Principles of Digital Communications
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Lecture - 55
M-ary Quadrature Amplitude Modulation (M-QAM)

In M-ary ASK and M-ary PSK the messages, which are patterns of n binary digits and where N is equal to log to the base 2 of M, where M is the number of messages are encoded either into amplitudes of phases of our sinusoidal carrier. Now, you will study M-ary quadrature amplitude modulation, popularly known has M QAM or just QAM.

It is a more general modulation that includes M-ary ASK and M-ary PSK as special cases. In QAM the messages are encoded into both the amplitude and phase of the carrier; QAM constellations are 2 dimensional and they involved 2 orthonormal basis signals.

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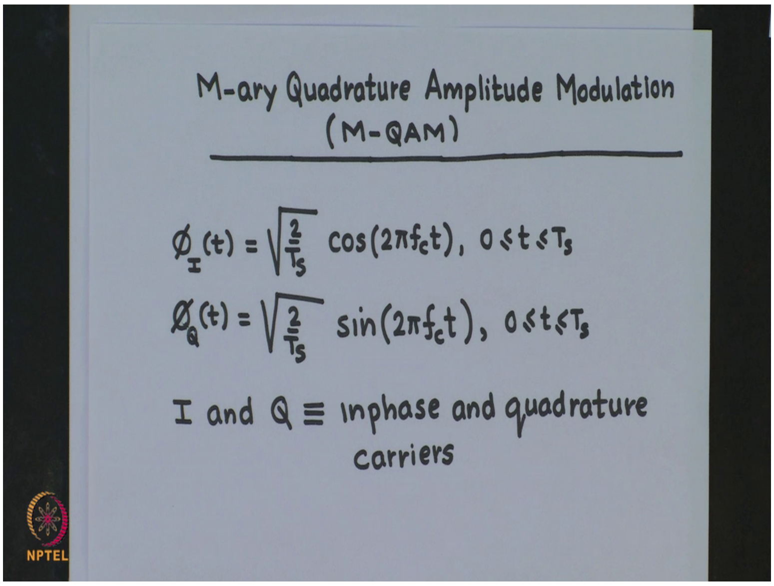

This orthonormal basis signals are as follows; phi it and phi Q t. This is known as in phase carrier and this is known as quadrature carrier. Both are defined over the simple duration that is T s, using this orthonormal basis signal we generate the QAM message signal set as follows.

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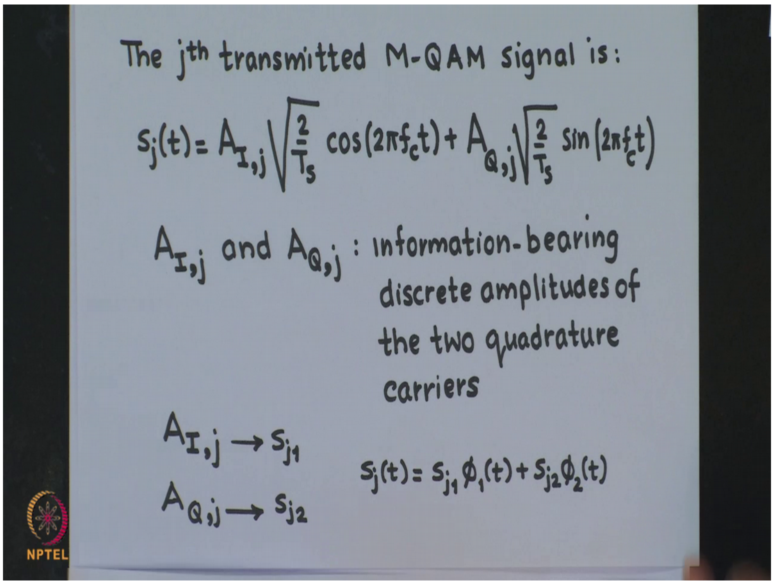

The j th transmitted M QAM signal is given by this expression; this is phi it and this is phi Q t, where this A I j and A Q j are information bearing discrete amplitudes of the 2 quadrature carriers.

This, shows that there are 2 phase quadrature carriers involved and each of them is being modulated by a set of discrete amplitudes and hence the name quadrature amplitude modulation. Also note that A I j and A Q j are precisely the coefficients S j 1 and S j 2 respectively, in the usual representation of the signal S j t in terms of the orthonormal basis signals phi 1 t and phi 2 t. The alternate representation of S j t is as follows.

