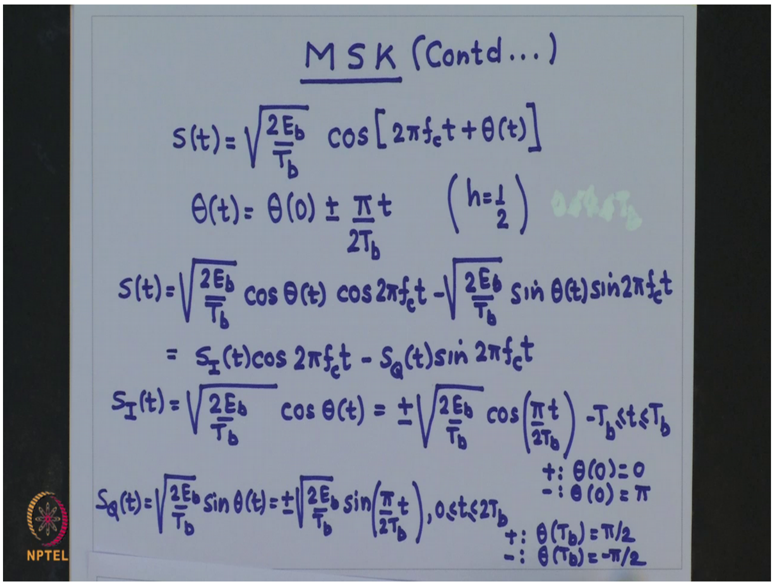
Principles of Digital Communications
Prof. Shabbir N. Merchant
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture - 51
Minimum Shift Keying - I

We will continue a study of Minimum Shift Key. Let us quickly recollect what we have done earlier. So, we said that minimum shift keying is a form of continuous phase shift keying modulation.

(Refer Slide Time: 00:39)

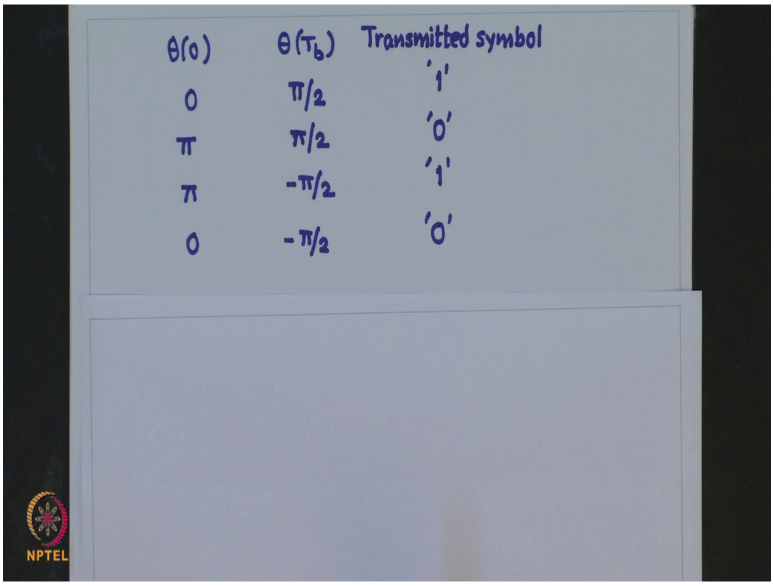


Where your modulated signal S t is given by this expression where theta t which is the phase of the modulated signal is specified by this expression, where the deviation ratio is chosen to be half. And on expansion of this expression we showed that it S t can be written as shown here, which can be written in the form of in phase and quadrature component. And we had also seen that S I t which is defined by this part of the in phase component is equal to the expression written here.

And we said that this expression is valid over the range minus T b to plus T b. For our discussion without loss of generality, we are assuming the bit duration between time t equal to 0 and t equal to T b. And S Q t, similarly we showed that it is equal to the half sine wave and the duration of that half sine wave is over the period 0 to twice T b.

