**Principles of Digital Communications**
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**Lecture - 48**
**Quadrature Phase Shift Keying - II**

So, in the previous class, we had studied the QPSK modulation scheme and we had derive the probability of message or symbol error. Now what we are interested is in the derivation of probability of bit error, and we will do that in this class.

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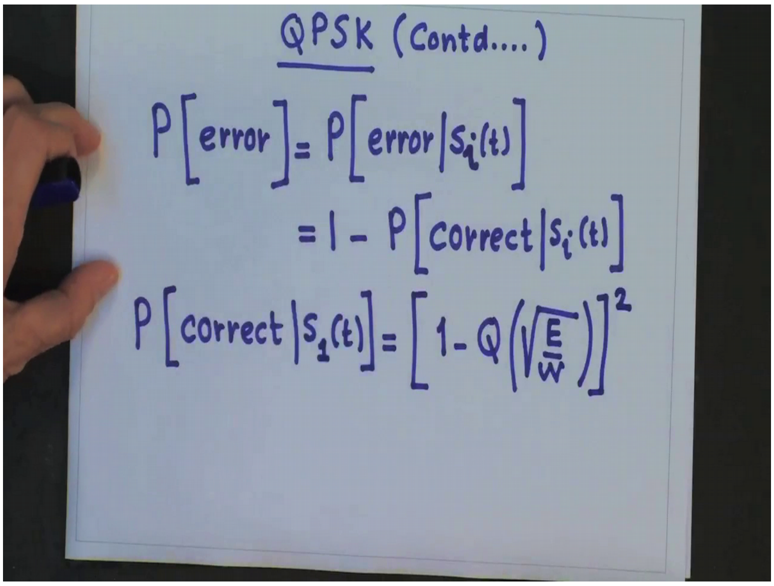


We had derived the probability of the symbol error to be as follows. So, given this we can find out the probability of symbol error.

Now, even though a message error has been made, it does not mean that the specific bit is in error, we took an example that if we transmit the message m 1 which is 0 0 bits and we decide in favor of m 2 which is 0 1, then in this case the only the second bit goes in error.

So, to determine the bit error probability, it is necessary to distinguish between different message errors.

Now, again because of the symmetry, it is sufficient to consider only a specific message say let us say m 1; then the different errors we will evaluate.

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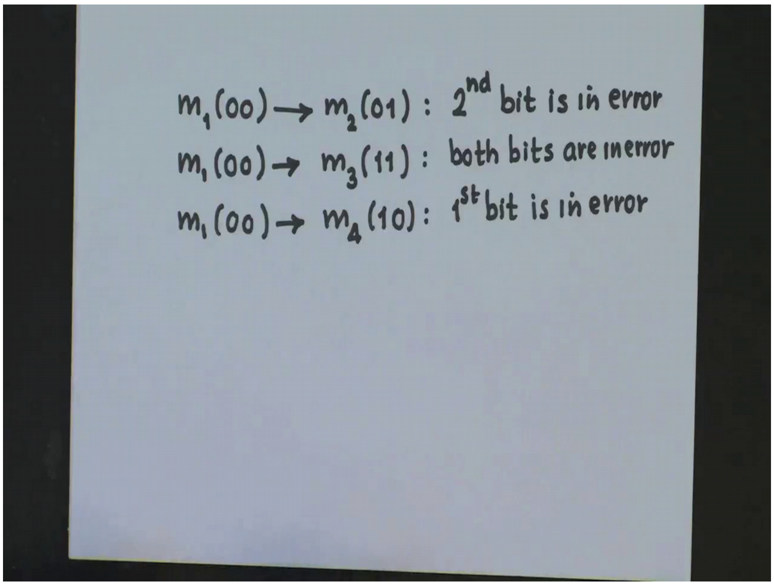


So, when we have got m 1, this could be decoded as m 2, in which case the second bit will be in error. If m 1 gets decoded as m 3 then both the bits will go in error. And if m 1 gets decoded as m 4 then only the first bit is error. So, the different error probabilities, which will get can be computed very easily.

