Information Theory, Coding and Cryptography
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Module - 32
Trellis Coded Modulation
Lecture - 32

Hello and welcome to our next lecture on Trellis Coded Modulation.

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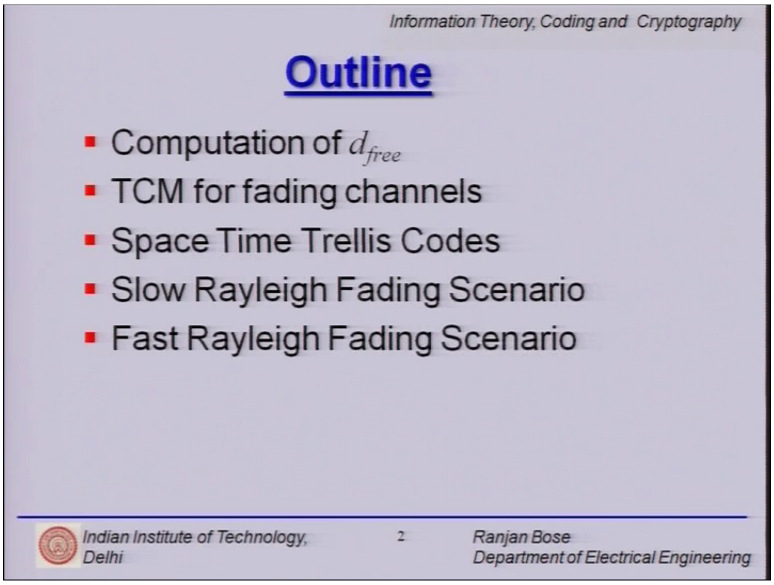

Let us start with a brief outline of today's talk. As we have seen that the d free which is the free distance is one of the single most important parameters to characterize our trellis coded modulation scheme and we would look at some methods to compute d free. Then we go to fading channels, commonly encountered in wireless communications, and we will see that the TCM design criteria for fading channels is different than a normal additive white Gaussian noise channels. Then we will logically shift to space time trellis codes, which is a logical extension going from TCM and we look at two distinct scenarios; slow Rayleigh Fading Scenario and fast Rayleigh Fading Scenario. We will look at the distance, product design criteria for these two cases.

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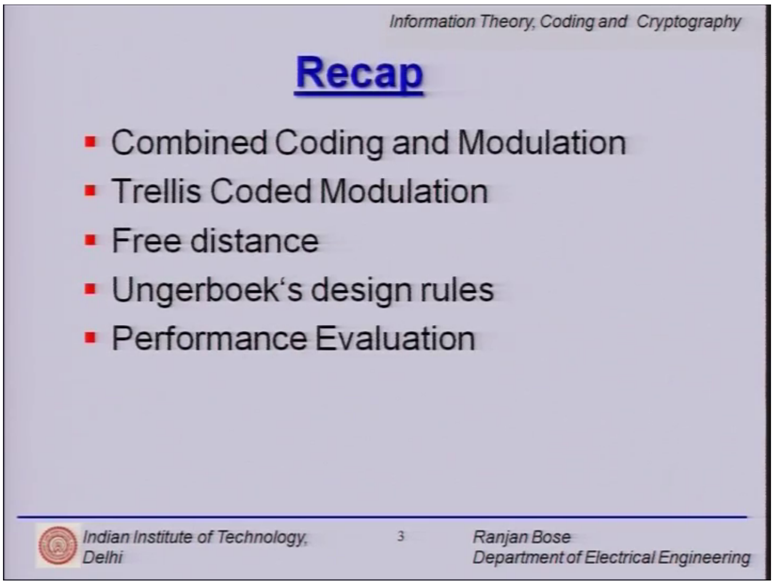

Let us do a quick recap; what we have learnt so far is the combined coding and modulation. Then we introduced the notion of trellis coded modulation, followed by what we mean by free distance; then we looked at Ungerboek's heuristic design rules and spend some time looking at the performance evaluation of designed TCM schemes.

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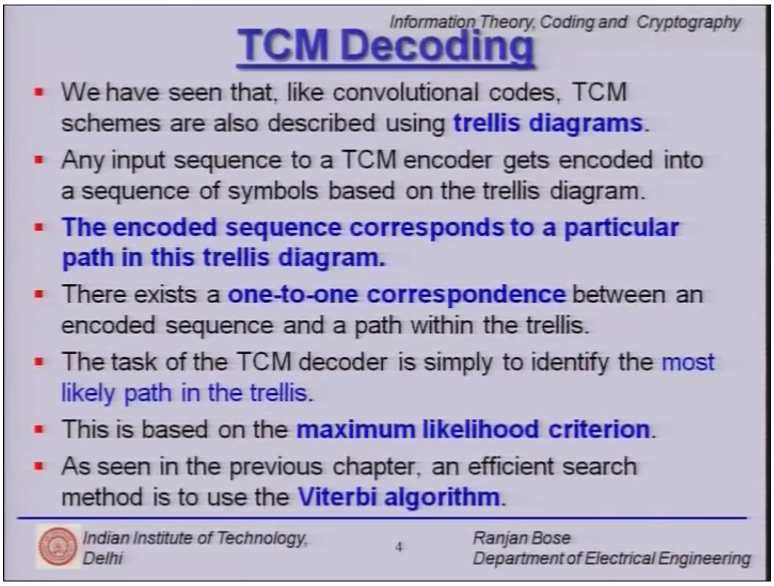

We can recollect the TCM decoding is done using the trellis diagram, and we use the notion that any transmitted sequence is actually a path in the trellis. And therefore, we have to find out the corresponding path in the trellis in terms of the decoding algorithm.

And we do this using the maximum likelihood decoding criteria and to do so, we use the very well-known Viterbi algorithm.

We are used earlier the Viterbi algorithm for convolutional codes, but remember TCM is non-linear as opposed to convolutional codes which were linear codes.

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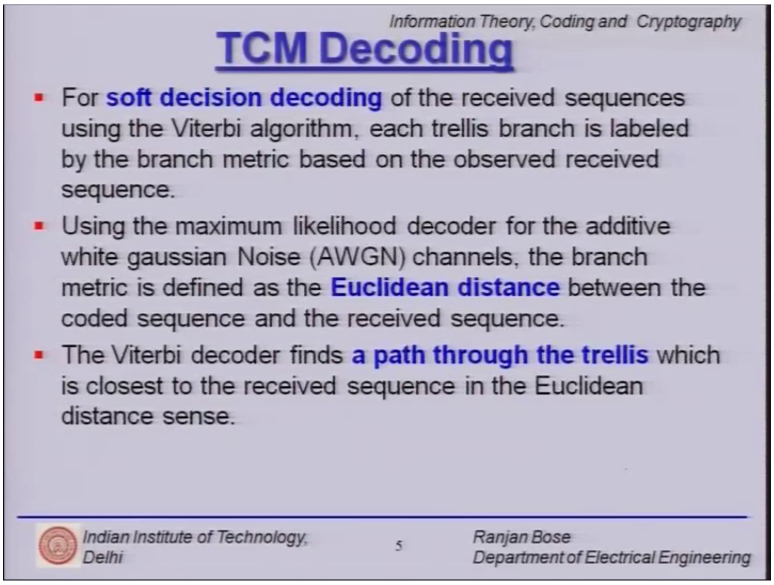

So for TCM Decoding, we are realized that soft decision decoding was used and we employed the Euclidean distance between the coded sequence and the received sequence, as a measure to find out the maximum likelihood sequence. And the Viterbi decoder tries this measure to find the most likely path in the trellis. So, we are using Euclidean distance as opposed to hamming distance, which was used for the convolutional encoder decoder case.

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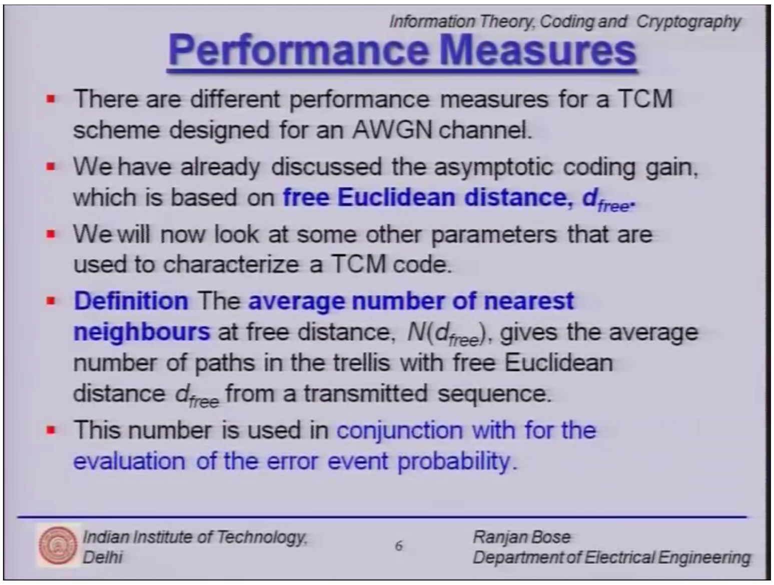

We looked at some performance measures and we talked about d free as a free Euclidean distance. And we also talked about the N free which is also a function of d free is in terms of the average number of paths in the trellis with free Euclidean distance d free. This number is used in conjunction with the evaluation of the error event probability.

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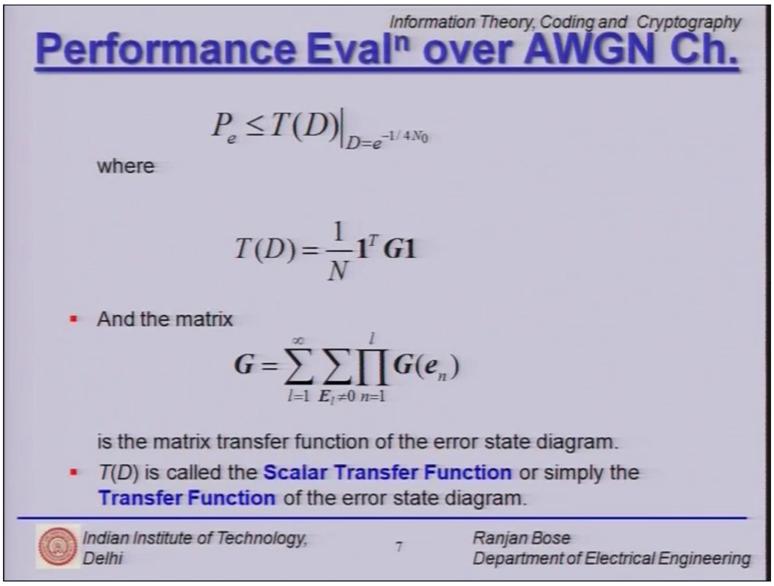

Then we went on to define this, probability of error in terms of the scalar transfer function T D. We have learnt how to calculate TD, using a modified state diagram and we figured out that finally we can upper bound the error event probability P by this following expression.

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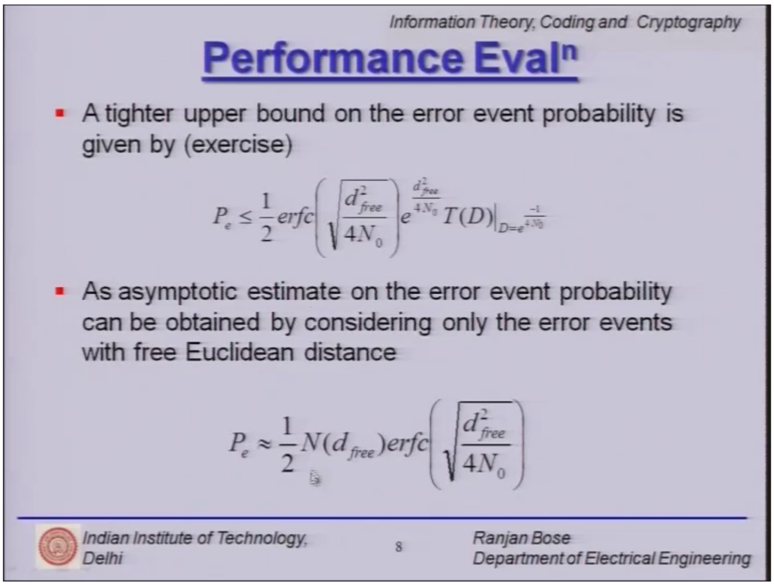

And if you look at the asymptotic estimate on the error event probability, we get the following expression; probability of error approximately half times N as a d function of d free error function complement under root d free squared over 4 n naught.

