Principles of Digital Communications
Prof. Shabbir N. Merchant
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 50
Continuous Phase Frequency Shift Keying

In QPSK modulation and it is variants, there will be phase discontinuity during the inter bits switching time. And this is also true of binary FSK modulation scheme. Now, such phase discontinuity give rise to high level of frequency contained, in the power spectral density and consequently this causes interference to the adjacent channels and, this is very much undesirable in communication system.

So, today we will study a scheme, where we can avoid this phase discontinuity during the inter bit switching time. And, we will see that this kind of modulation scheme has some kind of memory. So, the modulation scheme which we are going to study today is a modulation scheme, which has some kind of memory and it is a special form of frequency shift keying scheme.

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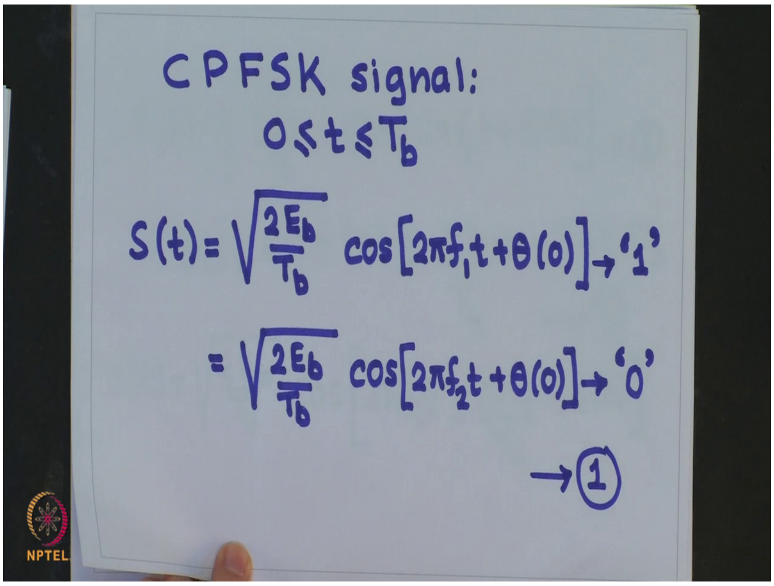

So, let us consider a continuous phase frequency shift keying signal, as follows this is over the interval 0 to T b. We have S t is equal to root of 2 E b by T b cos 2 pi f 1 t plus theta 0 and, root 2 E b by T b cos 2 pi f 2 t plus theta 0.

Now, E b out here is the transmitted signal energy per bit, T b is the bit duration and theta naught is the value of the phase at time equal to 0. So, theta naught sums up the pass history of the modulation process up to time t equal to 0. So, this theta naught will depend on the pass history of the modulation process. And we will transmit this signal for symbol 1 and transmit this signal for symbol 0. And both these signals are defined over the bit interval T b, because of this theta naught, we get the phase continuity between the inter bit switching time we will see this as we proceed. So, another way of representing this CPFSK signal is as follows.

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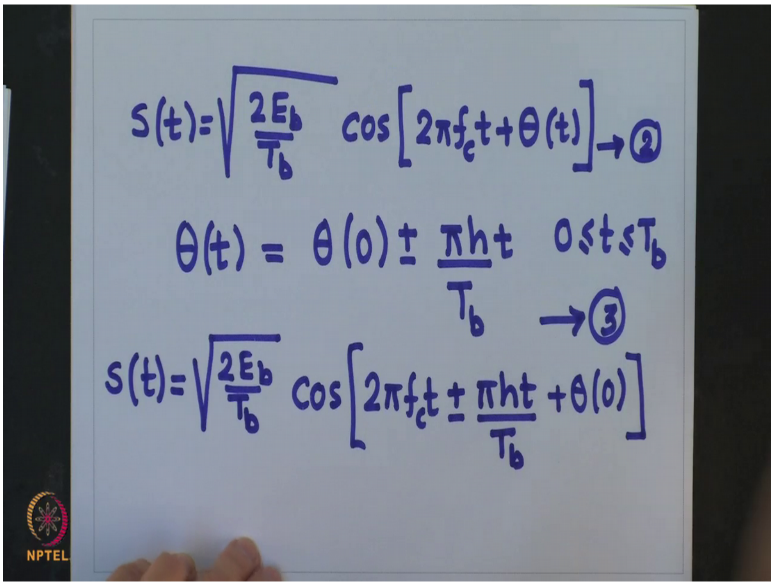

We can write S t is equal to root of twice E b by T b cos 2 pi f c t plus theta t, this form is the conventional form of angle modulation, where theta t is the phase of theta. So, when the phase theta is continuous function of time, we find that the signal S t itself is continuous at all times including the inter bit switching time. And let us select the theta t as follows.

