impractically large. That means, if I have a large M, then the average symbol energy that you need to invest is quite large and thus this M-PAM modulation scheme is not very interesting modulation scheme to use for very large values of M.

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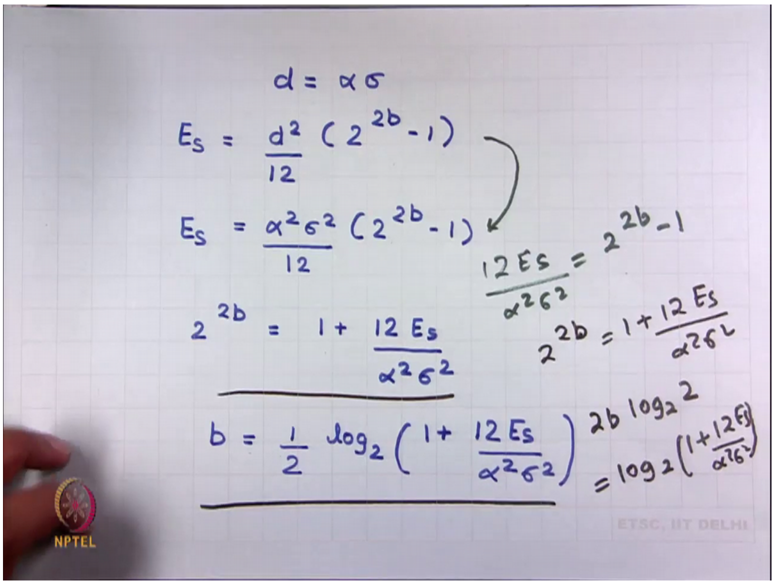

Let us look at one more thing. Suppose we have chosen d as some constant time sigma, we know that the distance between the symbols has to be chosen such that they are larger than the standard deviation of noise by some constant, then from this expression which we have already derived here substituting the value of d as alpha time sigma from this expression I can get this expression. So, d is just alpha time sigma. So, E s becomes alpha square times sigma square by 12 and this thing from this I can get 2 to the power 2 b is 1 plus this thing. So, what we are doing is, we can write this as from this expression we can get this expression.

And then it is straight forward I shift one to that side. So, I get 2 to the power 2 p is 1 plus 12 E s by alpha square sigma square and this is this expression taking the log on both side with the base 2. I will get 2 b log 2 is log 2 1 plus 12 E s by alpha square sigma square and log 2 2 is 1 and b 2. I shift to this side, I get this expression and this is very interesting expression.

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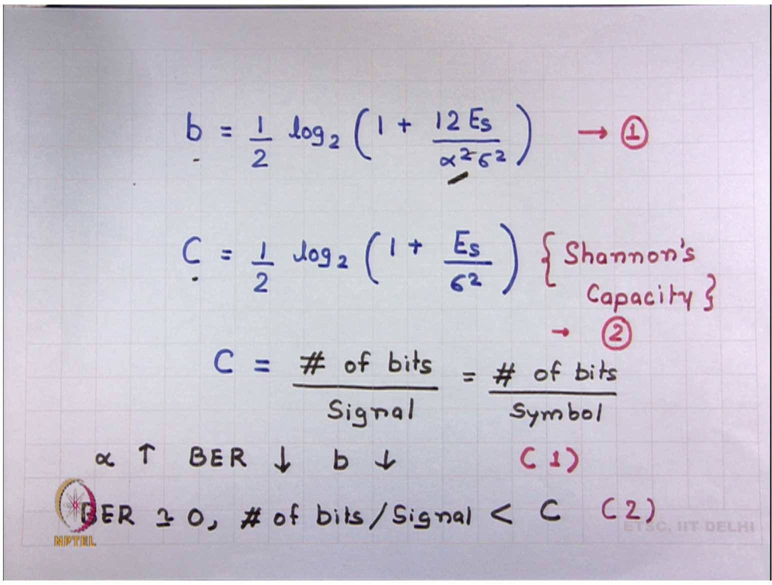

Once that we have derived here because it looks exactly like Shannon's Capacity Formula with some minor difference form is similar in Shannon's Capacity Formula. The c is number of bits that you can have per symbol or signal and here also in b is the number of bits per symbol; the minor differences in the interpretation of these two formulas.

This formula says that if you increase alpha, then you reduce the effect of noise because what are you doing is, you are increasing the distance between the two symbols. If you increase the distance between the two symbols, it is less slightly for the noise to create errors because noise amplitude is much than b much larger. So, by increasing the distance between the two symbols, we can decrease the effect of noise and thus when I increase alpha, I reduce my bit error rate, but then I also reduce my b. This is what this formula is saying.

So, if you want to reduce a bit error rate, the number of bits per signal also reduces whereas this formula says that if you are designing a modulation scheme or system via number of bits per signal is less than c, where c is the channel capacity and that is where we use the letter c, then you can make a bit error rate to go down to practically 0 of arbitrary, very small. This is what Shannon Capacity says.

So, if you make the number of bits per signal less than c, then you can achieve a bit error rate approximately of 0 or at least of a very very small value such that it does not matter.

So, this is all that we have in Pam. Remember the key things that we have set for PAM. The first thing that we have set as PAM is an example of a linear modulation scheme where b n or the symbols are restricted to be real symbols and once you are restricting yourself to real symbols, you can get to a signal constellation only by arranging your symbols on the real line. When you arrange your symbols on the real lines, what you want to make sure is that the minimum distance between the symbols is larger than d and where d is some constant times standard deviation of noise.

