Principles of Digital Communications
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Lecture – 21
Modulation Complex Baseband Representation of Passband Signals (Part -3)

Good morning. So, welcome to the third lecture on Modulation.

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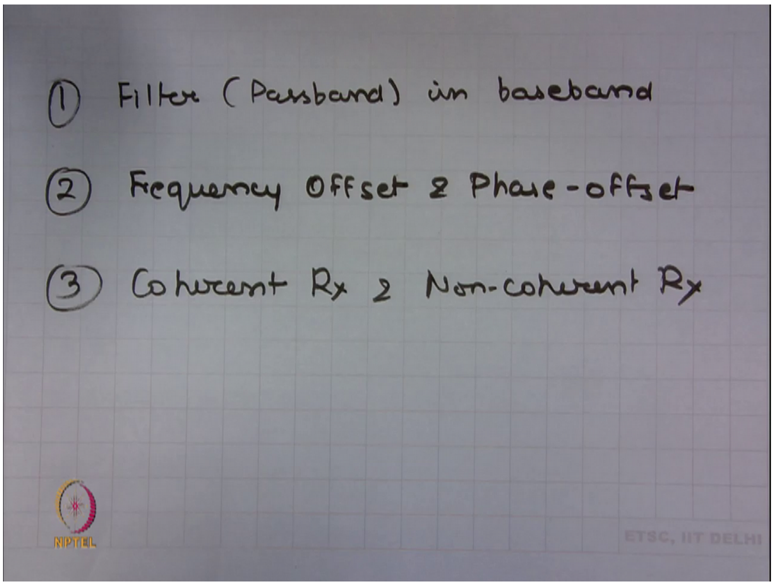

And last time when you finished our lecture we finished with these three points that we would like to do today. First we would think about how can we design a passband filter in baseband domain that is the first idea, that we would explore, the second thing that we would do is we would understand these frequency offset and phase offset effects and we would see how can we get rid of them in baseband domain ok, in basement processing. The third thing that we would do is we would like to think about the coherent receivers and non coherent receivers we have the entire unit on receivers. So, that would happen after modulation over here we would just talk about some ideas just getting started with receivers.

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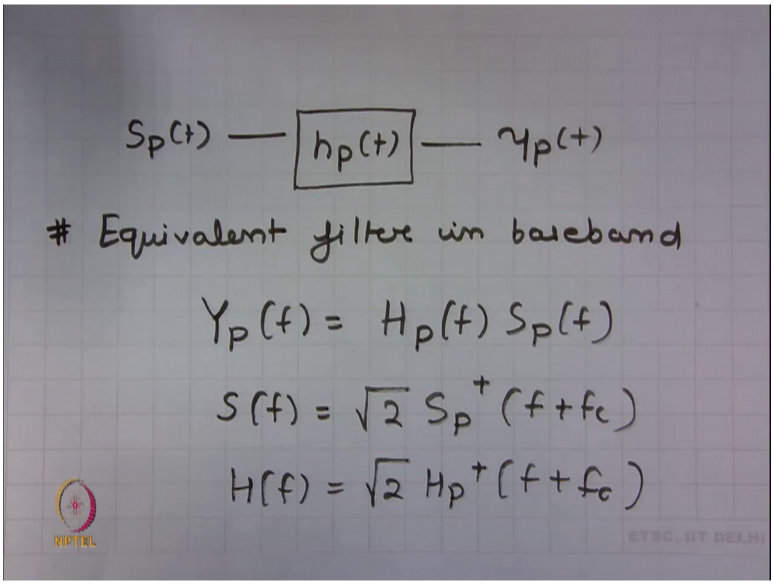

So, let us take the first problem had on and that problem is that if I have a passband signal S p t and this passband signal I want to pass it through a filter which is a passband filter h p t and what I get is Y p t which is a passband. Now I do not want to design this filter in passband domain right designing this would involve making analog demodulators or modulators and this would be complicated right and we would like to do everything in baseband domain. So, we would like also to make this filter in the baseband domain and today we will see how can we do that? So, the idea is equivalent filter in basement ok.

So, implementing a passband filter in baseband. So, if we look at this you know from basic again the cos and signals and systems then the spectrum output is spectrum Y p f is nothing but it is the spectrum of this filter let me call this as H p f and let us say that the spectrum of this signal S p t is S p f right. So, this is true in general if this filter is an LDI filter which we have assumed and this is a linear time invariant filter then I can write this Y p f is the product of the filter response and this signals the spectrum is very basic ok.

Now, we also know because we have derived equivalence between the frequency response of the baseband signal and the passband frequency response. So, for example, we have said that S f would be nothing, but it would be root 2 times S p plus f plus fc, this is a relationship that we derived in the last lecture. And similarly H f can be written as root 2 times H p plus f plus fc finally, what we would get is Y f would be root 2 times Y p plus f plus fc.

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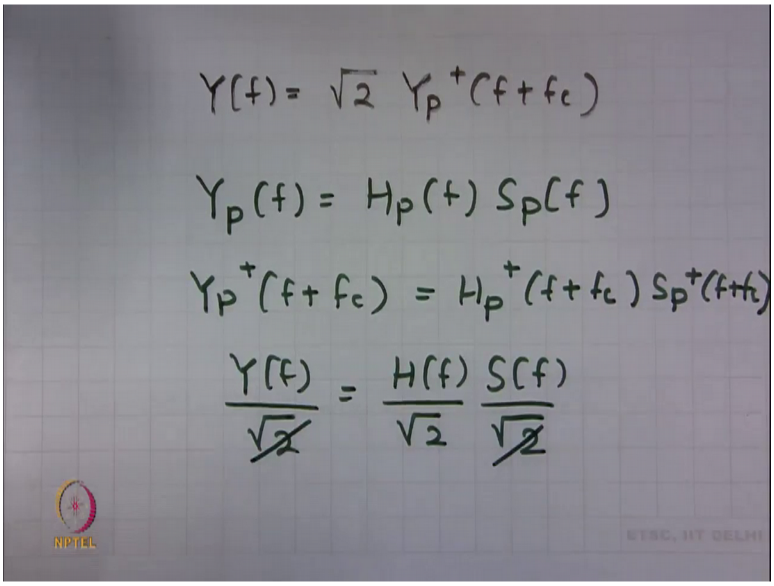

Now so, we know that Y p f is H p f times S p f. So, from this we can see that Y p plus f plus fc would be H p plus f plus fc times S p plus f plus fc. And now using the equivalence between the spectrum of a baseband signal and a passband signal I can write this as Y f by root 2 this is H f by root 2, this is S f by root 2. So, what I end up with is Y f is H f S f divided by root 2.