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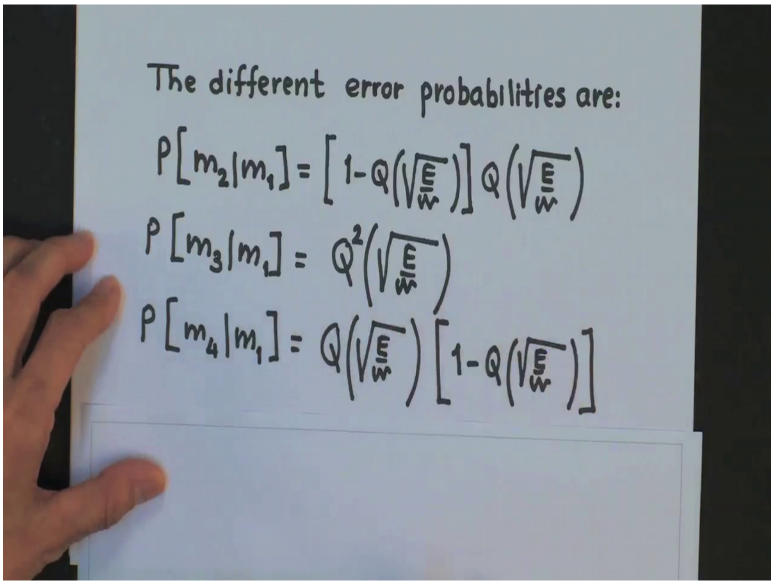

So, probability that we decode is m 2 given m 1 implies that, the first bit should be correct and that would be given by this expression and this is the probability that the second bit goes in error.

The calculation of these probabilities is very clear if you look at the signal constellation diagram which we have studied in the previous class.

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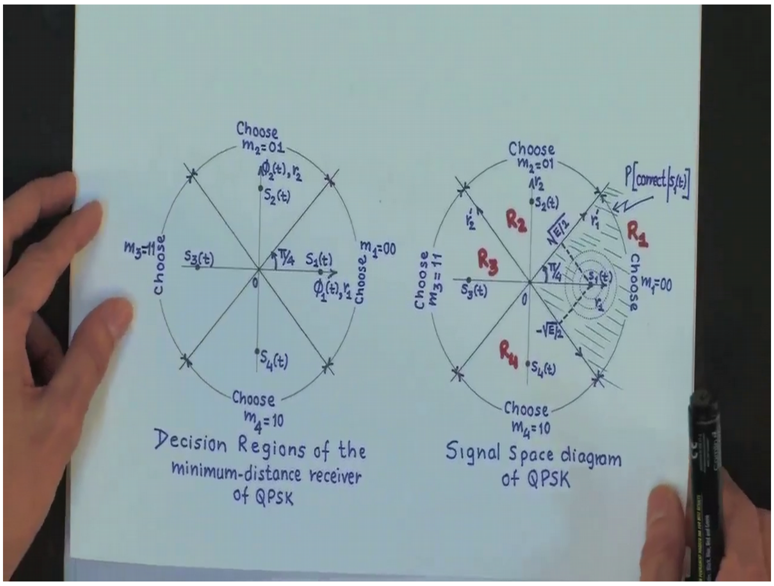


So, from looking at the signal constellation diagram, we can immediately write this probability of decoding as m 2 given m 1. Similarly for the probability of decoding m 1 as m 3 would be given by this expression, because we want both the bits to go in error. And probability of decoding as m 4 given m 1 would be that the first bit goes in error which is equal to this, and the second bit received is correct. So, it would be this value.

When the message errors of m 2 or m 4 occurs given that m 1 was transmitted, then the chosen bit is in error with a probability of 0.5. Where we presume that one of the 2 bits is chosen at random, that is with the probability of 0.5. Now when the message error of m 3 occurs then the chosen bit is certain to be in error therefore, it has a probability of 1.

With this philosophy we can write the bit error probability as follows.

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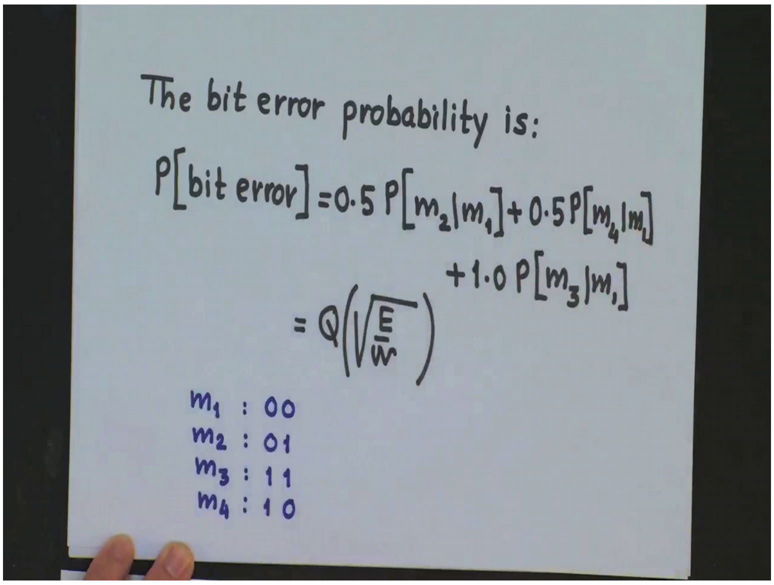


This is the probability given m 1 has been transmitted decoding as m 2, and you multiply this by 0.5, because the chosen bit is in error with a probability of 0 0.5 ok. And similarly for the case of m 4 being decoded as m 1 and in the case of m 3 given m 1, you multiply this by 1 ok. So, and then if we use these expressions, its easy to see that this will simplify to this expression out here.