And then we have when we have caught to that constellation what happens is that the average energy per symbol grows with the square of M or it varies as M square and thus for large values of M average symbol energy becomes impractically large and thus we do not use PAM modulation scheme for pre large values of M the better ways to go to multi level modulation schemes, then PAM and we will see them shortly.

Then the second thing that we have covered is we have got a very neat expression of the number of bits that you can afford over a symbol that depends upon what alpha you choose, where alpha dictates the distance between the two symbols namely the distance between the two symbols is alpha time sigma where sigma is the standard deviation of noise. So, we increase alpha. What we do as we increase the distance between the symbols making the fact of noise go down thus reducing the bit error rate, but the penalty that we have to pay for that is we cannot have more bits per symbol and we will see that why that is important ok.

So, PAM is basically baseband. So, S t that we have is a complex baseband signal. It is a baseband signal, isn't it. Once you are restricting b n to real, then S t becomes a real baseband signal.

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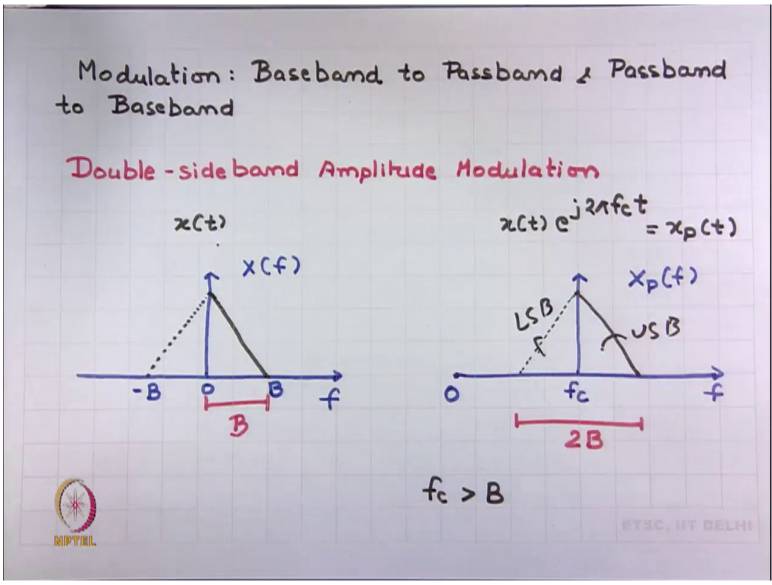

That is it, but as a baseband signal and what we want to do is, we want to go from baseband to passband and of course, at the demodulator you want to come from passband to baseband and there are various techniques by which you can go from baseband to passband. We have covered a lot about baseband to passband conversion and we will introduce few more ideas in this lecture about this baseband to passband conversion. Some of these ideas you might have studied in courses like in Analogue communication.

So, one idea to go from baseband to passband is to use what is known as Double Sideband Amplitude Modulation is the simplest way to go from baseband to passband and what you do is, you have a baseband signal and you multiply the baseband signal with this rotating complex exponential and you get a passband signal and you know that if the spectrum of the baseband signal is like this which is centered at around dc, the spectrum of the passband signal would be the same spectrum other than that. Now this is spectrum is shifted to the standard frequency of f c and we know that this f c the carrier frequency must be much larger than this B.

And we have looked into the definition of bandwidth several times and we know that by convention bandwidth is the range of positive frequencies present in the spectrum. So, in this case bandwidth is B because this is the range of positive frequencies and in this case the bandwidth is 2 B, all this spectrum is located on the positive side of the spectrum.

So, bandwidth in this baseband to passband conversion goes from B to 2B and that is bad.

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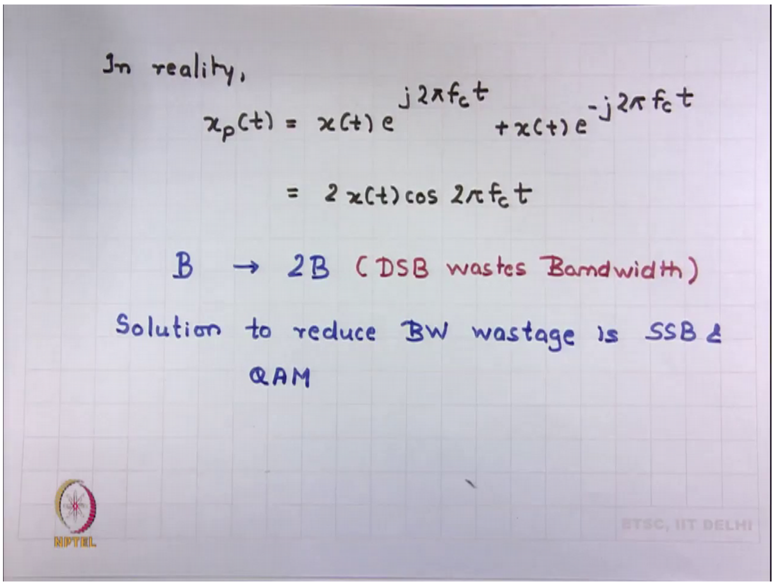

In reality what we do is, you cannot have a passband signal which is complex. This is a complex signal because of this thing. So, in reality what you want to have is a real signal. So, what you do is instead of multiplying x t with e to the power j 2 pi f c t, you also multiply x t with e to the power minus j 2 pi f c t and you add these two things up.