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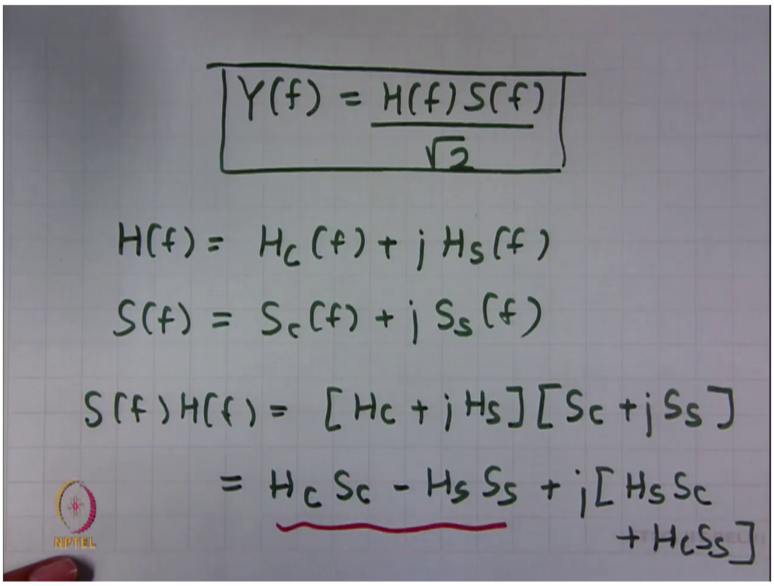

So, what we learn from this is if you have a baseband signal two baseband signals you can do this operation in a basement domain what you end up with is the spectrum of the resultant baseband signal right. So, Y f is the spectrum of the baseband signal of Y p t ok, let us see more about this.

So, H f we can write that this is nothing but this would be the a spectrum of H c f plus H s f. So, remember that is signal when we are talking about the baseband signal it is composed of the 2 parts cosine parts in the sine part and the spectrum of this H f would be nothing, but the spectrum of the cosine part in a spectrum of the sine part. Similarly S f I can write as S c f plus j times S s f, if I want to multiply these two things for simplicity again I ignore this f what I would have is H c plus j H s multiplied by S c plus j S s I have omitted the f part and then I multiply. So, this would be H c S c minus H s S s. So, plugging in the real parts plus j times H s S c, then I would have a term from there which is H c S s.

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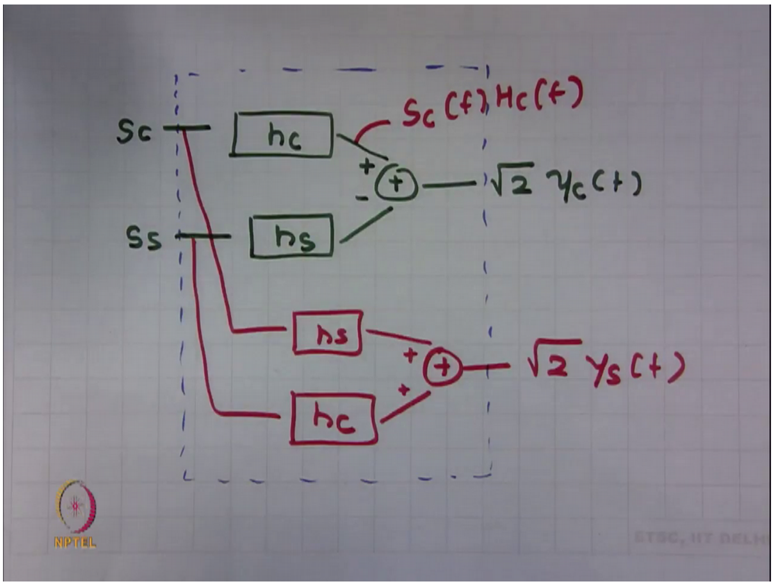

So, what this means is I can take S c you can start with S c I multiplied this at S c with H c, let me take a filter whose impulse responses h c t. So, let me have S s multiplied by h s and then I add these two parts this I had with positive, this I had with negative, and what I get is the root 2 times Y c t, if you like you can put t everywhere. So, what I am saying, if you look at this expression this would give me the cosine part of Y t ok, this when I multiply H f with S f I get Y f. So, when I do this corresponds to the a spectrum of the cosine part of Y f right, so, its inverse transform is Y c t. Now you may wonder that here I have been talking about the spectrum and here we are writing with the impulse response and the reason is very simple. So, if you look at this the output is S c f into H c f.

So, once you want to multiply S c with H c where these correspond to the spectrum this is exactly what we are doing right. So, when you multiply the spectrum what you need to do is you just have to pass the signal through a system whose impulse response is the inverse for a transform of H c f and you get the resultant spectrum at this point. So, this would give me route 2 Y c t. Similarly, what I can do is now I have to make this part. So, I have to multiply S c with H s. So, let me do this and I have to multiply s s with h c add these two things up, what I get is root 2 times Y s t ok.

So, basically what I do is I start with the baseband signal, if I have baseband signal and in this baseband signal what I do is I pass these baseband signals this cosine and sine parts of this baseband signal through this filter; this filter is completely a baseband domain. All these things all these impulse response are corresponding to baseband signals, what I end up with is sine and cosine part of Y t and this sine and cosine part of this Y t would correspond to a system to a passband system with which we have started with which would have been implemented like this. So, instead of implementing this filter h p t in passband domain we can implement this filter in the baseband domain.

So, that is that gives us some idea about what we have to do in baseband and how can we implement some of the filters in the passband and baseband. Let us now look at another interesting problem of frequency and phase offsets.

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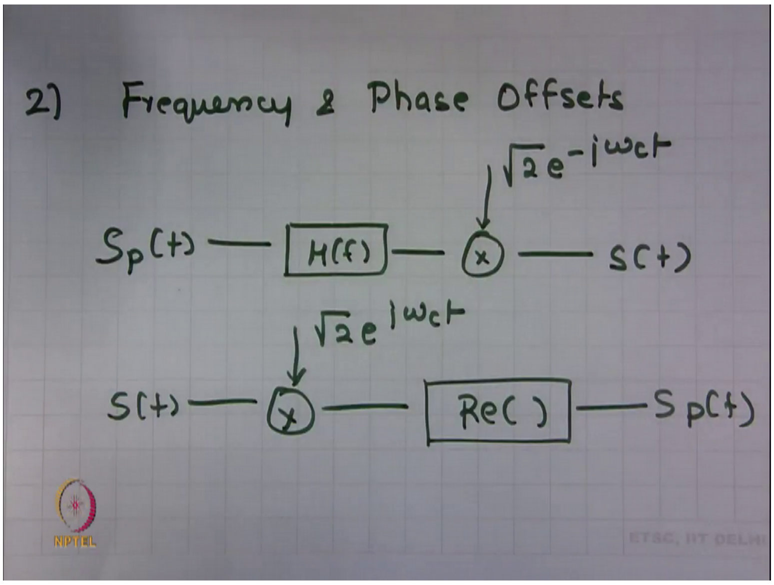

2nd important example that deals with frequency and phase offsets. So, let us look at this picture that we have seen several time which says that if you have S p t you pass it through Hilbert filter you multiply this thing with root 2 e to the power minus j omega ct and we have said you get S t. And if you have noticed that this rotating complex exponential should be compliant or should be synchronized to the rotating complex exponential that you used at the transmitter.

So, at this transmitter remember that we had S t which was multiplied by root 2 e to the power j omega ct, then we pass it through a system which takes the real part and it was giving us S p t. So, this was at the transmitter and this is at the receiver. Now if this e to the power j omega ct and this e to the power minus j omega ct does not have same frequencies for example, then what you would end up with is a different signal you would not recover S t ok.

So, when I say that I can recover S t, I assume that I have a signal available which has a frequency omega c and it is matched to the frequency of the modulator that I was using at the transmitter; in general that is not the case; because this modulator and this modulator or demodulator are at different locations and normally they would have some frequency in differences ok. So, we are trying to investigate now the situation in which the frequency of this carrier is different from the frequency of this carrier.

