Fundamentals of Power Electronics
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Lecture – 08
PN Junction

Hello and welcome back. Having understood the formation of these you know now modified semiconductors which are also called extrinsic semiconductors the p-type and the n-type, let us now see what happens when we try to bring you know these p-type and n-type semiconductors close to each other, ok. So, basically what we are saying is that let us take as an example of silicon crystal.

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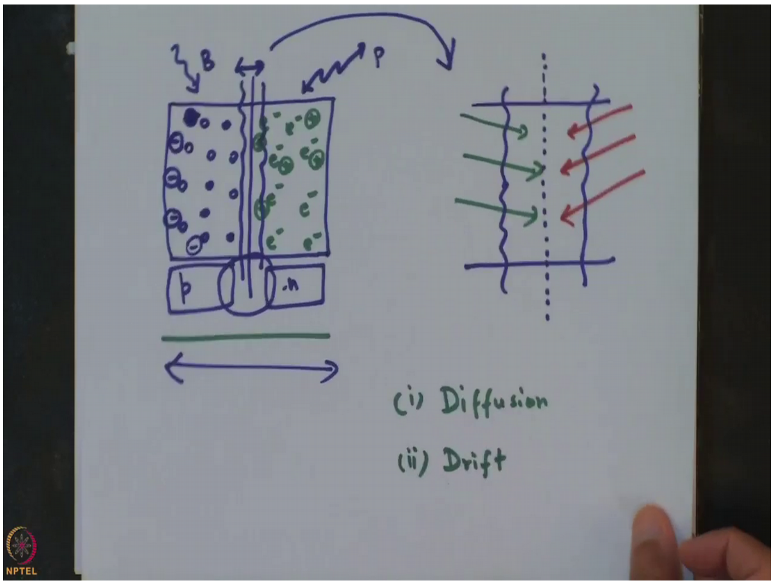

You actually bombard let us say one part, you bombard it with several trivalent atoms like the borons, you know aluminium and so on, those kinds of impurities. And as we know we will be ending up with several holes. These are the holes which are created, but of course we also have with each of them one negative ion, ok.

So, the overall material remains neutral. So, if it is a hole which is formed a corresponding negative charge has been introduced also, ok. So, this is let us say boron. And here let us say on this side we bombard with phosphorus atoms, let us say phosphorus atoms. And this gives rise to as we know these are the donor atoms so there are many electrons which are now available, ok, in this particular region. And of course, they also have their corresponding ions which are plus, they are positively charged ions everywhere you have this situation. The overall material is neutral no net charge, ok.

Talking about the P-N junction let us say we have a crystal on which on one side we are growing an n-type material and on the right side or the other side we are going the p-type material. So, let me just draw this energy level diagram for the n-type first.

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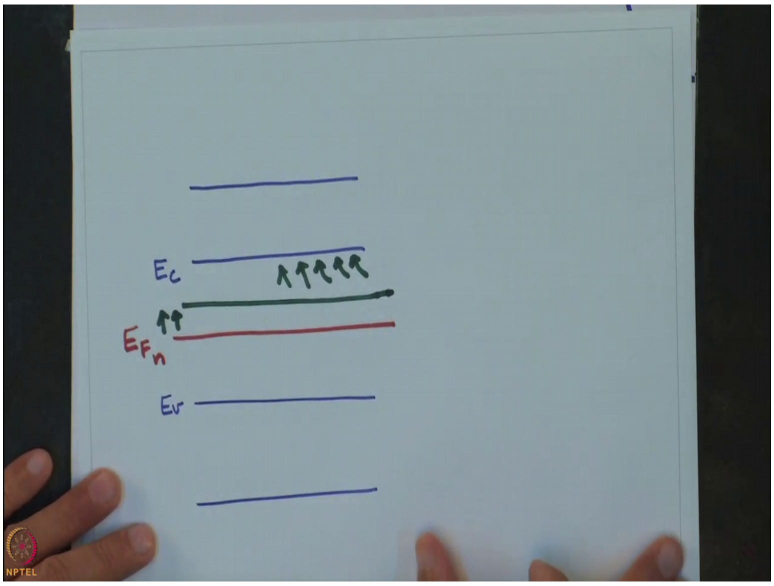

So, let us say these are the various energy levels, let us say that this is nothing but my conduction energy band level and this is my band gap this entire what you see here, this is the valence band level and so on. And let us say now let us make use of a very important parameter which is used in such systems which is called the fermi energy divided by E F. Now, what is fermi energy? Fermi energy is nothing but a chemical energy you know which is actually remaining constant for the material throughout you know in the equilibrium condition.

E F is approximately equal to the average of the conduction band energy level and the valence band energy level. So, we can say that it is equal to E c plus E v by 2, where E c is the conduction band energy level and E v is the valence band energy level, ok. So, which means basically that even when the two systems they merge with each other they end from the p side they are going to have a constant fermi energy, ok. And to do this we will see that how the energy levels shift. So, I am going to draw these diagrams further and actually I am going to explain that to you.

So, let me just draw the fermi energy level. This is the original level without any doping. So, let us me let me call this as E F for the n side, E F of n. Now, when we do doping what you will see is that we actually are doping with the donor atoms whose energy is very close to the conduction band. So, we are here. So, basically the net result of this is that you know my fermi energy shifts up. So, this E F n actually shifts up to a new value which is shown by this green colour, ok. Now, let us see what will happen correspondingly for a P-N junction, ok.

So, now let me just do the same thing for a for the p side materials. So, they are the other side of the crystal semiconductor crystal where I am trying to now grow p-type material. So, let me just draw again the energy diagram.

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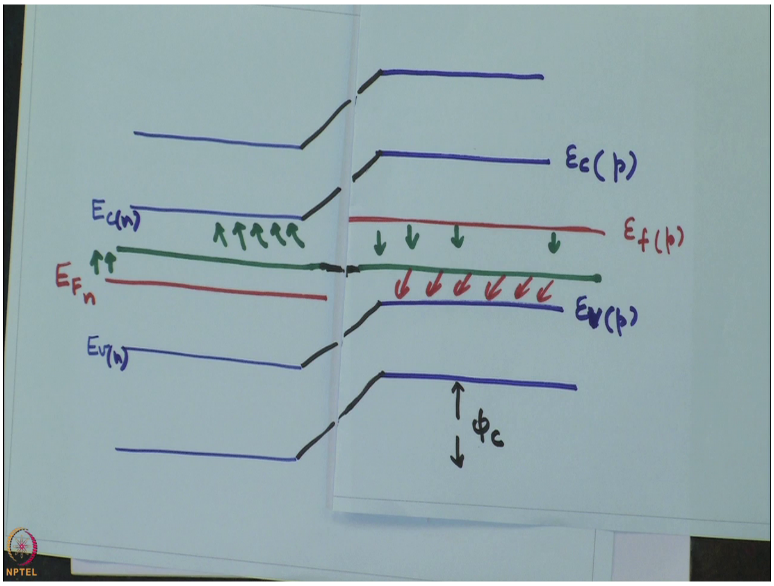

So, let us say this is the as before this is the conduction level and let me just say p here just to you know mention that this is the p-type and here also let me just say n just to show that this is the n-type in the previous diagram. And also let me just say E v of n here. Now, on this side if I further draw this I can say that this is my valence band E v p-type and like this.

So, what will be the fermi energy for just the p-type? It will be somewhere here, this is the fermi energy. Let us call this as for the p-type E F. Now, when we do the doping we actually bombard the acceptor atoms and as you know that the holes are associated with the valence band, so you are actually ending up you know or introducing these acceptor atoms or the holes on this side, which means that my fermi energy for this side p-type actually will shift down and you know it will it will come somewhere here. Let us say this is here. So, this is the new fermi energy.

Now, if you bring the two sides the p-type and the n-type together, I just keep them together and I just go by my definition, fermi energy should have the same value for the combined n p sides under the equilibrium condition, ok. Then, what we see is that there is a basically shifting of the other levels. You will see that the conduction band level, the valence band level on the p side they actually shift, they actually come on to this side and I am just trying to show you this. So, what you see here is that you know this is how things will be. So, basically I can just say that I can combine my green lines on the two sides which means that the fermi energy levels are same, while as far as the other levels are concerned they will all shift you know upwards as you can see I will have to just make this kind of a structure over there. This is how this P-N junction would look like.

So, we can actually see that there is a gradient, there is a potential gradient which has been created and this is what actually we denote as the junction potential phi c. So, we can see that there is a transition region between the p and the n-type material. As we will see later and analyze in due course that this is the depletion region or the space charge region of the P-N junction, ok.