So, what you get is basically x t times cos 2 pi f c t, but that does not do anything to the wastage of bandwidth that we have discussed. Double side band based modulation schemes simply converts basement bandwidth of B to passband bandwidth of 2B and this is bad from communication resource point of view and thus what we want to do is, we want to reduce this bandwidth wastage by using either single sideband modulation scheme of quadrature amplitude modulation scheme, ok.

So, let us look at single sideband modulation scheme. So, this is not very important for this course. This is important for the courses like Analogue Modulation in Digital Communication. We never use single sideband modulation and thus I will not talk about this a lot, but just for completeness.

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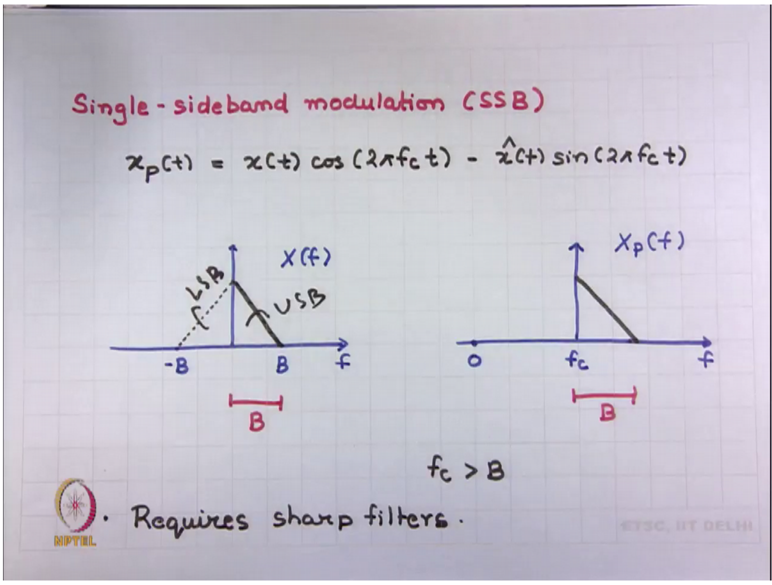

Let us see what is the single sideband modulation. You have taken the signal, multiply this with cos 2 pi f c t, you take the Hilbert transform of the signal and you multiply the Hilbert transform of the signal with sin 2 pi f c t and you produce a passband signal. This is what happens in single sideband modulation. Take signal, multiply with cos, take Hilbert transform of the signal, multiply with sin, subtract these two components and you get a passband signal. What it does is, it clips of one side of the spectrum. So, this is double sideband because it has upper sideband and a lower side band. When this signal is converted to a passband signal what happens is lower sideband gets clipped off.

So, what we end up with in passband is only with one sideband and this is the idea behind single sideband modulation. So, you only have a single side band. Remember in DSB double sideband we are having both sideband presence; upper sideband and lower sideband. In SSB we only have either upper or lower. In this picture we are just showing the upper sideband, but it might happen that you only have lower sideband and that is the name single sideband modulation scheme and if I look back to this double sideband modulation scheme, you will see that we have a dashed line and a solid line is simply says that the information in this part is redundant. Lower sideband has the same information as upper sideband because the signal is symmetric. Isn't it? If x t is real signal, the amplitude spectrum asymmetric and the phase spectrum is odd and thus from one side of the spectrum you can easily obtain the other sideband. So, from one sideband you can easily extract the other sideband.

So, in reality there is no need to transmit both sidebands lower as well as upper because it is redundant. One sideband is redundant and this is what we do in single sideband modulation. We clip off one sideband ok. We only have either lower or upper sideband and this is the idea behind single sideband modulation. Why it is not useful? This is not useful a lot because it requires some sharp filters to clip off one of the sideband and that is why we do not use this single sideband modulation technique.

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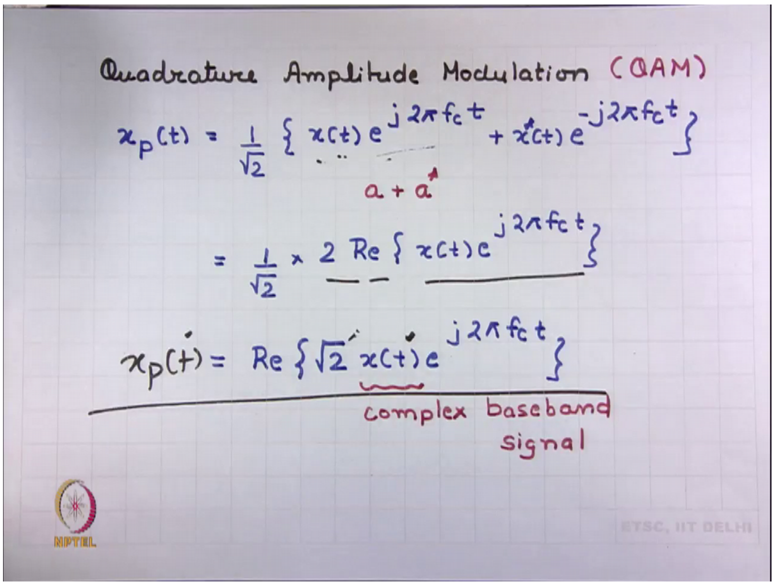

Let us now move to Quadrature Amplitude Modulation. In Quadrature Amplitude Modulation what we do is, we take in that signal. So, baseband signal multiplied by this rotating complex exponential and we add to this the conjugate of this thing. So, we have x t e to the power j 2 pi f c t and to this we add the conjugate of this. So, we get to a form like a plus a conjugate and we know what is a plus a conjugate is two times real part of this thing.