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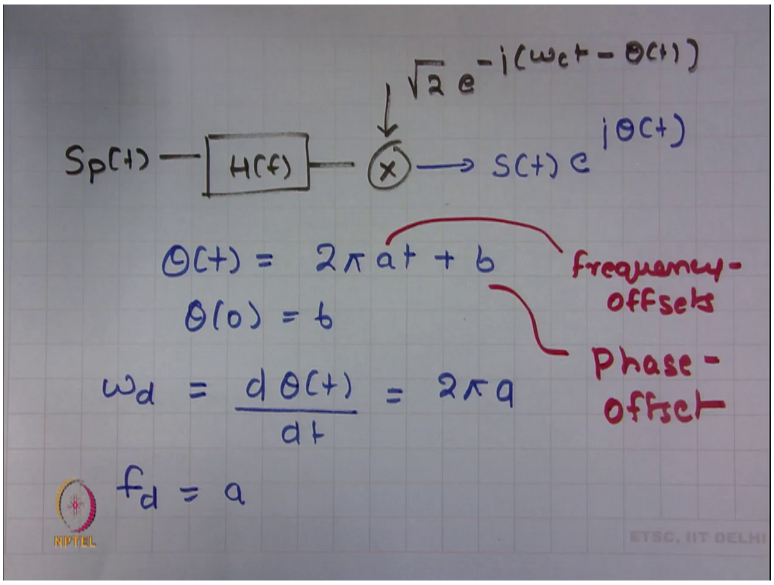

So, let us now assume that we have a situation in which we have S p t we pass it through a Hilbert filter that is good, but and this multiplier what we have is e to the power minus j omega ct minus theta t. So, this theta t would take care of any differences in the frequency and phase of this carrier with respect to the carrier that we used at the transmitter. And if you do some maths you can quickly see that the output that we will get in this case is S t e to the power j theta t. So, what we are getting is not S t, but S t multiplied by this complex exponential ok, let us see what can be a typical example of theta t; let us take theta t to be 2 pi a t plus b.

So, the phase varies linearly with time and there is some initialization value right. So, when you put t equal to 0 then you have b, otherwise the phase varies linearly with time ok.

Now if you look at this expression; if I want to find what is the frequency deviation what frequency deviation this would create? So, the frequency deviation of course, angular frequency deviation would be d theta t dt which in this case would be 2 pi a. If I am interested in the frequency deviation in hertz this is a radian per second I would get a. So, this a constant corresponds to the frequency deviation or frequency offset. So, a corresponds to frequency offsets and b corresponds to the phase offset. So, now, because of this theta t the phase of the carrier is different from the carrier which I used at the transmitter and also the frequency of the carrier is different and one simple expression like this captures both these effects. Now, let us see what happens because of these phase and frequency offsets.

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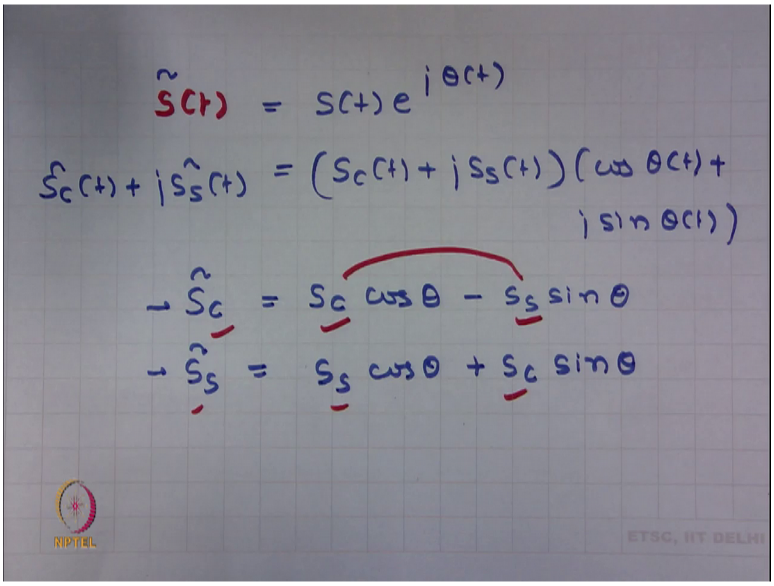

So, what I am receiving is S t let me call as S t tilde, because I am receiving something different from actually what was transmitted is S t e to the power j theta t I can write this as S c t plus j S s t times cos theta t plus j sin theta t and I can write this as S c tilde t plus j S s t tilde it is different from what was transmitted. So, in one go without any difficulty you can write that S c tilde is S c cos theta I am omitting t for again for typographical reasons.

So, I have need to multiply this with this and I can multiply this with this and as S s tilde would be S s cos theta then I would have S c sin theta. Now so, we end up with is we are receiving something different from what was transmitted? We transmitted S c and S s sine and cosine parts right, what we are receiving a different sine in cosine parts.

Now, how to deal with this situation? When you want to deal with this situation what you do is you assume some synchronization and the receiver. So, synchronization can mean different things depending upon the context here what I mean is the transmitter can solve this problem, he can transmit a known value of cosine and sine part of the signal. So, he can transmit the known value of S c and S s in certain duration. So, receiver knows when he is receiving the known value of S c and S s. So, when the receiver receives S c and S s tilde instead of S c and S s but he was expecting S c and S s. So, what he knows is this and this and what is receiving this in this. So, based on this what he can do is he can calculate theta ok and he can adjust the phase and the frequency of the carrier by knowing this value of theta that is one way. The second way and the way it is done is that instead of shifting the frequency and phase of the carrier it is difficult to do you take care of this effect in the baseband domain.

So, you do not touch your carrier that you are using in the receiver, but rather you are taking care of this effect at the signal processing level at the baseband level at the computer level ok. So, let us try to see how can we get rid of this at the baseband level.

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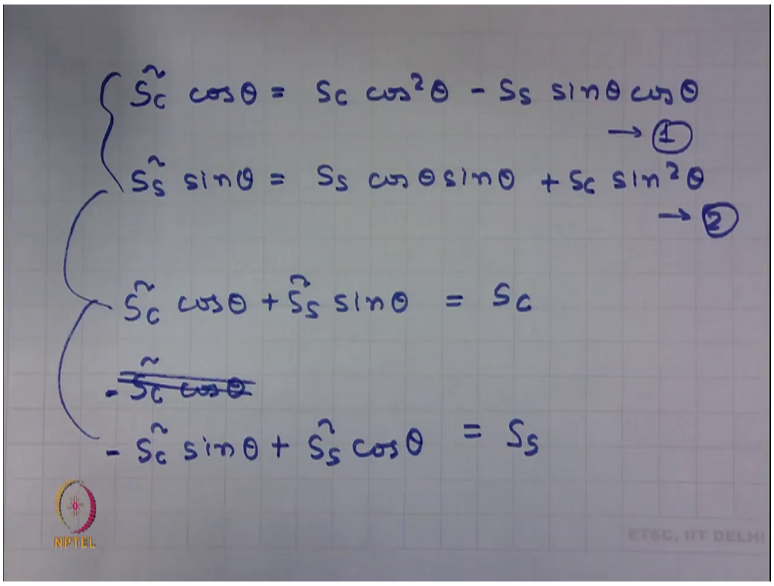

So, let me multiply S c tilde with cos theta I get S c cos square theta minus S s sin theta cos theta that let us equation number one I can multiply S s tilde with sin theta I get S s cos theta, sin theta plus S c sin square theta let us equation number 2 I can add these 2 equations I get S c tilde cos theta plus S s tilde sin theta and while adding these 2 up this term will disappear what you end up with is S c.