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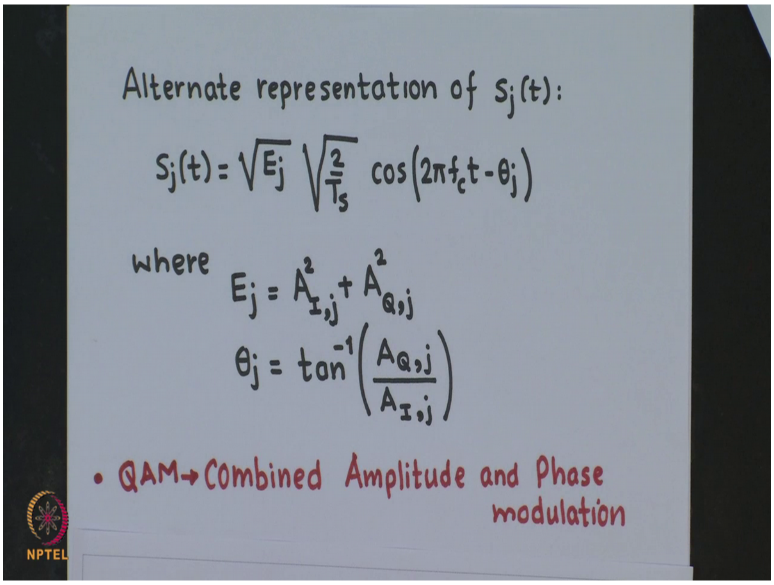

Now, in this representation E j is the energy, which is the sum of the squares of the in phase amplitude and square of the quadrature amplitude. And, the theta j is the angle which is tan inverse of A Q, j by A I, j. So, in this representation we see that our QAM signal is combination of amplitude and phase modulation. Now, we will assume that our message or signals are equiprobable our signal constellation is 2 dimensional. So, the optimum receiver would be the minimum distance receiver.

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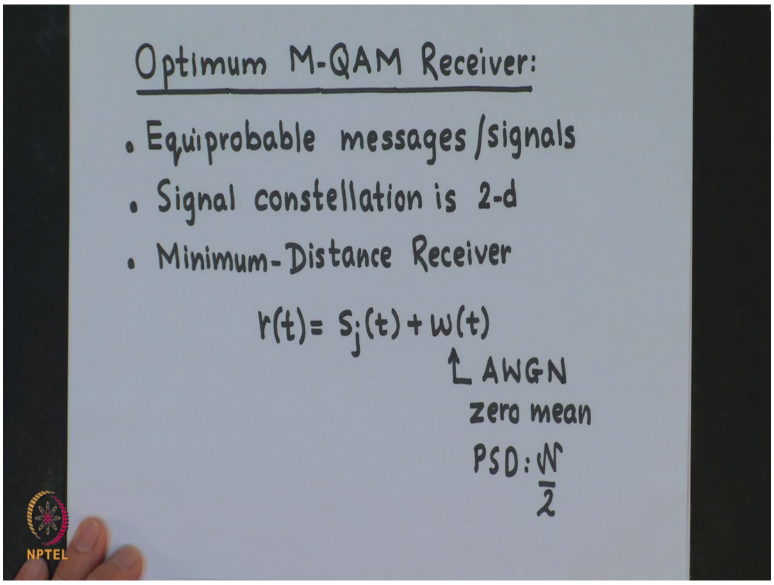

So, your receive signal.

So, your receive signal r t is equal to S j t plus the additive white Gaussian noise with 0 mean and the power spectral density as italic N by 2. Now, the minimum distance receiver can be implemented as shown in this figure.

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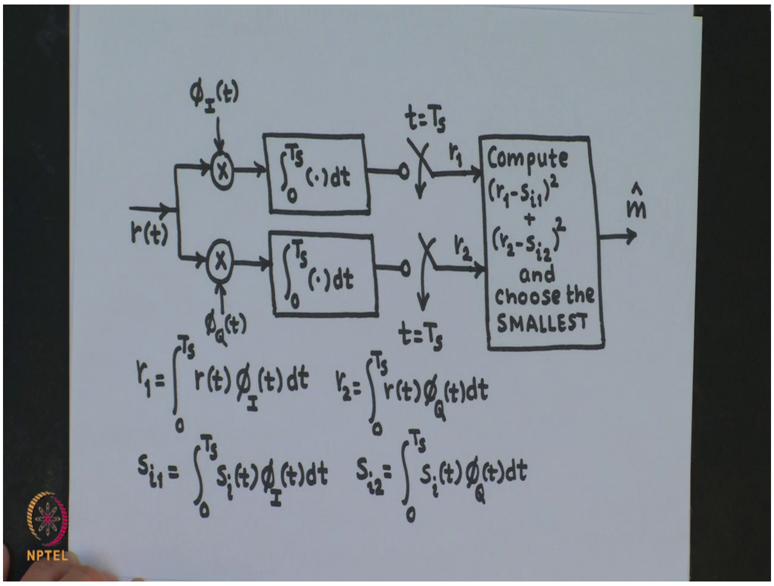