Now, let us just see here very close to this junction. Now, we know this is the p-type semiconductor we have created this is an n-type semiconductor we have created and let us just discuss this junction point where these two material they are just opposed against each other. So, let me just draw this again here. So, let me just draw this and now let me just say that this is the junction region. So, this region which was here has been now drawn here again.

Now, if I also see this region and analyze and see what is going to happen? What is going to happen is that now that there are the two types of charge carrier movements which will take place when this situation happens, ok. Now, one of them you know is what is called the diffusion we can see them in the left side that many holes, their concentration is much higher than their concentration on the right side. Likewise, the concentration of electrons is much higher on the right side compared to the left side, ok. So, there is a concentration gradient of holes. So, basically what happens that the holes try to flow because of this concentration gradient from left to right, ok. So, we have basically an influx of holes from left side to right side, ok. Likewise, if you see the right side which is rich in electrons and as a much higher concentration of electrons compared to the left side we will see that there would be a flow of electrons.

The second phenomenon which will cause the flow of charge carriers is the drift phenomenon, right. The I have just written the second type of you know phenomenon which will cause these charge carriers to flow as the drift phenomenon which is under the action of an externally applied field or a created or a generated electric field. We will come to that, ok. Now, what will happen? Diffusion is taking place, the holes are trying to flow into the junction area from left to right, and the electrons are trying to flow into the junction area from right side to the left side.

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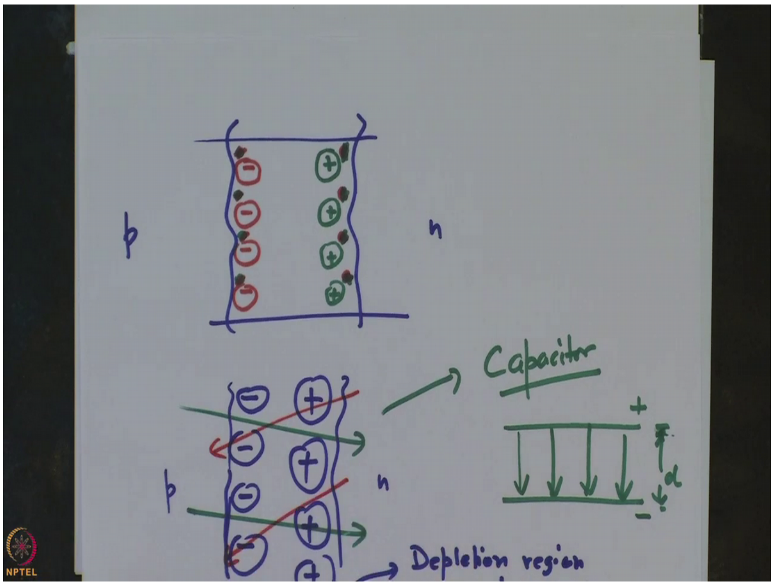

So, basically what happens if I just draw this again if I just draw the junction again that the holes moving in from left to right, ok, they would neutralize you know the some of the ions which are present along with their electron pair, ok. So, basically you know the situation will be that you have these ions and each of them you know these are positive ions each of them is balanced by an electron. So, basically when you are holes flow from left to right, they neutralize these electrons, we combine with them. Likewise, if you see on the left side of this junction you have negative ions, this is what we had particularly this is what is there here which are having holes, as pairs to neutralize them.

Now, when the electrons flow from right side to the left side they end up neutralizing these holes. So, basically what happens is that you end up with a region, so p-type, n-type and you actually end up with a small region between this p-type and n-type material, which is made up of these ions positive ions on this side and negative ions on this side, and there are no charge carriers in this region, ok. This region is what is called as the depletion region, no charge carrier, or space charge region.

Now, it is very easy for us to see that you know this kind of a structure is nothing but a capacitor. So, this is nothing but a capacitor. And we all know that a capacitor this is the let us say the positive plate and this is a negative plate it will have a field, and depending on this distance it would have a potential between these two plates, ok. So, this is what has happened because of this diffusion current which has flown in terms of holes and electrons. Formation of a depletion layer which is devoid of charge carriers which is having an electric field as a result of this electric potential.

Now, the question is how long will this diffusion current continue to flow. Now, the answer to this is that the diffusion current flows as long as this electric field which is created in the depletion layer or the potential that is created in this layer does not balance out the flow of the diffusion current. So, basically the tendency of this electric field that we have created is to basically cause a current that will flow now in the opposite direction to the diffusion current. So, if I was to say that my diffusion current direction net direction is this, assuming the direction of the holes to be the positive current and the actual current, ok. Now, what will happen is that drift current will try to flow the other way around? And therefore, this process of diffusion would continue till the drift current is equal to the diffusion current and balances it out, and that is where the whole process stops.

Now, that there are two types of currents which are flowing one of them is the diffusion current the other is a drift current. And you have seen that at the equilibrium point when the two currents become equal that we call as a thermal equilibrium condition. And if there is no external voltage applied then we find that this equilibrium is maintained, otherwise it is disturbed and a new equilibrium point is created, ok.

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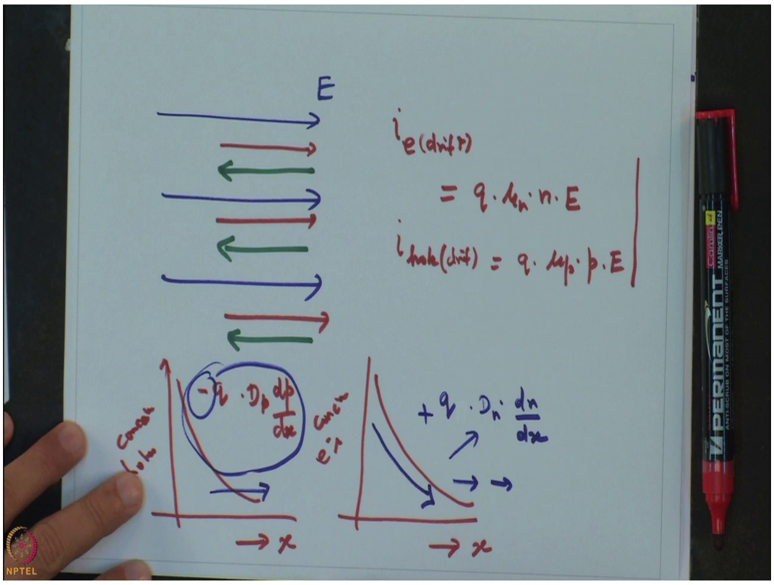

Now, if the applied electric field is on let us say towards the right this is the E field then we know that the holes, they are going to experience or they are going to cause a hole current in the same direction, the same direction is the field shown with a red colour. The electrons on the other hand will actually constitute a current which will be in the opposite direction to the applied field.

The net current due to the applied electric field is created because of the net result of the electron and hole current on account of the drift. And the expressions can be given for the electron drift current we can say that it will be you know i electron drift we can say that this is nothing but two times mu n into n into E, where q is the charge the electron, mu n is the mobility of the electron, n is the concentration which is the per unit volume concentration the number of you know the electrons per unit volume and E is applied field. Likewise, we have the hole current on account of the drift which is given by q times mu p into p and E, where q is a charge of the whole, mu p is a mobility of the hole, p is the concentration per unit volume of the holes and E is the applied electric field. So, it is the net sum of these which constitutes the total drift current.

Now, similarly if there is a gradient of the concentration from one side to the other. So, for example, if there are more holes on the left side compared to the right or there are more electrons on the right side compared to the left side they will actually flow from the high concentration area to the low concentration area. And I can actually show this with the help of these two, you know these two diagrams where I can say that the y axis is actually the concentration of holes and here also the concentration in the next plot y axis shows the concentration of electrons, and since we are doing the analysis usually in the x direction let us say this is the x direction. So, we can see that there is a there is a diffusion gradient that we see as we go from left to the right side both the cases.

And in the case of the holes we will find that the current that will be caused because of the diffusion of holes on account of diffusion will be given by you know minus of q into D p which is the nothing but the diffusion constant of holes into dp by dx. And for the electrons it will be given by q into D n which, where S n is nothing but the diffusion constant for electrons into dn by dx, ok.