So, based on what you are receiving? You are receiving different cosine sine part you already know what the value of theta is you have obtained this by doing some process of synchronization. A process of synchronization is simple receiver knows what that is expected to get transmitter sends the same signal in the desired period and based on the received signal it is anticipating the distortion and it can correct this as it would do now. So, we know that after the phase of synchronization this value of theta has been obtained at the receiver, now I have solved for S c and if I want to solve for S s this would be simple you can work this out this would be S c tilde cos theta with a minus sorry.

So, you can work out the value of S s yourself from these two equations. So, when we want S s it would be minus S c tilde sin theta plus S s tilde cos theta. So, using these two equations you can easily derive these two equations. So, we can find out the value of S c and S s the desired values of S c and S s from the received values of S c and S s and from the known value of theta, how would the receive a structure would look like? So, receiver structure you can use this 2 expressions.

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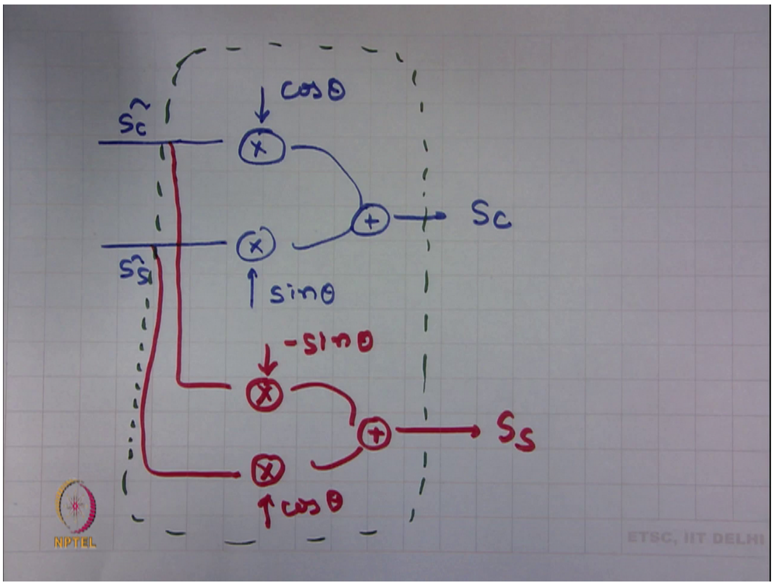

So, you take S c tilde whatever you are receiving you multiply this S c tilde with cos theta and you take S s tilde you multiply this with sin theta you add these two things up what you get is S c, what you can do then is you take S c tilde you multiply this with minus sin theta you take this multiply this with cos theta and you add these two things up what you get is S s. So, this is a kind of structure that you need to use to obtain the desired values of S c and S s from S c tilde and S s tilde by compensating the values of frequency and phase offset.

So, this you can implement a baseband domain without touching up your carrier which was working at the hospital, we have seen 2 interesting problem we are just trying to motivate you that certain complicated operations and the passband can be done fairly easily and the baseband I am saying fairly easily because processing comes at no cost right is it digital signal processing has become very cheap right. So, you are just changing or manipulating some numbers in a signal processor. So, that is almost no cost. So, all these complications that we had to deal with in analog domain before, now all these problems can be easily transported to baseband domain right and this is how most of the things are being handled today.

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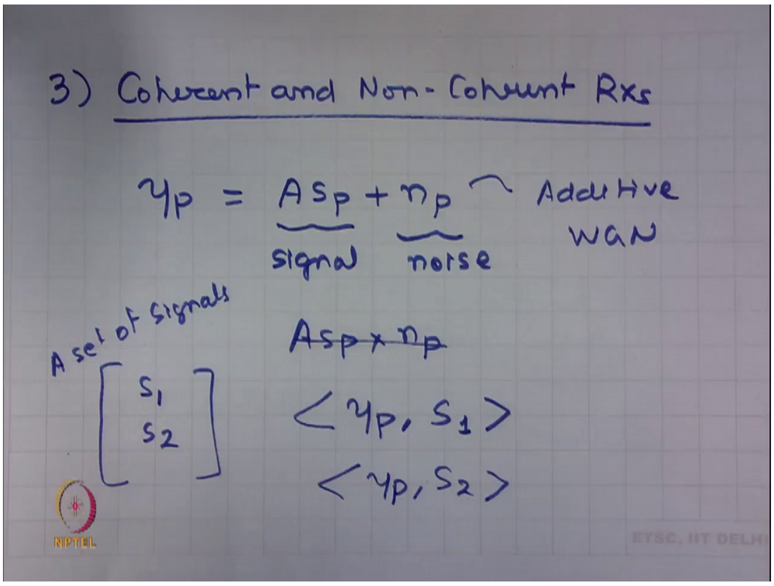

The third important point that we would like to talk about is coherent and non coherent receivers and as I have said we will like to talk about this coherent and non coherent receivers in great detail later on, but at this point it is really instructive to get some flavor of what are these quarantine non coherent receivers. So, the idea is very simple. So, what we are saying is we are receiving let us say we are receiving some A s p plus n p. So, this is the passband signal. So, we are receiving something that was transmitted that is sp, we have multiplied it with some constant A and to this what we add is a noise as you can see that noise adds to the signal. So, this is a signal and this is the noise.

So, what you would receive is signal plus noise; noise is adding to the signal and hence this is known as additive noise right. If noise would have multiplied with a signal we would have called it as multiplicative noise, but noise normally we assume to be additive and not to be multiplicative. Also we have seen in the lectures in the noise that the noises white Gaussian noise, that is the noise that we like to use for modeling the physical noise processes and hence this is also known as additive white Gaussian noise. Anyway at this point I am just trying to introduce you to a model in which we are using a signal and we have some noise addition and this is what you are receiving. Now in digital communication we normally have a set of signals right for example, let me take just 2 signals.

So, let me assume that I have a binary set. So, I have 2 waveforms that I can transmit either I can transmit S 1 t or I can transmit S 2 t and at the receiver my job would be to identify which waveform I have transmitted, whether I have transmitted S 1 t or S 2 t. So, this is the job of the receiver. So, what receiver does is whatever it receives it takes the inner product with this ok. So, let us say it receives Y p and it takes the inner product with a signal S 1 it receives Y p and it takes the inner product with signal S 2. So, this is normally what a receiver does.

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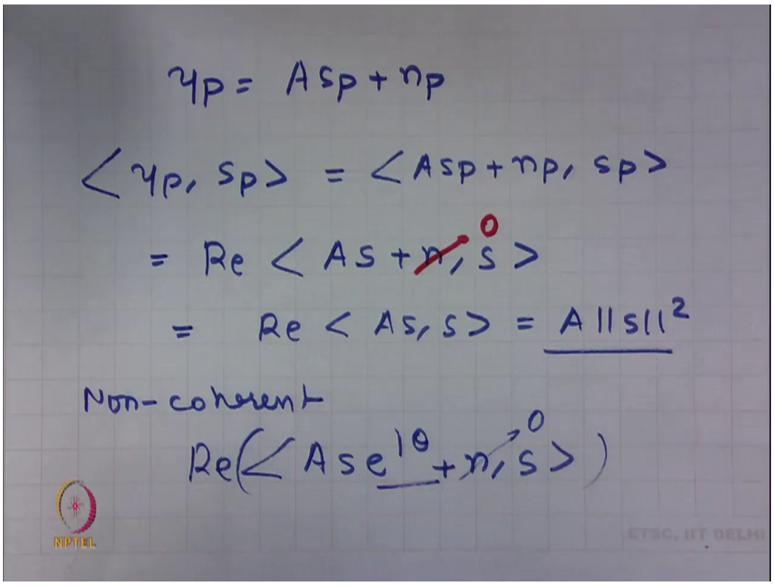

So, let me assume very simple case that whatever I am receiving let us assume it to be A s p plus n p and let us without worrying about whether that is an optimal thing to do or not let us just take the inner product of Y p and s p because s p was transmitted and let us see if the receiver does the inner product with the signal that was transmitted what do we get.