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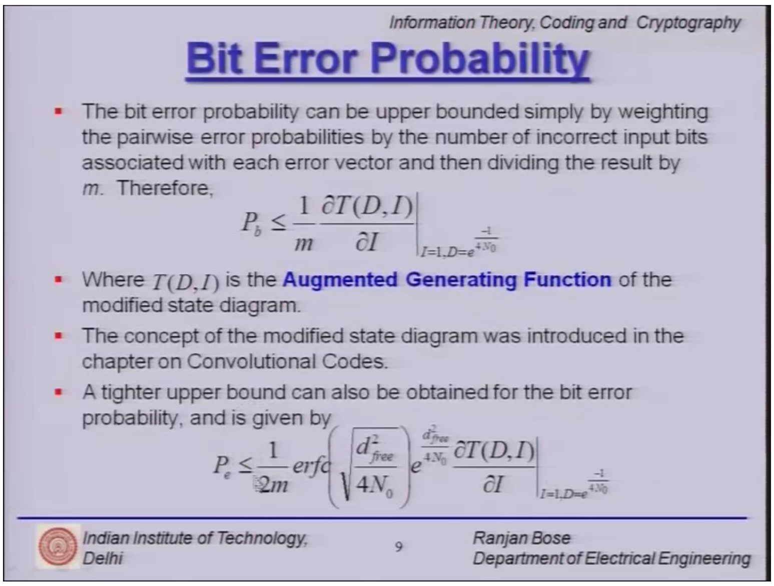

So, from that we went on to discuss, the bit error probability and we showed that, it can be written as a derivative for this modified T as a function of D and I. So, here T D comma I is the augmented generating function for the modified state diagram. Again when we had talked about convolutional code, we had introduced this function called augmented generating function.

Now what is important is to find out that the same function can be used to get an upper bound on the bit error probability. Here the derivative is evaluated I, I is equal to 1 and D is equal to e raise to power minus 1 over 4 N naught. A tighter upper bound can also be obtained for the bit error probability and is given by the following expression.

So, you see that d free is figuring prominently in all of these expressions, again establishing the fact that it is the single most important parameter. Of course, for the probability of error expression, the noise power also starts figuring in the expressions.

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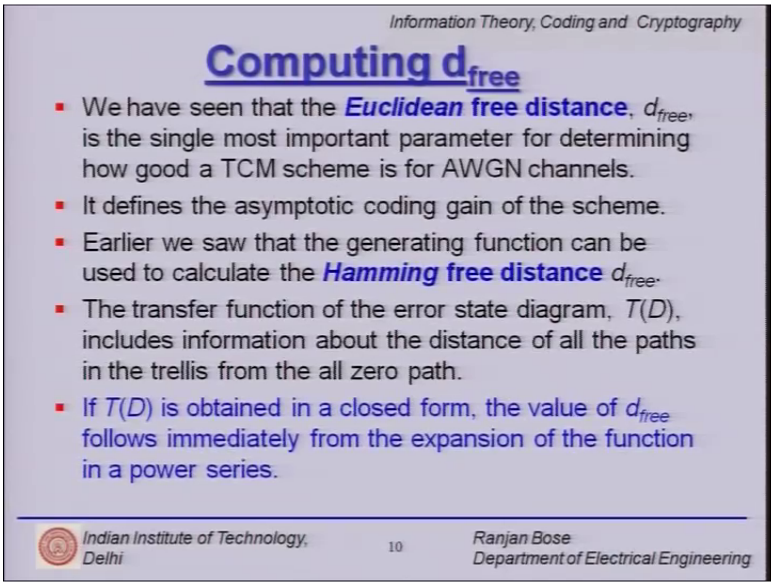

Now, since d free is such an important parameter, we spent a few slides talking about computing the d free ok. Now please note that this statement is correct uh, when we are talking about additive white Gaussian noise channels. When we go to fading channels, we will have a slightly different parameter that we will shortly talk about.

So, the generating function can be used to calculate the hamming d free distance earlier. Now the transfer function of the error state diagram TD also includes information about the distance of all paths in the trellis from the all zero paths. So, the question is, can we use that information to calculate d free for our case? The question is d free may or may not be available in a closed form expression all the time. In the event that TD is obtained in a closed form, the value of d free follows immediately from the expansion of the function in a power series. We have seen that earlier.

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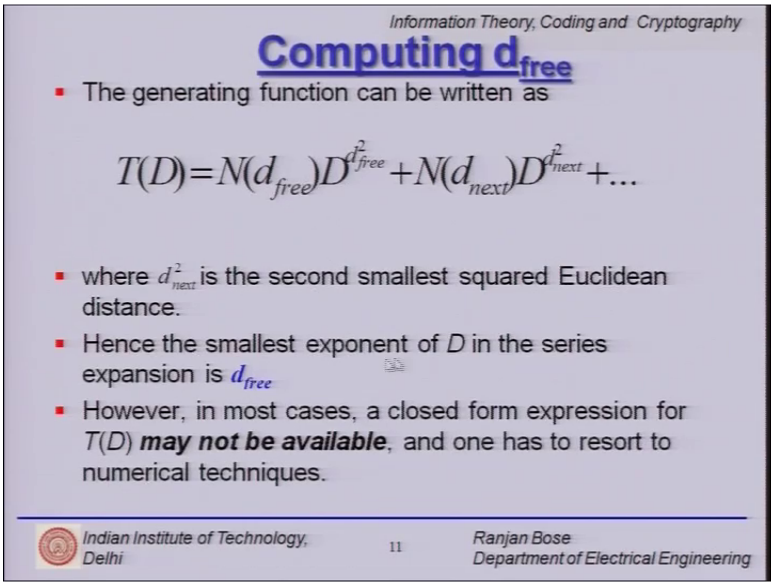

Now, if you see; if you do not really have a closed form solution, you can write as an expansion as follows. We had seen this earlier. Well, it comes directly from the closed form expression, but in general we can write like this. So, what we do is TD is nothing, but the number of paths with d free times, this shows that the d free is obtained for these paths and then there is a next expression where this is d free plus something. So, it is the d next and this N d next corresponds to the number of paths that contribute to those d next ok. So, d next is the second smallest squared Euclidean distance.

So, the smallest exponent of D in the series is in the expansion is d free, but we have made this point earlier that we may not have a closed form expression all the time and one has to resort to numerical methods.

Student: Sir, what is this d next exactly I mean relate this and d free plus 1 then it will be d free plus 2 or something.

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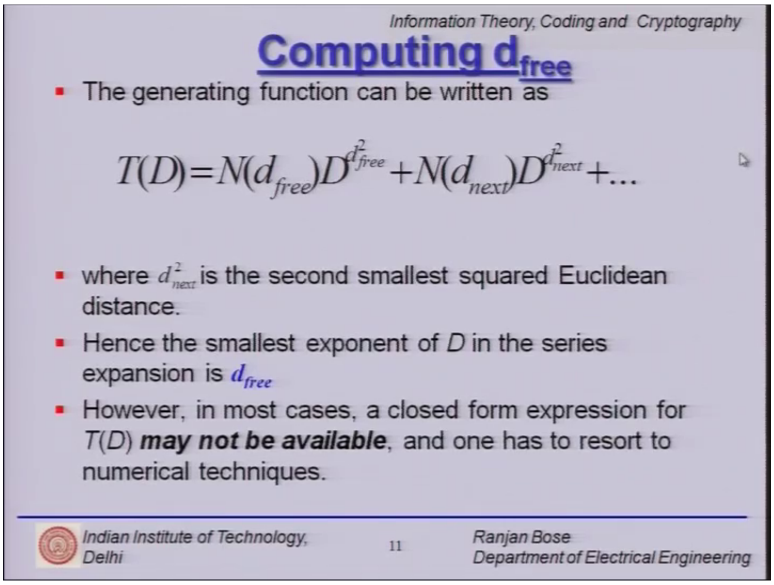

The question being asked is what is this d next? So, clearly d free is not an integer this time. Had it been, the hamming distance measured and we had taken in convolutional code, the generating function TD. Then this D raised to the power; if this was the 5, then d next could be 6 or 7. But in this case d free could be 3.722 and then the d next could be 4.312. Some value which is the next larger value.

So, we have no constraint on this being integers here, because we are talking about the free distance of a trellis coded modulation scheme. So, whatever it is. In general, there will be some larger distance and that should correspond to certain number of paths that lead to it. So those suppose, there are 4 paths which contribute a distance Euclidean distance which is d next, then n as a function of d next times d raise to power d next. And then there will be a next term, it will be d next plus next and so and so forth.

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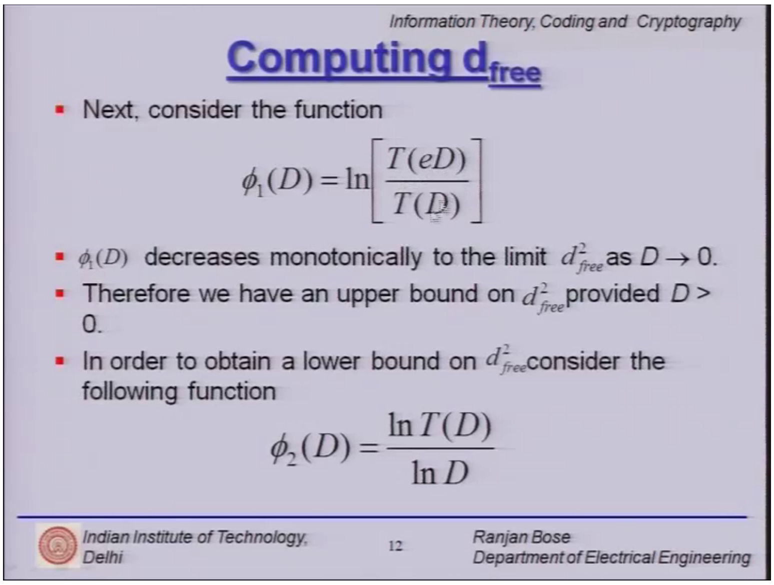

So, let us consider the function; phi 1 as a function of D as log of the ratios t e D over T D all right. Now if you see this, phi 1 D decreases monotonically to the limit d free squared as D tends to 0. Therefore, we have an upper bound on free d free squared provided D is greater than 0, in order to obtain a lower bound on d free squared.

Consider the following function phi 2 which is the ratio of log T raise to power S sorry log T as a function of D divided by log of D. So, we have this function phi 1 and phi 2 and we will use them to get an estimate of d free.

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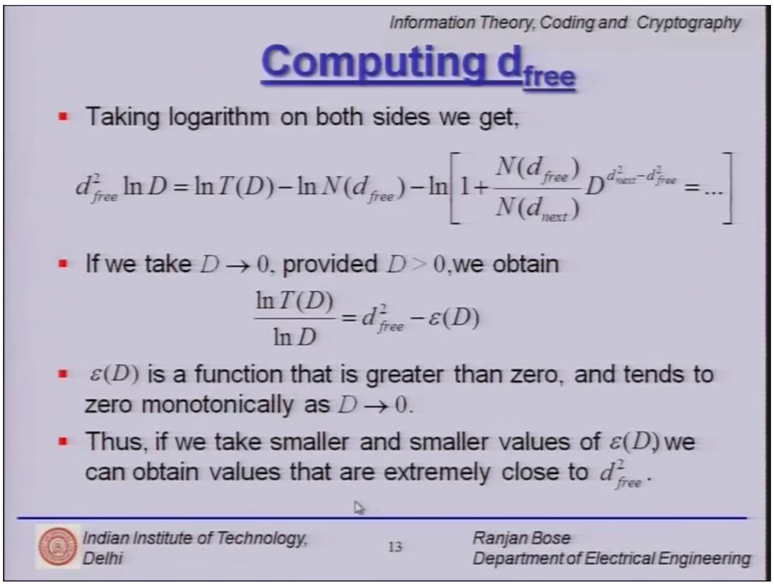

So, if you take log on both sides of the equation that we saw just now, then we have ln D and the d free squared comes in front and then we have this expansion going up next. So, if we take D tending to the 0 and D greater than 0, we would get roughly ln T as a function of D divided by l n D. This comes to d free squared minus epsilon D.