This is the correlation receivers in the in phase channel and this is a correlation receiver in the quadrature phase channel correct. And r 1 and r 2 are the projections of r t on phi 1 t and phi Q t respectively. And, similarly S i 1 and S i 2 are the projections of the signal sit on phi it and phi Q t respectively.

So, we compute the equilibrium distance and choose the smallest of, that depending on the number of possible symbols M and the set of amplitudes A I j and A q j a large variety of QAM constellations can be realized.

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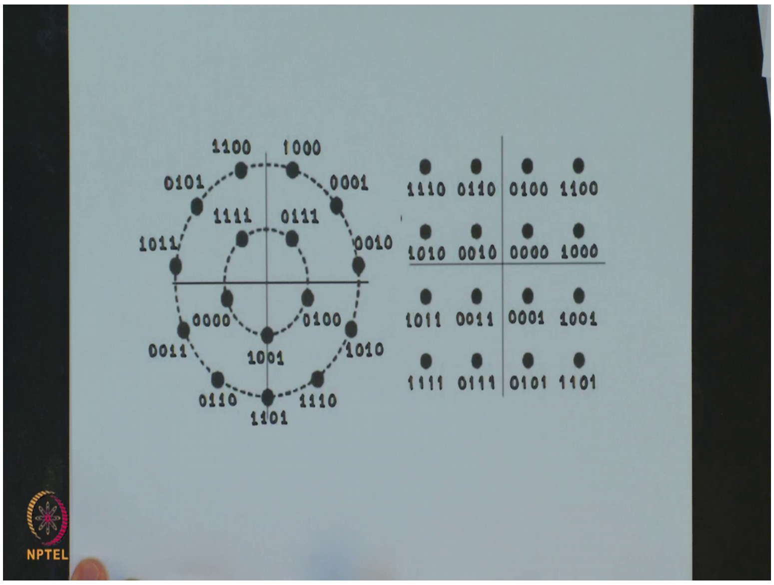

And this figure out here shows 2 of the signal constellations for M equal to 16 this is a rectangular constellation whereas, here you find it is 2 circles. Obviously, for a given M an important question is how to compare different M QAM constellations in terms of error performance?

Let us attempt to find an answer to this question, observe that for signaling over an additive white Gaussian noise channel the most likely error event is the 1, where the transmitted signal is confused with it is nearest neighbors. Therefore, in order to roughly maintain the same error probability the distance between the nearest neighbors in all the signal constellations should be kept the same. So, with this constraint the more efficient signal constellation is the one, that has smaller average transmitted energy.

The most important consideration of signal point is rectangular. More naturally has a rectangular QAM the signal points are placed on a rectangular grid spaced equally in amplitude by delta amount in each direction or dimension. Now in terms of energy rectangular QAM constellations are not the best M QAM signal constellations, but the average transmitted energy required to achieve a given minimum distance is negligibly greater than the average energy required for the best M QAM signal constellations.

So, for this reason rectangular or M QAM signals are those most frequently used in practice. The most practical rectangular QAM constellation is 1, which carries an equal number of bits on each axis that is N is even. So, you will have 2 4 6 8. And therefore, the number of messages that is M is a perfect square, which would be 416 64 256. The rectangular constellation is a square constellation.

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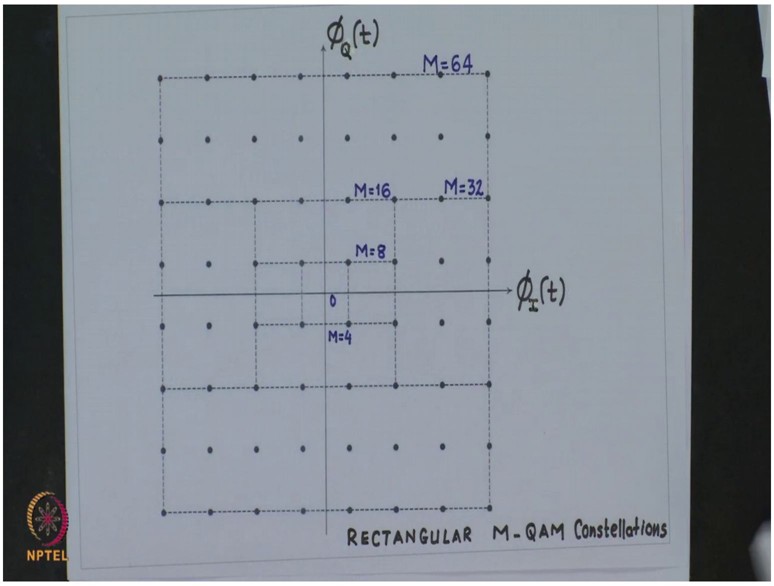