We are considering the first bit interval that is from 0 to T b ok. Now, this phase theta t increases or decreases linearly with time during the bit interval of T b second. So, this will be plus when we transmit symbol 1 and this will be minus when we transmit symbol 0. So, let us call this equation 2 and, this has equation 3 and we will denote this equation 1. Let us substitute equation 3 in equation 2 and see what we get so, if you substitute that.

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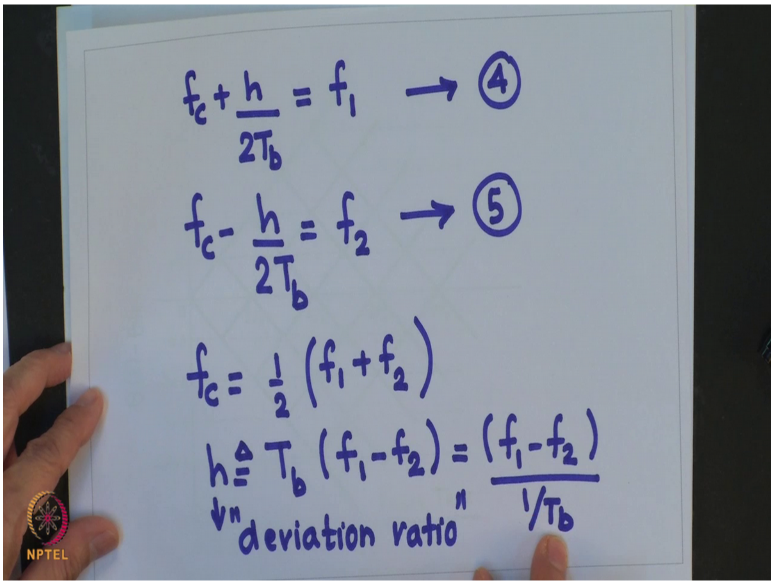

And now if we compare this equation, with this equation 1 it is very easy to see. That we can write f c plus h by 2 T b we will define h very soon. This is equal to f 1 and f c minus h by 2 T b; this is equal to f 2. So, solving 4 and 5, we will get f c is equal to half f 1 plus f 2 and h is equal to T b into f minus f 2. So, this is same as f 1 minus f 2 upon 1 by T b, 1 by T b is your bit rate. So, h is nothing, but the normalized frequency difference with respect to bit rate. And this is by definition known as deviation ratio, it is a dimensionless quantity.

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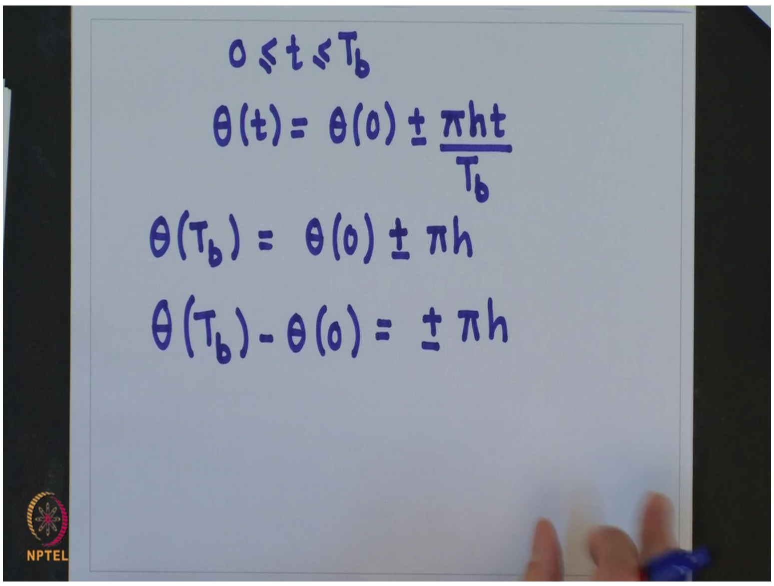

Let us consider the interval between 0 to T b and see how the phase varies. So, at t equal to T b what I get is plus minus pi h, or T b minus theta 0 is equal to plus minus pi h. So, this will be equal to plus pi h, which means that the phase will increase for the transmission of symbol 1 and it will be minus pi h which means that the phase will decrease for the transmission of symbol 0.

