Principles of Digital Communication
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Lecture – 15
Random Variables and Random Processes: Types of Random Process

Good morning. Welcome to the next lecture on Random Processes. So, so for what we have done is we have defined random process, we have looked at a spatial kind of a random process that is a Gaussian process, we have defined jointly Gaussian random variables, we have defined random vectors, we have defined Gaussian random vectors.

And today, we would be continuing with random processes and today we will cover two important concepts in random processes that is the linear functional of a random process and then, we will look at what happens when a random process passes through an LTI system, a Linear Time Invariant system.

So, let us get started with linear functional.

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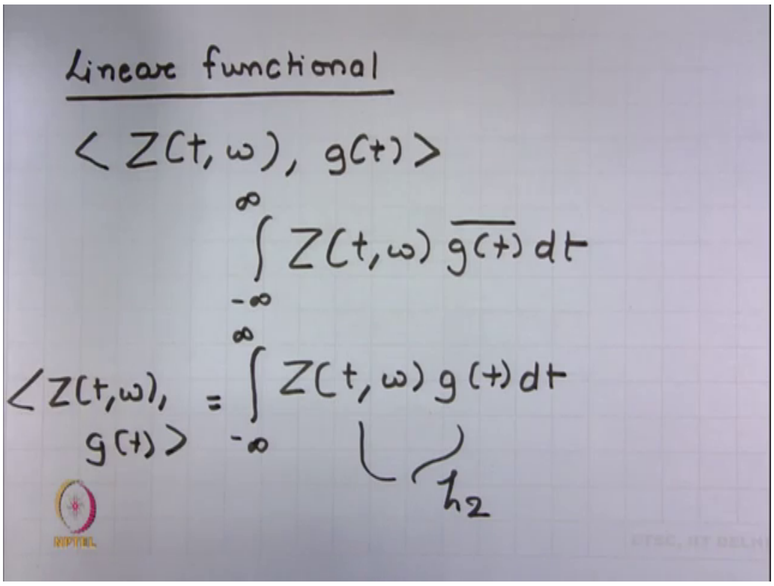

So, what is a linear functional? So, if you have a random process remember that a random process is usually denoted with a capital letter and it is a function of two independent variables time and outcome of the experiment. So, let us assume that we have a random process Z of t and if I take the inner product of this random process with a deterministic function that is the linear functional of a random process. So, linear functional of a random process is simple; linear functional of a random process is nothing but it is the inner product of the random process with some deterministic function g of t ok.

So, you know how to calculate the inner product. So, for calculating inner product, we have to evaluate this integration and usually as you know that we have to take a conjugate and one of the function, but because g t is a real function, we do not have to take this conjugate. So, the inner product of a random process with a deterministic function if everything is real can simply be calculated like this. So, this is the inner product of a random process with a function and this we say as the linear functional of a random process.

Now, some important points here before we move on to another section is that is this inner product always defined, it need not be and as we have seen from the vectors this inner product is always defined if Z t and g t are L 2 functions. So, what we are assuming as g t is an L 2 function and what we are assuming is the sample function of this random process is also an L 2 function. Because if they are L 2 function, then this inner product is always defined and we do not have to worry, then about the converging issues and so on and so forth. So, to simplify do not talk anything about the convergence, what we assume is the inner product is always defined because the sample function of the random process is an L 2 function and g t is also an L 2 function.

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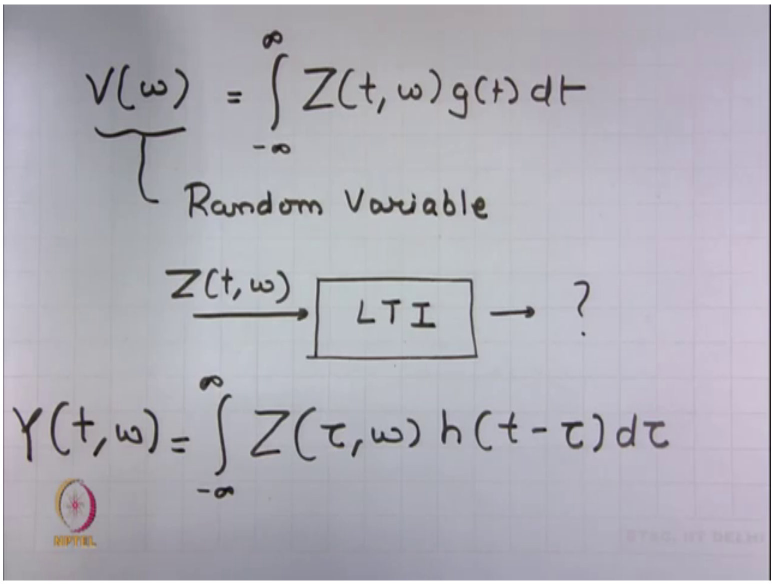

So, let me write that inner product again. So, this is the inner product of the random process with a function g of t. Now, let us look what is this quantity; what do you believe this quantity to V. This quantity if you see will not be function of t; why is this not a function of t? Because t is the running variable; so it will not be a function of t, but it would be a function of omega. So, what you would get is a real number V of omega is a real number and this number would depend upon omega. So, this V of omega is a random variable; is a random variable.

So, what we see now is that a linear functional of a random process is nothing but it is a random variable. Why this is a random variable? Because it will only be a function of omega; it will not be a function of t; t is the running variable, you are having a definite integration which are evaluating from minus infinity to plus infinity ok. So, we have looked into what is a linear functional of a random process.

Now, let us see what happens if you have a random process and what happens if you pass this random process through an LTI system ok. So, if you have a random process and if you pass this random process through an LTI system, let us see we get some output. How to evaluate this output? So, you know that this output could be computed simply by convolution. So, I need to take the convolution of this random process with impulse response and I have to integrate it from minus infinity to plus infinity. So, I have taken the convolution of a process with impulse response ok.

This you must know already now look at this, what this integration is a function of. It is not a function of tau for sure because tau is running variable; it is a function of both t and omega. So, we have got Y of t and omega it is a function of t and a function of omega. So, what you end up with is a random process ok. So, the output of an LTI system if you pass a random process through an LTI system, what you get is a random process ok.

So, let us summarize what have we said so far.