This figure shows rectangular QAM constellations for M equal to 4, M equal to 8, M equal to 16, M equal to 32 and M equal to 64.

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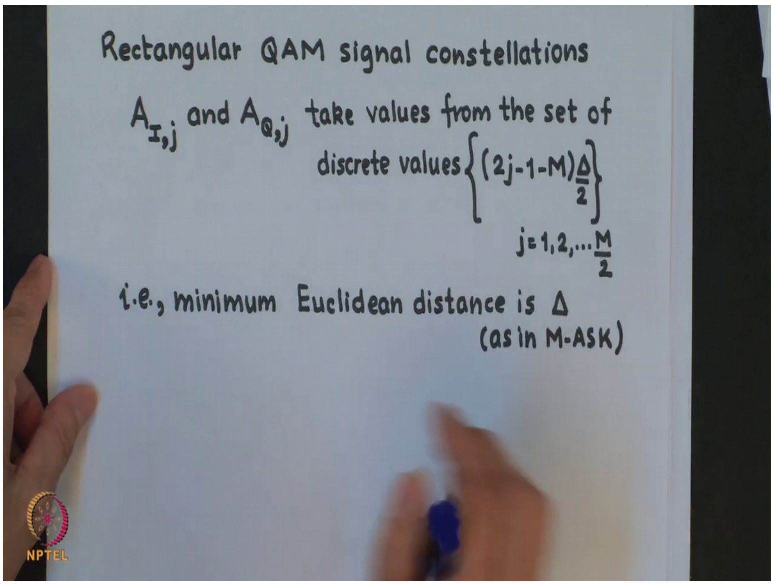

The signal components A I j and A Q j take values from the set of discrete values and the minimum Euclidean distance is delta as in M-ary ASK. The major advantage of rectangular QAM is it is simple modulation and demodulation.

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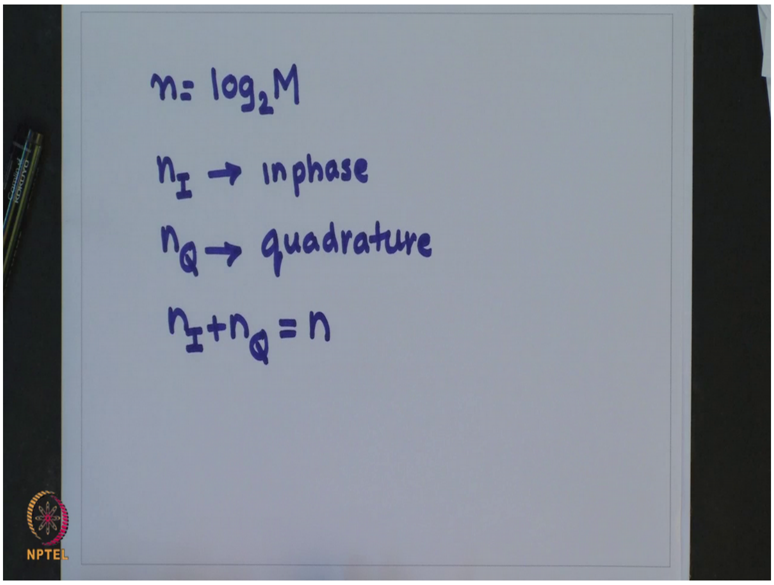

This is because each group of N bits can be divided into n I in phase bits and n Q quadrature bits, where n i plus n q is equal to n. The inphase bits and quadrature bits then modulate the inphase and quadrature carriers independently.

So, this generates inphase and quadrature ASK signals, which are then added to form the QAM signal before being transmitted. Now, at the receiver due to the orthogonality of the inphase and quadrature signals the 2 ASK signals can be independently detected to give the decisions on the inphase, and quadrature bits based on these discussions the modulator and demodulator of a rectangular M QAM is illustrated in this figure.