Now, there is a negative sign that will come along with the expression for holes that is because you know we have the concentration gradient for the holes which is in this direction, and which is negative, ok. So, we have a negative direct gradient of the diffusion as we go towards the right, but the conventional current is also in the direction of holes. So, wherever the holes are flowing that is the direction of the current, so there is a minus sign which actually remains there. But in case of electrons because the direction of the current is considered to be opposite to the direction of the electrons and the gradient of the diffusion gradient of the electrons is considered to be again negative in the in this x direction, that is why you get a positive q n into D n into dn by dx that expression. So, basically these two currents because of the holes and electrons diffusing from one side to another are added up and we get the net diffusion current across the P-N junction.

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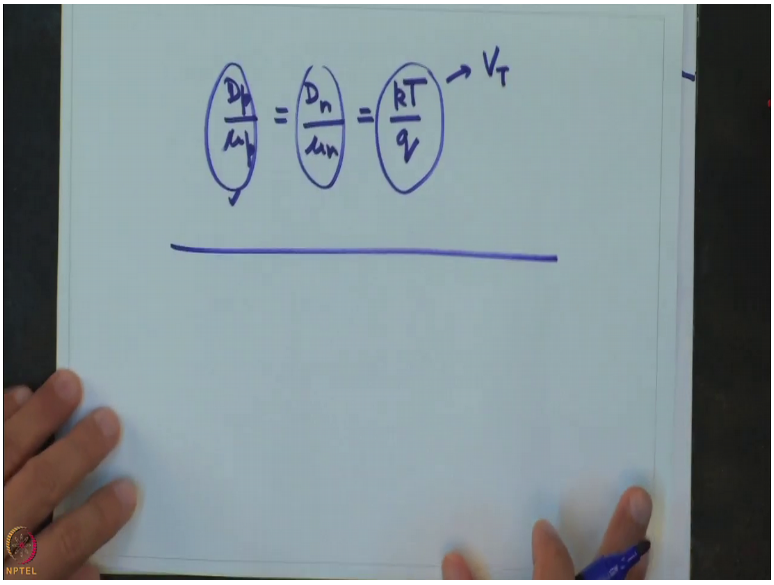

Now, this analysis part of the diffusion current drift current, it actually gets simplified if we use a very important relation which is called the Einstein's relation which is given by D p, the diffusion constant of holes by mu p the mobility of holes equal to the diffusion constant for electrons to mu n and this expression is actually given by kT by q. So, this ratio and this ratio these two ratios are equal to kT by q, where kT by q is often denoted by VT by some authors. So, this may be a very important relationship that we may be using in the future.

So, if you take a P-N junction which is formed by you know juxtaposing p and n-type of semiconductor material you actually end up with this kind of depletion layer, which will have a feel and a contact potential, ok. So, having understood the you know the formation of the P-N junction and the mechanism involved let us now see how a P-N junction works when an external voltage is applied across it.

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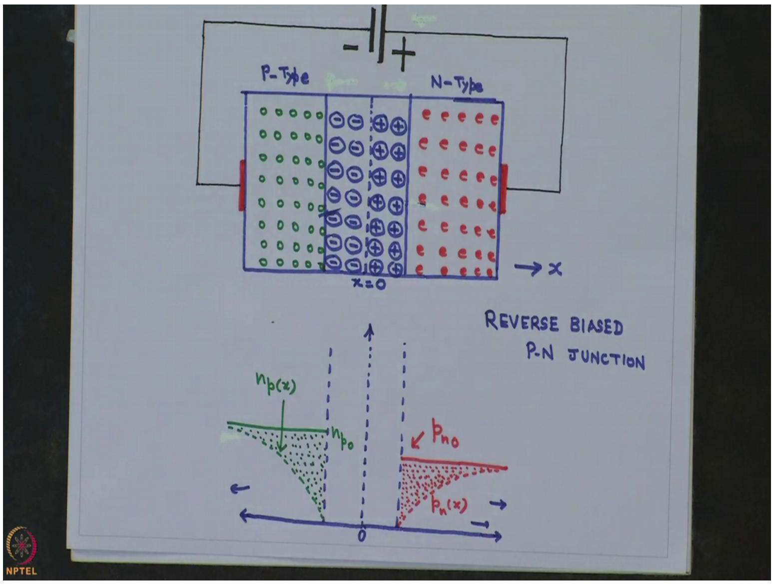

The P-N junctions are used as diodes which do not have a control terminal. Now, if you remember in my earlier lectures, we have grouped diodes under the category of type 1 switches, now they turn on and turn off depending upon the magnitude and the polarity of the voltage that gets applied across them you know by the circuit in which they are connected. You can see that you know there is a junction and on the two sides of that you can see the p-type and the n-type semiconductor. And you can also see in red colour the two metallic contacts which have been provided you know for connecting the wires.

Now, p-type material or the p-type side, it is actually called anode and it is denoted by a. While the n-type side it is actually called the cathode and denoted by k. Now, depending upon the polarity of the voltage you know a P-N junction may be subjected to either a forward bias where the voltage V KA is greater than 0, you know or it could be subjected to a reverse bias when a negative voltage that is V AK less than 0 is applied.

Now, what you see right now is the picture of a reverse biased P-N junction which I have drawn sometime back. Just below the P-N junction you can also see the distribution of the minority charge carriers. In reverse bias, the p side is connected to the negative terminal of the supply or the battery while the n side is connected to the positive terminal of the battery. For these conditions, the holes on the p side they move towards the left attracted towards the negative terminal of the battery.

Similarly, electrons on the n side they move towards the positive terminal of the battery thus both holes and the electrons they move away from the junction. As the holes in the electrons move away from the junction the depletion layer width increases. Now, the process will stop when all the holes will flock the negative terminal and all the electrons will flock the positive terminal. And the applied potential will be completely balanced by the potential developed by the depletion layer. Now, there is no current flow obviously, because there are no carriers and the only way these carriers could come is probably you know the p-type carriers, the minority which are in minority in n-type material that they actually cross the junction over the p side and then travel to the negative terminal, but you know there are too few holes on the n side.

Likewise, there are too few electrons in the p side, so this is not possible. However, thermally generated carriers holes in n-type and electrons in p-type do keep coming up and they move over the junction constituting a small current which is called the reverse saturation current it is denoted by i naught or i s. Now, this current does not depend on the value of the applied potential however, it shows a very strong dependence on the temperature.

Now, talking about the reverse biasing of a diode it is important to also highlight and mention the phenomenon of voltage breakdown or the device breakdown when the voltage in the reverse direction exceeds a certain voltage. Now, this will be reviewed again when you will look at the I-V characteristics of the diode. Now, when we talk about the breakdown, there are two types of breakdown mechanisms which are found in diodes. One of them is a Zener breakdown the device which happens if the breakdown voltage is in the range of typically 5 to 6 volts or less. The avalanche breakdown is the second type of breakdown mechanism which is absorbed in P-N junctions. This happens if the breakdown voltage is typically 6 to 7 volts or higher.

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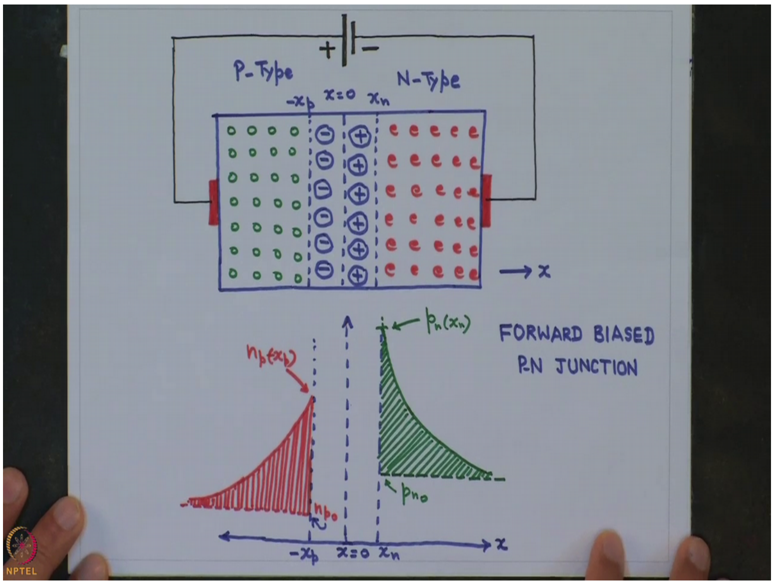

Now, what you see is a P-N junction which is connected in the forward bias manner. So, you can see that the p-type side or the anode is connected to the positive terminal of the supply and the n-type which is the cathode is connected to the negative terminal of the battery.