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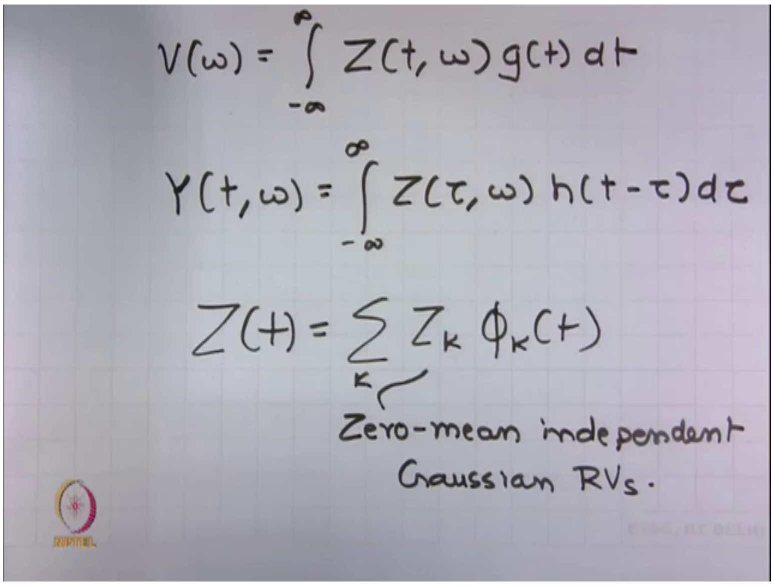

So, what we have said is a linear functional of a random process is nothing but it is the inner product of the random process with a deterministic function. That is one thing that we have studied. The second important thing is if you pass a random process through an LTI system, what you get is a random process itself given by this expression ok.

Now, let us see these things for spatial kind of a random process as the Gaussian process; we love Gaussian process because it is easy to evaluate linear functional and output of a Gaussian process and moreover, it is also very practical right. So, Gaussian process is easy as well as practical. So, let us take an example of a Gaussian process and we know that I can generate a Gaussian process through this orthogonal expansion function. So, I can consider a random process which is expressed in terms of orthogonal functions and the coefficients of these orthogonal functions are nothing but Zero-mean independent Gaussian random variables ok.

So, if you use this framework, this framework can be used to model all interesting Gaussian processes; it cannot model all Gaussian processes, but it can model all interesting Gaussian processes, now Gaussian processes which are of interest to us. So, let us revisit why can this generate a Gaussian process?

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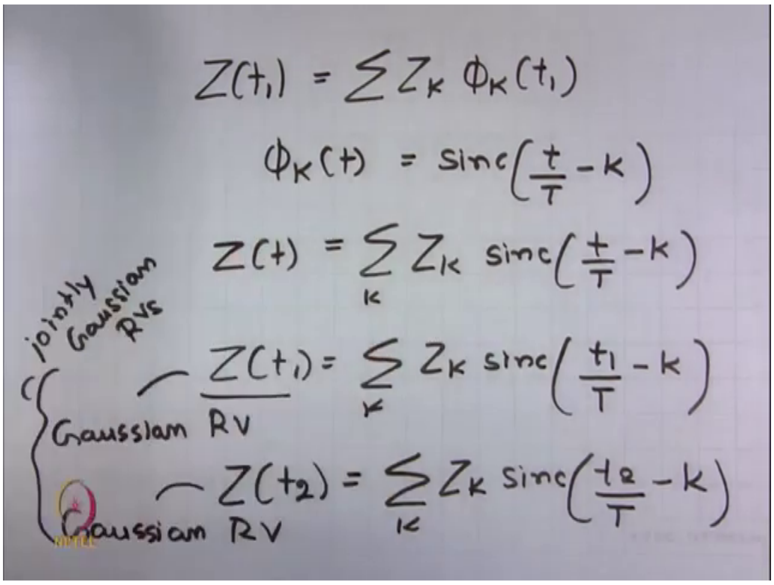

So, let us take a sample of that process and we have also said that most of the time we would take these orthogonal functions to be t spaced sinc functions ok.

So, let us now specify what these orthogonal functions are. So, the random process then can be written like this and if we assume a particular instance of t that is t 1 what I get is this expression. Now, as you can see what you get is a linear combination of independent Gaussian random variables. A linear combination of independent Gaussian random variables is nothing but a Gaussian random variable. So, Z t 1 is a Gaussian random variable and if you take a second sample; another sample of this process what you get is this expression.

So, what has changed? Nothing has changed. So, we are putting a specific value of t. So, in this case we have put t s t 1 in this case we have used t s t 2. Now what you get again is you are getting a linear combination of independent Gaussian random variable. So, Z t 2 is also a Gaussian random variable and what more, they are different linear combinations of the common underlying set of independent Gaussian random variables. So, Z t 1 and Z t 2 are also jointly Gaussian.

So, they are also jointly Gaussian random variables and if the samples of a random process are jointly Gaussian random variables with processes a Gaussian process. So, what I have convinced you again because it is so important that this process can be used to create and model Gaussian processes which are of interest to us ok.

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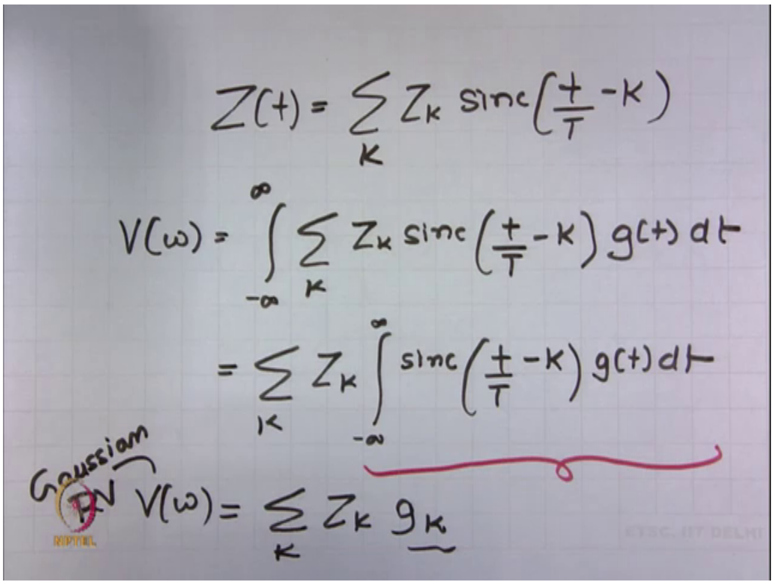

So, let us assume that we have taken such a Gaussian process and let us see what is the linear functional of such a process. So, linear functional is defined as the inner product. So, I have to take the inner product with let us say a deterministic function g t ok.

Now, pulling out this summation bringing in the integration as we usually do I can write this as ok, now what we can see from there is that this function this is a function of time; no, it is not a function of t because we have their running variable as t. It is a function of k right. So, I can write this as we are not interested in what is the value, but rather I can say that this is some function g k ok. So, it is a function of what is your g t and it is a function of k.