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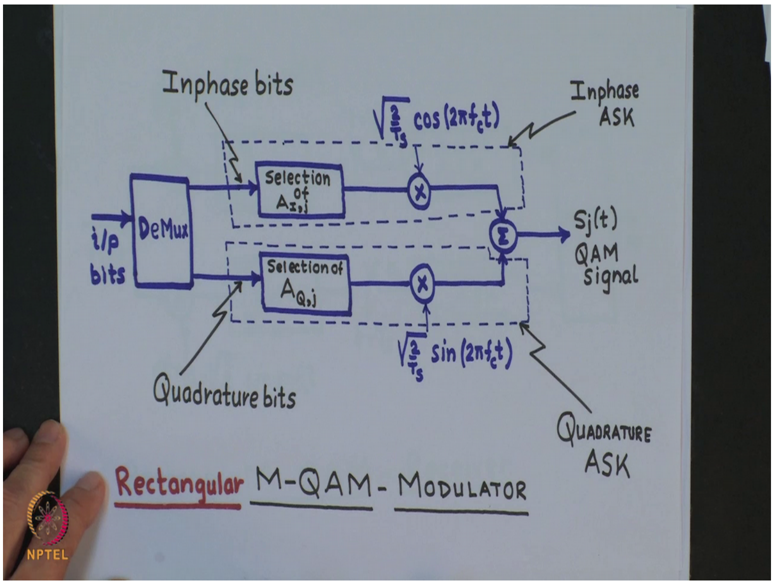

You have a input bits goes to the demultiplexer; we get inphase bits, quadrature bits, selection of A I j the number of A I j would be equal to 2 raised to n i where n i is the number of inphase bits. And the number of discrete values for A Q j will be equal to 2 raised to n q, where n q is the number of quadrature bits we get 2 ASK signal inphase ASK, quadrature ASK. And both are combined to produce the QAM signal S j t and based on this we get the receiver for the same as shown here.

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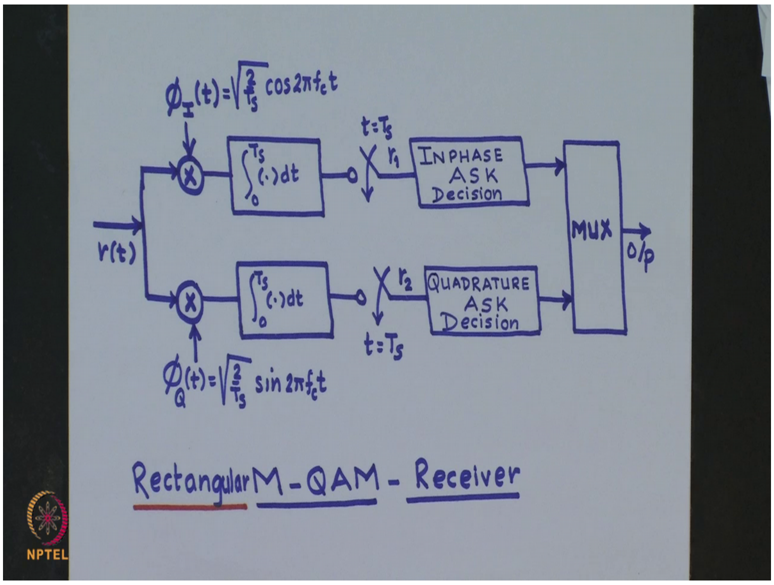

We have inphase ASK channel and quadrature ASK channel, both are combined with the help of a multiplexer to give the final output. Now, any rectangular QAM signal constellation is equivalent to 2 ASK signals modulating the quadrature carriers. So, the inphase carrier phi it carries n i bits and has 2 raised to n i signal points, while the quadrature carrier phi Q t has n Q bits assigned to it and 2 raised to n Q signal points. So, for square constellation each carrier will have root M that is equal to 2 raised to n by 2 signal points.

Since, the signals in the phase quadrature components can be perfectly separated at the demodulator the probability of error for QAM can be easily determined from the probability of error for ASK signals. So, let us do that.