So, the variation of the phase theta t with time t follows a path consisting of a sequence of straight lines, the slope of which represent frequency changes. So, this is becomes very clear from this figure out here.

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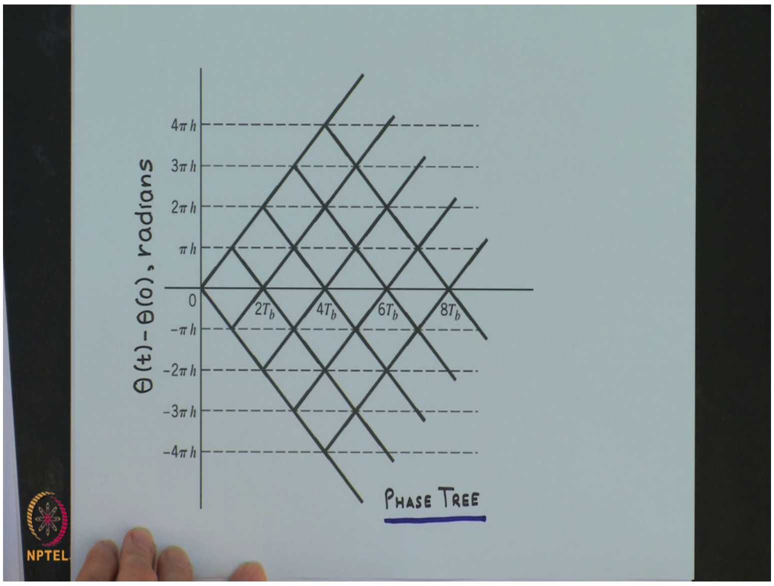

So, this figure depicts possible paths starting from time t equal to 0. So, a plot like this is known as a phase tree. This tree shows the transition of the carrier phase across interval boundaries of the incoming sequence of data bits. Now, from this phase tree, we can observe that the phase of the CPFSK signal is an odd, or even multiple of pi h radians at odd, or even multiples of the bit duration T b respectively.

So, for instances like 0 2 T b look here, this is going to be always either it will be 0, or it will be a twice pi h this will be 4 times pi h and that way. Whereas, that odd instances it will be the odd multiple of pi h correct ok. Now, let us take a special case of h is equal to 1. And this we will see basically will correspond to the case here, if I choose my h is equal to one then the difference becomes one by T b. So, this is a special case of FSK, which we had studied earlier and this is known as Sundays FSK.

So, in this case the deviation ratio h is unity. So, according to this figure the phase change over 1 bit interval, we will get is plus minus pi radians. But a change of plus pi radians is exactly the same as a change of minus pi radians modulo 2 pi. So, this implies that in the case of Sundays FSK, there is no memory that is knowing which particular change occurred in the previous bit interval provides no help in the current bit interval.

So, let us take a case where h is equal to half and this is the case of interest to us. Now, the phase can take only 2 values plus minus pi by 2 at odd multiples of T b and 0 and pi at even multiples of T b and, this is very clear from this figure out here.

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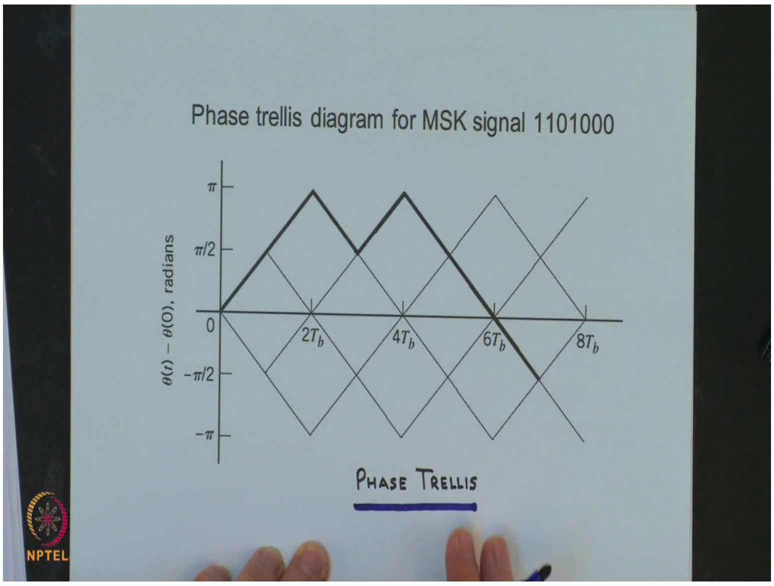