So, from the above derivation, it is clear that the way each information bit pair is mapped to a message or signal influences the bit error probability. Specifically the mapping determines how the probabilities 0.5, 0.5 and 1 are associated with the 3 error probabilities here correct. And recognize that probability of m 3 given m 1 is less than probability of m 2 given m 1, which is equal to same as probability of m 4 given m 1.

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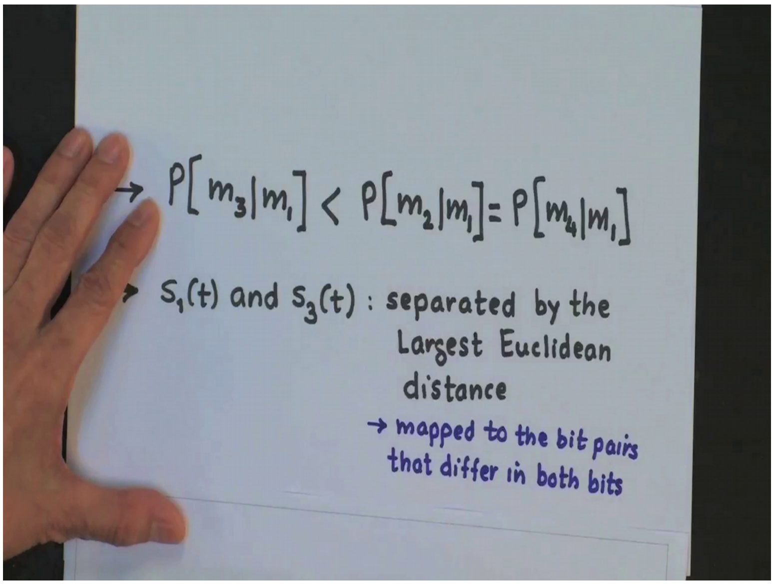


So, what is implies that it is desirable to associate the probability 1 with probability of m 3 given m 1. In order to minimize the overall bit error probability. So, what this implies that the signals S 1 and S 3 which are separated by the largest Euclidean distance should be mapped to the big pairs that differ in both bits.

So, equivalently the signals that are closest to each other that is the nearest neighbor should be mapped to the bit pairs that differ in only one bit. And such a mapping is called a grey mapping and this the way we carried out the mapping is a grey mapping.

Now, for a fair comparison with the error performance of the binary modulation schemes which we have studied earlier, it is necessary that we express the probability of the bit error in terms of the average energy per bit and this can be easily calculated as follows.

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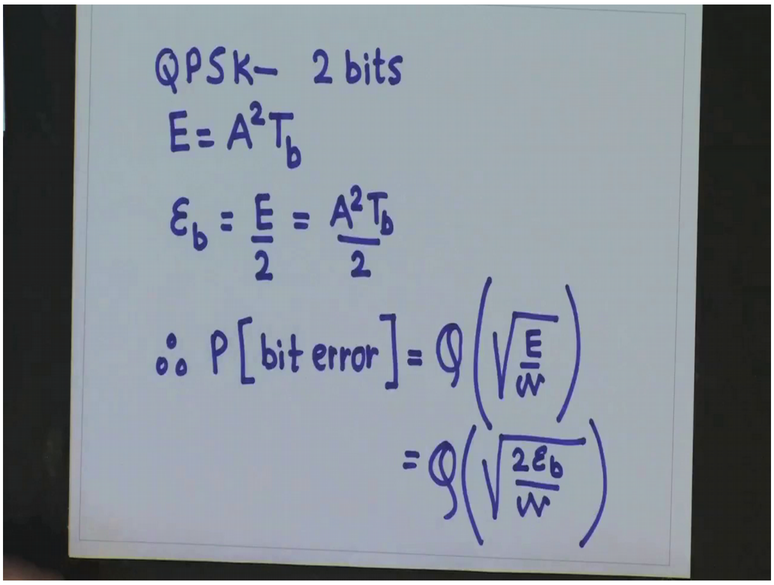


Now, QPSK signal carries 2 bits of information and the energy of each signal is equal to A squared multiplied by T b. So, the average energy per bit is equal to E by 2, which is equal to A square T b by 2. Therefore, we can write the probability of bit error in terms of the energy of the signal this is equal to.

So, what this expression shows that, the probability of bit error is the same as that of the binary PSK. So, this clearly demonstrates the advantage of QPSK over BPSK. With QPSK modulation the bit rate can be double without requiring any additional bandwidth or sacrificing the error performance. Interpreted differently to deliver the same transmission rate at the same bit error performance, using QPSK reduces a transmission bandwidth to half of that required by BPSK.