So, the point is if we can make this epsilon D as small as we can right. So, then, we can have a very good estimate on squared free distance. So, we take smaller and smaller values of epsilon D and we can values we can get our values which are extremely close to squared free distance. So, this is one way to determine or at least get an estimate of squared free distance from the generating function T D.

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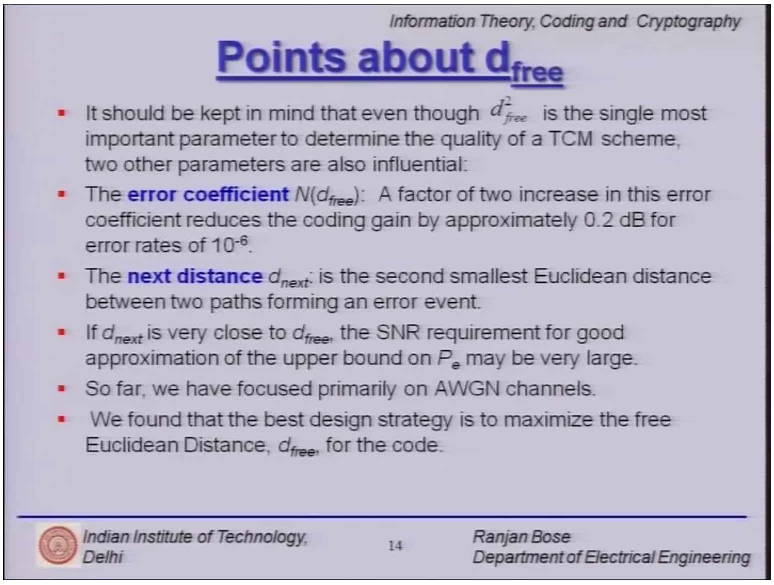

So, couple of things that we must keep in mind, while talking about d d free. So, the squared free distance is the single most important parameter to determine the quality of a TCM scheme. So, if you have to compare two schemes. The first thing we should do is to compute d free. Please note d free calculation does not have to be done from the all 0 path because we are dealing with a non-linear code here; however, two of the parameters are also important.

The error coefficient N as a function of d free which tells us that, how many paths which are there in the trellis which contribute to this d free? So, question is d free appears to be the weakest link in the chain. How many weak links are there in this chain? Ok and after d free, it is not always that we will always be in a bad shape; that we always get an error between the two closest paths. We can also have the d next coming into play.

So, the second smallest Euclidean distance between two paths forming an error event is also important as a second order effect. Now question is how far are d free and d next? If d next is pretty large with respect to d free, then d next should stop mattering; however, if d next is pretty close, why are we having this discussion while we did not have this for convolutional codes? That is because the d d next for the convolutional coder was always an integer a way. So, if d free was 2 or 4; then the d next would be at least 1 or 2 more, but we can have d free as 5.3 and d next could be 5.4.

So, they can be fairly comparable to each other. So, if d next is very close to d free and it can be as close as you want or your luck may have it. The SNR requirement for good approximation on upper bound on P e may be large.

Now so far, we have only talked about free distance and additive white Gaussian noise channels and we have found out of the best strategy is to maximize the free Euclidean distance for the code. But things may not be the same for fading channels. So, let us quickly talk about TCM design for fading channels. Before that let us take a minute to quickly recap, what do we mean by fading channels.

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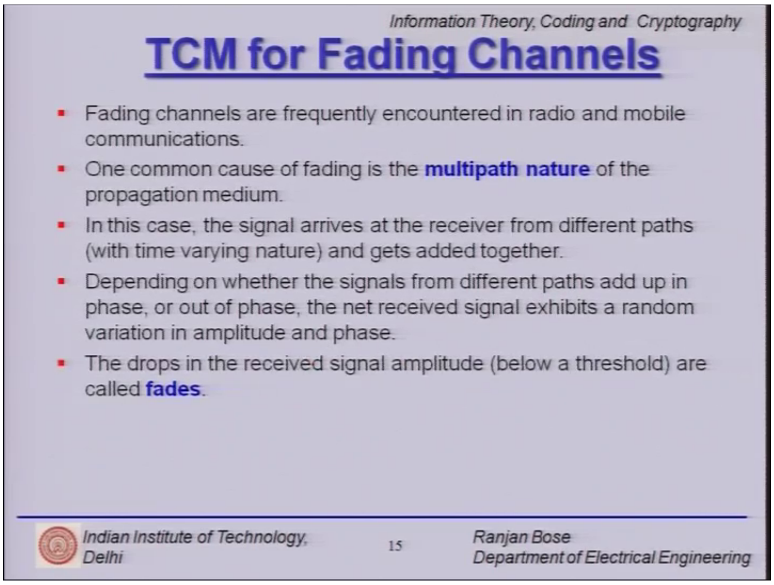

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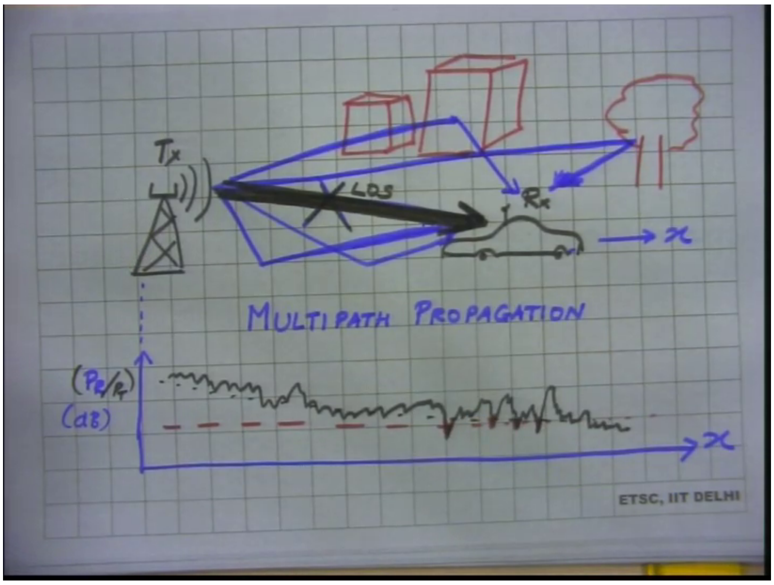

So, if you consider a transmitter and a receiver, then we have the signals going from the transmitter to the receiver; however, it is possible that it reaches the receiver through reflections and the reflections, it can come from many kinds of objects. So, you could have different buildings or other reflectors which will lead to the signal reaching from the transmitter to the receiver through multipath.

So, currently we have drawn a scenario we where we have multipath propagation. But the interesting thing is, when a signal reaches from the transmitter to the receiver through several paths, they add up at the receiver. Currently in phase out of phase, in such a manner that the resultant vector gives the actual value of the signal received. Now because of these paths reaching the receiver with delays because longer paths means more delay and hence a phase shift; that means, that they can add up and have a destructive interference also leading to the loss in the signal strength.

So, if you plot on this axis, the distance from the transmitter to the receiver and if you look at the power received and make a plot of the power received and if you do a logarithmic part you will have something like this. What it means is that; so if this is some kind of a plot in d B. So, we can normalize this.

So, if you see that even though as you move away from the transmitter and there appears to be a general downwardly trend. In the short term, there are major fluctuations in the received signal. And this is because that the signals received here add up destructively and the vector m gives a very low value.

So, this is typically of a profile of a fading channel and these are your fades. And if this is the receiver threshold over which the signal should work, then there would be many times when you go below the threshold and you may have a loss and communications.

So, there is the general nature of the fading channel and we would like to design a TCM scheme based on the characteristics of a fading channel. Now if there is a line of sight here, then we know that this is a Racian fading channel. However, if there is no line of sight, then we go to Rayleigh fading channel.

So, we will look at these scenarios separately. We come back to our slides now and we observe that fading channels are frequently encountered in radio and mobile communications and the cause of fading is the multipath nature as we discussed just now. And this is because the receiver gets signals from different paths with different path tellies. And if the signals add up in phase or out of phase, the neck received signal exhibits a random variation in amplitude and phase which are the sudden drops are called fades.

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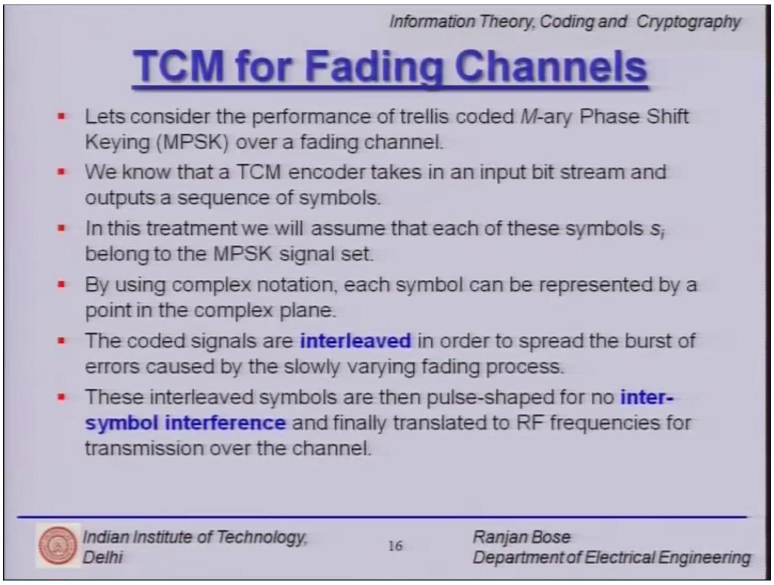

So, we now talk about TCM for fading channels. Let us consider the performance of trellis coded M-ary Phase Shift Keying MPSK over a fading channel all right. We know we know that a TCM encoder takes in an input bit stream and outputs a sequence of symbols. In this treatment, we will assume that each of these symbols s i belong to the MPSK signal set.

So, this is clear we have done this before and we will use a complex notation. So, each symbol is represented by a point on the complex plane. Now the coded symbols are interleaved in order to spread out the burst errors caused by slowly varying fading process and this interleaved symbols are then pulse shape for no inter symbol interference. So, those are the nitty-gritty details for communications, but will focus pretty much on the fading characteristic of the channel.

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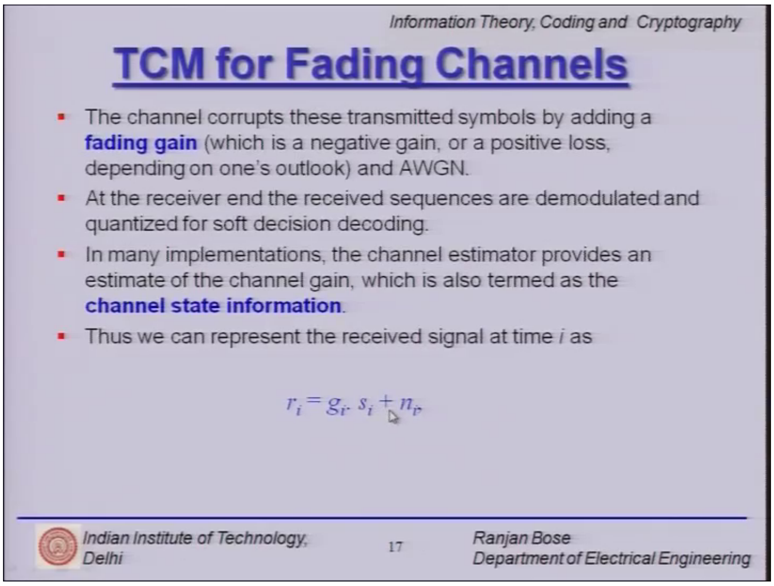

Now, the channel corrupts these signals by adding a fading gain right and it also had has additive white Gaussian noise. So, the fading gain is a multiplicative gain. So, at the receiver end, the received sequences are demodulated and quantized for soft decision decoding. And in many implementation, the channel estimator provides an estimate of the channel gain. So, we would either have the channel state information built into it or sometimes we will have to go without it for cheaper designs and then we say the CSI the channel state information is not present. So, they are two distinct scenarios.