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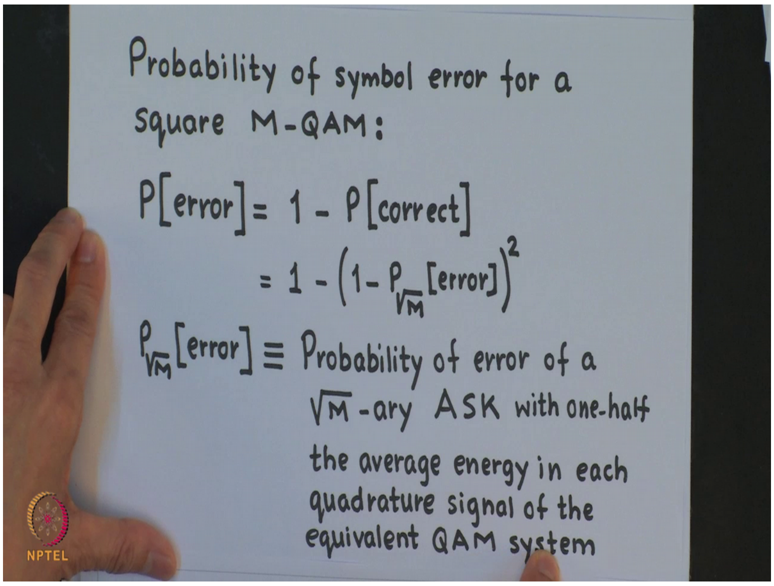

So, probability of symbol error for the square M QAM would be given as probability of error is equal to 1 minus probability of correct decision, probability of correct decision will happen if there are no errors in both the ASK channels, which means that probability of correct detection is equal to 1 minus probability of error in any 1 of the channel and then square of this expression.

So, this is the probability of correct detection in 1 of the channel. So, both the channel should correctly detect the source squared. So, the probability of this is the probability of error of root M-ary ASK we are assuming M to be square constellation. And, this will have 1 half the average energy in each quadrature signal of the equivalent QAM system.

Now, let us quickly recollect the MSK signal error probability, we know that probability of error.

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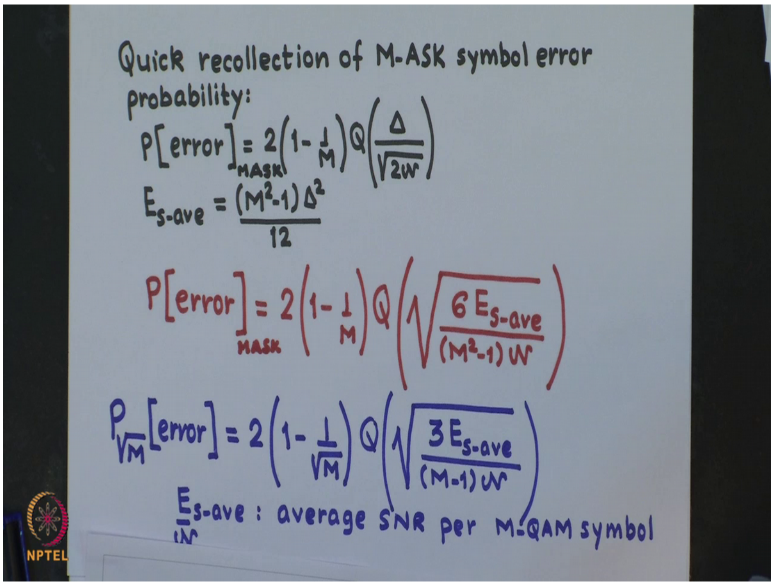

For the MSK case is given by this expression we also know that the delta is related to the average signal energy by this expression.

So, from this 2 expression we can get the probability of symbol error for MSK in terms of the average signal energy. Now, once we have this expression we can calculate the probability for our ASK channel remember in our case this M, now will be equal to root M and it is and the energy the average energy is going to be the half, that is why it become 3 and this M out here is now root M. So, from here we get the probability of error in the ASK channel in our model to be equal to this value correct.

So, E s average by root N is a average signal to noise ratio per M QAM signal. Now, using this and using this relationship we can find the upper bound. So, to the find the upper bound what we will do is we will neglect for large M s we can neglect this term. So, if we neglect this term this will be approximately twice of Q this value and if we use that as an approximation in this expression we will substitute that approximation.