So this is A s p plus n p with s p of course, because we have been talking about a lot of baseband you know that this inner product would be calculated in the baseband domain not in passband domain. So, how would we do it in baseband? So, this is nothing, but you have to take real part of A s plus n and s ok, where s represents the complex baseband equivalent of the passband signal, now if you do this what you get is let us now assume noise to be 0. So, let me put it to 0 for simplicity what do we get is real part of As with s and this is nothing, but A norm s square right, this we have already seen. So, now, what you get at the receiver is A times energy of s p.

So, at the receiver you would see it at most of the detection happens based on the energy of the received signal; so, this looks fairly ok. Now let us see what happens if I have a non coherent receiver instead of a coherent receiver. So, what would change? The thing that would change is instead of having A s in non coherent receiver what we would have is A s e to the power j theta plus n and s, why do we have this e to the power j theta?

Because the basic difference between coherent and non coherent receiver is; that we assume that the coherent receiver has the complete information about the frequency and phase of the carrier used at the transmitter or it can synchronize itself to the phase and frequency of the carrier or it can use a structure like this to get rid of this frequency and phase offset.

So, there are no errors introduced because of a mismatch in the frequency and phase of the carrier used at the receiver and the frequency in phase of the carrier used at the transmitter. So, a coherent receiver would employ a structure like this and when it employs a structure like this and the coherent receiver we can assume that we are having A s instead of A s e to the power j theta, because that part has been taken care of.

A non coherent receiver what we normally assume is that non coherent receiver does not use such a complicated structure. It simplifies the design by not using it, but then it has to pay some penalty right and this penalty we will see when we will talk about the performance of non coherent receivers at this point let us just see what happens if you do or you compute this inner product for a non coherent receiver of course, we need to have the real part as before.

So, if you do this again we can assume noise as 0, noise is no problem because we have not discussed the interaction of noise and so on so forth with the signals we will do this in receiver I do not want to bring in that complexity safely I am assuming that there is no noise.

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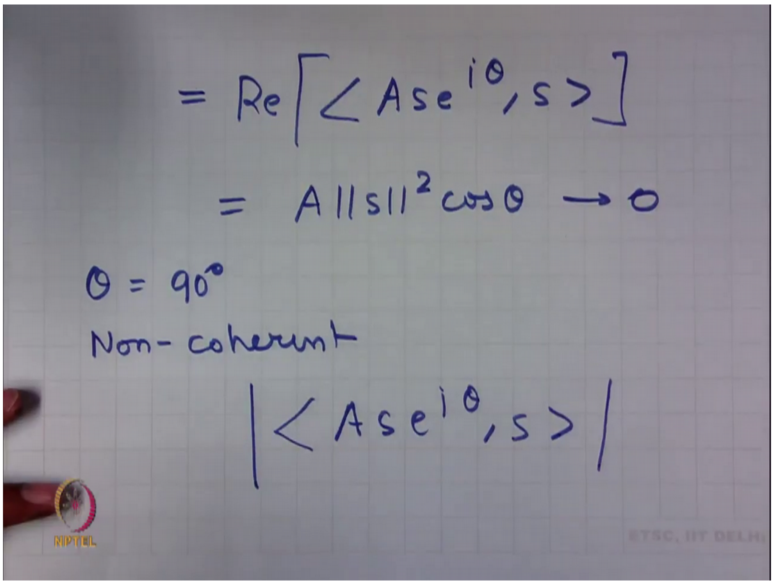

So, what this inner product operation boils down to its real part of inner product of a A s e to the power j theta and s. So, if you do this what you get is A norm of s square cos theta and now there is a trouble, because theta; theta can be anything right and if I assume theta to be 90 degree cos theta is 0; that means, this quantity goes down to 0. That means, even if there is no noise in the receiver because I am using a non coherent receiver I can get a flat 0 and getting a flat 0 means I would have lot of errors in this design is not going to work.

So, when you are having a non coherent receiver it is not a good idea to take the real part of these complex baseband signals instead what you should do is, you should take. So, in non coherent receivers what should we do is we should take the magnitude and that is important we should take the magnitude of this inner product once you do this its easy.

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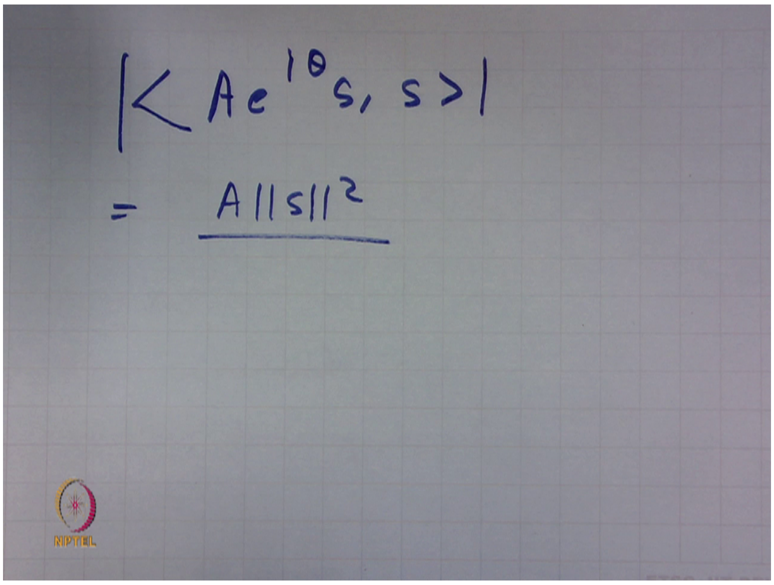

If you take the magnitude of A e j theta s with s what you would get is this is a constant it can be pulled out taking the magnitude of this would be 1. So, what we end up with is A norm s square same as what we got in the case of the coherent receivers, this is little bit misleading. Because we are claiming that the performance of non coherent receivers is same as a coherent receiver that is not the case this has turned out to be the same because we have not taken into account the non-linear interaction of noise with signal that happens in the non coherent receivers.

We will talk about all these issues when we discuss about the non coherent receivers. At this point the main message is that whether you should take the real part of the inner product or whether you should take the magnitude of the inner product depends upon the receiver design and both these operations are used. We take the real part of the inner product of the complex baseband signals where we are doing or we are in the regime of coherent receivers and we have to take the magnitude of the inner product of the complex baseband signals when we are talking about the non coherent receivers ok.

So, with this we have completed our discussion on the relationships between the complex baseband and passband signals and this is really an important equivalence that you should understand because the approach that we would take in this course is we will only talk about the modulation and demodulation in the baseband domain. We would not talk about the modulation and demodulation at the passband domain though there are certain books which attempted we would not do that. Because once we have understood everything about the basement modulation and demodulation and once we have understood the equivalence between passband and baseband that exercises little bit trivial and redundant.