So, at any instant, i time instant i the received signal r i is equal to g i times s i, s i is the transmitted signal g i is the multiplicative gain which is the fading coefficient plus the noise n i. This is a very simplistic model that we will be using for our case.

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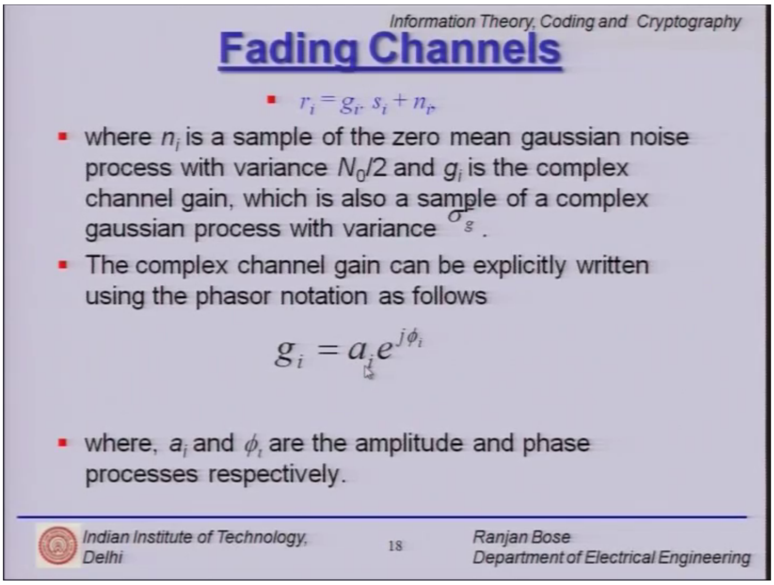

So, starting with r i is equal to g i into s i plus n I, we say that n i is a sample of the zero mean Gaussian noise process with variance N naught by 2; g i is the complex channel gain right which is also sample of a complex Gaussian process. So, the g i can be written as a i times e raise to power G phi i because g i is complex gain. It not only changes the amplitude, but also the phase remember, s i is complex.

So, complex into complex gives another complex. So, a i and phi are the amplitude and phase processes respectively and they can be modelled differently. For example, depending upon, whether you have a line of sight or not a i could be Rayleigh or Rician and phi i is typically uniformly distributed over 0 to 2 pi

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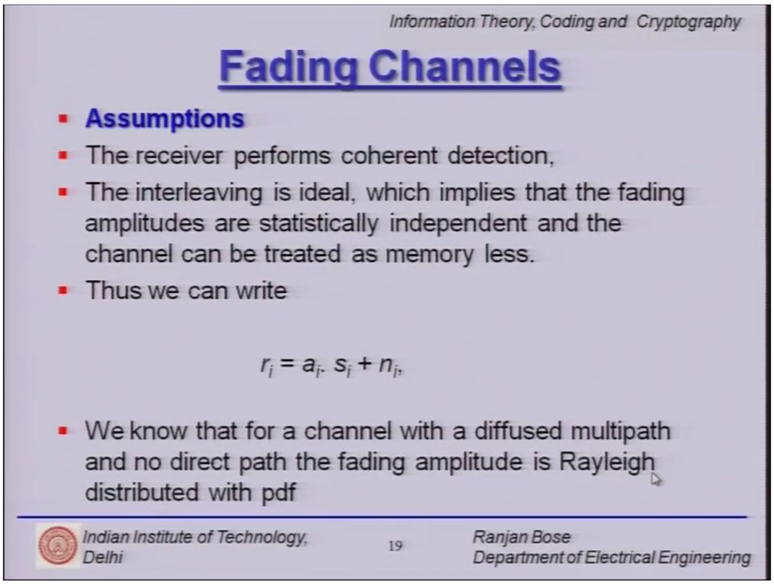

So, let us see; if we can start with some assumptions. Let us say that the receiver performs coherent detection, not a bad assumption. Interleaving is ideal. So, we have the fading and amplitude are statistically independent and the channel can be treated as memory loss channel. So, we have r i is equal to a i into s i plus n i and we know that for a channel with diffused multipath with no direct line of sight communication between the transmitter and receiver the fading can be rightly modelled as Rayleigh distributed.

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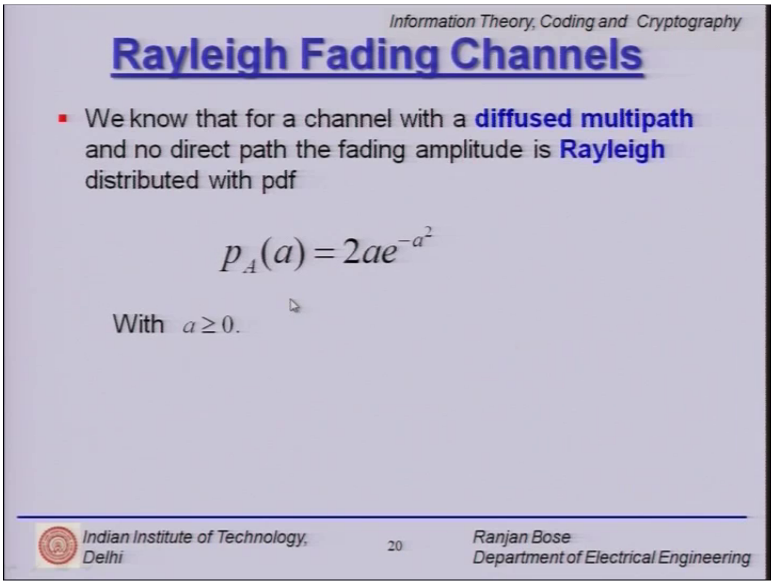

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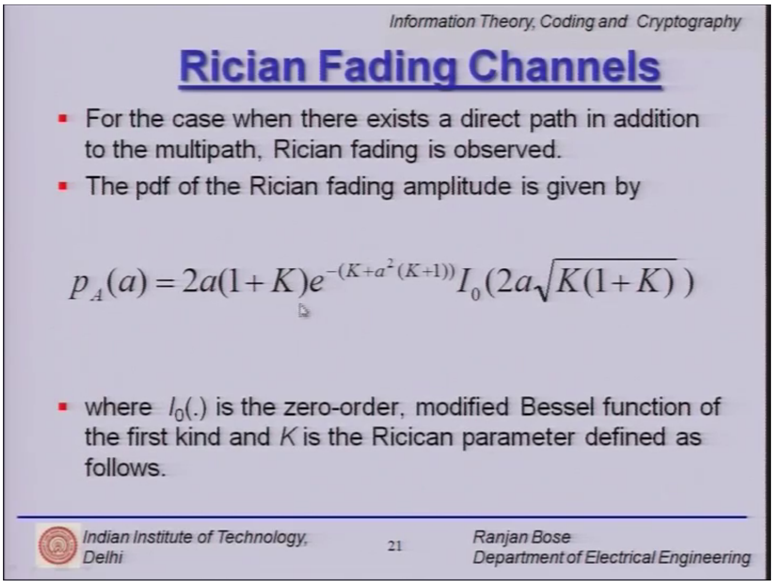

So, we write the pdf of the Rayleigh fading channel as follows. For the case where there exists a direct path in addition to the multipath Rician fading is observed and the following expression gives the pdf of the Rician fading.

Now K is the Rician fading parameter and we will define it shortly and I is the zero ordered modified Bessel function of the first kind. So, this K will play an important role and let us understand what K is.

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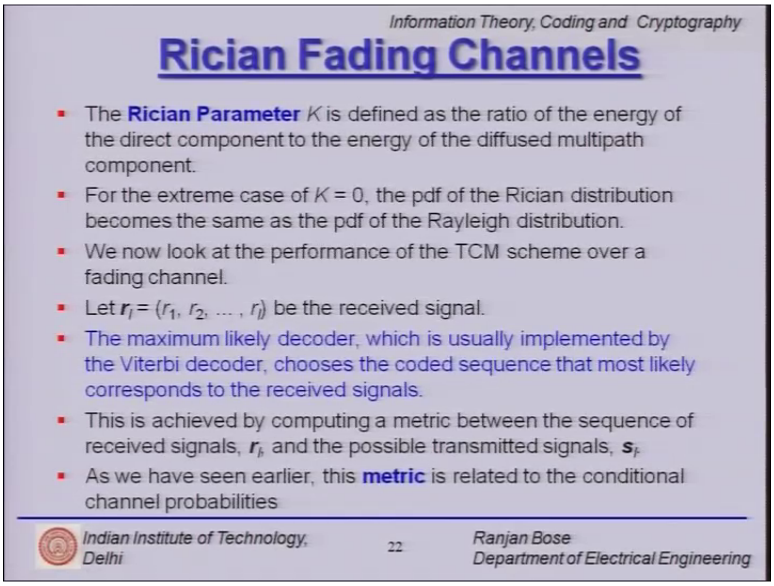

K is the Rician parameter. What is it? It is the ratio of the energy of the direct component of the signal to the diffused multipath component all right. So, if you go back to the diagram, we drew and we talk about the direct path. So, there is a certain amount of energy that is coming through the direct path call the line of sight. And then, there are so many other reflected paths which are carrying this energy to the receiver.

So, the ratio of the energy coming from the direct path to the non line of sight, the diffused path is the parameter K. So, clearly if there is no line of sight; then K is 0 and so, K is equal to 0 implies that we are looking at a standard Rayleigh fading channel. So, a special case K is equal to 0 comes back to the Rician Rayleigh fading distribution.

So, now let us look at the performance of a TCM scheme over the fading channel. Let us say that the received signal be r 1, r 2 up to r l. This is a vector. The maximum likelihood decoder which we will say, we will implement using the Viterbi decoder; chooses the coded sequence that is most likely to correspond to the received signals. So, this is achieved by computing a metric between the sequences of received signal r l. Here is the received signal and the possible transmitted signals s. And we have seen that metric is related to the conditional channel probabilities.

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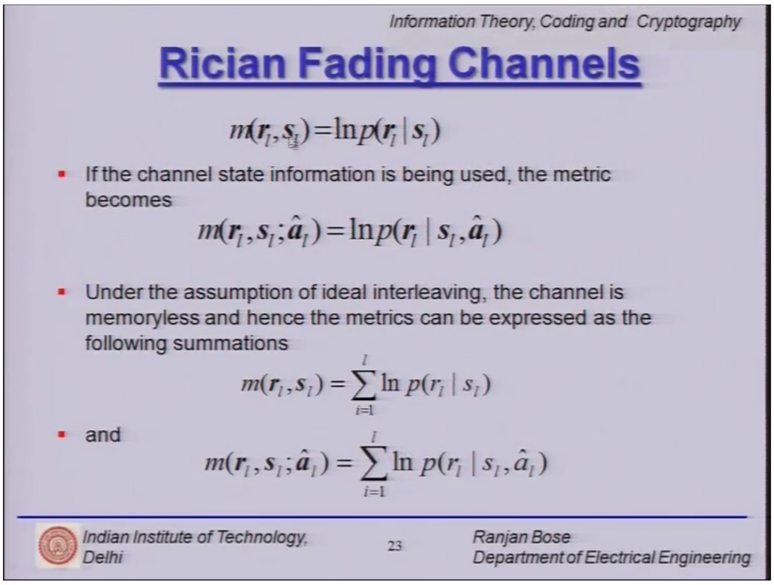

So this metric, we can take as log of the probability that r l; this vector is received given s l is transmitted. So, now, we can say that we have the access to the channel state information. So, the fading coefficients are known. So, at least the estimates are given. So, this metric also has r l comma s l semicolon a l hat right and this is the metric that comes in though.