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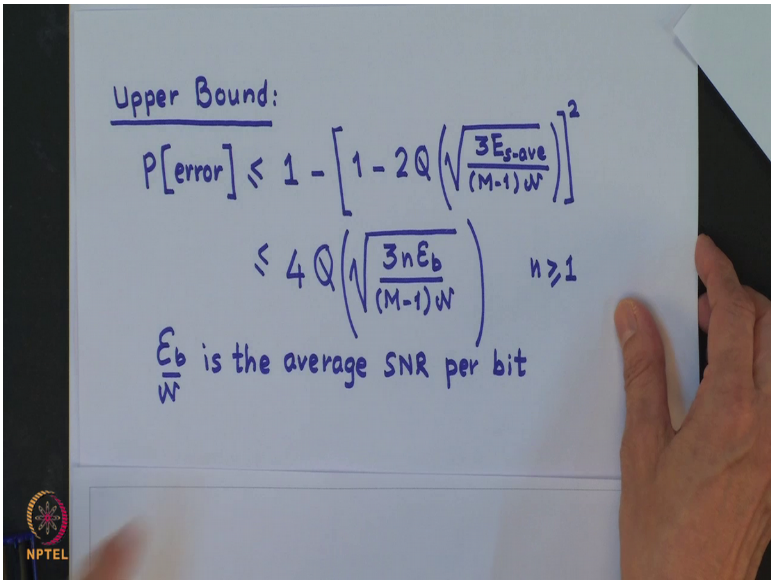

So, it becomes less than or equal to and on expansion I neglect Q squared factor I get this equal to 4 times Q this is valid for N greater than equal to 1.

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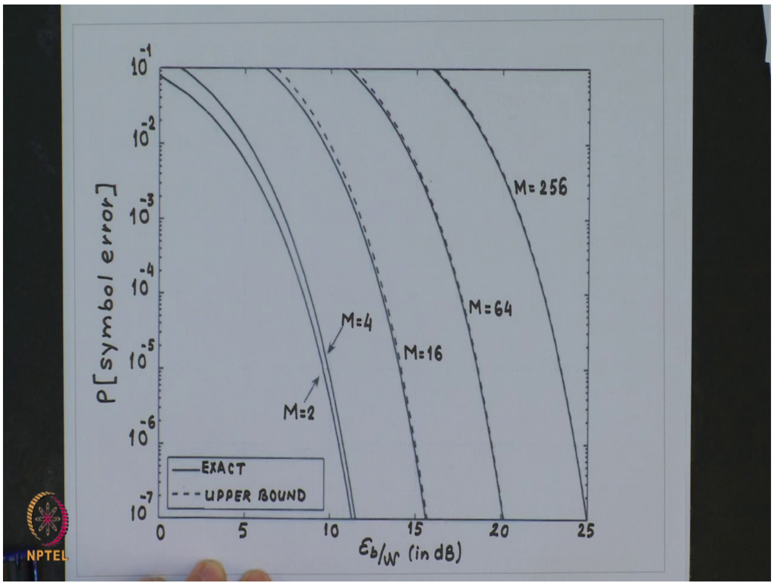

Now, in this E b by root n is the average SNR per bit. So, the plot of the symbol error for M QAM is as shown in this figure here; as a function of e b by root N, for various values of N. The upper bounds plotted in this figure are based on this equation and can be seen to be very tight had high SNR. So, the exact and upper bound they merge for high SNR value this is true for all M equal to 16 M equal to 64 and M equal to 256. Since, both

QAM and M-ary PSK are 2 dimensional signal sets it is of interest to compare the error performance for any given signal size M.

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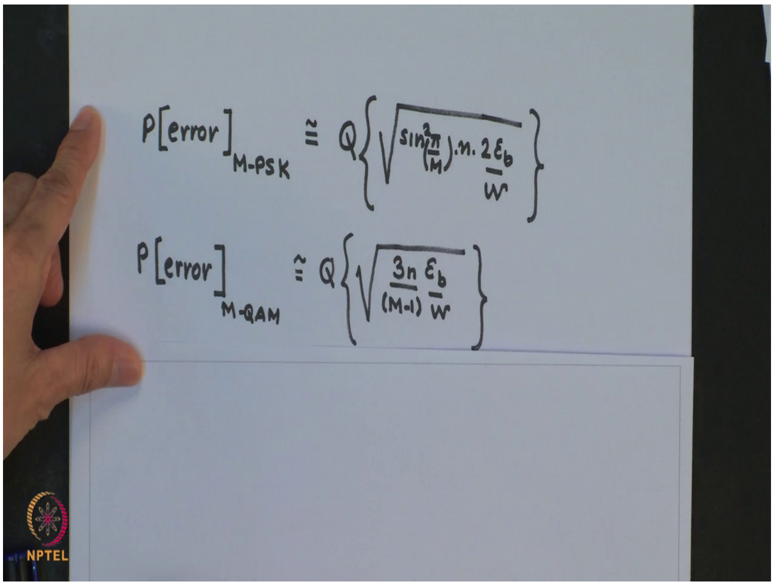

Now, let us do that and to do that let us recollect the probability of symbol error for M-ary PSK, which was given approximately by this expression. And the probability of symbol error for M QAM has been just derived to be equal to this value. Now, remember here, that average signal energy is equal to n times the average bit energy and that is how we got this expression?.