So, what we have done and we have emphasized upon this equivalence is also because of this reason, because we will not discuss the passband modulation and demodulation schemes. And that is also the case means all this modulation and demodulation happens at the baseband only in the practical communication systems and what you then do is you just multiply it with this carrier and you go to the passband domain. And from the passband you can also come back to baseband domain by using this passband to baseband converter ok.

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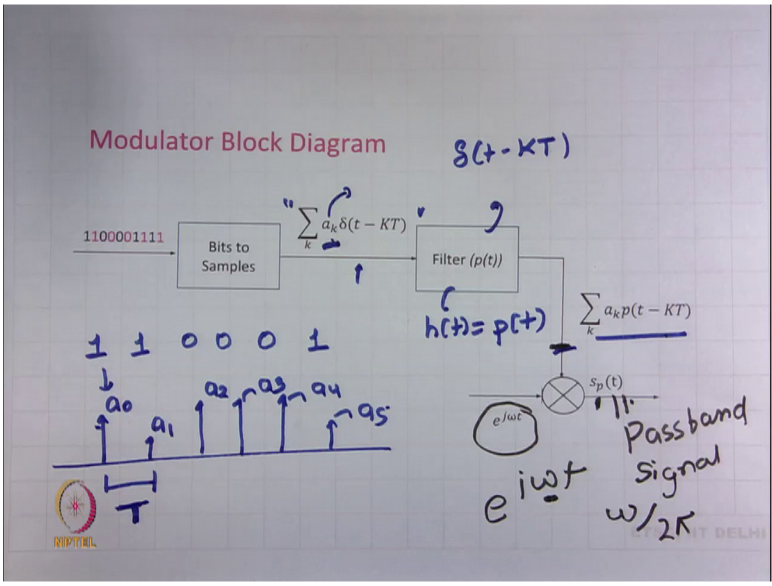

So, let us see what we have discussed so far. So, up to now we have clearly understood the relationship between this baseband signal and this passband signal. And now from today onwards we will be trying to investigate what happens when you convert a binary sequence to these samples and how is the modulator impacted by the impulse response of this filter.

So, first thing that we will start investigating today is the influence of this a ks and this p t on the bandwidth occupancy of the modulation scheme; that means, how you choose the weights for this binary sequence and how do you choose the impulse response of the filter. One of the important parameter in digital communication system is the bandwidth occupancy of the modulation scheme, because the operators have to pay for that right. So, what we want to do is we have to minimize we want to minimize the bandwidth requirement for the given transmission of information and we will see that the bandwidth occupancy is a very strong function of how you choose this a ks and how you choose this p t ok?

So, this is what we are up to in this lecture and over next couple of lectures to study about this spectral description of sources. Remember that the binary sequence that is available to this modulator is a random sequence and it is a stationary sequence. So, the ideas that you simply take the Fourier transform of this to get the spectrum and the bandwidth occupancy of the signal will not be useful. Because as we will see that this sequence if it is a stationary might have infinite energy and thus we cannot take the Fourier transform of this random stationary sequence easily and as we have to do something else to evaluate the spectral description of this random source.

So, we would be using the ideas that we develop in random processes we will be thinking about autocorrelation function and so on so forth. So, this is exactly what we are up for in this second unit that we have started in modulation trying to study about the effect of a ks and p t on a spectral description of sources.

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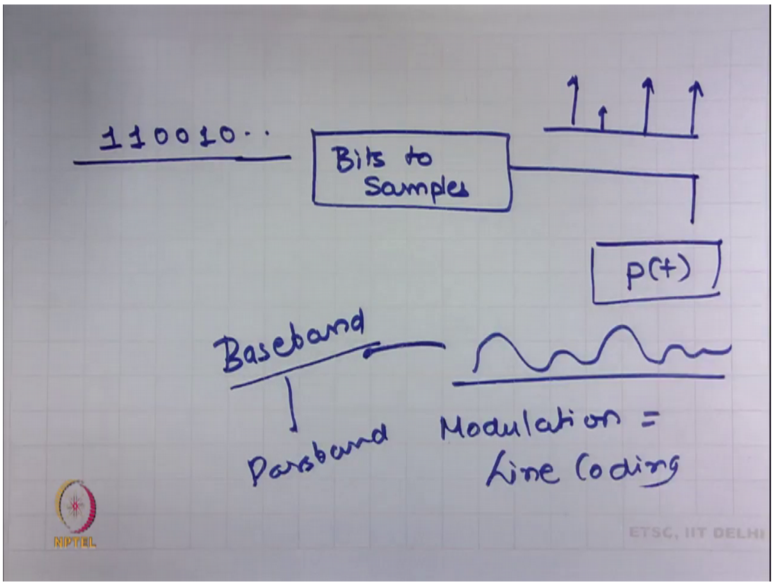

One more point that you have to be careful in the use of definitions again. So, if you have this binary sequence whatever this binary sequence is as we have already said that the first step is to go from this bits to samples.

So, at this point we have a weighted train of impulses and then from this weighted train of impulse when you pass this through a filter with impulse response p t you get an analog waveform. And as I have said that this is a baseband waveform and we have already said that we can study everything in baseband domain without going to passband, because you can go back and forth between baseband and passband without losing out information and this conversion is straightforward and easy thus if we confine our self to the discussion in the baseband domain it would be sufficient.

Converting a basement signal to passband signal has already been covered by us and thus we will confine ourself in rest of the modulation that we discuss to baseband signals, we would stop ourself at this point because this to this has been understood thus we will study about modulation only in baseband domain from now onwards.

And the second point is that if you have a baseband signal available at the output of the modulator. So, modulation is referred to as line coding because now the job of a modulator is just to produce the waveforms which are suitable for baseband channels. So, modulator is same thing as a line coder if we are just confining ourself to this step.

So, instead of saying that you have a modulator we can also say this as the line coder. So, what is the line coder? It just have 2 elements first there is a bits to sample converter and then there is a filter ok and this is a line coder it produces baseband analog waveforms which are suitable for transmission over baseband channels. So, line coder is a part of a modulator ok. So, sometimes we also call this as line coding schemes ok.

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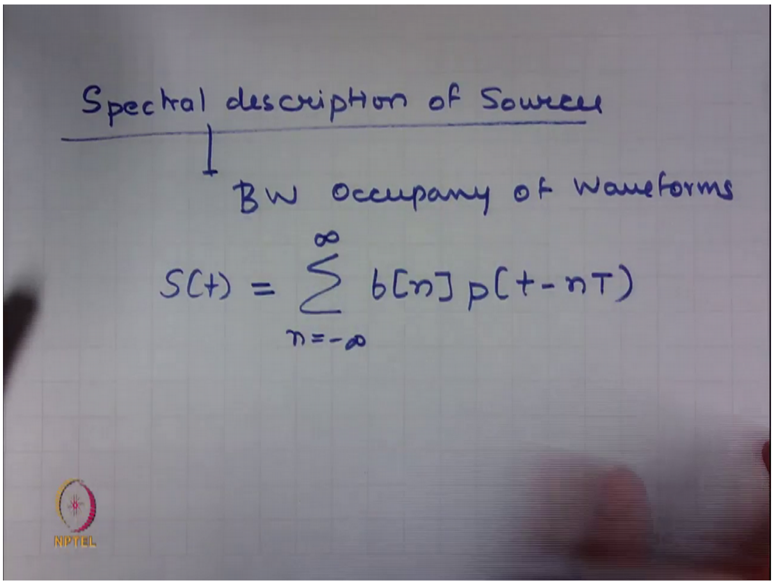

So, next thing that we will do is to talk about a spectral description of sources and the big idea in this topic is we would like to see what is the bandwidth occupancy of my waveforms and this is bandwidth is a really important thing because bandwidth comes at a price and you want to minimize the bandwidth occupancy of your waveforms. So, let us start with a source let us say that the waveform the baseband waveform that I have is summation b n p t minus n T n going from minus infinity to plus infinity.