We have the probability of r l given s l and I have got the estimates right. So, under the assumption that the ideal interleaving is happening, we can assume that the channel is memory less. So, each symbol do not have any implication on the subsequent symbol. Therefore, we have this product because if they are independent, then the probabilities would multiply, but since there is a log. Now that is a summation there.

So, we have, now i is equal to 1 through l; l n p r l given s l because we have declared that each symbol received is independent of this earlier symbols. And if you have the channel state information, you can write this summation i is equal to 1 through l; l n p r l given s l comma a l hat. So, these are the two matrix with which we will work.

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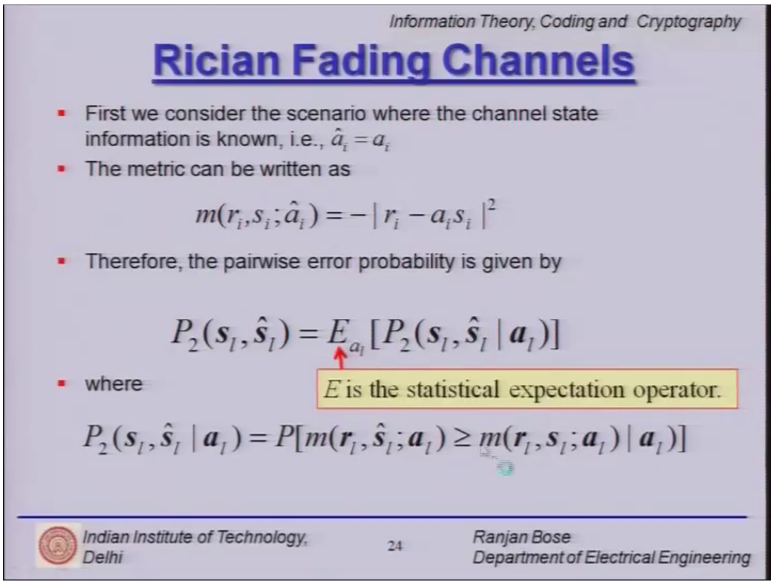

Now first we consider the scenario, when the channel state information is known correctly all right. So, the metric can now be written as minus r i minus a i s i because a i is known right. So, received and we have to see whether s i into a i and this is the distance measure, we have. So, the pair wise error probability P two between s l and s l hat is given by E a l. This is the expectation operator; statistical expectation operator and this is P 2 s l comma s l hat given a l. On what does this P two stand for?

Well, it talks about the sequence s l and s l hat which is estimate; given the channel sate information as the metric of s l hat right with r l greater than or equal to between r l and s l. So, how do I choose the most likely? So wherever, which maximizes the metric given of course, this one. So, it is pretty straightforward, if you write it out.

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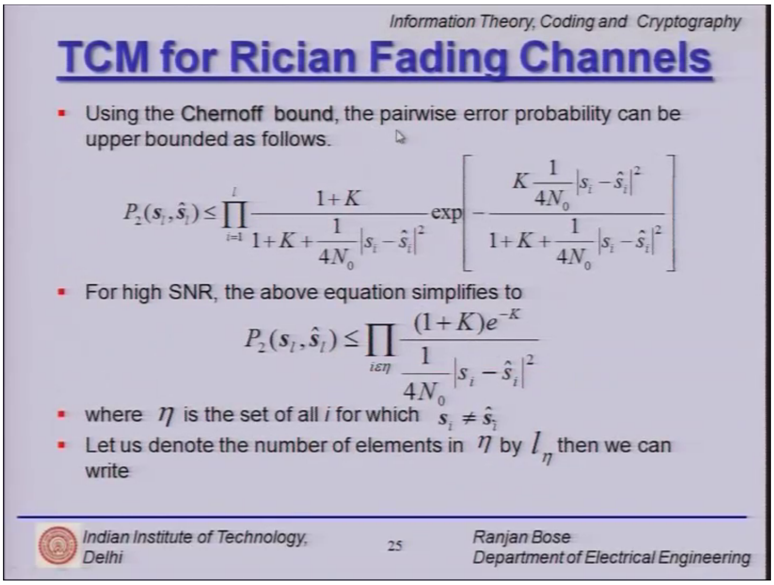

So, we use Chernoff bounds and we can write the upper bound on the pair wise error probability as follows. So, this is the pair wise error probability between s l and s l hat. This is the expression that comes out where K is the Rician parameter. So, if you see for high SNR scenario right; so well, this is low. So, this expression is large and you can neglect this, when the denominator you are left with this. So, you can write out P 2 s l comma s l hat as a product where here; this eta is the set of all i for which s i is not equal to s i hat. So, that is why we are talking about the error probability.

So, let us denote the number of elements in eta by l n; so, l eta. So, we are defining this term l eta. What is l eta? Where is the number of elements in this? So, number of cases where s i is not equal to s i hat. So, error happens.

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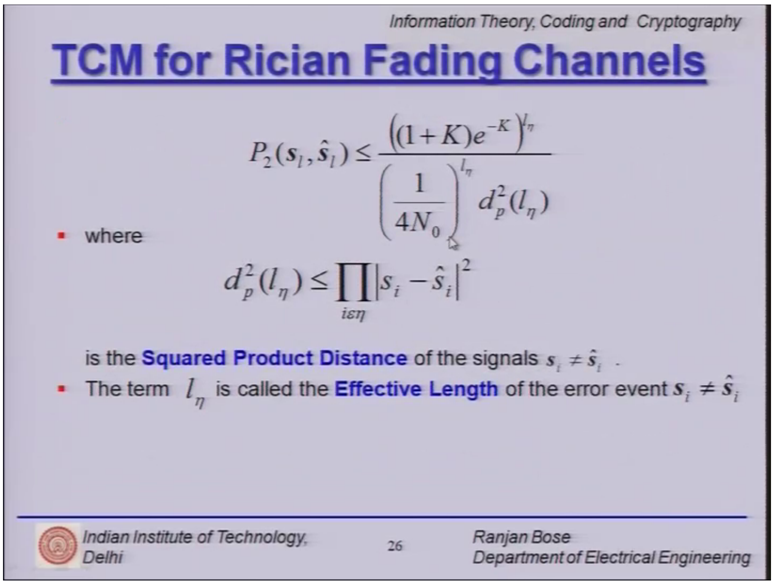

So, if you write it down, then from here you can see in terms of the product i is an element of eta. Therefore, you get here raised to power l eta because only those number of terms multiply. So, you have got this expression now.

So, it brings us to a very interesting conclusion. You can see that we have introduced this term, dp squared as a function of l eta. What is d p squared? d p squared is the product of this squared Euclidean distance; where i is an element of eta. All those paths where you do not have s i is equal to s i hat. So, this d p squared; is the squared product distance of the signal s i not equal to s i hat. This will be a very very important parameter for fading channels; specifically we are talking about Rician fading channel.

So, the term l eta is called the effective length of the error event s i not equal to s i hat. We will use these two extensively for the next few slides, but it is so happens that from the derivation, we have come to this product; the squared product distance.

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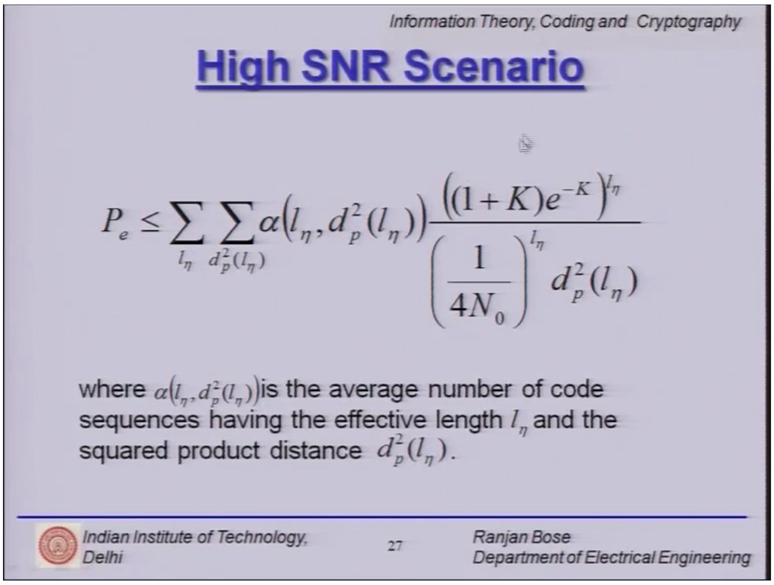

So, if you talk about the high SNR situation, you can write in the upper bound as P e less than or equal to this summation over this, effective length and summation over all the d p squared with effective length. And then you have this expression where this first term, alpha as a function of effective length and squared product distance is the average number of code sequences having the effective length l eta and the square product distance d p square.

So, please note, we are doing sequence by sequence decoding and we have come across a set of sequences which have the effective length l eta and a corresponding square product distance d p squared.

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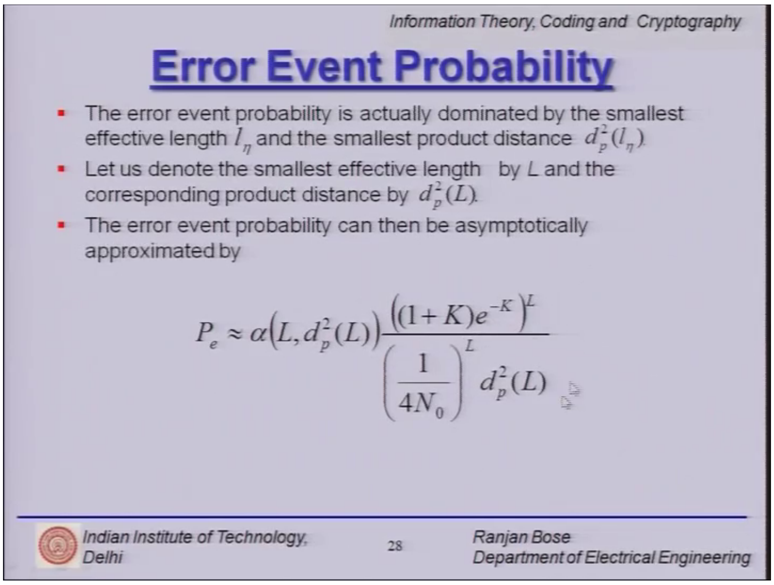

So, the error event probability is actually dominated by the smallest effective length l eta and the smallest product distance d p squared. So, this is an important observation. So, let us denote this smallest effective length by l and the corresponding product distance by d p square l ok. Since, these are the two important parameters which dictate the error event probability. Then asymptotically the error event probability can be written as approximately P e alpha, we have talked about this. And in the numerator, we have 1 plus K e raised to power minus K raise to power L. This corresponds to the noise power and squared product distance.