So, as you can see in the forward bias condition the applied potential is in a position with the contact potential of the junction. This results in the lowering of the height of the potential barrier across the junction which causes the holes from the p side to diffuse across a junction or to the n side. And likewise, the electrons can diffuse from the n side to the p side across the junction.

Now, under the equilibrium condition I have plotted the concentration of the minority carriers holes on the n side and the minority carrier electrons on the p side let us just consider the n-type side. Now, because of this concentration profile that you see there is a diffusion gradient and the holes they tend to flow from left to the right, as they flow from left to the right they are actually neutralized by electrons. To maintain this equilibrium profile which you can see here in the green colour electrons are replenished by the source negative terminal and there is no problem with it because there is no dearth of electrons now. So, there is no dearth of the supply of the charge carriers to compensate this.

Now, similar explanation holds for the electrons on the p side. On the p side we will see that the positive terminal of the supply will actually be replenishing the holes which will be used for recombining the electrons which are trying to diffuse towards the left side. So, you can see that this could actually result in a continuous process where the electrons and the holes can be continuously supplied. And it is very much unlike the reverse bias condition that we seen before. Hence, it is a forward biased configuration which is used for the conduction of the diode.

It may be noted that the current during the forward bias condition is induced by the minority charge carriers. They follow the mechanism of diffusion. Since both electrons and holes are involved in the process of current flow that is why this device is called a bipolar device, such devices are also called minority carrier devices because the current is induced by the minority charge carriers.

Now, let us derive the analytical expression for the diode current, you know which is actually called the diode current equation which will be applicable during both the forward bias and the reverse bias conditions that we discussed just now. Let us look at this diagram which I have drawn just a few minutes ago to save some time. So, what you can see is here the excess minority carrier or distribution you know in the end in the p-type.

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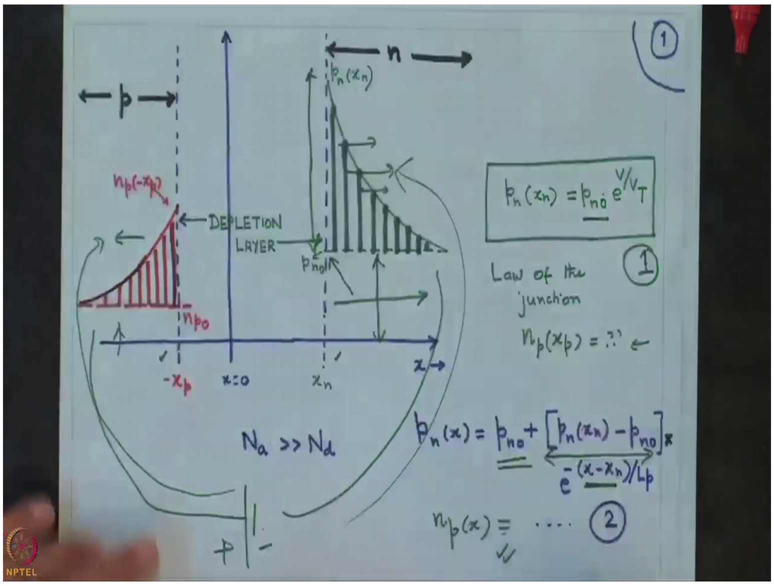

So, you can see the see the distribution of the p-type you know carriers here on the n side and you can see the distribution of the n-type minority carriers in the p-type region here. And here in this is the depletion layer, and we have just marked the two edges of the depletion layer as minus x p and x n as before as we have done in the previous analysis and here x is equal to 0 it actually is the metallurgical junction.

Now, there is a very famous law which is used in the semiconductor physics, very extensively which is the law of the junction and what it does is that it actually gives an expression for the concentration of the minority carriers. Like for example, if you see here at x is equal to x n which is the edge of the depletion layer, so it actually the law of the junction it gives the concentration of the minority charge carriers at the edge you know of the depletion layer.

And so, you can get see that the holes in n the subscript, n actually denotes that it is now at about n side. So, p minority carriers p in n at a location x of n is given by this expression which I have numbered as equation number 1 here as you can see. So, this is actually equal to p and 0, where p and 0 is nothing but the thermal equilibrium value of the minority charge carrier concentration at this point at this edge of the depletion layer. So, this is basically you know the point we are talking about, and this entire thing this height is nothing but the thermal equilibrium concentration of holes in the n-type region. So, I can write a similar equation you know for n-type charge carriers in p, at x of p I can write down a similar expressions there, ok. So, now, we know the concentration of the minority carriers at the two edges. So, I can we have it at x n and we also have it at minus x of p.

Now, as we go towards the right side into the you know the n-type or the p-type material you know there is this relation which is given by 2, again a very standard relation which we are given here for the holes that shows the how the concentration of the holes in the n-type it exponentially decays, ok. It starts from a p n 0 which is the steady state offset due to the thermal equilibrium value. And as per this relation now what is this relation here? This is nothing but this particular height. So, this into, so this is just an into sign here into this exponential of x minus x n divided by L p. Now, x minus L x n is actually the variable which is taking us on to this side of x. So, we are just called it as expression 2. And it is not difficult for you to see that a similar expression can be found for n, minority carrier n in the p-type as you can see on the left side and we can find a similar relation almost same expression with proper substitution of the variables we will get the corresponding expression for n p.

Now, in this expression 2, what you see is L p is nothing but the diffusion length of holes. The diffusion length of electrons is denoted by L n. And what it actually shows is that how long into the n-type can the holes penetrate before getting recombined with electron or getting neutralized by electrons, ok. So, this actually is a very important parameter smaller is the value of L p, faster would be the decay of this exponential curve on the right side and the same thing with electrons in the p side, ok.

Now, if you look at the this distribution, minority carrier distribution again in this forward bias junction and what happens is that as holes move due to diffusion towards the right side they combine with electrons. And to maintain the equilibrium condition the external source, which in this case is negative here and positive here which is connected, ok, so the negative source here must supply the majority carriers electrons, depending on how many electrons are used up in neutralizing the holes the minority carriers which are trying to diffuse onto the right side. And similarly, on the right side we can see that as electrons are neutralized by the holes the holes have to be supplied by this, so that equilibrium profile is maintained.

This diffusion length L p, it is actually related to the lifetime the minority carrier life time. So, what is the lifetime of let us say holes in the n-type if we denote it by tau p, then we can say that the diffusion length of the holes in p-type is actually given by D p into tau p, where D p is nothing but the diffusion coefficient of the holes, ok. That we have seen when we were doing the diffusion, diffusion current, and tau p I just told you is the lifetime of the holes in the p in the n-type material, ok.

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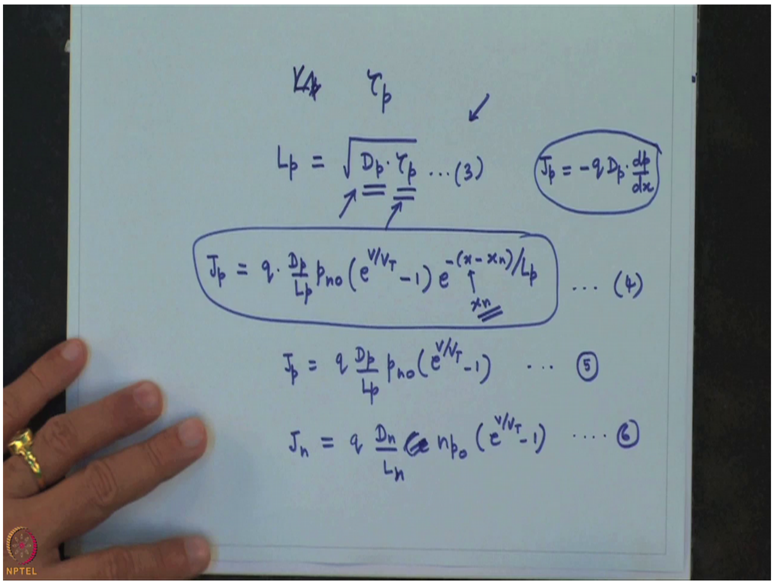

~~L_p~~ τ_p

$$L_p = \sqrt{\underline{\underline{D_p}} \cdot \underline{\underline{\tau_p}}} \ldots (3)$$

$$\boxed{J_p = -q D_p \cdot \frac{dp}{dx}}$$

$$\boxed{J_p = q \cdot \frac{D_p}{L_p} p_{no} \left(e^{V/V_T} - 1\right) e^{-(\underset{\underline{\underline{x_n}}}{x} - x_n)/L_p}} \ldots (4)$$

$$J_p = q \frac{D_p}{L_p} p_{no} \left(e^{V/V_T} - 1\right) \ldots ⑤$$

$$J_n = q \frac{D_n}{L_n} n_{p_o} \left(e^{V/V_T} - 1\right) \ldots ⑥$$

So, these their standard values are actually available which from many sources which can be referred to. So, basically now these minority carriers these holes they actually give rise to diffusion. Current density J of p equal to q times D p by L p, p n 0, e raise to power V over V T, where V T is nothing but kT by q that we have seen before minus 1 e raise to power minus of x minus x n by L p. And in arriving at this equation at this equation you know we have of course, made use of you know the equation that we just wrote the equation number 2. And the other expression we have used is the expression for the diffusion current which is nothing but just to kind of quickly recall for your ready reference minus q times D p into dp by dx, ok.