Let me explain this from where we get this expression though we have discussed it several times, but it will be really nice that I revise this for you and what happens actually is that you have some binary sequence available you map these binary sequence in numbers.

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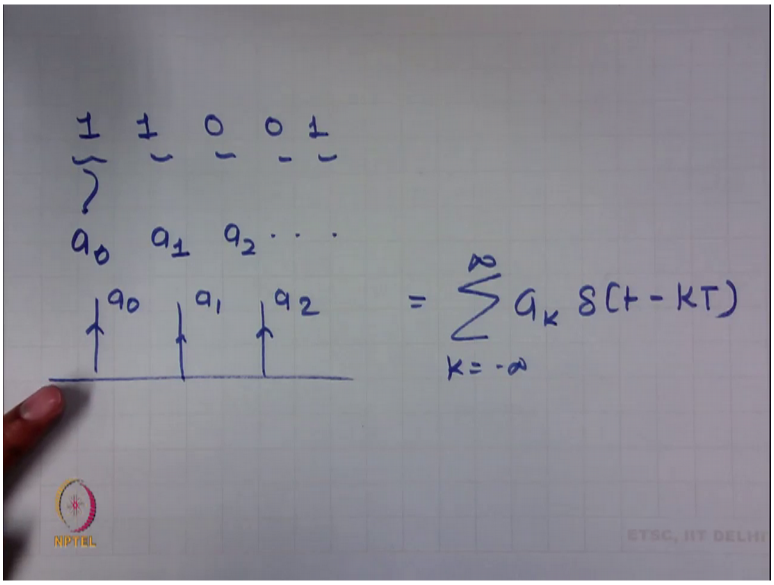

So, I map this lets say to some number a 0 this a 1, a 2 and so on so for I map these a logical sequence to some numbers of electrical voltages then I put I create an impulse train and the weights of this impulse train is decided by these numbers. So, what we have is summation a k delta t minus KT. So, you take a binary sequence you find some real numbers corresponding to a sequence you create an impulse train weights of the impulse train is decided by these numbers then you create an impulse train this is an impulse train impulse located at 0 T, 2 T 3 T and the weights of the impulse is given by a ks.

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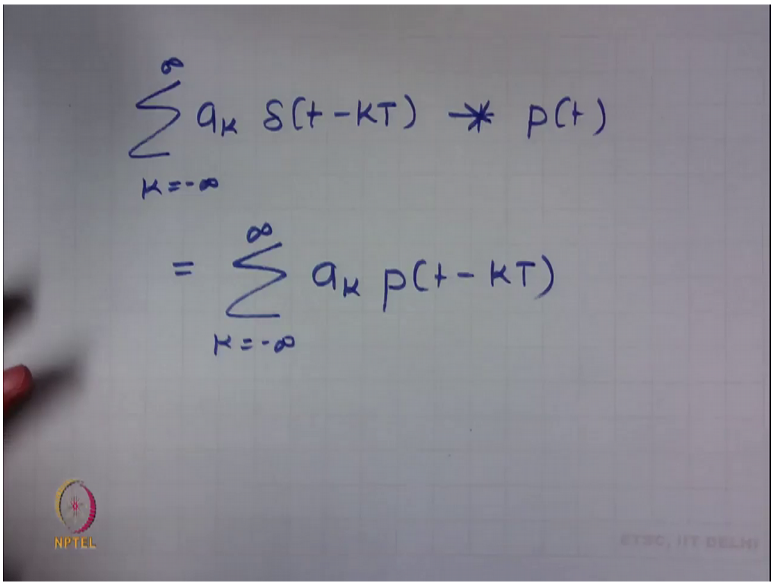

Now, then you pass this impulse train through filter. So, passing through a filter; that means I need to convolve this with the impulse response of the filter let us assume that the impulse response of the filter is p t.

So, what you get is K going from minus infinity to plus infinity ak n pt minus K T as I have said these are typical steps that happen at the modulator not all modulators can be described using these simple steps.

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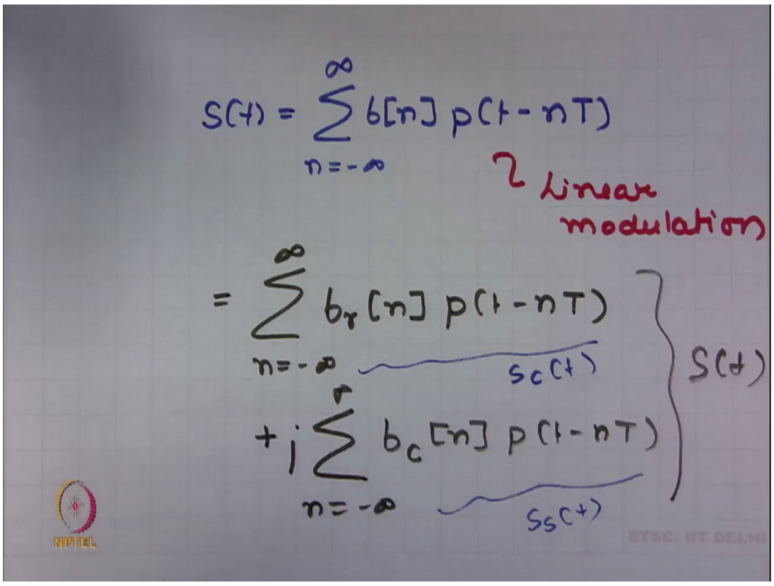

So, now let us go back to the equation that I have written. So, I have written S t is summation b n p t minus nT either you use n or k does not matter n going from minus infinity to plus infinity. Now if we see this equation this is an example of what we call as linear modulation; linear modulation means the pulse that you are using is common. So, all real numbers or all numbers are modulated using the same pulse. In fact, we are just concentrating on real numbers, but this can be complex as well.

So, this b n can be complex let us generalize this to complex if this is complex what I have is b r n pt minus nT n minus infinity to plus infinity plus j times b c n pt minus n T, and then you can consider this as S t the complex basement waveform ok. So, it is a simple. So, this would then denote this is all analytical S c t and this is S s t for example, ok. So, we can assume these numbers to be complex numbers and if we assume these numbers to be complex number you see that this is actually S t this is a complex waveform assuming that this is real. If this is complex this is real you will eventually get a complex thing and so, this will be complex baseband waveform it can be composed into 2 real waveforms.

So, one you can treat as S s t S c t and the other one you can treat it as S s t, so, eventually you can assume b n to be real as well as complex. Now anyway let us not digress to this whether its real or complex let us keep focus on the point that I mentioned before that we are treating we will be treating this or this is treated as a linear modulation you are using the same pulse shapes for all numbers. So, do not understand that linear modulation is only the kind of modulation we also have non-linear modulation ok, in which you have different pulses to transmit different signals this is linear modulation and for some time we would just focus on linear modulation ok.