And this figure is known as phase trellis. So, why this is known as trellis is because, the trellis is a tree like structure like we had for the phase trellis, but with reemerging branches correct. So, that is why it is known as trellis.

Now, each part from left to right through the trellis of this figure corresponds to a specific binary sequence input. So, let us take one example. So, here this bold lines which have been shown, this correspond to the binary sequence 1 1 0 1 0 0 0 and let us see how do we get this. So, we start with theta 0 equal to 0 and with h is equal to half, we find that the difference between f 1 and f 2, now is going to be r b by 2. So, half the bit rate here we start. So, when we get 1 we increase by pi by 2, then again we get 1 so, we increased by pi by 2. So, it becomes pi then we get a 0. So, it reduces to pi by 2 again it becomes 1. So, it goes to pi and it continues this way.

Now, remember that for h equal to half, since the frequency difference turns out to be half the bit rate. This is the minimum frequency spacing that allows the 2 FSK signals representing symbols 1 and 0 to be coherently orthogonal. In the sense that they do not interfere with each other, we had studied earlier that for 2 carriers at frequency f 1 and f 2 for coherently orthogonal case the minimum frequency difference required is 1 by 2 T b whereas, for non currently orthogonality condition, we require the difference to be 1 by T b.

So, this MSK case is the special case of FSK, where the difference between the 2 frequency that is f 1 and f 2 is minimum 1 by 2 T b. And that is why this version of CPFSK, where the deviation ratio is chosen half is known as minimum shift keying or also known as fast FSK.

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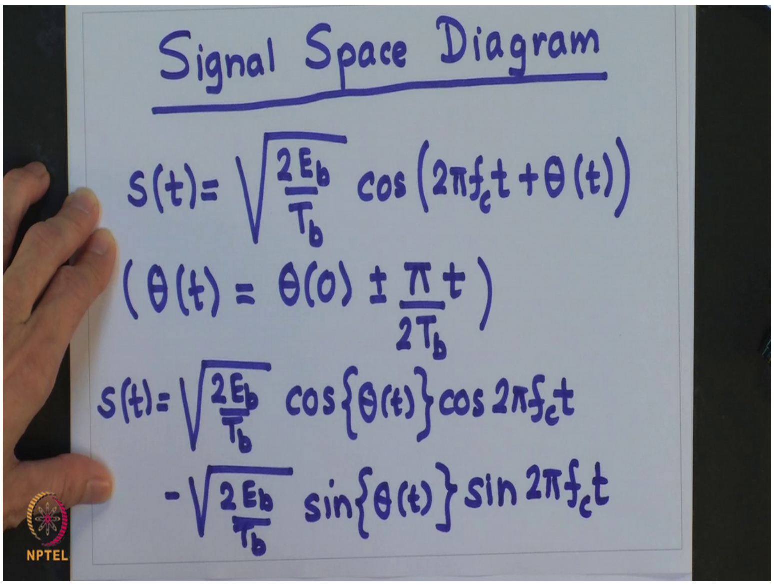

Now, let us look at the signal space diagram for this MSK case, signal space diagram will help us to design the optimum receiver and, also help us to calculate the probability of error. And it will also help us in understanding the modulation scheme for MSK.

So, let us we have this S t given in the form, 2 E b by T b cos 2 pi f c t plus theta, where theta t is of the form theta 0 plus pi by h. So, in our case we will choose h to be half. So, we will write it as 1 by 2 equal to t so, this is our MSK. Now, this can be expanded this S t can be expanded as follows. So, this form can be written as follows.