Now, g k is a real number right. So, it is some real number depending on the value of k. So, what you end up with is a linear combination of independent Gaussian random variables and we have seen that a linear combination of independent Gaussian random variables leads to a question random variable. So, V omega is a Gaussian random variable. So, what we have learned is a linear functional of a Gaussian process is a Gaussian random variable.

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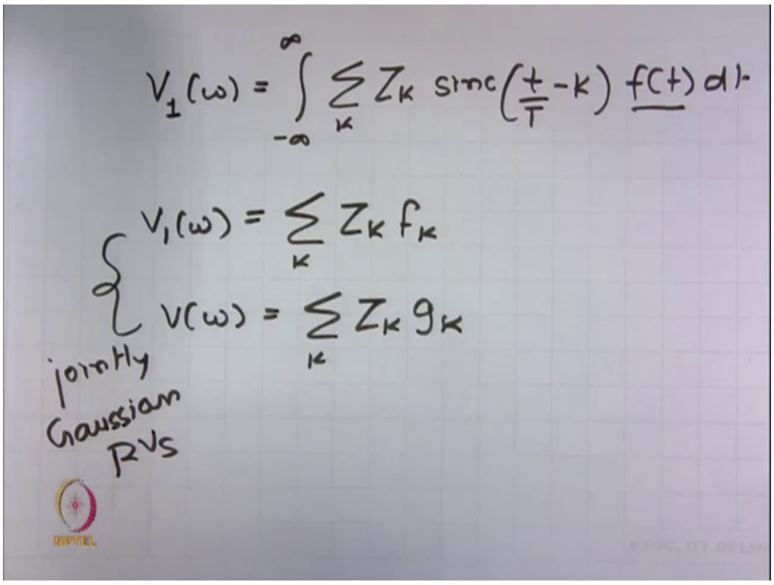

Let us let us take a linear functional let me write this as V 1 omega for some other deterministic function, for the same Gaussian process. Let me take it for f of t d t. So, now, I have to replace g of t with f of t; f of t is another deterministic function. So, similarly you can show that this would be nothing but using the same analogy we get that V 1 omega there is again linear combination of independent Gaussian random variables, where value f k is decided with the k and what function you have assumed.

Now, if you see that V 1 omega and V omega, they are the different linear combinations of the common underlying set of independent Gaussian random variables and thus, these are jointly Gaussian random variables. Why are we studying all this is because most of the time, what you deal with is how does a random process interacts with the digital communication systems with your receivers and so on and so forth and in that context understanding about linear functional will be going to help us a lot and thus, this idea of linear functional is important.

Now, let us see what happens when a Gaussian process passes through an LTI system.

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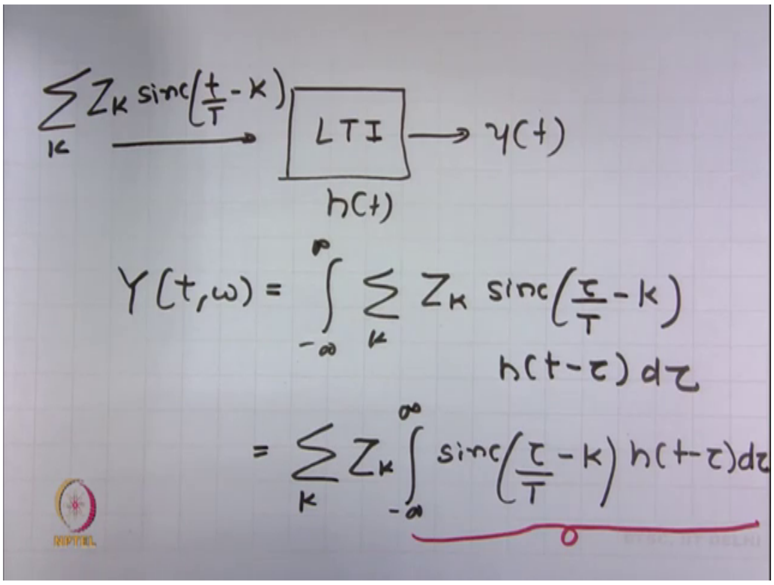

So, let me assume that we have an LTI system and let me assume that the impulse response of this LTI system is h of t and let me assume that at the input of this LTI system we have a Gaussian process and as you would have known by now that Z k for this to be a Gaussian process the Z k has to be independent Gaussian random variables.

So, let us ask the question what is Y t? So, first we know that it should be a random process itself and this should be summation k Z k sinc tau by t minus k into h of t minus tau t tau ok. Then, pulling the summation out putting integration in what we get is Z k ok. Now from this, what we can write if you see this carefully now what is this? This is a function. So, that we are not interested as before; we are not interested in what this integration evaluates to, but we are more interested in what is this quantity.

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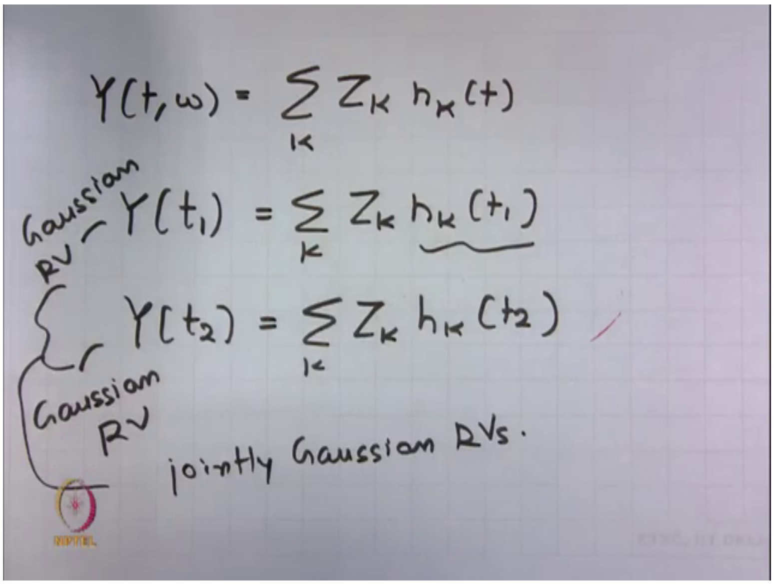

So, if you see that let me first write and then explain. So, we would have Z k and this quantity would be a function now, it is a function of t ok. It is a function of t and this is also a function of k. So, what we would have is let us say we would have h k t because it is a function of time. Now, this is a random process because it is a function of time.

Now, if you take the sample of this process and let us say specific time instants t 1, what we get is Z k h k t 1. If you take the sample of this process at time t 2, what we get is Z k h k t 2 right; just substituting t s, t 1 and t 2 and what you see now is what is this again, now this is some number some coefficients. So, again Y t 1 is a linear combination of independent Gaussian random variables.