So, the effective length L and the squared product distance, prominently figure out in the error probability expression and so, we should have our designed criteria based on these

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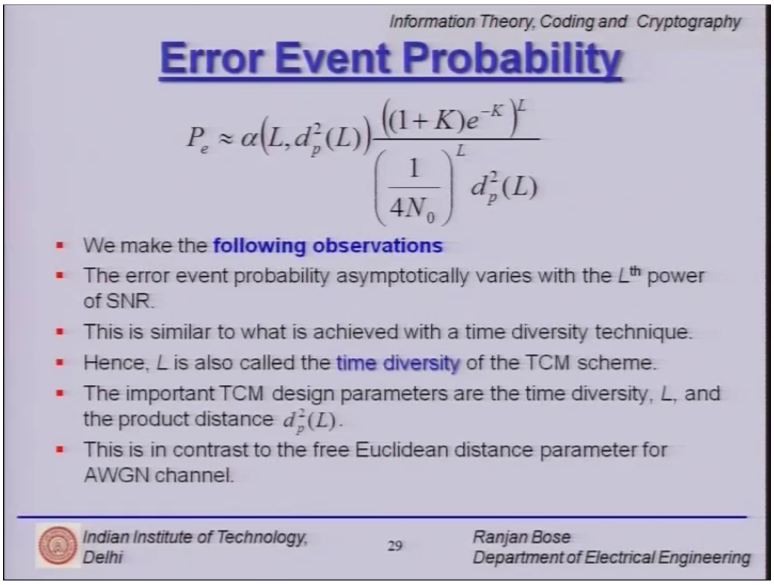

So, we have taken down the same expression on this slide and we make the following observations. So, the error event probability asymptotically varies with the L'th power of SNR here; this term L'th power of SNR. This is similar to what is achieved by time diversity techniques. This is very important.

So, this observation tells me that here, we are getting a diversity gain because of the TCM scheme right. So, L is also called the time diversity of the TCM scheme. We are not explicitly using any time diversity schemes. If you remember, time diversity means that you can transmit the same information in different instances in time; assuming that the spacing between these times is not correlated and you get time diversity gain.

Here L is the time diversity of the TCM scheme; simply because we have an effective length L and so, a design criteria should be such that we maximize this effective length L so as to gain time diversity in terms of the TCM scheme.

The important TCM design parameter are the time diversity L and the product distance d p squared. So, we will use them for our design or comparison of two TCM schemes in fading channel. Please note, we are not talking about d free. There are two different parameters that we have come across; one is a time diversity L and the other is the product distance d p squared. This is in contrast to the free Euclidean distance parameter for the additive white Gaussian noise channel. So, what works for fading channel, may not work for AWGN channel and vice versa.

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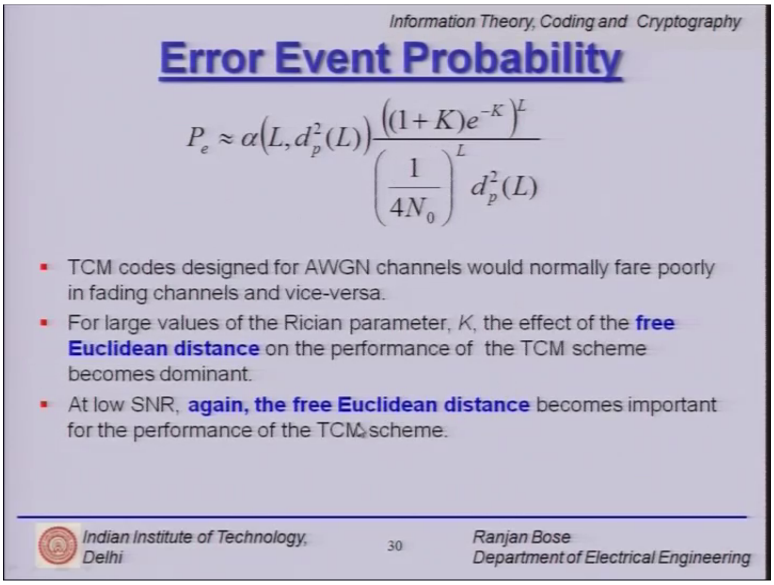

So, this observation we have made and for large values of the Rician parameters K, the effect of free distance on the performance scheme becomes dominant; that is because if you go to very large value of K, then it starts resembling the AWGN channel; the diffused components whether away and it there is pretty much a line of sight, direct line of sight between the transmitter and receiver. And their Gaussian noise plays dominant role and therefore, free Euclidean distance becomes important. You can also see that when K becomes large this terms kinds of vanishes and we are pretty much left with the Euclidean distance part.

Now at low SNR, things again change. Because now at low in SNR part, this component right, we will play start playing a role and then you will have; even though the value of K is not large, then the SNR starts playing this role. So, again the free Euclidean distance becomes important for TCM designing.

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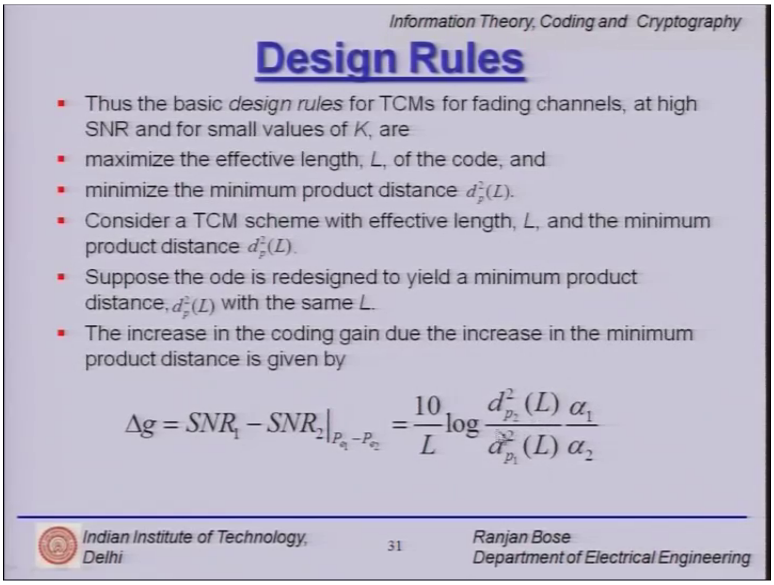

So, let us look at the basic design rules for TCM for fading channels at highest SNR's and small values of K ok. Why small values of K? If the K becomes too large, then it is no longer really making a fading channel; that are the design rules? Maximize the effective length l of the code and minimizing the minimum product distance d p square ok. These are the two designs.

So, if you consider as TCM scheme with effective length L and their minimum product distance d p squared; suppose that the code is redesigned to yield a minimum product distance d p squared with the same L. Then increase in the coding gain due to the increase in their minimum product distance is given as follows. So, this is the coding gain and we have the same effective length, but the product distance changes. So, if you have been able to decrease the product distance. That means, your gain will improve and therefore, you can have a relative coding gain as follows.

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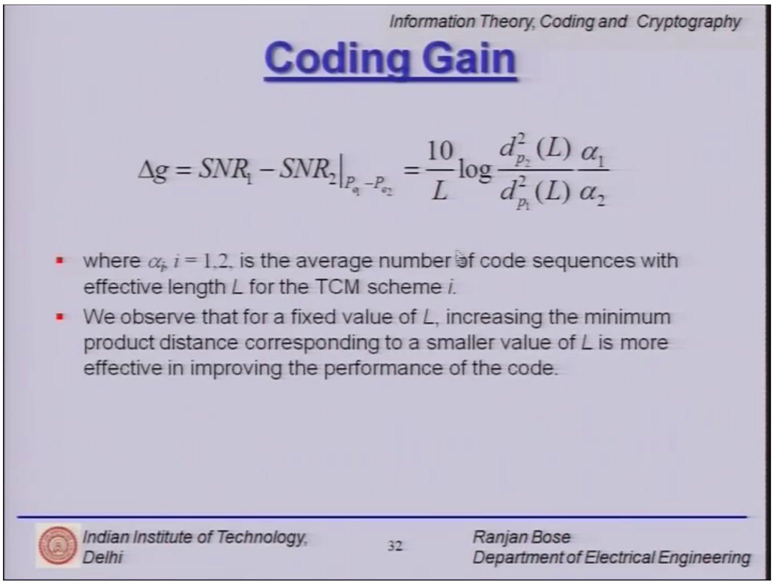

So, here the parameter alpha; alpha 1 and alpha 2 are the average number of code sequences with effective length L. So, it also matters how many sequences in the trellis diagram have that effective length. So, we observe that for a fixed value of L, increasing the minimum product distance corresponding to a smaller value of L is more effective in improving the performance of the code because here the smaller value of the L will give you a better.

So, at on one side you have to increase L, but when you have tweaking the code; then the gain will be more, if the increase is for a smaller L because L is also here. And please note that the product distance squared is in the logarithm whereas, this is inversely proportional.

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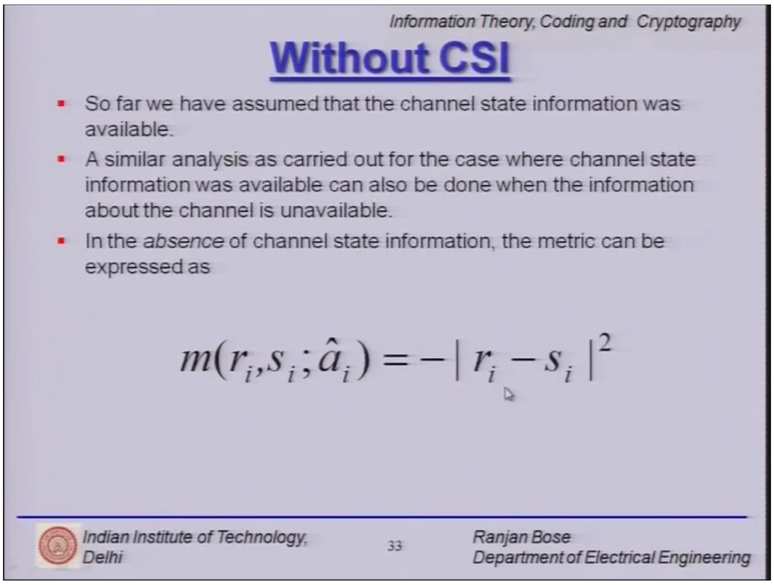

Now so far, we have assumed that the channel state information was indeed available, but we can carry out a analysis when the CSI is not available. So, what will change is the metric? Earlier we had alpha here, the gain parameter. Now, we do not have. So, the best way, it is r i minus s i. So, this is the metric we define.

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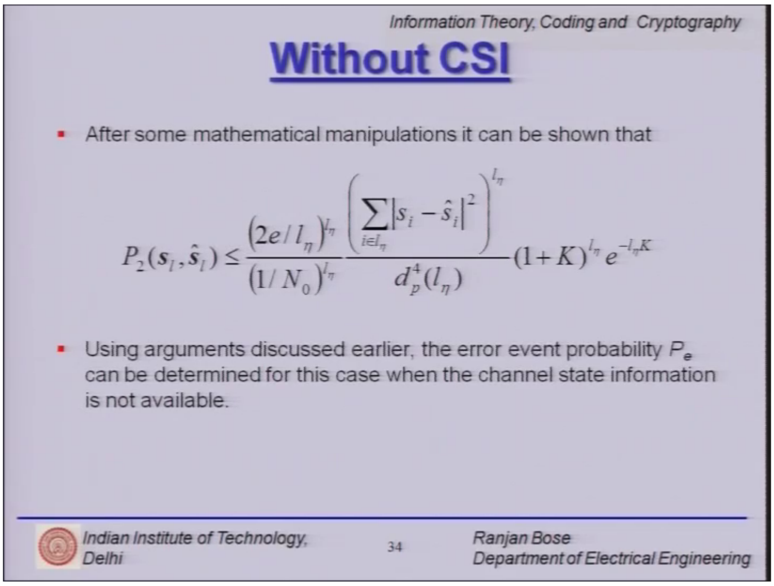

So, we can do similar mathematical analysis and we can get this pair wise error probability upper bounded by the following expression ok. Again the effective length and K show a prominental prominently in the expression. But here please note that we have d p raise to power 4 ok. So, it is even more sensitive here. So, the error event probability P can be determined for this case, when the channel sate information is not available.