So, when we actually do a differentiation of expression tool and we actually use this in the diffusion current equation along with you know the and when we use this, we actually get this expression. So, let us call this as expression 4, and let me just call this as expression 3.

Now, clearly the number of electrons supplied by the external source should correspond to p n of x n which is the maximum concentration of holes on the n side. Hence, using x is equal to x n in equation 4, the current density due to the holes is given by J p equal to q times d p by L p, p n 0, e raise to power V by V T minus 1, ok. Likewise, a similar analysis can be applied on to the p side material and we can actually write J n you know equal to q times D n by L n, D n is nothing but the electron diffusion constant and L n is nothing but the diffusion length of the electrons, and E raised to power sorry there would be a n p 0 term into E raised to power V over V T minus 1. So, let us call these two equations as 5 and 6.

Now, we know that the net current that will flow across a P-N junction, would be a sum of these, ok. And moreover, if A is the cross sectional area through which this current is going to flow from the density, the current density J we can actually get the absolute value of I. So, if we can write I is nothing but A times q D p into d into p n 0 divided by L p plus q times D n p 0 divided by L n, e raised to power V over V T minus 1 and let us call this as 7.

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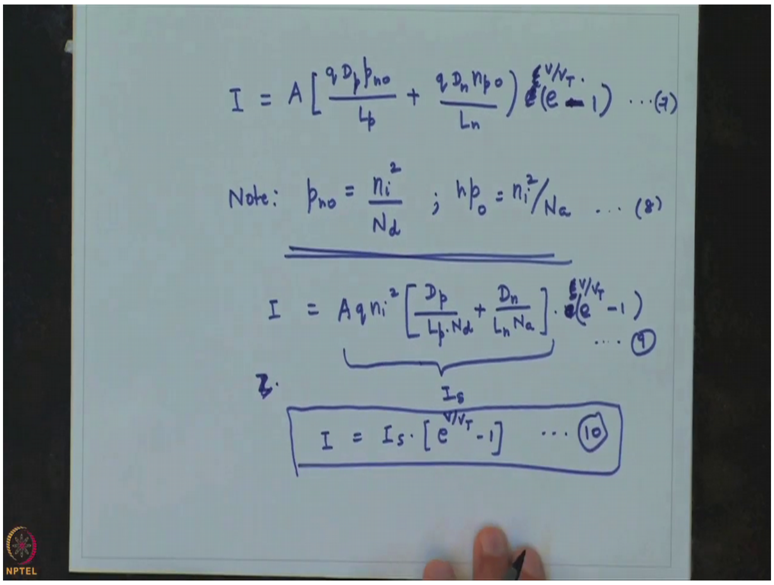

$$I = A\left[\frac{qD_p p_{no}}{L_p} + \frac{qD_n n_{po}}{L_n}\right)\left(e^{V/V_T} - 1\right) \quad \cdots (7)$$

Note: $p_{no} = \frac{n_i^2}{N_d}$; $n_{p_o} = n_i^2/N_a$ … (8)

$$I = Aqn_i^2\underbrace{\left[\frac{D_p}{L_p \cdot N_d} + \frac{D_n}{L_n N_a}\right]}_{I_s}\cdot\left(e^{V/V_T} - 1\right) \quad \cdots ⑨$$

$$\boxed{I = I_s \cdot \left[e^{V/V_T} - 1\right] \quad \cdots ⑩}$$

So, this is the total current on account of the electrons and the holes which are constituting the current as minority carriers, ok, the explanation that we have just seen, ok. Now, a further understanding into this expression can be obtained by noting that p n o which is the thermal equilibrium value of holes in the n-type region can be given by n i square the intrinsic divided by N d and we can say n p o is equal to n i square divided by N a. So, these are like when we have seen the doping, at that time we have obtained these relationships and we have in fact, given some standard expressions and relations which are very useful you know for our further analysis, works.

And when we do this substitution then we actually get I you know the current due to electrons and holes the total current that is A q into n i square, in the bracket it is D p the diffusion constant for holes divided by L p into N d plus the second term within this bracket which is D n over L n, the diffusion length of the electrons into N a the acceptor atom concentration into e raised to power V over V T minus 1. This is the expression that you know we will get. There is a small change that you can do the bracket actually should come like this not here it would come like this and e should be inside. So, it is e raised to power V over V T minus 1 here, it is e raise to power V over V T minus 1 here and they are in the bracket just for easy readability.

So, the final current we can if we recognize that you know this dome here denotes the saturation current, ok, the current that will flow for example, when the P-N junction is reverse biased let us say if this is I s, then I straight away get I is equal to I can just write I s and I can say e raise to power V over V T minus 1. So, if I just number these previous equations as let us say 8 and this one as 9, then I get this as a 10; question number 10 which is nothing but the famous diode equation which is used extensively in the analysis of P-N junction.

The ideal characteristics of the diode based on this diode current equation that we derived you know it can be drawn as follows.

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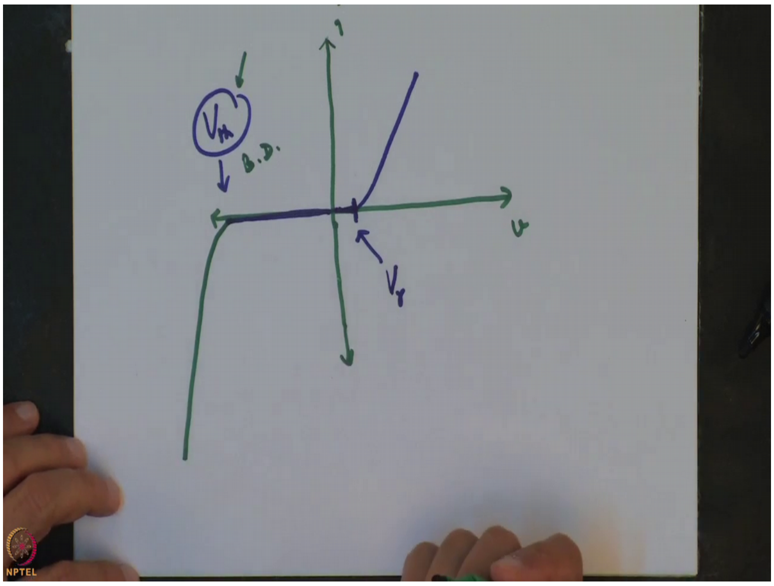

What we will see is that let me just say this is the x axis and this is the y sorry this is the y axis, let us say I is plotted here and this is the x axis or which the voltage across the P-N junction is plotted.

Then what will we observe in the case of the forward bias is that there would be an extremely small drop. So, there will be a small cut in potential here which is usually defined by V gamma and its typical value as I said is physically it is a contact potential, and the applied voltage that appears across the diode in a forward bias case must be greater than this voltage which is V gamma or also called as its threshold or the cutting voltage. And after that so there is no current till that time, because there are no charge carriers which are able to jump. Till they have at least got energy to overcome this potential contact potential, and after that it actually rises the current rises like this.

Now, in the reverse biased case where we are actually having a negative voltage which is applied across a P-N junction, as you can check from the polarities of the battery which are connected there you will find that there is no possibility of any current flowing, so it remains 0, current remains 0. After a certain threshold voltage of the negative side there is a sudden current that builds up in the negative direction, this actually follows the same phenomenon you know that we discussed some time back about the breakdown of the P-N junction when a large reverse voltage is applied.

After a certain threshold point, let us call it as thresh threshold for now a very large negative voltage, so you are increasing the negative voltage across the P-N junction slowly and a time comes you know when large current flows in the negative direction, ok. So, because of this threshold voltage it is more common to call as a breakdown voltage, because what has happened is that the diodes, the diode has basically broken down the mechanism has broken down now there is a reverse current which has taken over and which is actually a very high value. So, it a huge current would flow unless it is limited and would actually destroy the P-N junction.