So, Y t 1 is a Gaussian random variable and Y t 2 again it is a linear combination of independent Gaussian random variables. So, Y t 2 is also a Gaussian random variable. What more is again they are different linear combinations of common underlying set of independent Gaussian random variables. So, Y t 1 and Y t 2 are jointly Gaussian random variables. So, everything is very similar to the last time. So, they are also jointly Gaussian random variables. So, what we can say now is Y t is also a Gaussian process right.

Similarly, you can do it for any samples and you would see that the random variable created by looking at this process at a specific time instants would nothing but it would be a different linear combinations of the common and the length set of independent

Gaussian random variables and thus, you can say every sample created by looking at this specific order by looking at this process will be a jointly Gaussian random variables and hence you can conclude from this is Y t omega is a Gaussian process.

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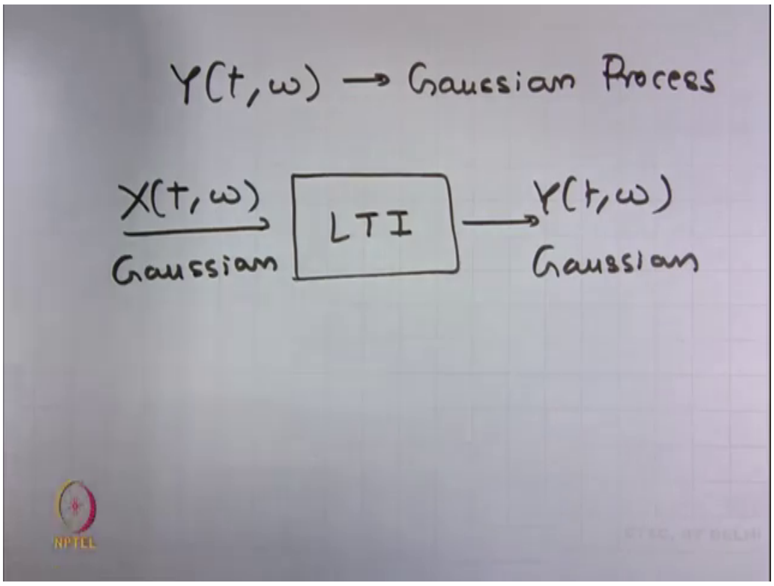

That is a very important and useful observation that if you have an LTI system, linear time invariant system; if you have a Gaussian process at the input of this LTI system, the output process is also Gaussian. So, if input is a Gaussian process, output is also Gaussian process and we say this by saying that linearity preserves Gaussianity right. So, this can be any linear system linear time invariant system. So, if you have a linear time invariant system, what happens is input is Gaussian process; output is also Gaussian process; it is a very useful result ok.

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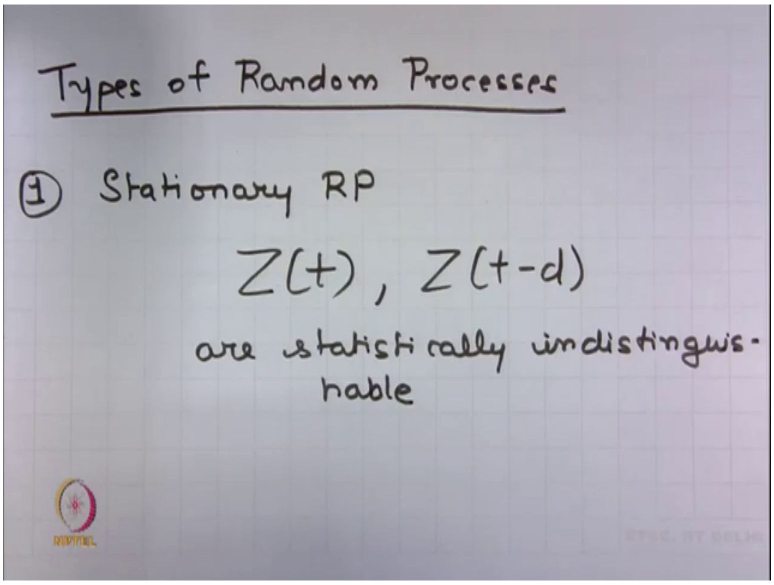

Now, let us define different types of random processes. Let us define different types of random processes and the first interesting class of random process is a Stationary. Stationary random processes; what is the stationary random process? Let us try to first understand it intuitively. So, suppose if you have a random process Z of t and if you delay this random process by some amount, let us say if I delay this with d unions right.

Then, these processes Z t or Z of t minus d are statistically they are statistically indistinguishable ok. That means, you take a random process, you delay it by a anytime units you want; then the process that you create is statistically indistinguishable from the original process. If that is the case then the random process involved is known as the stationary random process; that means, it does not care where your time origin is right; does not care what is what is the value of t ok. So, such processes are known as stationary random processes.

So, let us see mathematically how can we define this more concretely.

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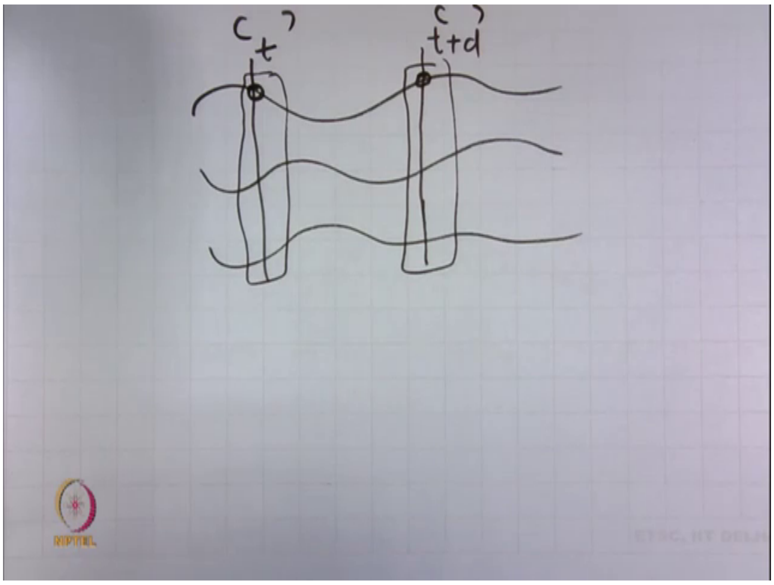

But before taking that mathematical definition let me warn you do not think suppose I have a random process which is a collection of several sample functions. So, let us look this process at time instants t and let me look this process it time instants t plus d. So, I am looking this process at two different time instances. What I am saying is if this process is stationary, then it statistically this sample is same as this sample; statistically.