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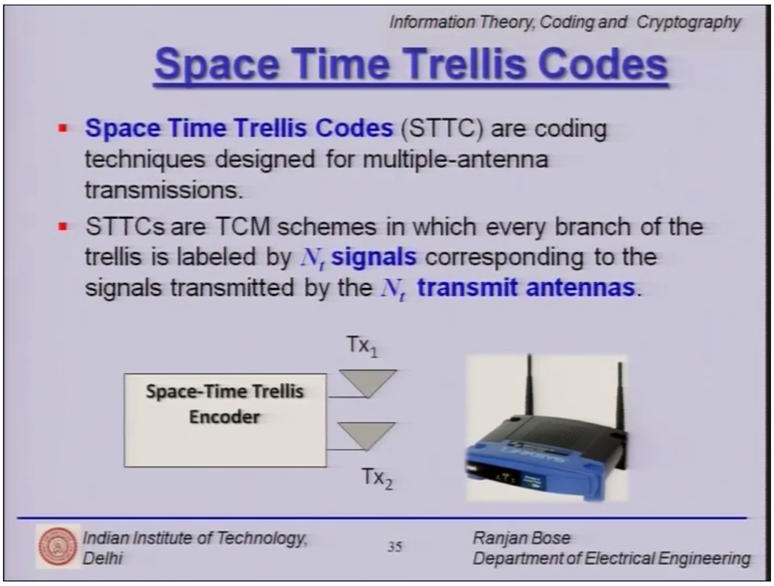

So, with that we kind of lay grounds for a next step which is called the space time Trellis Codes; which is in one way an extension of trellis coded modulation schemes. But what we do is we now look at expressions and cases where we have two transmit antennas.

So, if you see, earlier we had only a 1 antenna to be the trellis and input sequence came in, we looked at that trellis on each branch. We had a single symbol labeled and the transmitter transmitted at. What if we label every branch with a pair of symbols? So, we have now two transmitters. So, symbol one is sent from transmitter 1 and symbol two is sent from transmitter 2. It is a very logical extension and I can extend it to n antennas, N t antennas; standing in front n transmit antennas.

So, what happens, if we do this ok? This kind of a scheme is called space time trellis codes because we also not only use the space coordinate, but also time coordinates. So, in different time instances, we have different pairs being sent from transmitter 1 and transmitter 2. So, we are curious to finding out, what happens.

So, now, we go to multi antenna systems. So, we have TCM schemes where every branch is labelled with N t signals. In this example, there are two transmitter antennas.

So, every branch will have two symbols, if you have N t transmitter antennas; I will have N t symbols.

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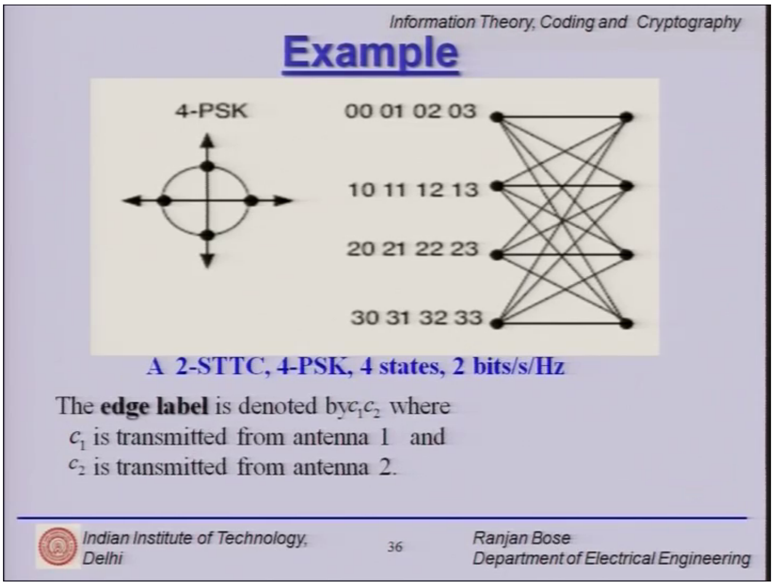

Let us look at a very simple example. So, there are four states in this trellis and it is sufficient to draw the first portion of the trellis diagram because it will repeat infinitely after this. There are 4 emanating branches from every node. So, there should be 4 labels here. Now this is symbol S; S 0, S 1, S 2, S 3. So, this is the 4-PSK constellation, QPSK.

Now in a normal TCM schemes each branch will be labelled with either s 0 or for a short 0 S 2 or 2 S 4. Or here it is QPSK, so S 0 S 1 S 2 S 3; so, they will be labelled here. But here you see there is a pair. So, 0 0 means, if this branch is to be traversed; then transmitter antenna 1 should sent S 0, transmitter antenna 2 should sent S 0.

If we go on this branch, then transmitter antenna 1 should send S 0, transmitter antenna 2 should send S 1 and so and so forth. So, this is a mapping scheme. So, if the label is c 1 c 2 for any edge, then c 1 is transmitted from antenna 1 and c 2 is transmitted from antenna 2; a very simple extension of the TCM scheme.

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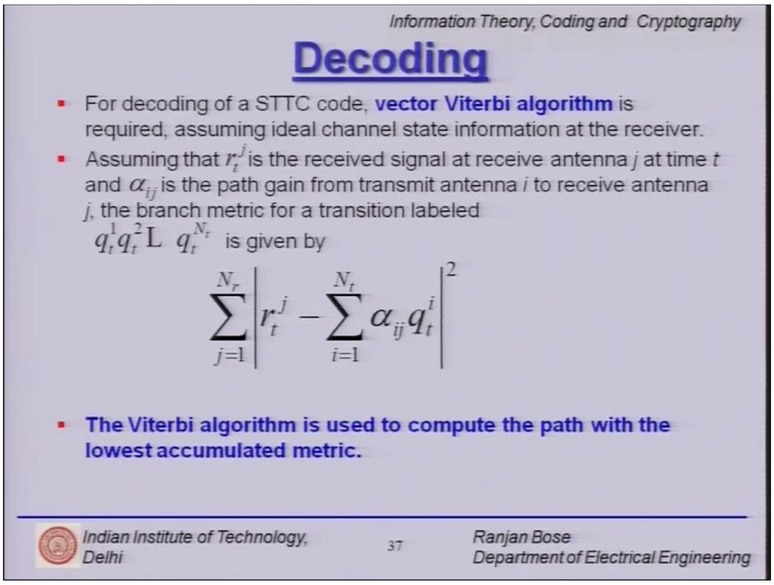

So for decoding, we would use vector Viterbi decoding right and we can assume that channel state information is present at the receiver. And let us see, what how do we define it. So, assuming that r j is the received signal and the received antenna j at time t and alpha i j is the path gain from the transmitter antenna i to receive antenna j and branch metric of the transmission is labelled as follows.

So, this is q 1 t, q 2 t dot dot dot up to q t. So, then we have all the path gains in the transmission path times the received signal and minus r t j. So, the Viterbi algorithm is used to compute the path with the lowest accumulated metric. So, please note, what we are doing is defining the metric slightly differently taking into consideration both or all the N t signals that are transmitted

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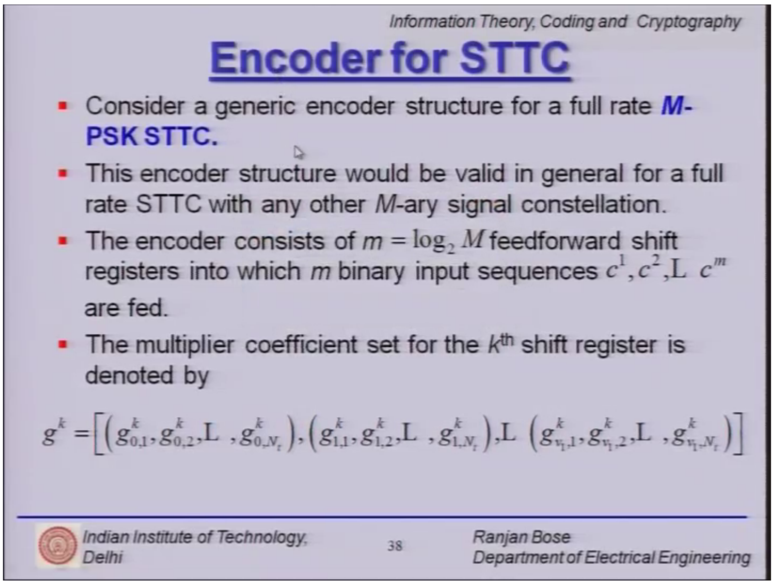

So, consider a generic encoder structure for a full rate M-PSK STTC; space time trellis code right. So, what we will do is in the next slide we will look at the generic structure of this and it will be better to start with the circuit diagram and or a block diagram of this encoder first.

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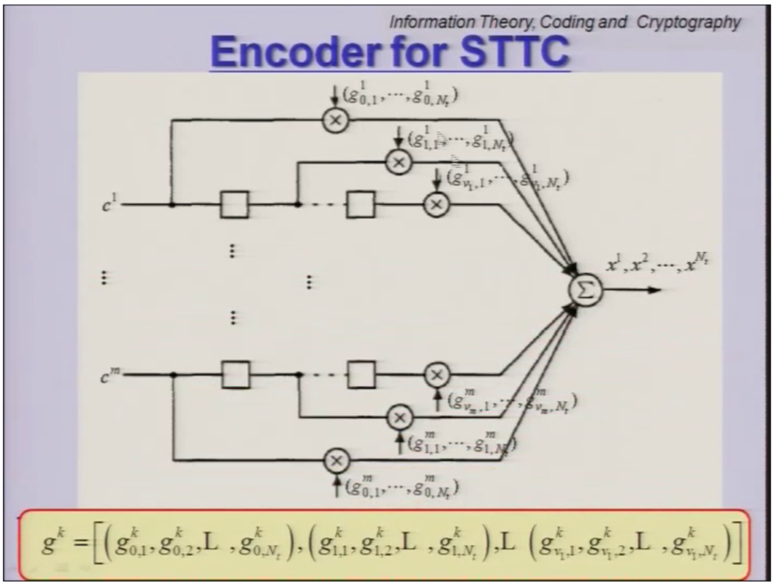

So, we have c 1, c 2 up to c m coming in and then we have these coefficients that we just now looked at. They are multiplied and then they add and then this is what is sent out.

This g k is represented by as follows. This is the general encoder structure for a space time trellis code.

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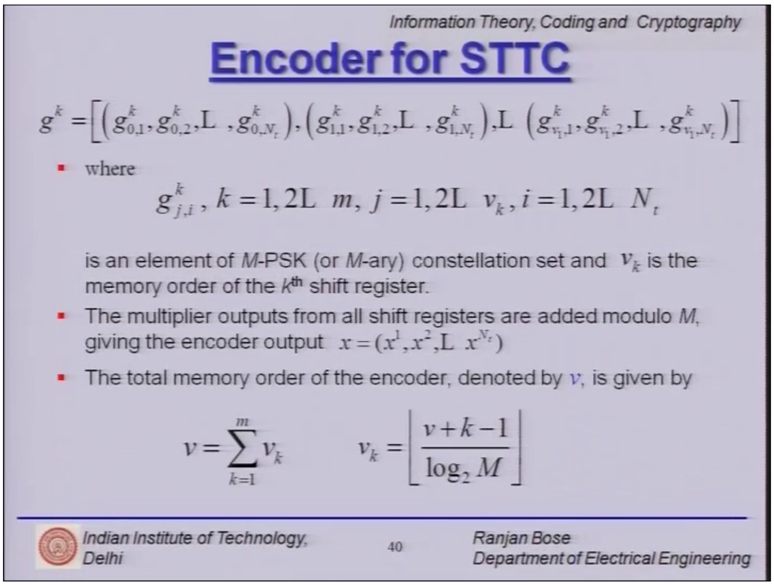

So the what we can do is, we can use that diagram to understand that the multiplier outputs from all the shift registers are added modulo M and we will give the encoder output as follows x 1, x 2 dot dot dot up to x n raise to power N t and the total memory order of the encoder is denoted by nu is as follows.