So, from this we get this. This thing is nothing, but two times real part of this quantity and this can be arranged as real part of root two times x t e to the power j 2 pi f c t and when we have said the passband to baseband conversion actually this is the equation that we used and because this is the most important way of conversion of baseband signal to passband signal because that is using the ideas behind quadrature amplitude modulation.

So, even though we have not mentioned it, but if your passband signal can be expressed like this, actually we are in the regime of quadrature amplitude modulation. This is the way of conversion of baseband to passband signal, and we have read a lot about this that how can you think about a complex baseband signal in terms of passband signal or the other way round how can you think about the passband signal in terms of complex baseband signal and we have already seen several equivalence between this complex baseband and passband signal, namely that the inner product of passband signals is the real part of inner product of baseband signals.

We have seen that the energy of the passband signal is the same as the energy of the complex baseband signal and to make that equivalence we have introduced this scaling factor of route 2. So, if you have forgotten about this complex baseband to passband conversion, I request you to look again at lecture number 19, 20 and 21 where we have dealt in detail about this equivalence of passband and baseband signal.

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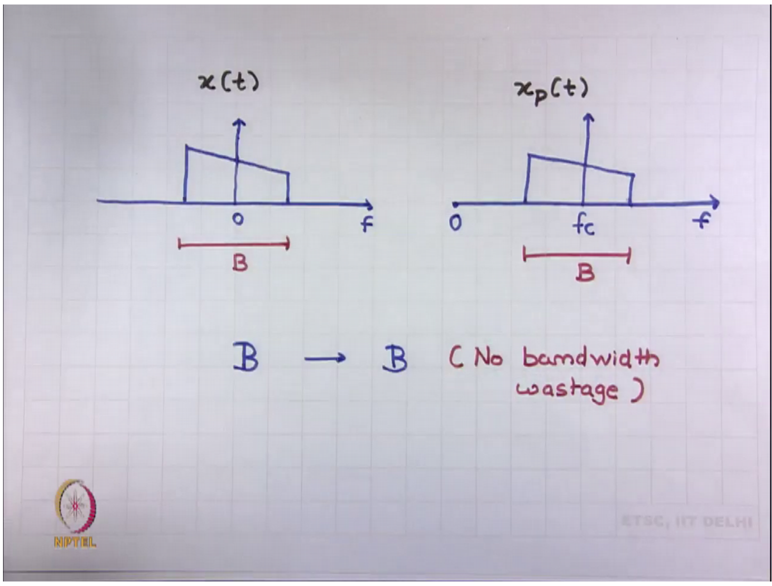

The one picture that is important in this context is if I have a complex baseband signal, it is complex. That is why the spectrum is not symmetric and because this spectrum is not symmetric, it has no redundant information present, right. It's upper sideband is completely different from lower sideband and this the information is present in both sidebands and thus for bandwidth we have also seen this the bandwidth of this signal should be considered as the total support of this spectrum including both positive and negative side of the spectrum.

And when this is converted to passband signal, there is no bandwidth inflation because bandwidth of this passband signal is also B. Thus if we have a baseband signal of bandwidth B, you can go to a passband signal also bandwidth B. There is no bandwidth wastage. We can also think about this in some other ways.

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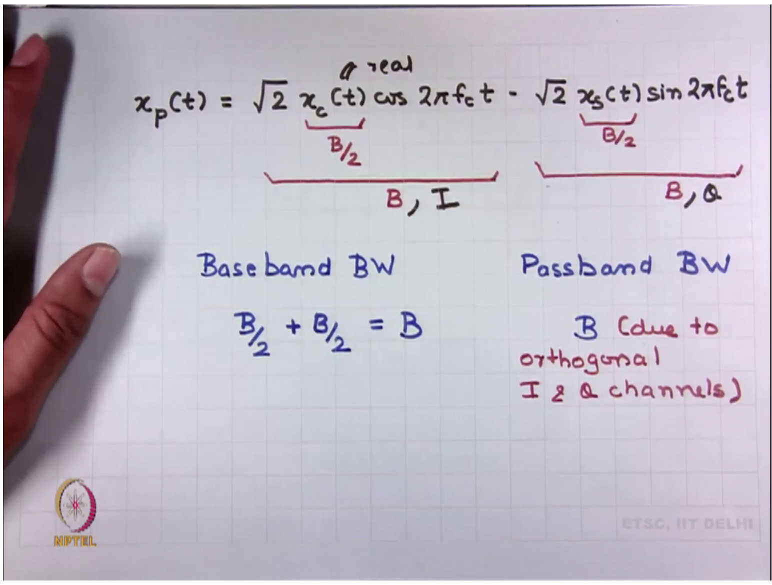

For example let us say if I have a signal x c t which is a baseband signal and let us take it to be a real signal actually x c t and x S t are signals and let us assume that the bandwidth of x c t is B by 2 bandwidth of x S t is also B by 2 bandwidth of this. I channel is B because there is a double sideband modulation. The bandwidth of this Q channel is also B for the same reason.

But the total bandwidth of the signal is also B. It does not add up. Why? Because I and Q are orthogonal components and thus the total bandwidth of x p t is B and the baseband bandwidth, the baseband components are not orthogonal is B, ok. So, this is one idea why we like to use quadrature amplitude modulation in short the quadrature amplitude modulation allows for two orthogonal channels, isn't it. And, thus you can have double the rate compared to the information in one channel and thus you reduce bandwidth wastage. So, there are many ways in which you can understand this idea of Quadrature Amplitude Modulation.