It does not mean that this value is same as this value I do not mean that right. So, you have to really understand what stationary random processes; do not get confused that the sample functions are completely deterministic, they are not right. Statistically if you see this random process at a time instants t or at a time instants t plus d, the statistically these samples are same; that means, they have the same mean they have the same probability density function and so on and so forth ok. So, that is the meaning of statistically indistinguishable.

Let us try to see how can we define mathematically stationarity. So, suppose I have a random process.

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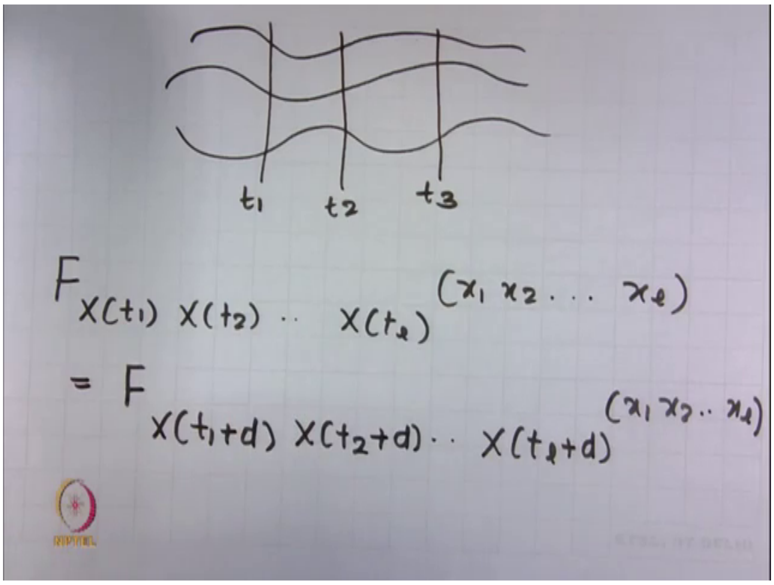

So, let me draw this. So, that you get a grip of what is a random process and I sample this at different time instances. Now when a sample a random process the different time instances what I am going to get? I am going to get certain random variables and if I get certain random variables, I can define what is known as the joint CDF right. So, let me define the joint CDF; so this is how we define a joint CDF of this random process.

Now, you know what is the joint CDF; it is saying that what is the probability that random variable X t 1 takes an argument less than or equal to x 1 and random variable X t 2 takes in a value less than or equals to x 2 and so on and so forth random variable X t l takes in a value less than or equal to x l. So, we have already discussed what joint CDF is; just we can define a joint CDF for this random process.

Now, if this random processes is a stationary random process; what should happen more is that if you delay these samples by let us say some d times. So, instead of t 1 you collect the random variable at t 1 plus t similarly then if a process is stationary the joint CDF that you obtained by sampling the process at time instants t 1, t 2 and so on and forth up to t l is the same joint CDF if you sample the process at time instants t 1 plus t 2 plus t and so on and forth up to t l plus t. Of course for all f X 1, X 2, X l and for all values of d right; so if this equation satisfied, then we say that the process is a stationary process ok.

So, as you can now see that stationarity checking stationarity is also very difficult right. It is not easy to prove that a joint CDF of l random variables involved does not change when you shift the sampling time by d units and you have to do it for all values of l right 1 or 2 would not be sufficient you have to do it for all possible positive integers. So, doing this would be difficult.

Now, let us see some interpretations of what stationarity can mean.

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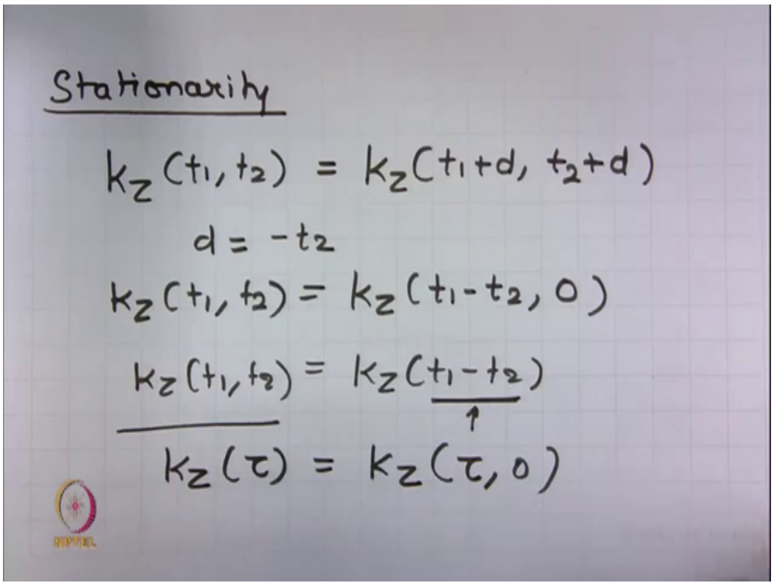

So, stationarity will mean that as we have already said the insight is that if you shift a random process by d units; if you delay a random process then this random process remains is statistically indistinguishable and we have also said then the process does not care where the time origin is. So, let us say that we have covariance function; we have already defined what a covariance function is. So, let us assume that we have obtained a covariance function by sampling the process at time instants t 1 and t 2.

Now, this covariance function should also be same as this covariance function; that means, if you shift the t 1 by d units and t 2 by d units, then you should get the same covariance function because the process is stationary. This is not always true this is only true when involved processes is stationary random process; that means, the sampling or random process does not care what the absolute values of time instances are right. So, it does not care whether you have t 1 and t 2 or you have t 1 plus d and t 2 plus d ok.

Now, I can choose any value of d. This must be true for all values of d. So, let us choose d as minus t 2. So, if you choose d as minus t 2 what you get is k z t 1 minus t 2 and 0. Now, this is interesting. So, for a stationary process, the covariance function obtained by sampling the random process is times t 1 and t 2 is nothing but it is the covariance function where one time instants of t 1 minus t 2 and the other time instants is 0 ok. Now, this 0 is kind of redundant. So, I can write this as k z t 1 minus t 2. Now, see what we have done. So, this covariance function it is a function of 2 arguments right, but this covariance function is a function of only one argument t 1 minus t 2.

So, when you see the covariance functions with one argument; what you should interpret? You should interpret that this covariance function should be of a stationary process right. So, whenever you see a covariance function of one argument like in this case, it should hint to you that this is a covariance function of a stationary process of stationary random process. Now, furthermore what it could imply is that the argument here t 1 minus t 2 should be nothing but it should be the difference.

So, how to relate this from this; so, this t 1 minus t 2 you should interpret that this should be nothing but it is the difference of 2 arguments in the case of the covariance functions with 2 arguments. For example, if I have k z tau and this is a covariance function of a single argument. So, you should interpret this as that this should be same thing as when if I am writing this in terms of 2 arguments, then what we should have is tau and 0 ok.