Now, having seen the working of a P-N junction. Let us now analyze the P-N junction, ok. Let us try to see how the electric field and the potential they vary you know across the junction length.

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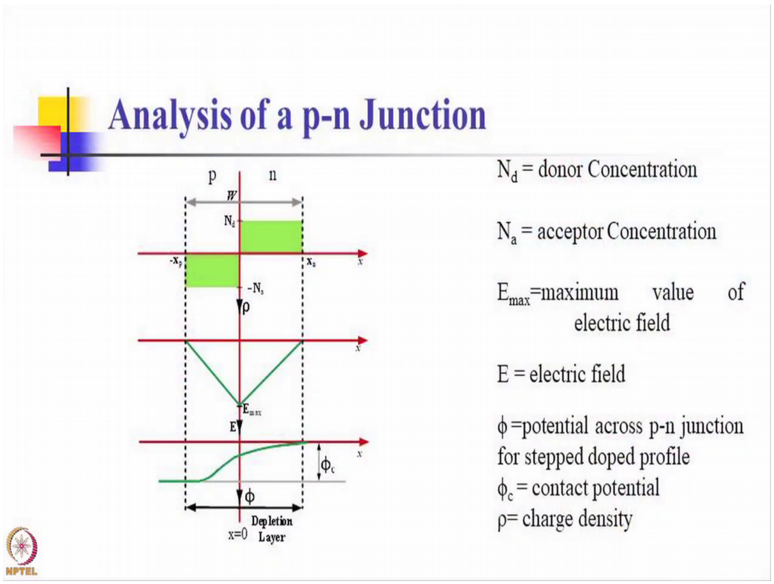

Analysis of a p-n Junction

N_d = donor Concentration

N_a = acceptor Concentration

E_{max}=maximum value of electric field

E = electric field

ϕ =potential across p-n junction for stepped doped profile

ϕ_c = contact potential

ρ= charge density

Now, what you see on the slide is you know a plot of the charge density on the two sides of the metallurgical junction defined by x is equal to 0. It is assumed that the P-N junction has a step doping profile. Further it should also be noted that thermal equilibrium condition has been assumed. We have also plotted the variation of the electric field and the potential. Now, what we are going to do is we are going to actually get analytical expressions for the electric field and the potential, which we will later on use in the further analysis and understanding of what we can do with the devices where we use the P-N junction.

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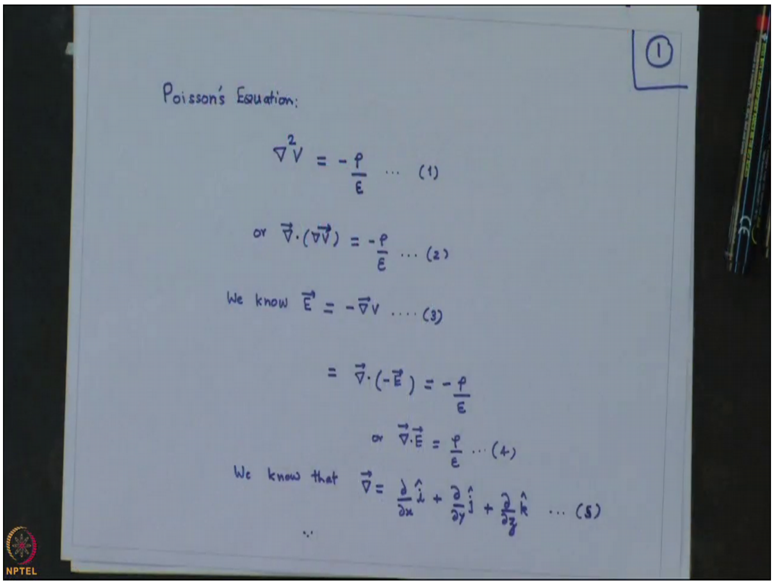

①

Poisson's Equation:

$$\nabla^2 V = -\frac{\rho}{\varepsilon} \quad \ldots (1)$$

or $\vec{\nabla}\cdot(\nabla\vec{V}) = -\frac{\rho}{\varepsilon} \quad \ldots (2)$

We know $\vec{E} = -\vec{\nabla} V \quad \ldots (3)$

$$= \vec{\nabla}\cdot(-\vec{E}) = -\frac{\rho}{\varepsilon}$$

or $\vec{\nabla}\cdot\vec{E} = \frac{\rho}{\varepsilon} \quad \ldots (4)$

We know that $\vec{\nabla} = \frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k} \quad \ldots (5)$

∴

We begin by the famous Poisson's equation. As you can see del square V is equal to minus rho by epsilon is the Poisson's equation, where rho is the charge density. So, what we have done is we have just written some of these equations beforehand just to save time. So, you can see that the Poisson's equation may be written you know as del dot of del V equal to minus rho of epsilon. Now, knowing that E is nothing but the negative gradient of V, we can actually further simplify this and get del dot E equal to rho by epsilon.

Now, we very well know as you can see in equation 5 how we define del. Del is defined you know as given by this expression in terms of the partial derivatives of the unit vectors i, j and k.

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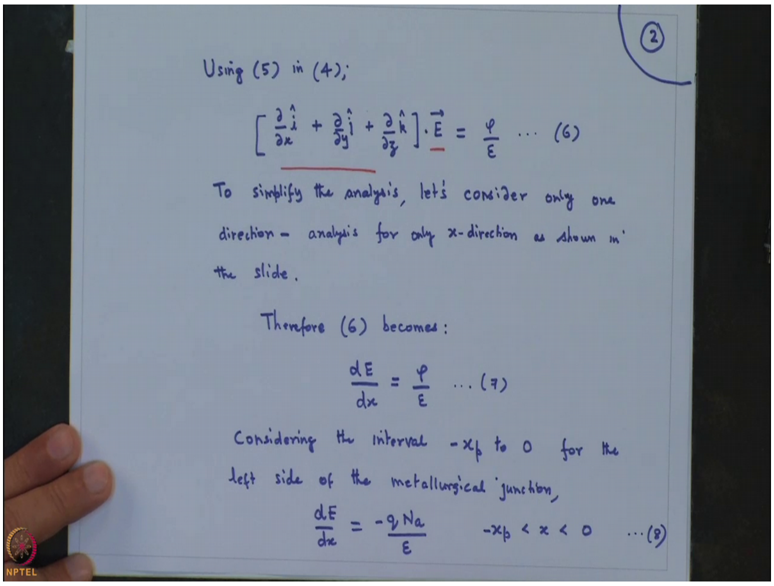

②

Using (5) in (4);

$$\left[\frac{\partial}{\partial x}\hat{i} + \frac{\partial}{\partial y}\hat{j} + \frac{\partial}{\partial z}\hat{k}\right]\cdot\vec{E} = \frac{\rho}{\varepsilon} \quad \cdots (6)$$

To simplify the analysis, let's consider only one direction - analysis for only x-direction as shown in the slide.

Therefore (6) becomes:

$$\frac{dE}{dx} = \frac{\rho}{\varepsilon} \quad \ldots (7)$$

Considering the interval $-x_p$ to 0 for the left side of the metallurgical junction,

$$\frac{dE}{dx} = \frac{-qN_a}{\varepsilon} \qquad -x_p < x < 0 \quad \ldots (8)$$

Now, if we use 5 equation 5 in equation 4, and we just put these partial derivatives expression there dot E as you can see here, so this dot E you get a rho by epsilon. Now, if we take this 3 dimensional analysis then it will get a bit complicated and difficult to understand. So, to make a life simple let us just consider the analysis only in the x direction, ok, in which case the expression 6 you know instead of using these partial derivatives in the x, y and z direction we can just take now in the x direction and this partial derivative I can write as a full derivative dE by dx is equal to rho by epsilon which is our equation 7. And we can treat this as a starting point of our analysis.

Now, let us consider the interval minus x p to 0. You know you can refer to the picture on the slide and you can see that this is the interval towards the left of the metallurgical junction into the p region, ok. So, now, if we use the data for this left side interval in equation 7. We can say that dE by dx is nothing but minus q N a by epsilon where minus q times N a is equal to rho which is nothing but the charge density. The negative sign comes because of the fact that the charge is negative for the interval minus x p to 0. Now, this can be further simplified as per equations 9 and 10; you know and we can actually get a very nice expression for the electric field in the left interval to the left of the metallurgical junction.