So, what is Quadrature Amplitude Modulation or sometimes it is known as QAM, QAM short. So, let us just contrast PAM with QAM.

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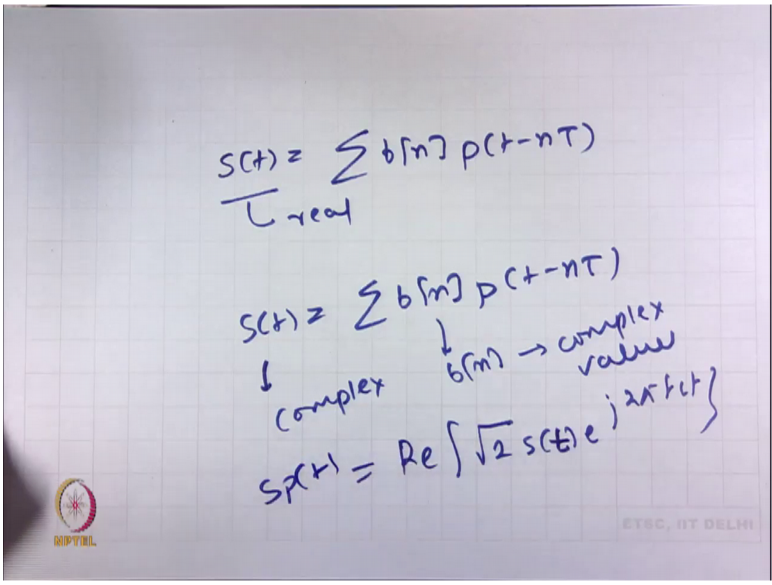

So, in PAM we have a signal which is restricted to be a real signal in QAM. We have a signal which is complex signal otherwise there is no use of QAM. If S t in the case of QAM is a real signal, then QAM is same as PAM ok. So, in QAM the signal S t has to be complex signal which can happen when this b n takes in the complex values.

And once you have a complex baseband signal what you can do is, you can use this relationship to construct passband signal from the complex baseband signal ok. This is expression we have seen before lecture 19, 20, 21. We have seen all about this. I channel, Q channel orthogonalities of I and Q channel. So, it would be a nice idea to revise them because actually we were discussing this passband to baseband conversion in the context of QAM.

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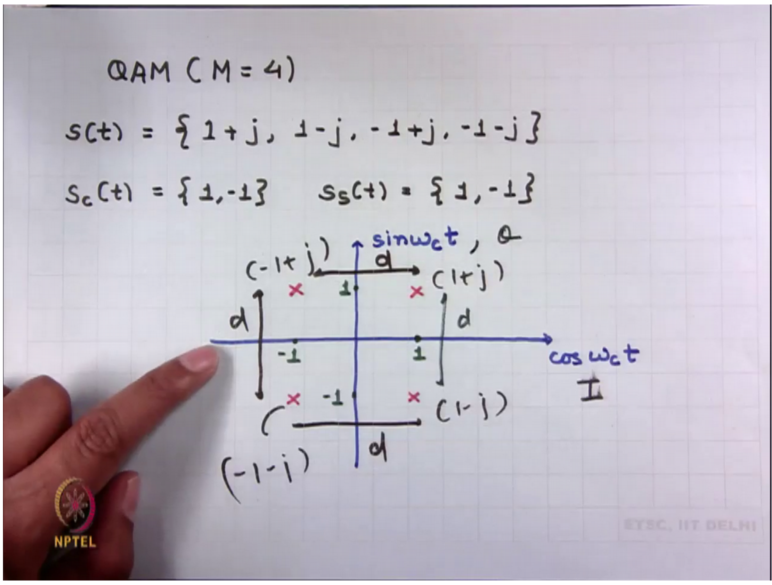

Let us look at the signal constellation diagram of QAM. In this case I have QAM with M equals to 4. That means, the multi levels in a QAM is 4. That means, I am having four symbols present. The distance between the two symbols has to be again some constant d. So, you have to make sure that the distance between any two symbols is at least d, ok. So, in this case we are making sure that the distance between any two symbols is at least d, right.

So, we have four symbols. One symbol can be 1 plus j. So, this is 1 plus j, this is 1 minus j, this is minus 1 plus j and this is minus 1 minus j. So, I am allowing for four complex numbers let me write that expression again.

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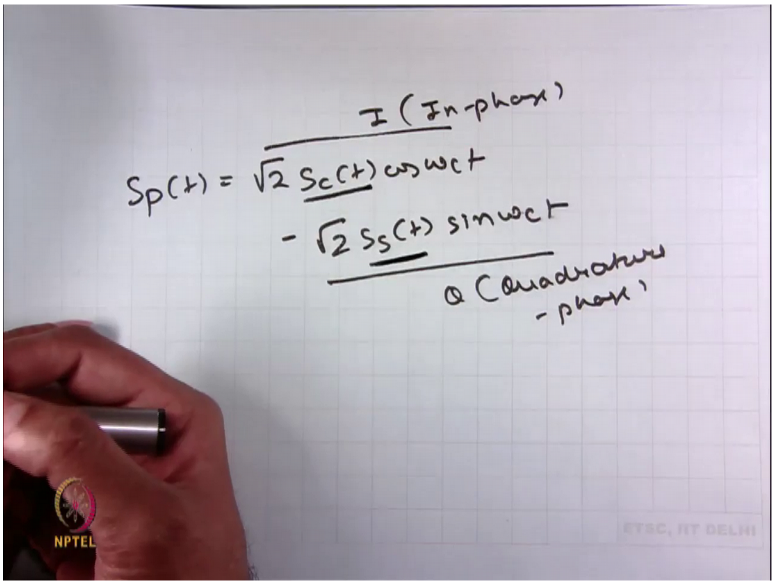

So, the passband signal is given by. So, this is I channel, Q channel or this is also known as in phase and quadrature phase component. Q stands for Quadrature and I stands for In-phase.