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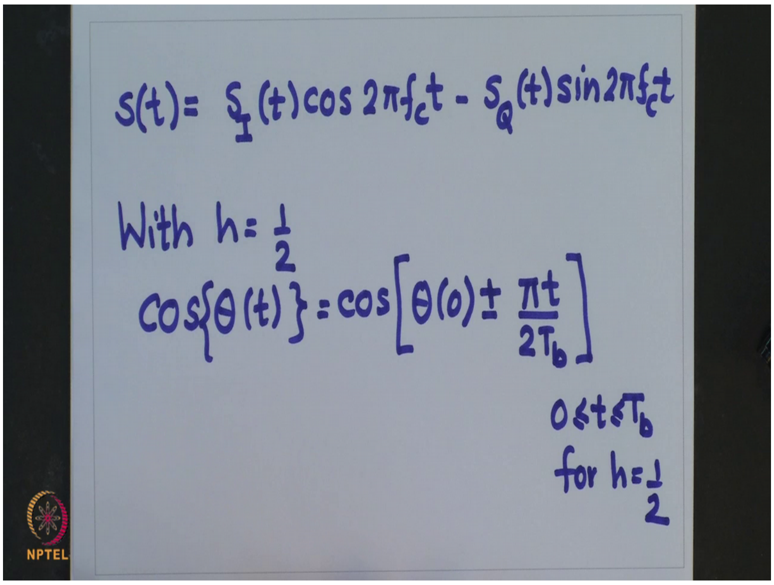

This is a standard form for writing band pass signals. So, we write in terms of the in phase component of cos 2 pi f c t and S Q t forms the quadrature component sin 2 pi f c t. So, here this will become your sit and this will become your S Q t. So, it becomes quadrature modulation. With h is equal to half our cos theta t, we said is the of the form, let us consider the bit interval 0 to T b to start with remember plus is for the transmission of symbol 1 and minus is for the transmission of symbol 0.

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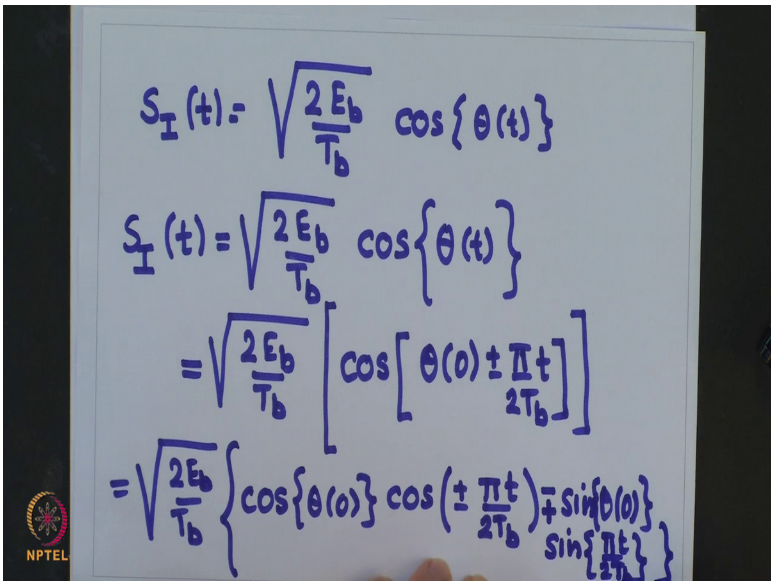

Now, note that your S I t is of the form root 2 E b by T b cos theta t. So, this is a cos function out here. Now, between 0 to T b theta t at t equal to 0, we will have value either 0 or pi, depending on the past history correct. And at t equal to T b the value will go to 0 because, theta T b is either going to be plus pi by 2, or minus pi by 2 in which case cos will turn out to be 0. Now, similarly if we try to evaluate this quantity at theta equal to minus T b, we will find that it goes to 0.

So, what it implies that this sit is some kind of half cosine function, with the maximum value at t equal to 0 correct what we have written here is also valid between minus T b to 0. So, cos theta t this relationship can be extended between minus T b to plus T b correct. The only thing is that the algebraic sign is not necessarily the same in both the intervals ok. So, if we take that so, the polarity of this cos theta t will depend on theta 0, regardless of the sequence of 1s and 0s transmitted before or after t equal to 0. And remember theta 0 can take only 2 values that is 0 and pi.

So, your S I t is going to be a half cosine pulse. And let us evaluate that pulse so, this I can write it as cos of theta 0 plus minus pi 2 T b t, which I can rewrite it as cos theta 0 cos of plus minus pi t by 2 T b first term I will get ok. So, and the other term would be minus plus sin of theta 0 multiplied by sin of pi t by 2 T b. Now, remember theta 0 can take values 0 or pi. So, this quantity will go to 0.