So, this is what stationarity leads to. A stationarity converts a covariance function which is normally with 2 arguments in the covariance function with single argument. With a single argument denotes that difference between the time instants of the covariance functions with 2 arguments ok. What should happen to the mean? So, normally mean as we have illustrated before whats the mean of the random process?

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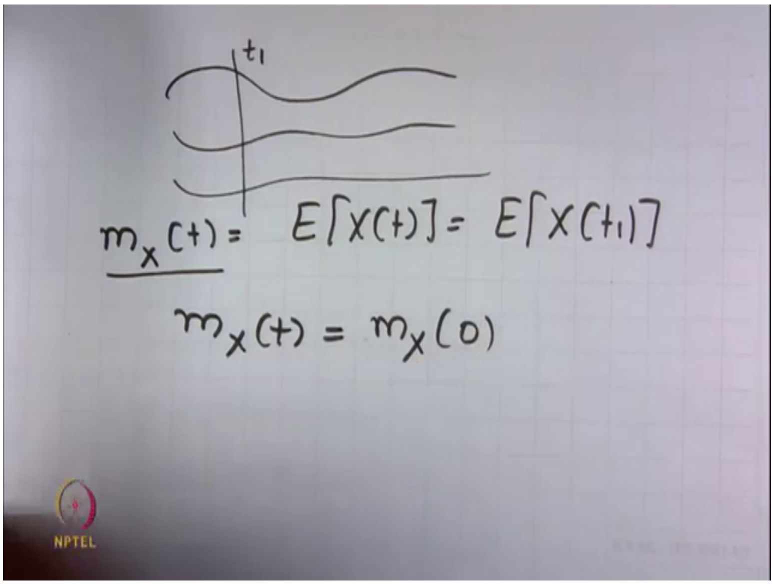

So, you sample this random process at a time instants t 1, then the mean of the random process can be understood as the mean of this random variable X t 1 X t 1 is a random variable. So, we calculate the in sample average of this random process or you calculate the expected value of this random variable X t 1. And similarly you can calculate the mean of this random process by varying your t from minus infinity to plus infinity.

So, mean is generally a function of time; is not it? So, because at every time instants, you get a random variable and then you calculate the expected value of that random variable right. So, mean is a function of a time for ordinary random processes, but now if I am talking about the stationary random processes and if I am saying stationary random processes does not care where the time origin is; then, what it would mean is that mean should not be a function of time, it should be constant right.

So, mean of a stationary random process should be a constant; that means, you can choose any value of time for example, have chosen the time to be 0. So, m x t is nothing but m x 0. In fact, you do not have to specify what the timing instance is, it would be independent of the time. So, mean of a stationary random process is a constant right. It is not a function of time ok.

So, what we have learnt is that when we are talking about the stationary random process things becomes even simpler right; simpler in the sense that the covariance function which is normally covariance is a function of 2 arguments reduces down to a covariance function of a single argument and the mean remains constant. So, these are some effects of a stationary random process.

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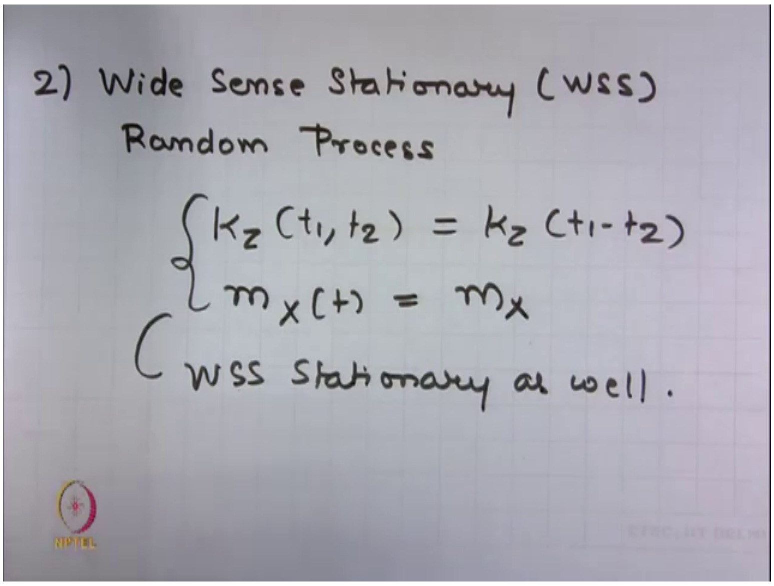

Now, let us define another class of random processes which are known as wide sense stationary; Wide Sense Stationary in short WSS wide sense random processes and what are they right. So, of course, they are stationary random processes in some sense in wide sense right. So, what we say is a process a random process is a wide sense stationary, if the covariance function two argument based covariance function is nothing but you can convert this into a single argument based covariance function and the mean of a random process is a constant ok.

So, these are the effects of the stationary process that we have already discussed. So, if this process is stationary, it needs to satisfy these two conditions right; it needs to satisfy the two argument based covariance function can be reduced to a single argument based covariance function. This we have already proven and what more we say that the mean is constant is not a function of time anymore for a for a stationary random process.

Now so, if these two conditions are satisfied, we call that processes a wide sense stationary process as well. Now so, you can imagine that every stationary process is a wide sense stationary process right, but other way around is not true if a process is wide sense stationary; that means, these two conditions are satisfied that does not mean that strict condition of a stationarity is satisfied. What is the strict condition of a stationarity?

A strict condition of stationarity is this. So, you have to prove a stationarity, you have to prove that this equation is satisfied for all apples for all arguments for all values of d right. So, this is the strict condition of the stationarity, but these are kind of weak conditions right. These are weak conditions that you can say that the processes a wide sense stationary process if only these two conditions are satisfied ok.

So, every stationary process would satisfy these two conditions. So, the every stationary process is a wide sense stationary process, but every wide sense the stationary process will not be stationary process. So, there is some confusions in case of the language used. Sometimes the wide sense the stationary process is also referred to as a stationary process right.