So, in the In-phase component I have S c t and in quadrature phase I have S s t. So, here I am drawing two orthogonal axis one along cos omega c t which corresponds to I channel and one along sin omega c t which corresponds to Q channel and then, this I channel or cos omega c t is modulated by S c t and S c t can take two amplitude levels 1 or minus 1. Similarly S s t can take two amplitude levels 1 or minus 1.

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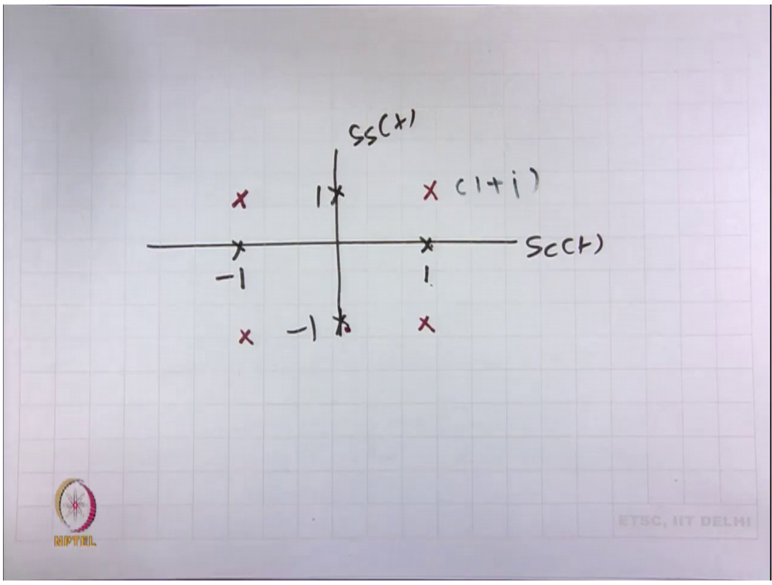

So, if I have S c t taking a two amplitude levels 1 and minus 1, so S c t is along this S s t can take two amplitude levels 1 and minus 1. So, what can happen if S c t takes in a value one S s t takes in a value one, so I have 1 plus j I get to this symbol. If this take this value this take this value, I get to this symbol if this take this value, this take this value, I get this and similarly if this S s t take this value S c t take this value, then I get to this symbol. So, in short I get to four amplitude levels.

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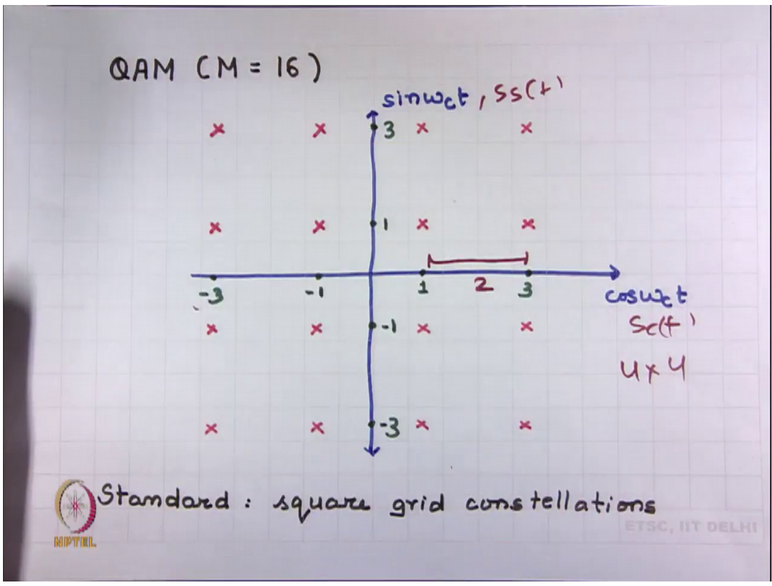

Now, I can look at QAM where M is 16. So, again you can think of this as S c t taking and the four amplitude levels. Again the distance between the amplitude levels is 2 and this distance is kept constant and similarly S s t can take in the four amplitude levels. Now based on the combination of the amplitude levels of S c t and S s t, you can have 16 amplitude level so, 4 times 4, 16 combinations.

So, you have the 16 symbols and the minimum distance between any two symbols is also same and constant in this case and this is the constellation structure of 16 QAM and if you look at this constellation grid, you can see that this is a square grid and this is the standard constellation structure of QAM, ok. So, QAM constellations basically are a square grid constellations, right.