So, S I t and S Q t wave forms are displaced or offset by T b seconds and the polarity of this half cosine wave and half sine wave is decided by the phase state at t equal to 0 and at t equal to T b. So, if theta 0 is equal to 0 this quantity will be positive and if theta 0 is pi then this quantity becomes negative, and similarly here when theta T b is equal to pi by 2, this quantity is positive and when theta T b is equal to minus pi by 2 or 3 pi by 2 modulo 2 pi this quantity is same as that. So, then this quantity becomes minus.

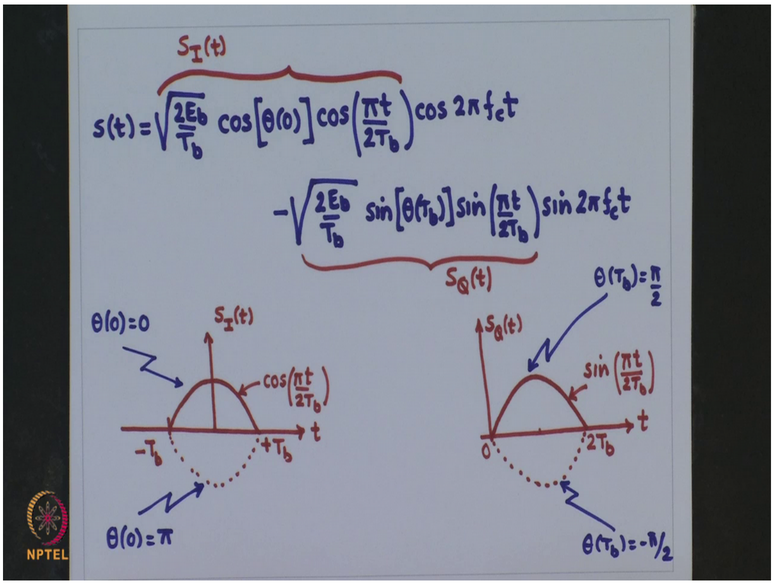
(Refer Slide Time: 03:34)



So, we said that there are 4 states which it could have; the transmitted symbol information is contained in the phase states theta 0 and theta T b. So, we will have 4 phase states then it is 0 pi by 2, we know that one has been transmitted when it is pi pi by 2 we know 0 is being transmitted and similarly for the other states.

Now, what we want to do is to represent this signal S t in terms of orthonormal basis signals. And to do that let us try to understand how the information is being transmitted. The information is being transmitted using this signal S t which has 2 parts or 2 components.

(Refer Slide Time: 04:36)

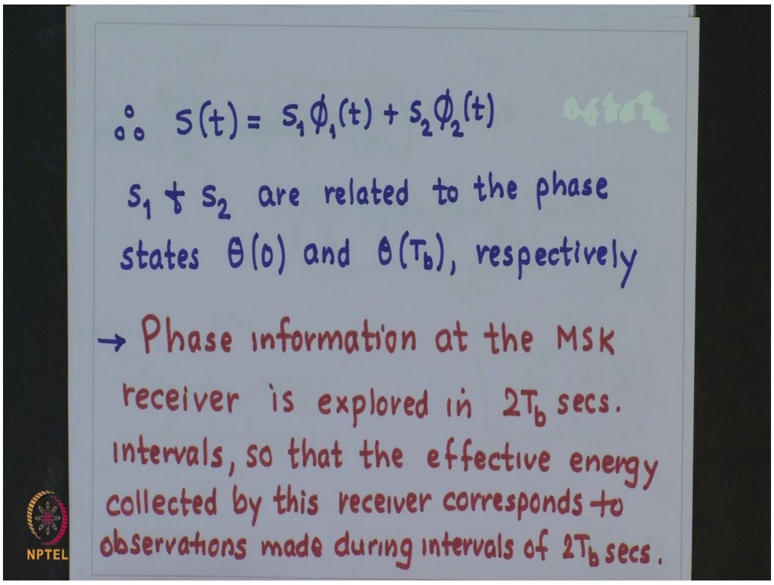


One is a S I t component which modulates cos 2 pi fct, and we have S Q t component which modulates sin 2 pi fct. The information of the signals which has been transmitted is contained in this phase theta naught and theta T b and the waveform corresponding to this states are S I t and S Q t respectively. And we have seen that this waveform S I t is of the form shown here, which is half cosine and the polarity of this waveform is decided by the phase state theta naught.