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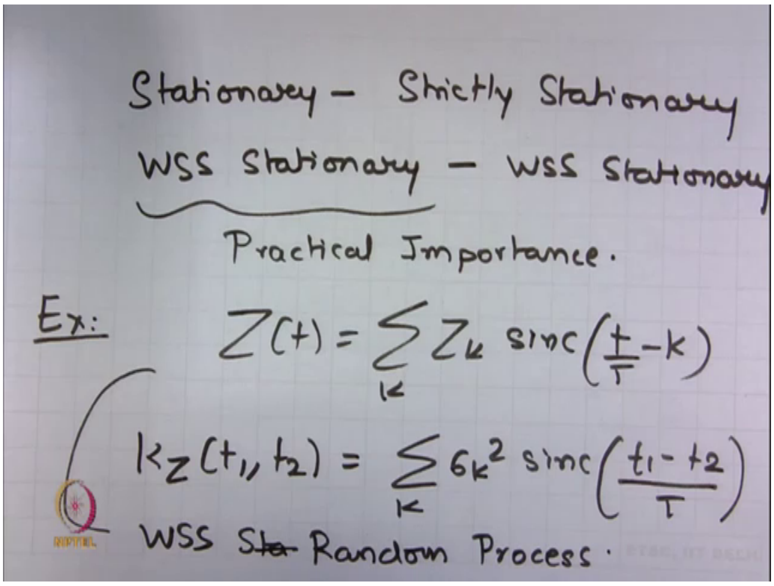

To avoid any confusion in the words, sometimes these stationary processes are also referred to as strictly stationary processes and these wide sense stationary processes are referred to as wide sense stationary processes. It is also important to know that these words are not used consistently in literature. So, remember that when we say stationary or a strictly stationary process, we mean that joint probability density function of the random process is invariant to time shift; whereas, when we say wide sense stationary process it simply mean that covariance function and mean is invariant to time shift.

So, normally as you can imagine thinking about the stationary process is or a strictly stationary process is really difficult. So, we will stick to wide sense stationary processes only right. So, these wide sense stationary processes or processes of practical importance; the one that can be easily defined and can be used ok.

Now, let us take some example right. So, let us take the good old example of the random process that we have been using so far. So, what does this produces if it is a zero-mean independent Gaussian random variable. Zero-mean is not important, but we are saying it because we have been defining them to be like this always.

Then you see that it produces a zero-mean Gaussian process right. Now, we have already seen that what is the covariance function for such a process? If you remember the covariance function for such a process can be calculated like this. We have already proven this. So, you refer to one of the classes in which we have calculated the covariance function of this random process and what we observed then is that the covariance function evaluated at t 1 in t 2 is not depending upon the absolute values of t 1 and t 2 rather it was depending upon the difference of these timing instances right.

So, hence this process this framework that we have been using can be used to model wide sense stationary processes only. So, it models wide sense stationary processes or wide sense stationary random processes; that is one insight. So, together with modeling a Gaussian process it only models Gaussian processes that have wide since of stationary Gaussian process that is one thing.

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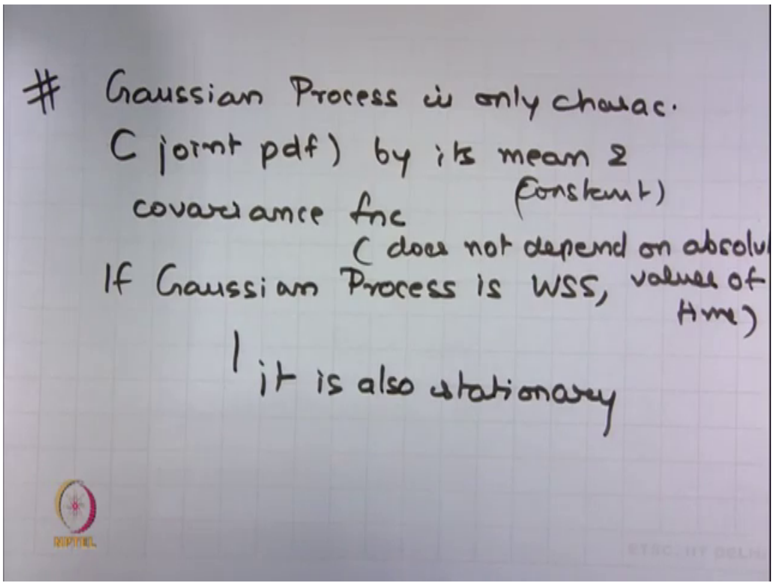

The second thing is now we know that a Gaussian process is only characterized is only characterized means when we are talking about this joint p d f we already determined what is the joint p d f of a Gaussian process, there we saw that this is only characterized by it is mean and covariance function.

Now, if it is characterized by it is mean and covariance function and if the Gaussian process is wide sense stationary, if Gaussian process is wide sense stationary that means, it is covariance function does not depend upon the absolute values of time instances that means, this joint p d f also does not depend upon the absolute values of timing instances because joint p d f is only a function of mean and covariance right.

So, let us say that if the processes wide senses stationary that means, the mean is constant and covariance function does not depend on absolute values; absolute values of time right and hence this joint p d f is also. So, does not depend upon absolute values of time and hence if a Gaussian process is wide sense stationary it means that it is also stationary.

So, everything simplifies for a Gaussian process right. So, we have said in general it is quite difficult to prove whether a processes stationary or not because we have to write a this joint p d f, but for a joint p d f in the case of a Gaussian processes is easy. Because it is completely characterized by it is mean and covariance function. So, if we say that the process is wide sense the stationary that means, it is mean is constant and this covariance function does not depend on absolute values of time. From this, we can conclude that is joint p d f is also time invariant it is. It is not depending upon the absolute values of time and hence, the wide sense of stationary Gaussian process is also a stationary. The two more points that we will like to point out at this point is because we are mostly talking about zero-mean processes right, the covariance function.

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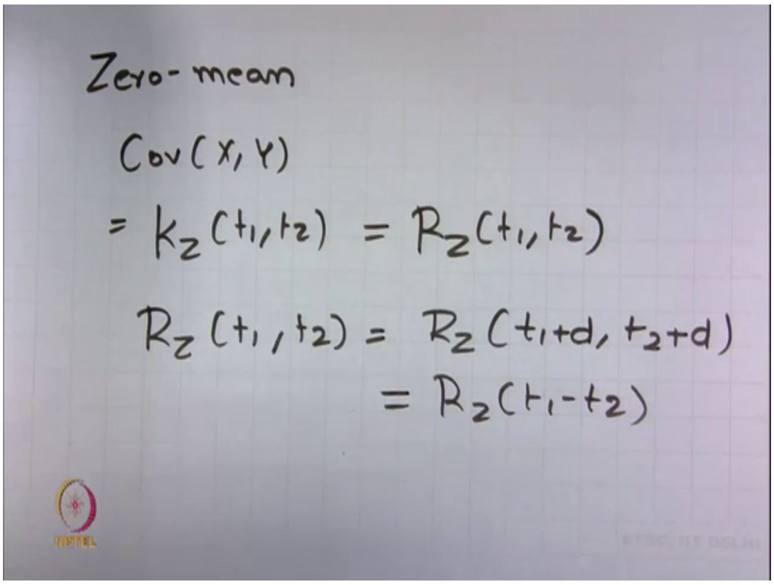

Let us say the covariance function of 2 random variables are the covariance function of a random process very sample the random process at 2 time instants is nothing but it is the same thing as it is autocorrelation function right because it is a zero mean. The difference between covariance and correlation only comes in terms of mean if something is zero-mean then either you can talk in terms of the covariance function or you can talk in terms of the correlation function it means one and the same thing right.