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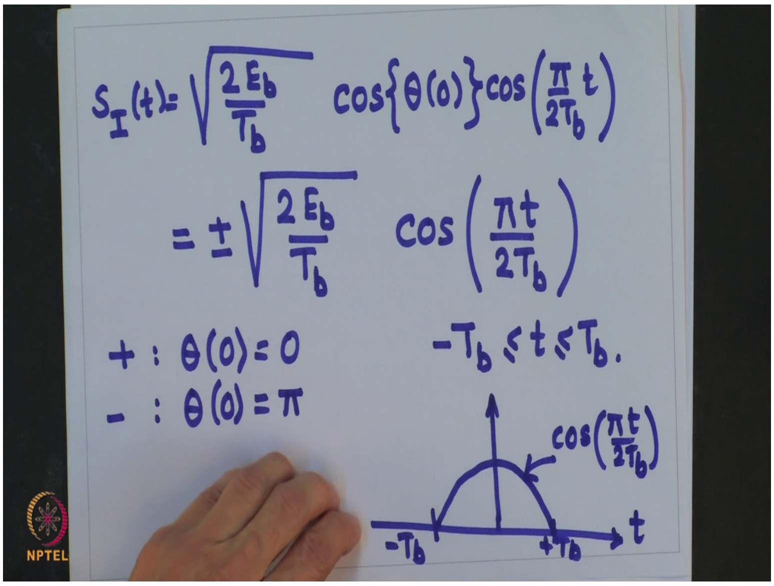

So, what we will get from here is my S I t is going to be of the form 2 E b by T b cos theta 0, cos is a even function. So, this I will get it as cos of pi by 2 T b theta t correct. So, depending on what is theta naught 0 or pi what I am going to get here is plus minus 2 E b by T b cos of pi t by 2 T b. So, I will get plus sign when theta naught is equal to 0 and minus sign, when theta naught is equal to pi. And, remember this basically S I t is going to be valid over this interval.

So, you will have something like this. So, this could be positive or negative. So, this is minus T b this is plus T b and this is your cos pi t by 2 T b. So, this is assuming that theta naught is equal to 0, if theta naught is equal to pi then this will be the inverted. So, polarity will depend on theta naught fine.

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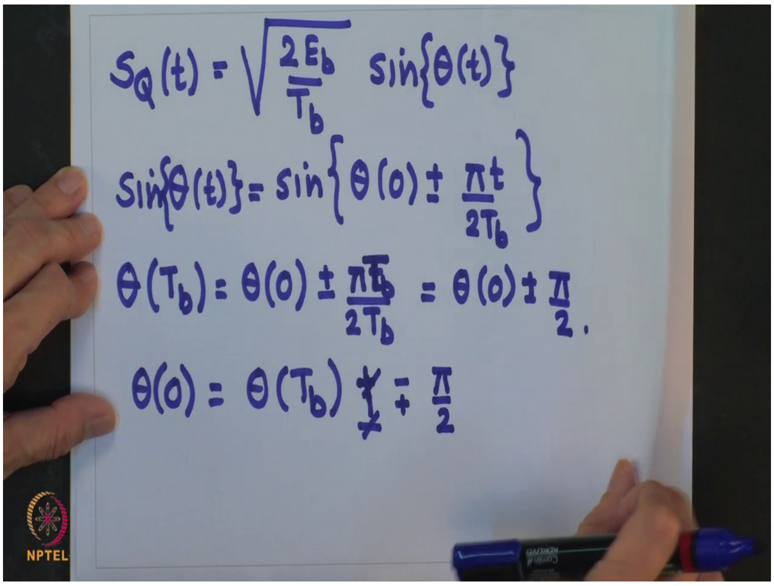

Now, let us look at the quadrature component, which is your S Q t that is equal to root 2 E b by T b sin of theta t.

Now, look here if you look between 0 to T b this value at t equal to 0 will go to 0, at t equal to T b it will be maximum it will be either plus 1 or minus 1 depending what is the value at of theta at T b. If it is plus pi by 2 this will become 1, if it is minus pi by 2 it will become minus 1 and at theta equal to 2 T b, this will again go to 0. So, what this implies that it will be a half sin wave between 0 to 2 T b, the maximum value will be at equal to t equal to T b. And the polarity of that will depend on sin of theta T b fine.