So, if theta naught is 0 I get this positive waveform and if it is pi, I get it to be negative. So, what this means that we are transmitting 2 states of theta naught either theta naught can be 0 or it can be pi. And to do that we are using these 2 waveforms of opposite polarity and similarly the argument can be extended to S Q t. We are transmitting the phase information by using these 2 waveforms and what are the phase information theta T b equal to either pi by 2 this waveform or theta T b is equal to minus pi by 2 given by this waveform.

So, now this S t is composed of 2 waveforms this waveform modulates cos 2 pi fct. So, there will be cos 2 pi fct in this and there is sin 2 pi fct in this. So, your S t is composed of one waveform which goes from minus T b to plus T b and there is another waveform, which goes from 0 to 2 T b. If you understand this then it is not difficult for us to see that our signal S t can be represented in terms of 2 orthonormal basis signals as follows.

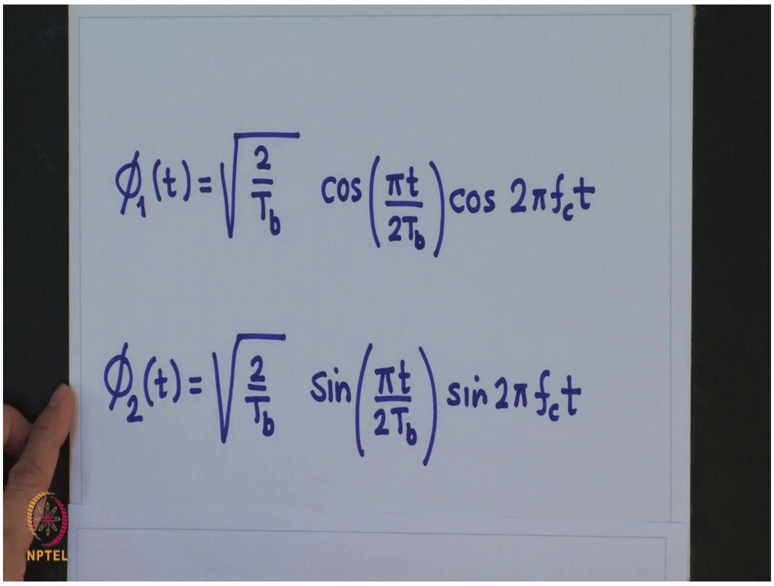
(Refer Slide Time: 07:41)



So, my task is to write S t has S 1 times phi 1 t and S 2 times phi 2 t. S 1 and S 2 are related to the phase states theta naught and theta T b respectively.

So, phase information at the MSK receiver is explored in 2 T b seconds in terms. So, that the effective energy collected by this receiver corresponds to observations made during intervals of 2 T b seconds. The only difference is that those 2 waveforms which we have they are offset by T b. So, given this argument it is not very difficult to see that we could choose 2 orthonormal basis signals as follows phi 1 t is chosen as this signal.

(Refer Slide Time: 08:44)

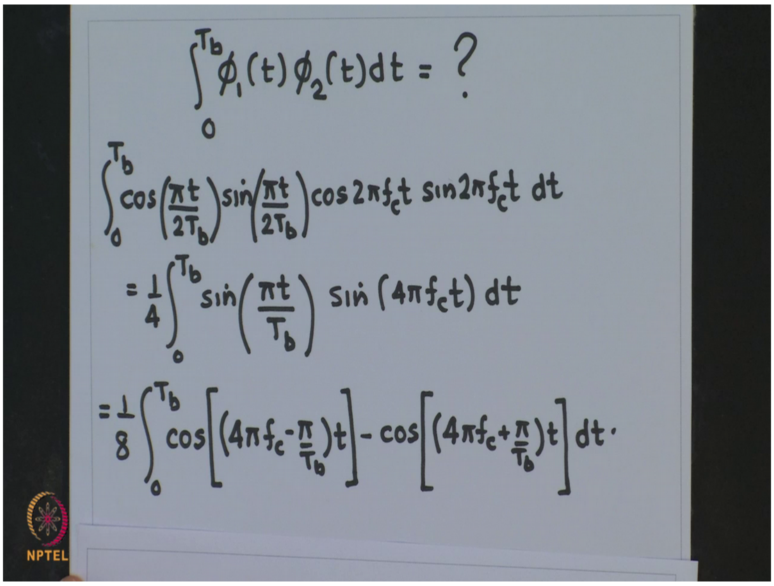


And the duration of this phi 1 t will be between minus T b to plus T b correct.