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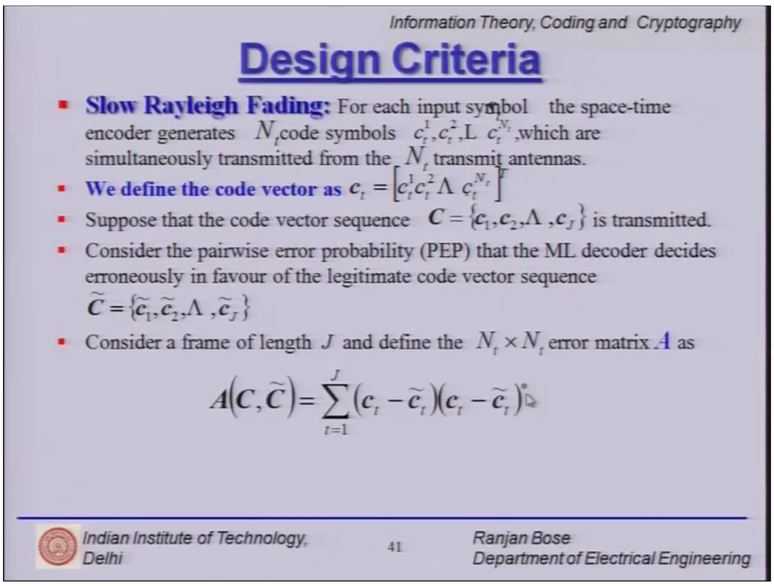

So, we have 2 conditions; Slow Rayleigh Fading and fast Rayleigh fading. So, if you consider the slow Rayleigh fading, so for each symbol the space time encoder generated N t code symbols c 1, c 2 t up to c N t t which are simultaneously transmitted from the N t transmit antennas that we have seen.

Now we define the code vector as follows right. So, this is the code vector sequence which is transmitted and the ML decoder decides to erroneously find the C tilde as follows. So, transmitted sequence, received sequence. So, frame of length J is considered. So, we have now an error matrix as follows which is an N t cross N t error matrix A and we will use this for design criteria.

What is A as a function of C and C tilde? C was sent C tilde was received. It is we define it as c t minus c tilde times c t minus c tilde complex conjugant.

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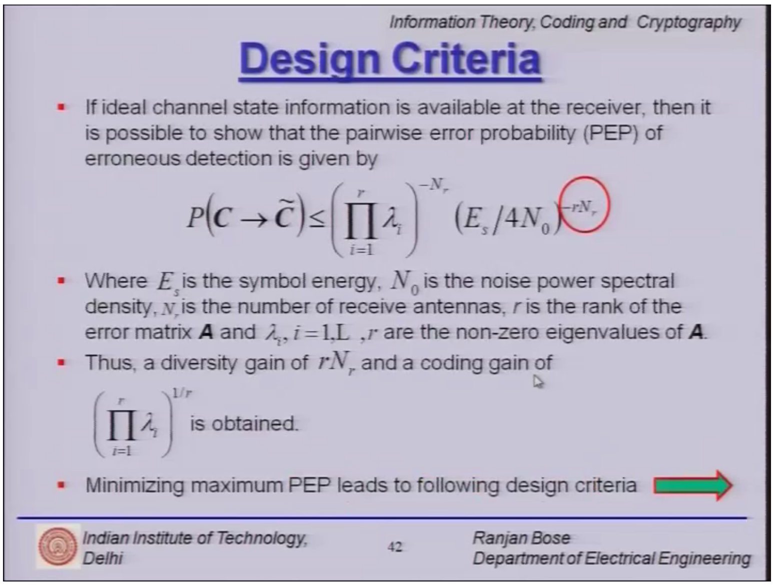

So, if ideal channel state information is available at the receiver, then it is possible to show that the pair wise error probability of their erroneous detection signal is given as follows. So, we do not go for the derivation, but this is the pair wise error probability that C was sent and C tilde was received, at least decoded incorrectly as follows. And we find very interestingly that the Eigen values of the a matrix come into picture. N r is the number of received antennas right and r is a rank of the error matrix a which we are defined previously.

So, based on this to minimize the PEP which is the pair wise error probability, we have to play with the N r and r the rank right. So, rank has to come into picture. So, please note that the diversity gain is given by so, this is the effectively the SNR. So, diversity gain of r N r is obtained and the coding gain is given by the product of all the lambda i's which the Eigen values of a 1 over r. So, this r is playing a major role in terms of the diversity gain and the coding gain.

So, for the first time, it is very explicit that a space time trellis code is going to give you both a diversity gain and coding gain. So, diversity gain tantamount to the slope of the (Refer Time: 49:50) curve and coding gain is a kind of the left shift of the curve. So, we get both diversity gain and coding gain from space time trellis code. And now with this expression we can formulate the rank determined criteria

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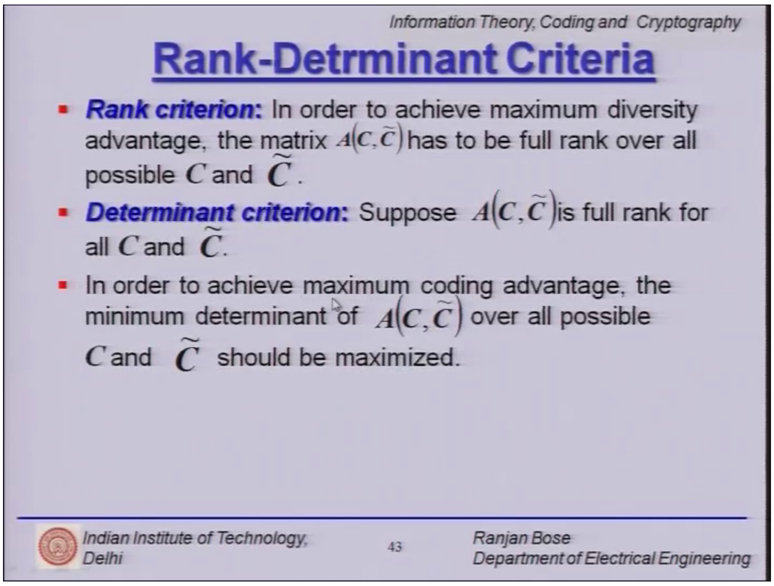

What does the rank criteria say? In order to achieve the maximum diversity advantage, the matrix A C comma C tilde has to a be full rank because r was a rank. So, we have to find this C and C tilde. We should design such that we should have this to be full rank and suppose A is full rank, then in order to achieve the maximum coding advantage, the minimum determinant of A over all possible C and C tilde should be maximized. So, this is known as the rank determined criteria that was so slow fading channel

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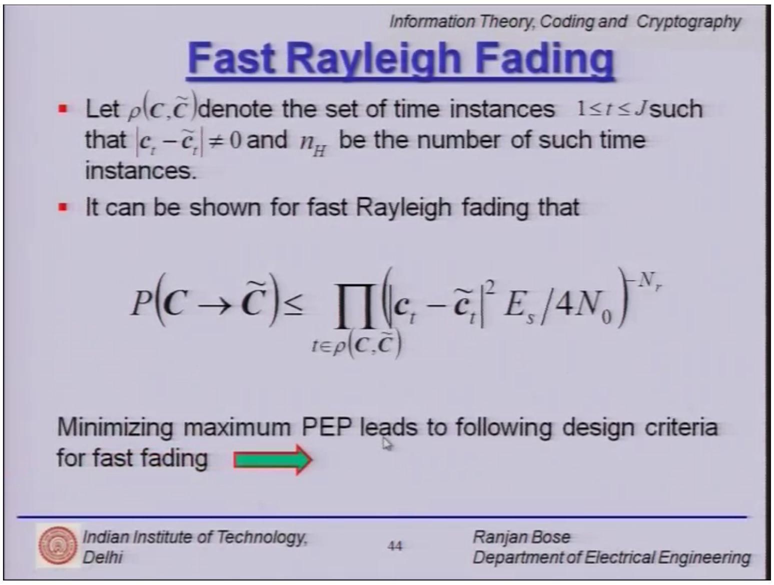

And if you do a similar analysis for fast fading channels, you get a parallel set of design criteria. So, you can first show that: what is the pair wise error probability for fast Rayleigh fading channel. The expression comes out something like this. So, the rank r term is missing here. Now the minimizing the PEP leads to the following design criteria for fast fading channels.

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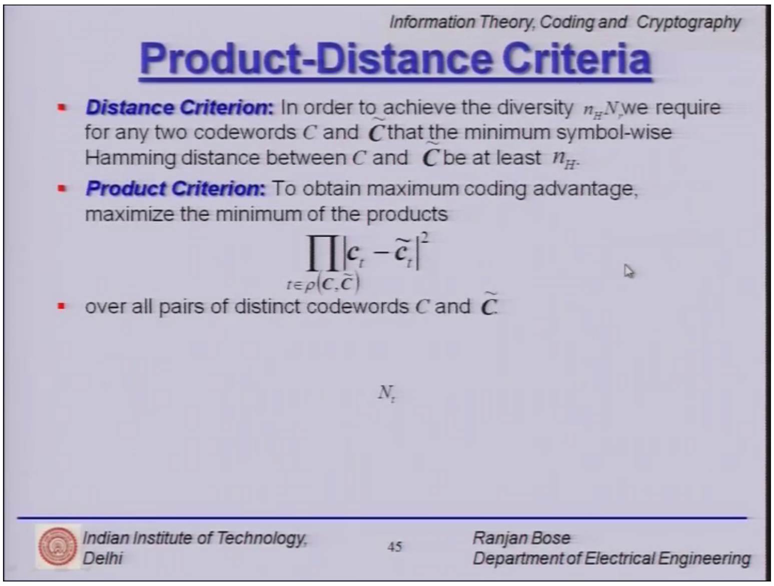

First we have the distance criteria and then we have the product criteria. Therefore, they are called the product distance criteria for fast fading channel. In order to achieve the diversity in n h N r we require any two code words of C and C tilde such that the minimum symbol wise hamming distance between C and C tilde must be at least n H.

So, this will give you the diversity of n H N r; that is number 1 and the product criteria again coming from the PEP expression is to obtain the maximum coding advantage, we have to maximize the minimum of the products as follows. So, this can be done by simple search algorithms to find out good mapping. Finally, it comes down to the mapping. How do we assign the labels, the edge labels of the trellis? So, these this is the product distance criteria, with that we come to the end.

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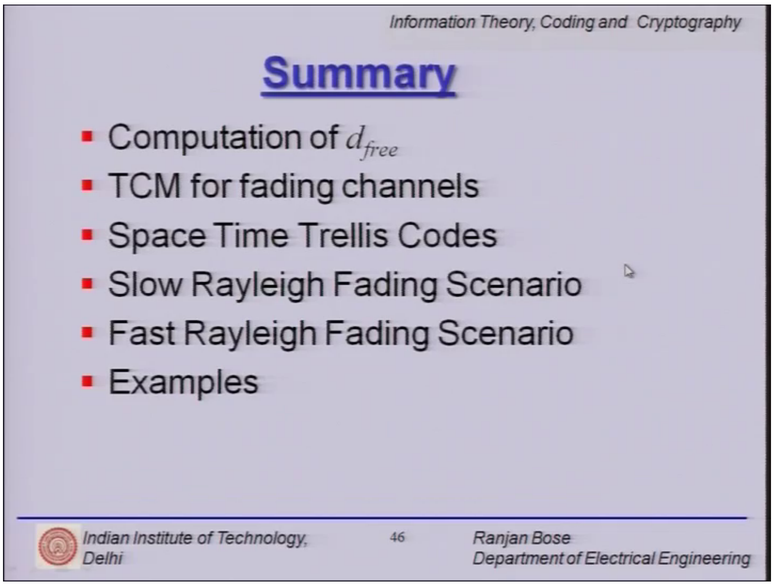

So, let us summarize. We have looked at how to compute d free then we went to study TCM for fading channels. Then logically we went into space time as codes, and looked at 2 scenarios slow, and fast Rayleigh fading channels, and we looked at a couple of examples.

With that we come to the end of this lecture.