Now, let us expand sin theta t so, this I can write as sin of theta 0 plus minus pi t by 2 T b. Now, we will use this relationship this is T b please. So, theta T b is going to be this so, what it implies that this is equal to theta 0 plus minus pi by 2. So, what this implies that theta 0 is equal to theta T b the, this should be naught plus minus this should be minus plus pi by 2 correct. So, I am going to use this theta T b equal to minus plus pi by 2 into this and let us see what we get ok.

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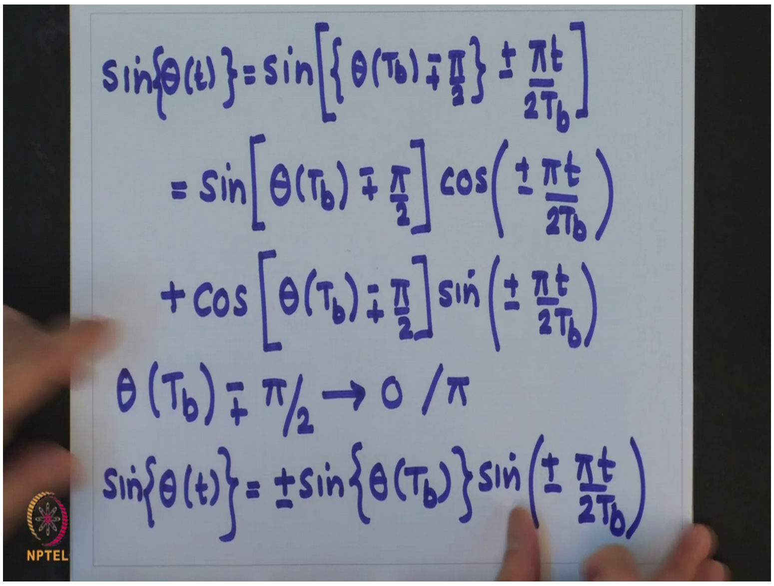

So, sin theta t we can write it as follows in place of theta 0, we write this quantity plus minus pi t 2 T b. This we expand it sin of theta T b minus plus pi by 2 cos of plus minus pi t by 2 T b plus cos of theta T b minus plus pi by 2 sin of plus minus pi t by 2 T b.

Now, note that theta T b minus plus pi by 2 can take values only 0 or pi because; this can take values plus or minus pi by 2. So, what this implies that this quantity when substituted here, this quantity will go to 0. So, we will get our sin theta t to be equal to this quantity and this quantity if you expand it now, you will get it as so, cos theta b, cos pi by 2 that will go to 0 so, you will get plus minus sin of theta T b sin of plus minus pi t by 2 type T b. So, this being odd function.

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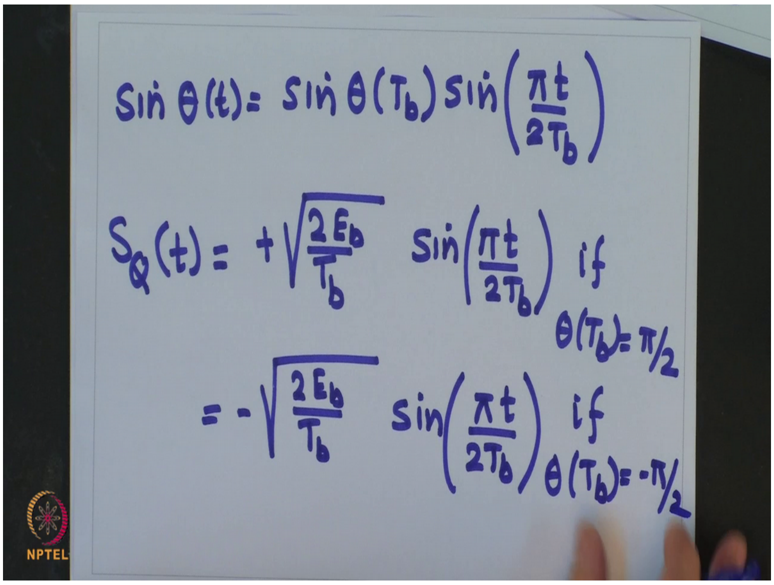

Now, if this is simple we will get it as sin theta t would be equal to sin of theta T b multiplied by sin of pi t by 2 T b correct. So, your S Q t now is going to be either plus 2 E by T b sin of pi t by 2 T b, this will be if theta T b is equal to pi by 2 and this would be equal to minus 2 E b by T b sin pi t by 2 T b, if theta T b is equal to minus pi by 2.