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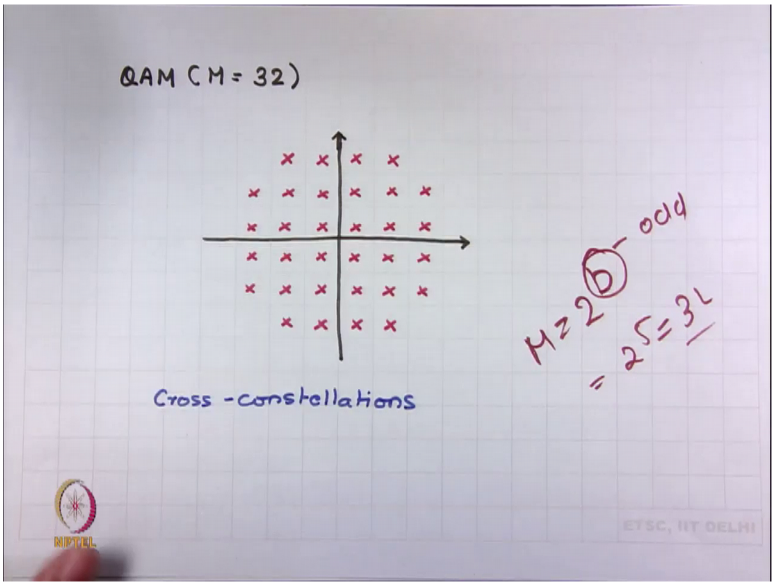

Somewhat non-standard constellation is where M is 2 to the power b and if this b is odd. For example, if b is 5 you get to 2 to the power 5 which is 32. So, in those kind of modulation schemes, you cannot have a square grid constellation. For a square grid constellation, you need to have b. As even if b is odd, we cannot have a square grid constellation and then, you have cross constellations something like this, ok. So, in QAM we can have square grid constellation which is the standard constellation for QAM and you can have other constellation structures for QAM like cross constellations and so on and so forth.

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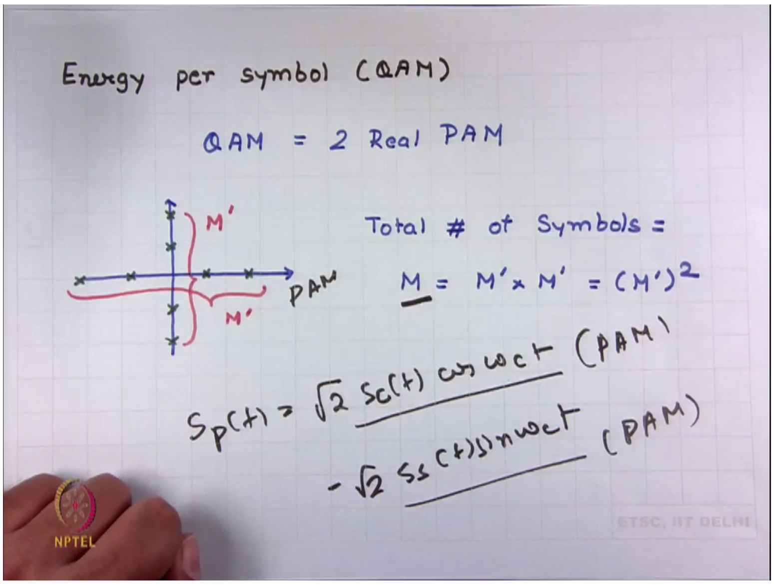

Let us now look at the energy per symbol in QAM and the way we want to do this is we want to interpret QAM as two real PAMs. So, if you look at the waveform for QAM, you can think of this as two PAM. So, there is a PAM here, this is the waveform corresponding to PAM and there is a PAM running in here isn't it. So, PAM which is double sideband modulated.

So, PAM is always real. So, I can think about the quadrature amplitude modulation as two real PAM and if in this PAM I have M dash symbols and in this PAM I have M dash symbols, then the total number of symbols in QAM is M dash times M dash which Is M. M is the total number symbols. So, M is nothing, but it is the square of M dash. That is one important idea that if you want to think quadrature amplitude modulation as two real pulse amplitude modulation systems.

If pulse amplitude modulation systems are built to support M dash symbols, then this QAM would have the total number of symbols as M dash times M dash. So, total number of combinations would be M dash times M dash which would decide the number of symbols in QAM.

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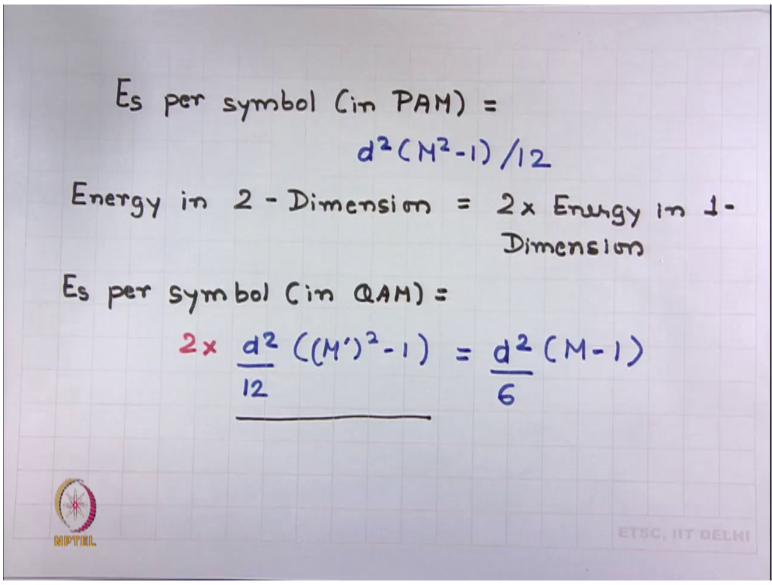

From this we can get energy per symbol in QAM where first considering what was the energy per symbol in PAM. Energy per symbol in PAM was d square times M square minus 1 divided by 12 because in QAM I am having two PAM systems energy. In QAM would be two times the energy in PAM, isn't it. Thus the energy per symbol in QAM would be 2 times because the 2 PAMs running in the energy of 1 PAM energy of 1 PAM would be given by this 1 PAM is supporting only M dash symbols.