And we will choose another orthonormal basis signal, phi 2 t and its duration is going to be from 0 to 2 T b. Given this 2 basis signal of this duration I should be able to write my S t as this form and consequently what I am doing is I am trying to represent my signal S t, which is given by this expression in terms of orthonormal basis signal. If we choose this has the orthonormal basis signal let us see whether these 2 signals are orthogonal and let us try to find out the energy of each of this signal.

So, first let us try to find out the orthogonality and the common interval between these 2 signal is 0 2 T b whereas, between minus T b to 0 phi 1 t exists, but phi 2 t does not exist and similarly phi 2 t exists between T b to 2 T b, but phi 1 t does not exist over that duration. So, the only common interval for these 2 signals is 0 2 T b let us see what happens between that duration. So, let us try to calculate this integral phi 1 t phi 2 t over the duration 0 to T b.

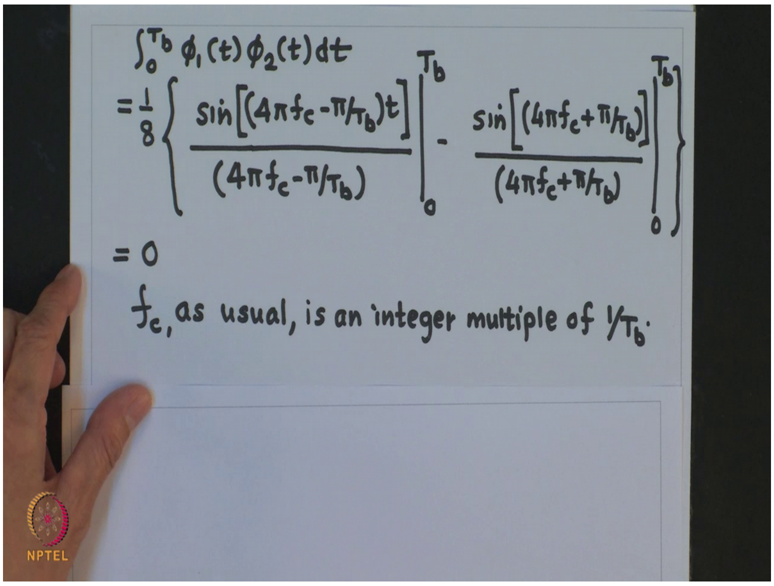
(Refer Slide Time: 10:36)



So, I will ignore the factor of root 2 by T b associated be each of this waveforms without loss of generality and if we write this expression using the trigonometric identities, I can rewrite this expression as 1 by 2 of this and similarly this I can write it as 1 by 2 of sin of 4 pi fct that is why we get 1 by 4.

Now, again we use the trigonometric identity and simplify this product to this summation of 2 cos terms and if we integrate each of this individually a straightforward integration.

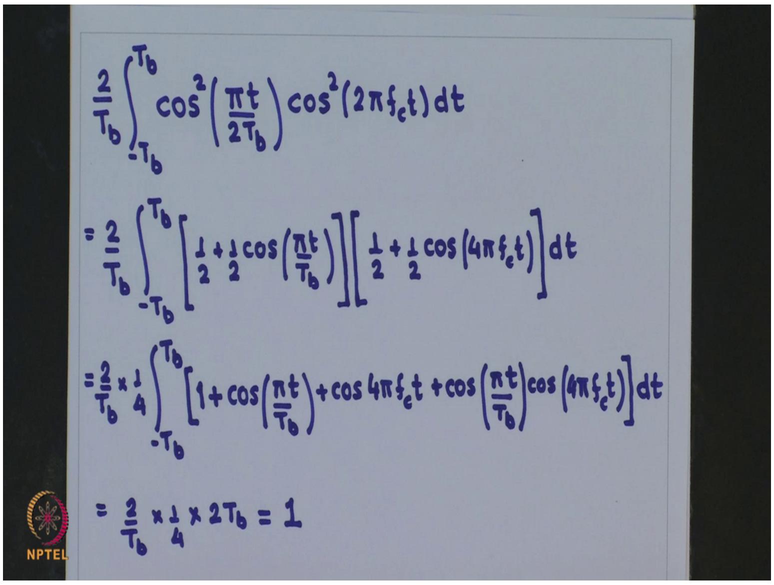
(Refer Slide Time: 11:38)