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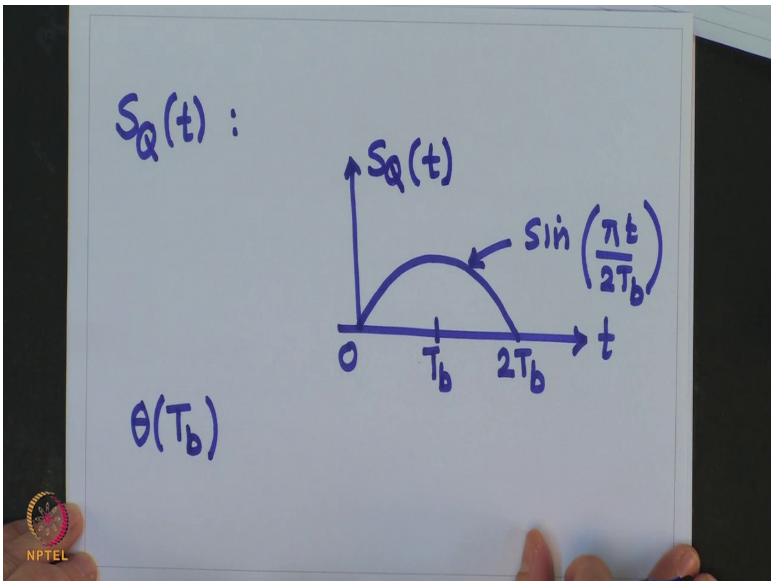

So, your S Q t will be half sin function, which will look something like this. So, the polarity of this half sin function will depend on the value of theta T b. So, when if this is equal to pi by 2 this will be positive and, if it is minus pi by 2 it will be negative. So, this is your S Q t. So, from our discussions we see that theta 0 and theta T b each can assume 2 possible values.

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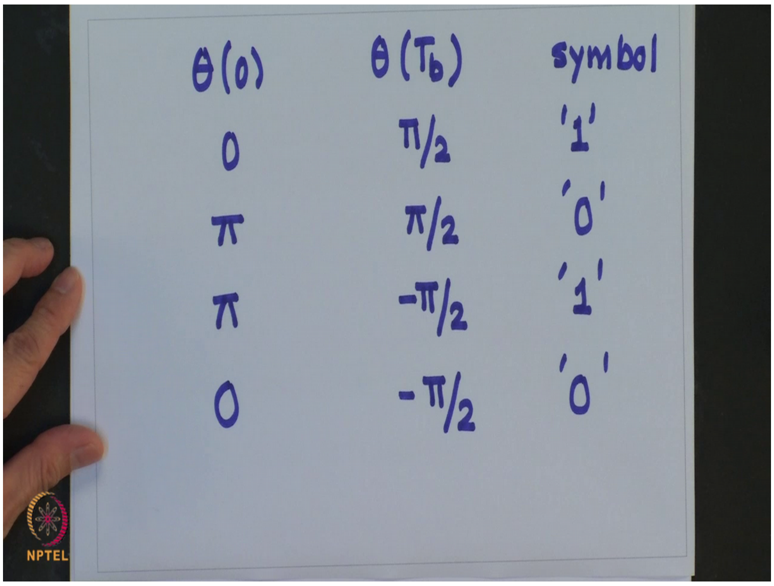

So, we have theta 0 and theta T b this could take the value say 0 and this could take value say pi by 0. In this case the transmission is of symbol 1, or we could have theta 0 equal to pi and theta T b could be equal to pi by 2, in which case we have transmitted symbol 0, or we could have this theta naught to be pi and theta T b to be equal to minus pi by 2, which is same as 3 pi by 2 modulo 2 pi this implies that there is a transmission of 1. And theta 0 equal to 0 and this equal to minus pi by 2, it means I have transmitted symbol 0.

So, what this means that MSK signal may assume any one of the 4 possible states, depending on the value of theta naught and theta T b. Now, based on this study we will find out the orthonormal basis signals which are needed to represent this MSK signal and, then obtain the optimum receiver and calculates its performance in terms of probability of error. This we will do it next time.