So, whatever we have set for the covariance function also remains true for the correlation function. So, for example, if I want to say about the wide sense stationary process, I can say wide sense stationary process is a random process which has covariance function which is time invariant that means, it does not depend upon the absolute values of time right and this boils down to t 1 minus t 2 using the same ideas as we have used for the covariance function. In short if it is a zero-mean process whatever is set for covariance function can be set exactly for the correlation function not a correlation function that is one point.

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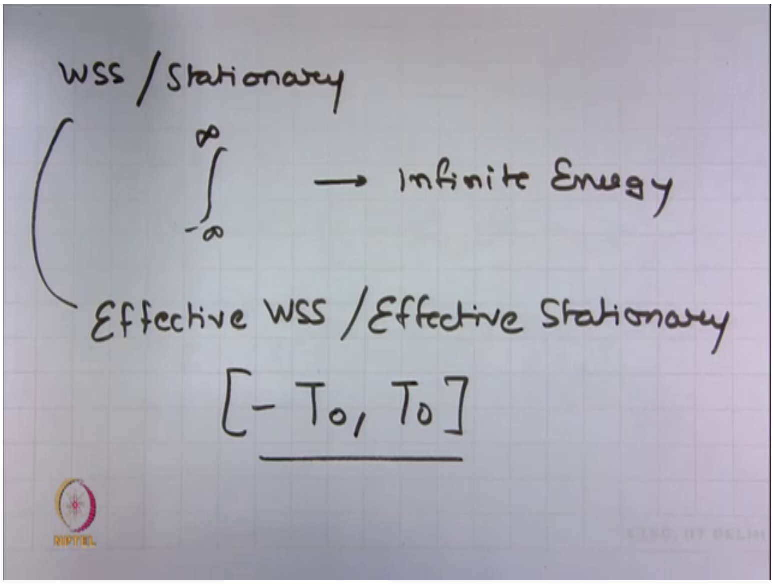

Second point is let us see if the process is wide sense stationary all that is stationary; that means, we do not talk about the time origin right. So that means, when you are defining this process you have to define it from minus infinity to plus infinity because it is not a function of time. So, in terms of the time this process should exist from minus infinity to plus infinity right. Now, if something exists from minus infinity to plus infinity that thing is going to have infinite energy; think more about this. So, anything that exists from minus infinity to plus infinity if the sample function is the stationary of the random process is stationary existing from minus infinity to plus infinity; then, it must have infinite energy.

Now, if it has infinite energy all this oil to theory that we have developed does not work because L 2 L 2 theory is for the sample functions with finite energy right. So, in order to get rid of this problem, sometimes you talk about the effective wide sense stationary or effective stationary process meaning that the process is not completely stationary, but it is a stationary in the timing limits minus t naught to plus t naught ok. In the processes of stationary only within this time interval and after that the process does not exist it becomes 0 whatsoever, then if you consider the process to be effectively wide sense stationary, it would have the finite energy because this process exists only between minus t naught 2 plus t naught right.

So, any effective wide sense stationary or effective stationary process would be a random process with finite energy in all this L 2 theory will work and how do we choose this t naught and minus t naught as long as they are pretty large, it does not matter right. For example, how does this random process behaves in very far future or how it behaved in the past before minus t naught if t naught is pretty large is not going to impact today right.

So, these limits does not matter as long as the value of t naught that is chosen is pretty large and what does this help us with is it makes the process with finite energy. So, this idea is also sometimes used. So, we will not use it, but in certain text books you would find another class of wide sense stationary and a stationary process is known as effective stationary process; that means, they are only stationary within a time window right and this time window is chosen to be pretty large. So, that it does not impact the present ok.

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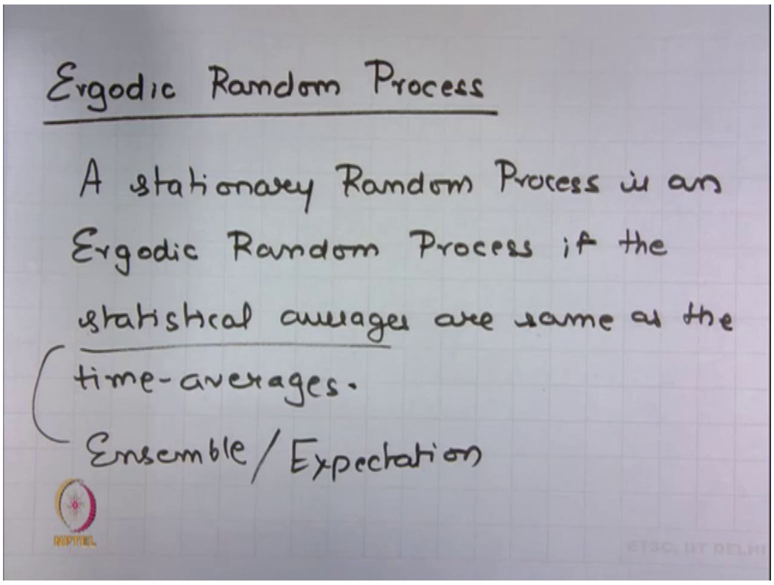

So, let us now define another process which is Ergodic random process. So, what is an ergodic random process? A stationary process a stationary random process is an ergodic random process, if the statistical averages are same as the time averages ok. So, we have defined what is an ergodic random process. So, the census first of all it has to be a stationary random process ok. If it is a stationary random process and if it is statistical averages are same as the time averages, we say that the processes and ergodic random process. Now what are the statistical averages?

So, statistical averages are the same as an ensemble averages or they are same thing as taking expectation ok; taking expectation. So, these are one and the same thing and the time averages are one that you have already studied right; you have already studied how can we take in time average of a quantity. So, ergodic random processes for an ergodic random processes these averages are same. So, we would choose to take an average which one is more convenient ok.

So, it is an important class which we will talk about in the next lecture. So, what we have seen today. We have defined what are the linear functional's and we have got some insight into how does a random process behaves when it passes through in LTI system and more or less we have defined various important classes of random processes.

We have defined what is the stationary random process; we have defined what is a wide sense a stationary random process and now we are going to define what is an ergodic random process and remember that most useful random processes of wide senses stationary random processes and if a processes a Gaussian random process and if it is wide sense is stationary that means, it is also a stationary random process.

We will see more of this in the next lecture.

Thank you.