We get these 2 terms and they have to be evaluated over the lower limit and upper limit of the integral and its simple to see that, when I do this both of this terms go to 0 and I will get to 0. Here when I am doing this it is assumed as usual that fc is an integer multiple of 1by T b ok. So, we can say that phi 1 t and phi 2 t are orthogonal over the periods of interest.

Now, let us calculate the energy of phi 1 t and phi 1 t exists between minus T b to plus T b.

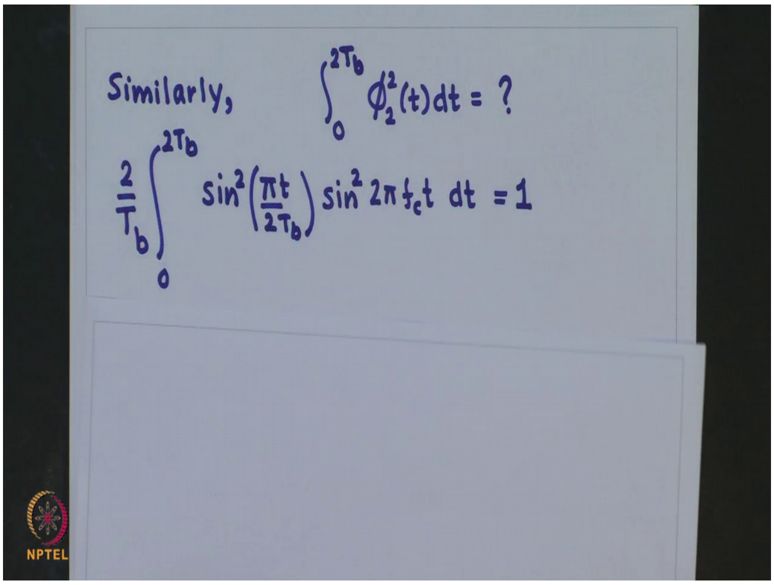
(Refer Slide Time: 12:28)



So, we will evaluate this expression this expression can be rewritten using the trigonometric identity as this expression and similarly cos squared 2 pi fct can be written using trigonometric identity has this expression, and then we take half out of both. So, I get one fourth and then multiply these 2 terms, I will get all these terms

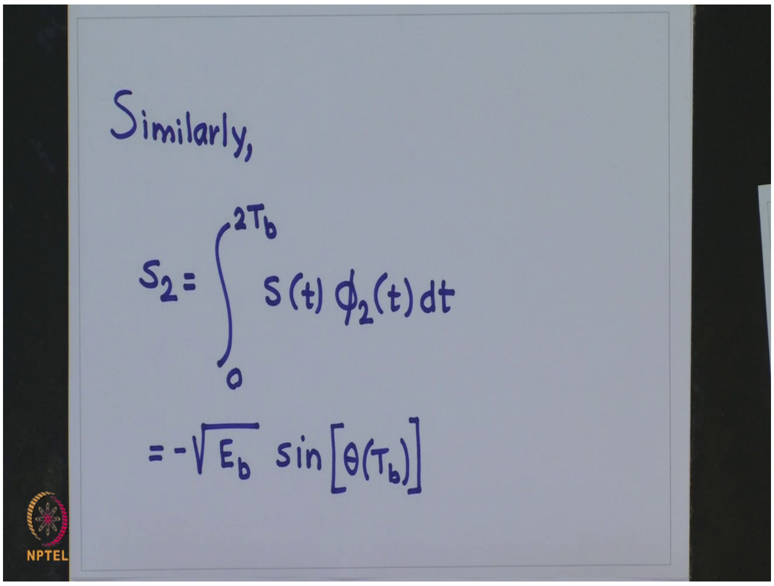
Now, when we integrate each of this terms these three terms over this period minus 3 be sorry I repeat when we integrate this terms over the period minus T b to plus T b all these three terms will go to 0 correct. And we will get the integration of only this the constant and that will be equal to 2 T b and that is how I get 1. And similarly now it is very easy to show that even phi 2 t has unit energy over the duration 0 to 2 T b correct using the same similar procedure you will get this value ok.