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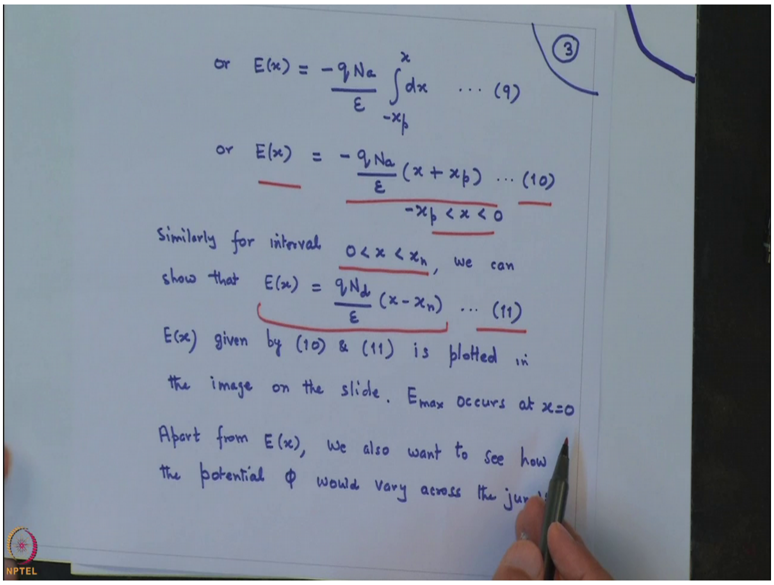

③

or $E(x) = -\frac{qN_a}{\varepsilon}\int_{-x_p}^{x} dx \quad \dots (9)$

or $\underline{E(x)} = \underline{-\frac{qN_a}{\varepsilon}(x+x_p)} \quad \dots \underline{(10)}$

$-x_p < \underline{x < 0}$

Similarly for interval $\underline{0 < x < x_n}$, we can show that $\underbrace{E(x) = \frac{qN_d}{\varepsilon}(x-x_n)} \quad \dots \underline{(11)}$

$E(x)$ given by (10) & (11) is plotted in the image on the slide. E_{max} occurs at $x=0$

Apart from $E(x)$, we also want to see how the potential ϕ would vary across the jun

Now, we can similarly find an expression for the right side of the metallurgical junction which is which we can find and which will be valid for this interval and this is the expression.

Now, E x which is given by 10 and 11 is what you see plotted actually on the slide. So, you can see that in the left interval the electric field starts from 0 and goes down all the way to the maximum value E max and then in the right interval it again goes down all the way to 0. So, the maximum value of the electric field occurs at x is equal to 0 which is nothing but the metallurgical junction. Now, apart from this electric field variation we would also like to see how the potential phi would vary across the junction length.

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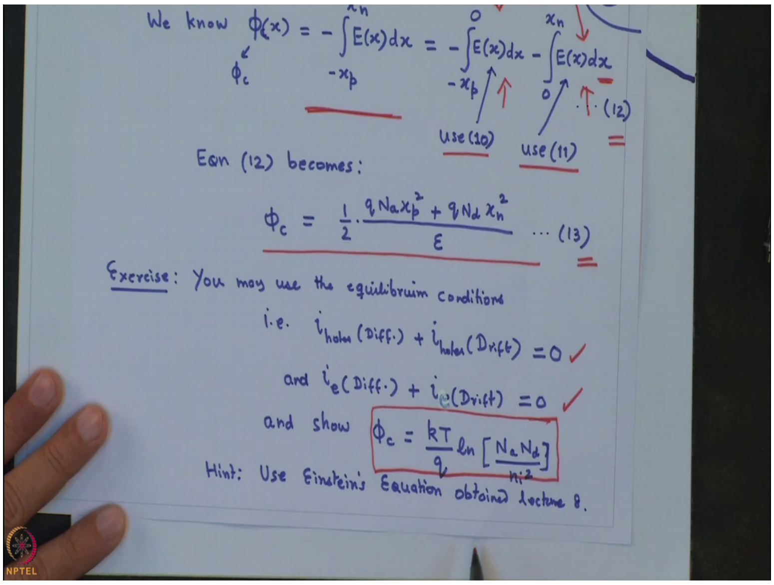

We know $\underset{\phi_c}{\phi(x)} = -\int_{-x_p}^{x_n} E(x)dx = -\int_{-x_p}^{0} E(x)dx - \int_{0}^{x_n} E(x)dx \quad \cdots (12)$

use (10) use (11)

Eqn (12) becomes:

$$\phi_c = \frac{1}{2} \cdot \frac{qN_a x_p^2 + qN_d x_n^2}{\varepsilon} \quad \cdots (13)$$

Exercise: You may use the equilibrium conditions

i.e. $i_{holes}(\text{Diff.}) + i_{holes}(\text{Drift}) = 0$ ✓

and $i_e(\text{Diff.}) + i_e(\text{Drift}) = 0$ ✓

and show $\boxed{\phi_c = \frac{kT}{q} \ln\left[\frac{N_a N_d}{n_i^2}\right]}$

Hint: Use Einstein's Equation obtained lecture 8.

Now, we know that phi of x you know it is given by the minus integral of E x with respect to x if you are considering only x direction analysis. So, we can actually split this integral on the right side into two parts one for the interval minus x p to 0 and the other for the interval 0 to x n, and then for these E x values within this and this these two components left interval we use expression 10, the expression that we have obtained for E of x and we use 11 for the right side expression of E of x, ok. So, when we actually substitute these expressions for E x using 10 and 11, we actually get this equation 12 which on simplification. We can see gives us this expression.

Now, one interesting thing that you can try is you know is that you may use the what are called the equilibrium conditions where we say that the current due to holes across the junction because of diffusion and drift will add up to 0 under equilibrium condition. Similarly, we can say that the electron current due to diffusion and drift will also add up to 0. So, you can see these two expressions and once you actually fill up the values and you equate them to 0, you can actually show a standard expression that we get for the contact potential phi c. You may use the Einstein equation obtained earlier in this lecture.

Now, during the forward bias the applied voltage across the diode or the P-N junction would actually oppose the junction potential phi c. So, it is not really very critical for us to know what happens to the deplay depletion layer width, or the drift layer width. But during reverse bias where the applied potential adds to the contextual contact potential that time the depletion layer expands and it is important for us to evaluate what is the value of W analytically and that is what we are going to do now next.

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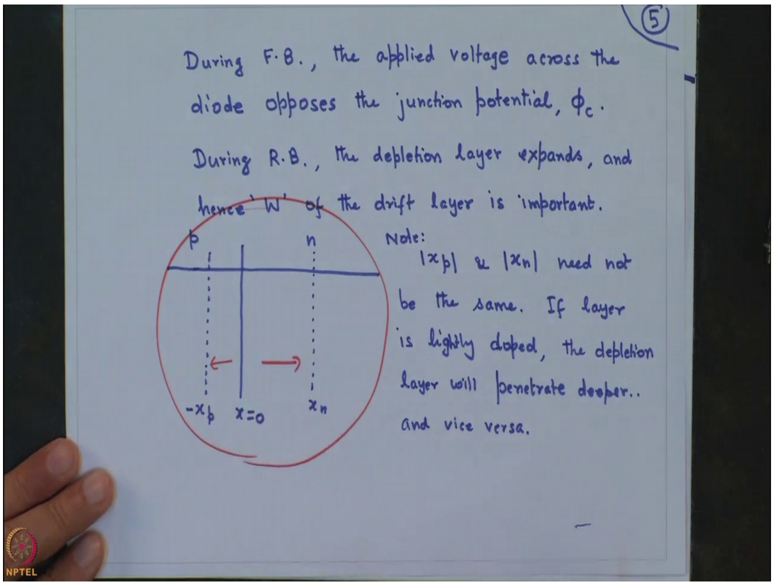

⑤

During F.B., the applied voltage across the diode opposes the junction potential, ϕ_c.

During R.B., the depletion layer expands, and hence 'W' of the drift layer is important.

Note:
$|x_p|$ & $|x_n|$ need not be the same. If layer is lightly doped, the depletion layer will penetrate deeper.. and vice versa.

Note that if you see this picture which I have drawn here, I have shown that x p you know the depletion layer into the p region and x n the limit of the depletion layer into the n region they are not same, and they are not required to be same also it will actually depend on the doping levels of the p-type and the n-type, ok. So, this is something that is true actually generalize the analysis that we are presenting.