And based on this expression this was plotted. Now, since the error probability is dominated by the argument of the Q function; one is simply compare the squared arguments of this Q functions for the 2 modulation formats, and the ratios of this 2 argument is defined as K M.

So, we will take the square of this and divided by the square of this quantity if we do that we get this quantity and we call this to be as K M.

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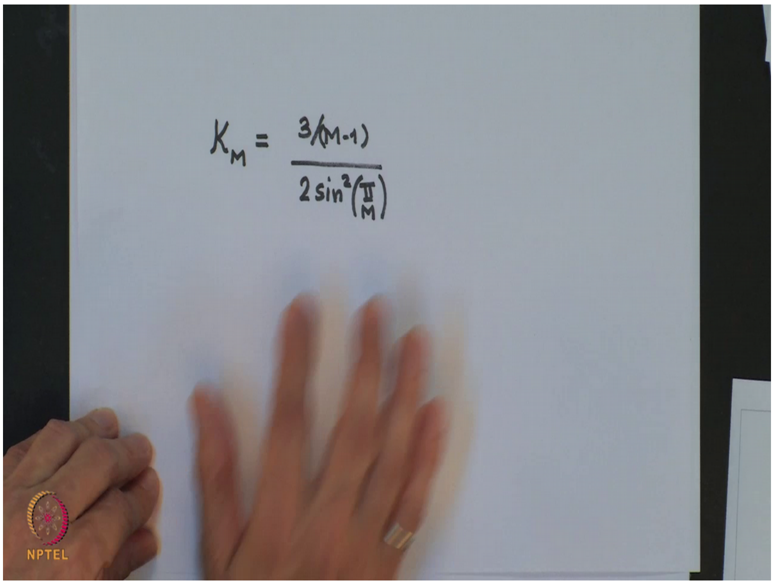

So, in this case when M is equal to 4 K M turns out to be 1. So, what this shows that, 4 PSK that is Q PSK and 4 QAM are the same modulation format correct. This is not very surprising. However, when M is larger than 4 it is seen that K M will be greater than 1, which means that M QAM yields better performance than M-ary PSK.

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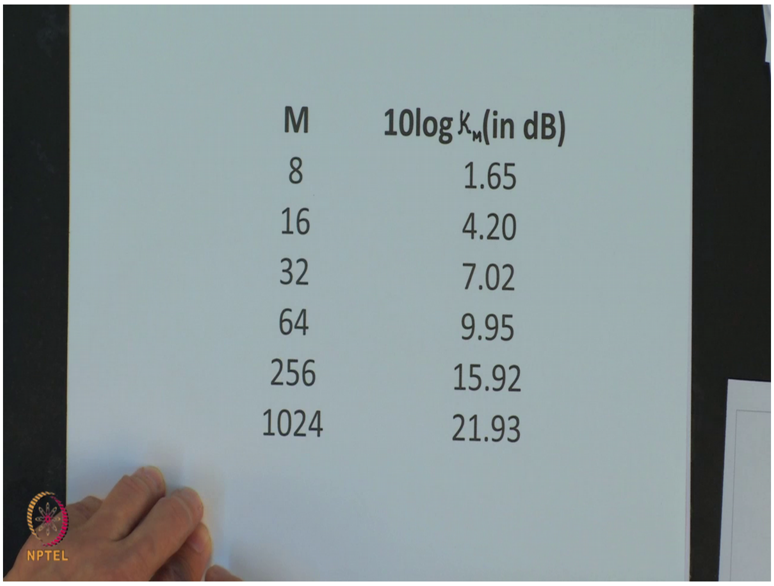

M	$10\log K_M$(in dB)
8	1.65
16	4.20
32	7.02
64	9.95
256	15.92
1024	21.93

So, the table here illustrates a signal to noise ratio advantage of M-ary QAM over M-ary PSK for several values of M.

For example, 64 QAM has about 10 decimal signal to noise ratio advantage over 64 PSK modulation, but it is important to remember that this advantage of course, is gain at the expense of increased sensitivity to amplitude and phase degradation in the QAM transmission. Now, we have studied M-ary ASK M-ary PSK and the hybrid of M-ary ASK and M-ary PSK, in the form of M-ary QAM or simply QAM.

Now one more M-ary modulation scheme is left out and that is M-ary FSK M-ary phase shift key. We will see that this modulation scheme utilizes the signal constellation, which is very different from the modulation schemes studied so far and this we will do next time.

Thank you.