(Refer Slide Time: 13:49)



So, now we can use this phi 1 t and phi 2 t as our orthonormal basis signal. So, we will get the coefficients S 1 and S 2 they can be evaluated easily. So, S 1 would be obtained by taking the projection of S t over phi 1 t and its very straightforward to see that, your s t is given by this quantity.

(Refer Slide Time: 14:13)

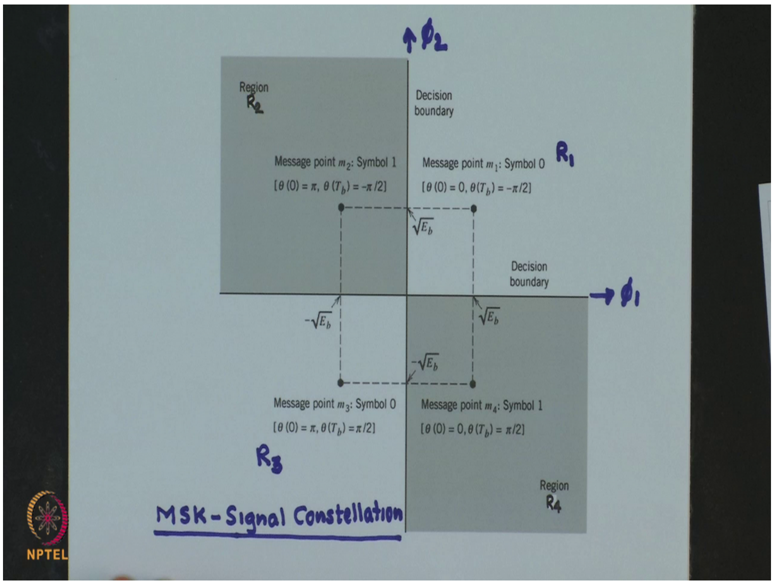


And if I integrate this quantity complete after multiplying by phi 1 t its simple to see that the phi 1 t is orthogonal to this term out here, it will go to 0. So, you will get the contribution only from this term, and you will get root E b cos theta naught. So, this is what we get that will be my projection of s t or phi 1 t.

And similarly we can obtain the projection of s t over phi 2 t and I will get this quantity out here. Please note this presence of minus sign because your S Q t was without minus sign and that is why that minus sign comes with S 2 there is only difference, and important to note that both the integrals for S 1 and S 2 are evaluated over the duration of 2 T b, but the lower limit and the upper limit are offset by duration T b correct.

So, we see that the signal constellation for this is going to be the 2 dimensional one and it will be as shown in this figure out here.

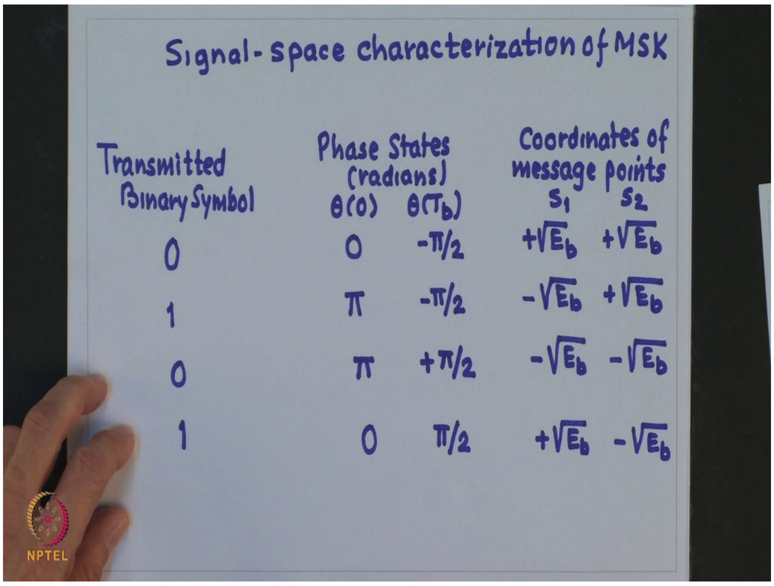
(Refer Slide Time: 15:54)