So, energy of 1 PAM is this the number of symbols in PAM is M dash. Thus from this I can easily get the energy per symbol in QAM M dash square is M and the energy per symbol in QAM thus reduces to d square by 6 times M minus 1. So, we have easily obtained the energy per symbol in QAM. Let me do one exercise before we conclude this. Let me take a simple QAM modulation.

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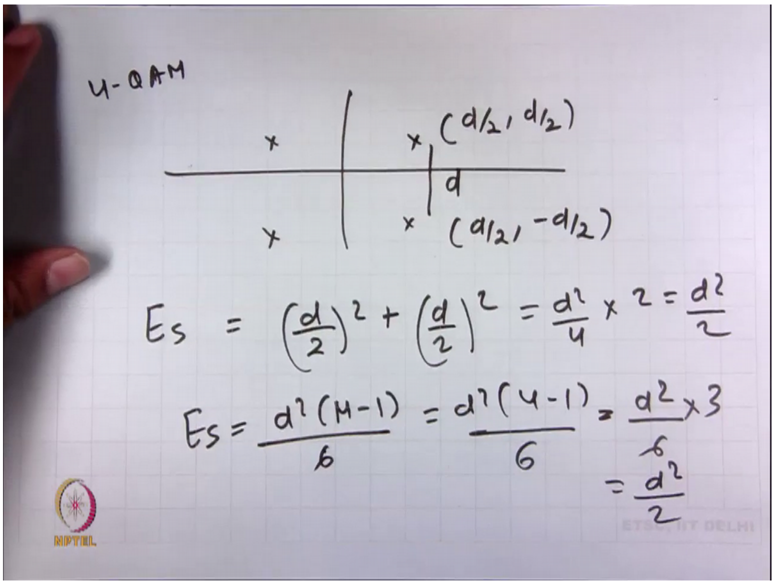

Let us say 4 QAM and let us take four symbols because this 4 QAM. Let me assume that the distance between the symbols is d. So, this symbol would be d by 2, d by two. This would be d by 2 minus d by 2 and similarly you can obtain the values corresponding to other symbols.

Energy of all these symbols must be same. So, if I calculate the energy of one symbol that would be same as the average symbol energy, all the symbols have the same energy. This is square grid constellation where M is 4. So, energy of one symbol is d by 2 square plus d by 2 square which is d square by four times 2 d square by 2. So, you can quickly estimate the energy in the QAM systems.

And if I use the formula which we derived which is d square M minus 1 by what was it 6. So, we get M as 4, we get d square by 2. That means, our formula is working and is correct and it can be used to estimate the energy per symbol in even higher modulation QAM systems where calculating energy will be not so simple as in this case. So, let me try to summarize this lecture by presenting the key ideas that we have developed today.

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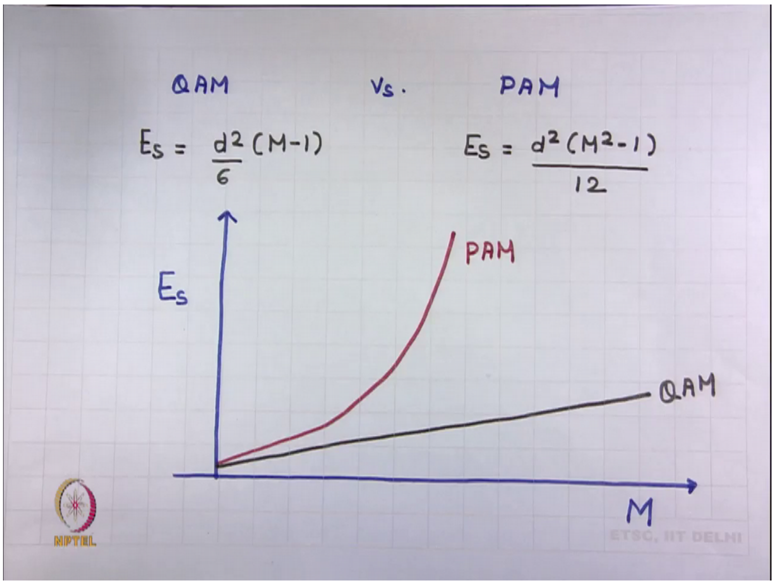

One thing that we have said about PAM and QAM is that PAM restrains itself to having real symbols. What happens to this PAM when we do this baseband to passband conversion using ideas like double sideband modulation scheme, there is a bandwidth inflation by a factor two. You can avoid this bandwidth inflation by using the modulation technique known as QAM where you allow the baseband signal to be a complex baseband signal and you modulate the In-phase component of the baseband signal with cos carrier and you modulate the quadrature phase component of this baseband signal with sin carrier that will avoid the bandwidth inflation, ok.

The second idea that we have developed is we have covered the average symbol energy that is required in PAM versus QAM and if you see this, it will be extremely clear why this PAM should not be used for large values of M. In PAM the average signal energy scales as M square where is in QAM the average signal energy scales as M. It linearly increases with M, but not as a square of M and this is the significant saving.

So, if you want to minimize the average symbol energy, you should use QAM where it scales only linearly with M, where is in PAM it increases as the square of M and this is not good. In next lecture, we will continue with these differences between QAM and PAM and we will introduce some other modulation schemes as well.

Thank you.