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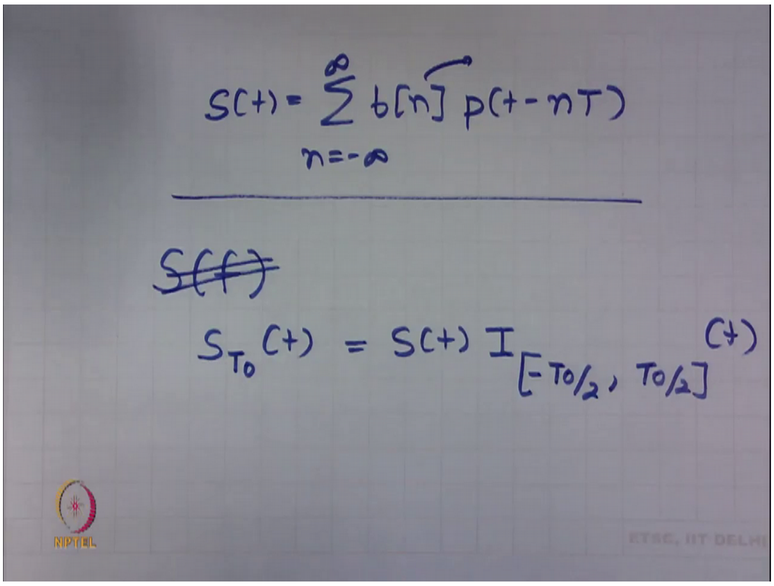

So, we are trying to start with this expression when there are so many things that we want to say about this simple expression again we will revise what are the important things that we have said; we have said that this is an example of a linear modulation because we are using the same pulse for transmitting all real numbers then we have said that this b n can be complex in that case S t would be complex baseband waveform and you can understand this to be composed of two real baseband waveforms ok.

Now let us try to define the power spectral density of the sequence to find out the power spectral density the first thing that we need to do is to find out the Fourier transform. So, let us try to find the Fourier transform of S t which I call as S f. Now if you want to find the Fourier transform of the sequence you would end up in trouble and the reason is because this b n goes from minus infinity to plus infinity.

So, the energy of this sequence would be infinite ok, if I assume that the sequence is stationary for example, if I assume this b n to be stationary; that means, b n should be defined from n going from minus infinity to plus infinity then what happen is that the energy would definitely be infinite with probability one and in that case the Fourier transform may not be defined right.

So, when we are trying to find out the flourier transform of this expression it would not be possible right. So, rather than finding the Fourier transform of this expression, we modify our equation. So, we say that we are not having S t going from minus infinity to plus infinity rather we have truncated it to t naught. So, let us say we are using S t and I have truncated it for a duration t.

So, now what I am saying is I am having a signal which spans only from minus T naught by 2 2 plus t naught by 2 if I am truncating a signal for a finite duration the energy of that signal will also be finite. So, this allows that now we can calculate the Fourier transform of this signal ok.

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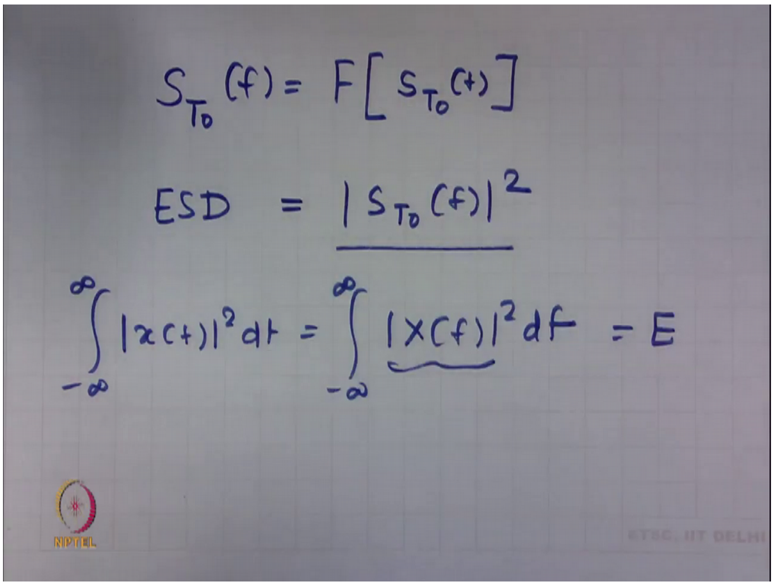

So, Fourier transform of this signal can easily be calculated which I say ST 0 f it is the Fourier transform of S T ok. Now if I define the energy spectral density energy spectral density of this signal is nothing, but S T 0 f n mod square. Let us see why this is energy spectral density because you know from basics that the energy of the signal is this and again from Parseval's theorem you can think about the energy as this and if this is the energy this quantity will be energy spectral density because you need to take energy spectral density integrated for a band of frequencies and then you get the energy.

So, this quantity is thus energy spectral density. So, energy spectral density of S T naught t would be this, what would be its power spectral density.

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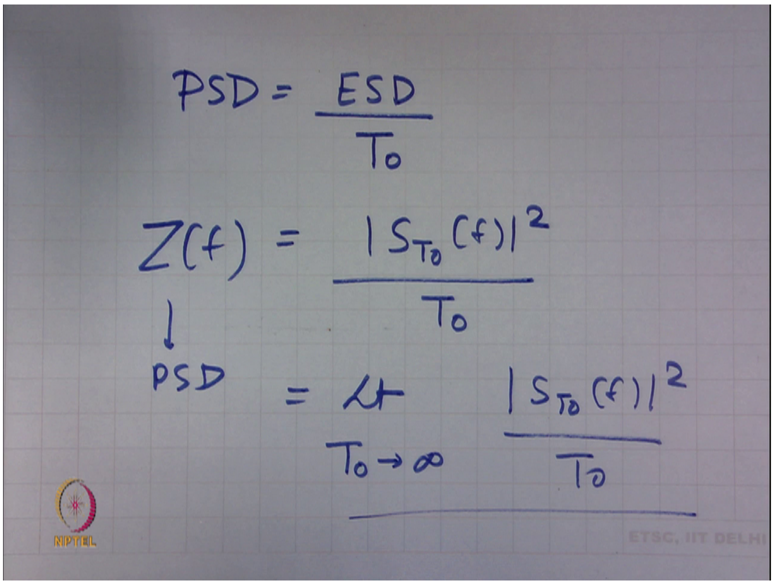

Power spectral density is nothing, but it is the energy spectral density by the time duration right, and the time duration of the signal is T naught. So, power spectral density is mod S T 0 f square divided by T naught right and because we do not have the good notation we would denote power spectral density with Z of f right we earlier have used S of f, but now S of f is we are using for the spectrum of a signal S T. So, let us use Z of f to denote the power spectral density.

Now this is the power spectral density for a signal which runs for a duration of T 0, now I can probably calculate this quantity by taking the limit of T 0 to infinity and this is how you define the power spectral density of S T 0 t, first you truncate the signal for a duration of T naught. So, that the energy of the signal is finite. So, that the Fourier transform is defined otherwise you cannot take the Fourier transform of the signal which has infinite energy and then you divide it by T 0 and then you take the limit of T 0 to infinity.

So, now, we have derived just basic expression of how can we calculate the power spectral density of our signal and we will work upon these equations we will develop some interesting equations of the power spectral density of certain waveforms and we will see that certain waveforms occupy more bandwidth than the other waveforms. So, there is lot of interesting things to happen in the next lecture, see you.