. So, in this case n is equal to capital N, which is the dimension of the signal space will be equal to 2, we have 4 points here out here. So, there are 4 message points. So, this signal constellation looks very similar to QPSK case, but the difference is that in the QPSK case, all the 4 message points correspond to 4 different signals. Whereas, for MSK signal constellation though we have 4 message points only there are 2 messages to be transmitted because this 2 message points correspond to symbol 1 and this 2 message point correspond to symbol 0.

Now, given this based on our definition for S 1 and S 2 we can calculate what are this values out here correct. By choosing appropriate theta T b which can be plus phi by 2 and minus phi by 2 I know S 2 can be either minus root E b or plus root E b and similarly for your S 1 plus root E b or minus root E b. So, we will get this 4 message points and we can have the signal space characterization of the MSK has given in this table shown here.

(Refer Slide Time: 17:35)

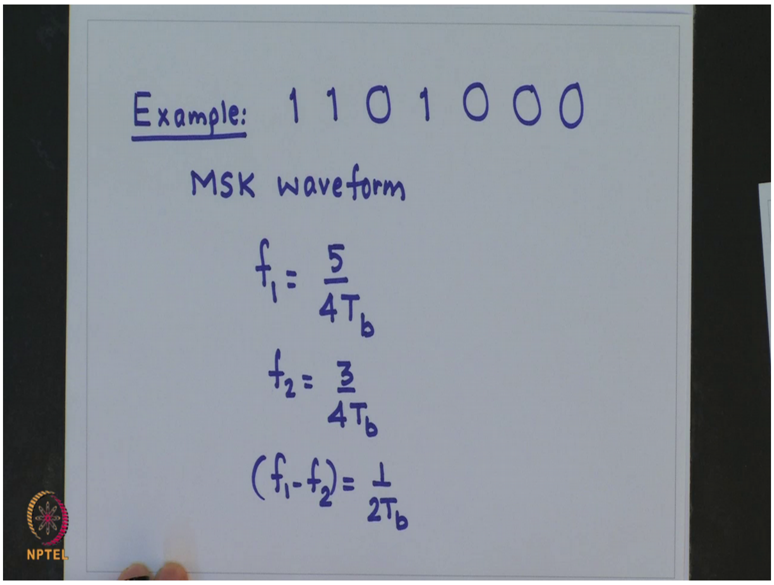


So, we have 4 phase states 0 minus pi by 2 pi minus pi by 2 and corresponding to this 4 phase states, we have the corresponding transmitted binary signals. So, see 2 states this and this correspond to one binary signal that is 0 and this 2 states correspond to another binary signal that is 1 and corresponding to this phase states, we get the coordinates of message points S 1 and S 2 as shown in this table.

So, this table summarizes your MSK signal constellation correct. So, given the input data sequence, we can use the entries of the table to derive on a bit by bit basis there are 2 sequences of coefficients required to scale phi 1 t and phi 2 t and once I know how to scale them; that means, I am obtaining the coefficient S 1 and S 2 then I can get my MSK signal as S 1 phi 1 t plus S 2 phi 2 t ok.