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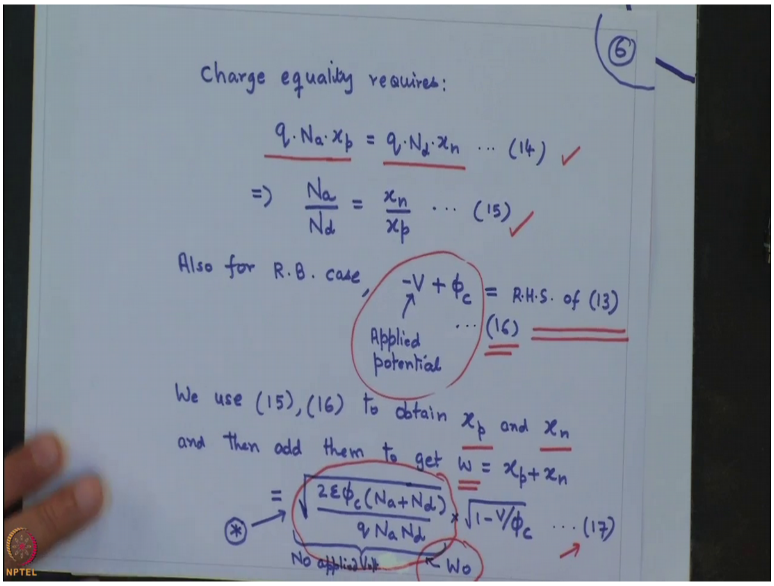


⑥

Charge equality requires:

$$\underline{q \cdot N_a \cdot x_p} = \underline{q \cdot N_d \cdot x_n} \quad \cdots (14) \checkmark$$

$$\Rightarrow \frac{N_a}{N_d} = \frac{x_n}{x_p} \quad \cdots (15) \checkmark$$

Also for R.B. case,

$$\boxed{\underset{\substack{\uparrow \\ \text{Applied} \\ \text{potential}}}{-V} + \phi_c} = \text{R.H.S. of (13)} \quad \cdots \underline{\underline{(16)}}$$

We use (15), (16) to obtain $\underline{x_p}$ and $\underline{x_n}$ and then add them to get $\underline{\underline{W}} = x_p + x_n$

$$\circledast \rightarrow = \underbrace{\sqrt{\frac{2\varepsilon\phi_c(N_a+N_d)}{q N_a N_d}}}_{\text{No applied Volt}} \times \sqrt{1-V/\phi_c} \quad \cdots (17)$$

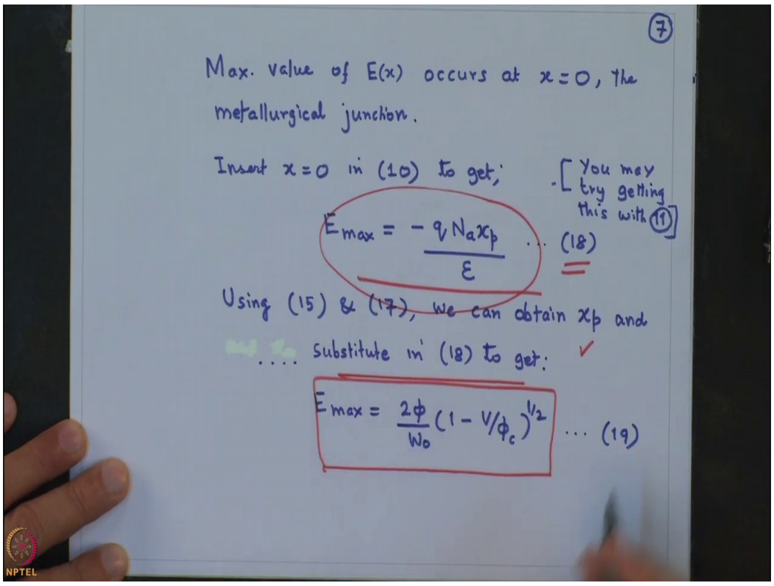
$\leftarrow W_0$

Now however, in spite of this a symmetry, the charge equality requires that two times the accepted atoms concentration into x p. Please refer to the picture on the slide. Must be equal to two times the donor atom concentration into x n, and let us call this as 14. And 15, so you can see N a by N d is equal to x n by x p.

Now, as we have said before that for the reverse bias case the applied potential minus V, it actually adds up to the contact potential. So, basically you know this whole thing it becomes the net potential across the device and this we can equate the expression we obtained in 13.

Now, if we use equation 15 and 16 we can then obtain x p and x n, and then if we add x p and x n if you again refer to the image, we can get the expression for W, which is the width of that depletion layer. And we can show that very simply that it will actually given by this expression 17, now this left side term this term the one which have encircled with red, ok, this let us denote it by W z 0 and this is actually the width when there is no external voltage bias applied. Now, we would like to see what is the value of the maximum electric field that will occur, ok. So, that we can get very easily by using an expression 10, you might use even expression 11 if you want to try both will give the same answer put x is equal to 0 and actually you end up with this relation. So, this is the maximum E feel.

(Refer Slide Time: 54:15)



Max. value of $E(x)$ occurs at $x = 0$, the metallurgical junction.

Insert $x = 0$ in (10) to get; [You may try getting this with ⑪]

$$E_{max} = -\frac{q N_a x_p}{\varepsilon} \quad \dots (18)$$

Using (15) & (17), we can obtain x_p and .... substitute in (18) to get: ✓

$$\boxed{E_{max} = \frac{2\phi}{W_0}(1 - V/\phi_c)^{1/2}} \quad \dots (19)$$

By using 15 and 17 we can obtain the x value of x p, and substitute in 18 to get this final value of E max.

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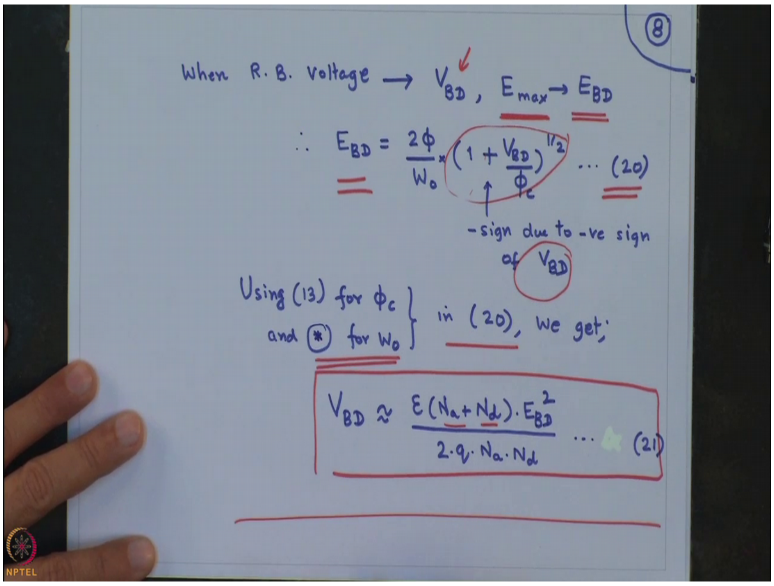

⑧

When R. B. Voltage → V_{BD}, $E_{max} \rightarrow E_{BD}$

$$\therefore E_{BD} = \frac{2\phi}{W_0} \times \left(1 + \frac{V_{BD}}{\phi_c}\right)^{1/2} \quad \ldots (20)$$

- sign due to -ve sign of V_{BD}

Using (13) for ϕ_c and ⊛ for W_0 } in (20), we get;

$$\boxed{V_{BD} \approx \frac{\varepsilon (N_a + N_d) \cdot E_{BD}^2}{2 \cdot q \cdot N_a \cdot N_d} \quad \ldots (21)}$$

Now, when the reverse bias voltage becomes so large it goes to all the way to the breakdown voltage, then at that time my electric field maximum value also goes to the breakdown electric field value, and then I can actually get you know this final expression for the breakdown electric field. And you can see that there is a negative sign here which was there if you see equation 19, but that has got actually accommodated because there is a negative sign associated with V BD also because there is a negative voltage which we are applying.

Now, let us call this bring down electric field expression equation to be equation 20. Now, using expression thirteen for phi c and the equation I have numbered as star in 20 we get this final value of the breakdown voltage across the P-N junction. So, you can see that we can actually say that this approximation will have to be applied to simplify the analysis and we can actually get this V BD approximately equal to epsilon into N a plus N d into E BD square divided by 2 q N a into N d. So, this is the final expression that we get and later on we will use them to further enhance our understanding about different aspects of the P-N junctions, ok.

So, next time we will see certain more aspects of these. I hope that I will be able to cover some very interesting things in the coming lectures.

Thank you very much for your attention.