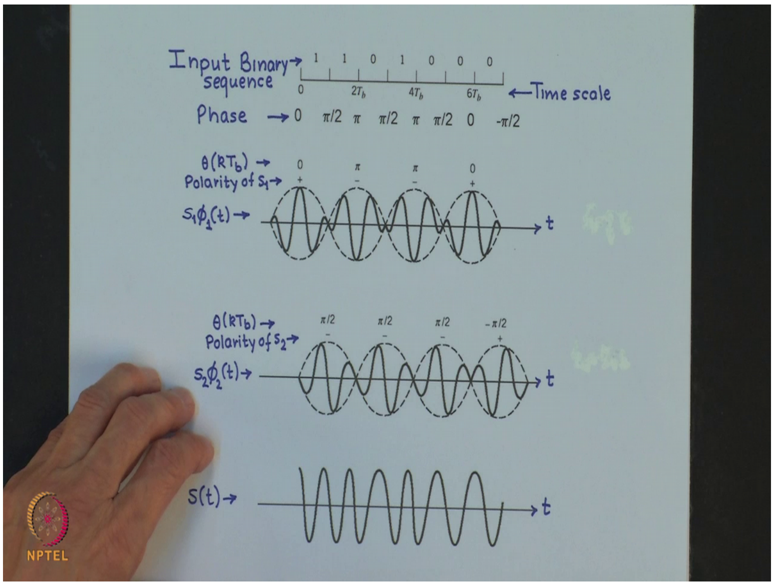
Let us take one example to help us to understand the concepts which we have studied so far. So, I will take one example of binary sequence transmission as 1 1 0 1 0 00 and we want to generate the MSK waveform for this.

(Refer Slide Time: 19:27)



F 1 has been assumed to 5 by 4 T b and f 2 is this. So, the difference between the 2 is 1 by 2 T b, which is the condition to be satisfied by an MSK signal and for this case we will generate the MSK signal as follows.

(Refer Slide Time: 19:58)



I have my put binary signal corresponding to this input binary signal I have the phases specified here.

So, when I transmit I will assume that at time T equal to 0 of my phase value is zero. So, at this duration T equal to T b since I am transmitting 1 my phase will increase by pi by

2, again I am transmitting 1. So, my phase will increase by pi by 2 it becomes pi, I transmit 0 during 2 T b to 3 T b. So, my phase decreases by pi by 2 and so on we will get the phase states of the sequence at each time instances.

Once we know this phases we can calculate the polarity for S 1 and S 2 as follows. So, we require to the theta for finding out the polarity of S 1 we need to know the phase state theta 0, theta 2 T b, theta 4 T b and theta 6 T b. So, if we look at theta 0, phase is assume to be 0; sp cos 0 is 1. So, the polarity of S 1 is going to be positive. Then at 2 T b we have pi cos pi is minus 1. So, the polarity of S 1 is going to be negative and similarly for 4 T b and 6 T b we can find out the polarity to be minus and plus respectively.

Given this polarity remember now we can find out our S phi 1, I know my S 1 polarity I calculate my S 1 value the magnitude of S 1 is root E b correct fine. So, only the polarity has to be known. So, here it is positive. So, we choose this waveform; so your cos function starts from here. So, this is positive like this, here in this case your polarity has become negative; so your cos function which we choose half cosine is negative. So, your cos 2 pi fct becomes negative correct and similarly this is negative and then again this is becomes positive fine.

Now, we find out similarly for the polarity of S 2. So, we have to find out the phase state at theta T b, theta 3 T b, theta 5 T b and theta 7 T b ok. We know the phases now sin of pi by 2 is 1, but remember when we are calculating S 2 there is a minus sign before that. So, this becomes negative. So, here also the state is pi by 2 sin pi by 2 is 1, but there is a minus sign. So, negative. So, we get the polarity of S 2. So, here it is going to be negative negative negative and positive and we choose this. So, your sine wave this here it will start like this similarly it will start like this, here also it will start like this correct and here it becomes positive.

So, I am just saying one half. So, similarly is true for the other half of this ok. And then we once we know this S 1 phi 1 t, I know s phi S 2 I add them together I get my S t and notice that your S t signal now is a continuous signal without any phase discontinuity. So, we have achieved what we desired that we want phase continuity during the inter bit switching time correct because we do not want the spread of the power spectral density ok.

Now, we will try to find out the modulator for the MSK case, look at the receiver and calculate the probability of error for the same. And finally, we will also evaluate the power spectral density of the minimum shift keying signal. And this will do in the next class.

Thank you.