Now, we look at the alternative implementation of the QPSK modulation and we will do this with the help of an example.

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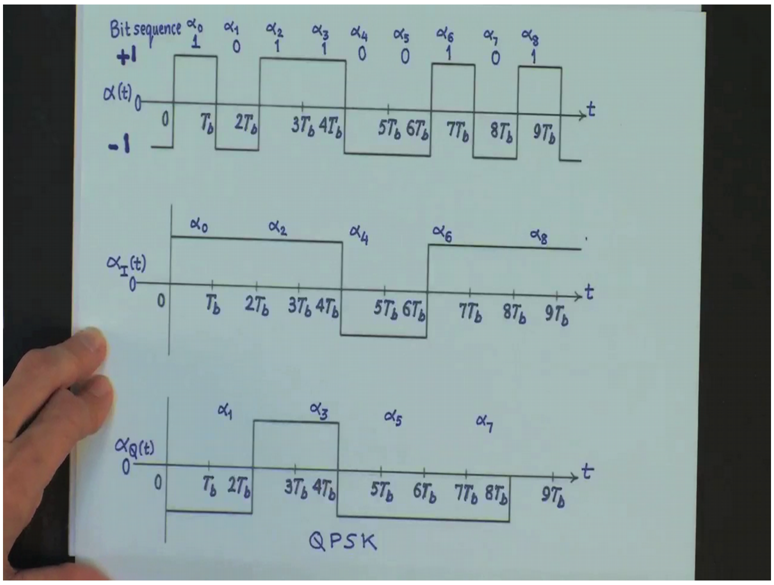


This alternate representation of QPSK helps us to implement the QPSK easily in practical applications and the principal operation is as follows. So, given a bit sequence 1 0 1 1 0 0 1 0 1; the information bit stream is first converted to a non return to 0 waveform with plus minus 1 levels. So, these are levels are plus 1 and minus 1 and this waveform we call it as alpha t. This waveform alpha t is then demultiplex into even and odd waveforms of bit streams, where which we denote as alpha I t and alpha Q t. So, I and Q here are mnemonics for in phase and quadrature respectively.

So, the individual bits in each stream occupy the duration of T b seconds which we call it as T s second, and each of this modulate the in phase carrier and quadrature carrier. So, in this case the in phase carrier would be a cos 2 pi fct and in this case the quadrature carrier would be a sine 2 pi fct. So, it i s easy to see this I take the first bit alpha 0, and it remains there over the period 2 b then we take alpha 2 and that also remains over 2 T b duration. And for alpha Q t we pick up the odd bits from here alpha 1, alpha 3, alpha 5, alpha 7 and get this waveform for alpha Q t correct.

So, each of these even and odd bits are picked up from the original bit stream and they are extended over a duration of 2 T b right. So, the QPSK modulator will look something like this.

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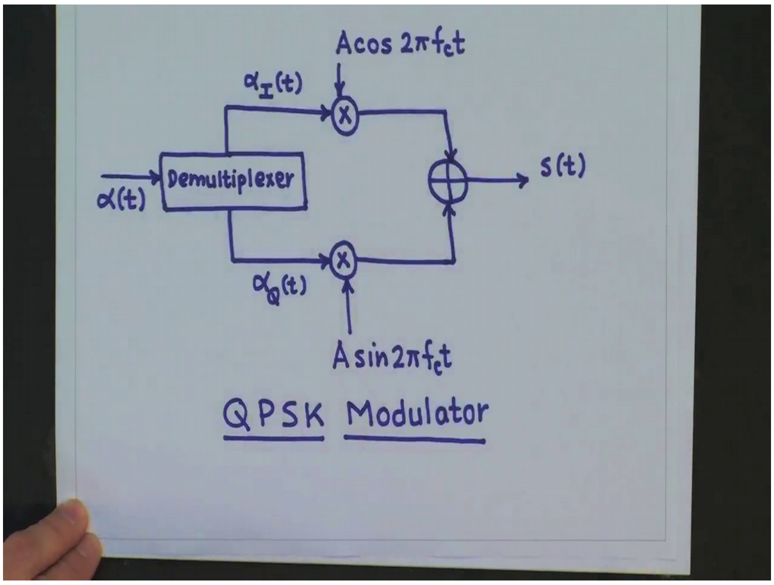


I have my alpha t the input and then it gets demultiplex into 2 bit streams, this is the in phase and this is the quadrature bit stream. And each of this bit stream get modulated this gets modulated by cos 2 pi fct, this gets modulated by sine 2 pi fct and st is the QPSK signal which is generated.

So, the transmitted signal is as follows.

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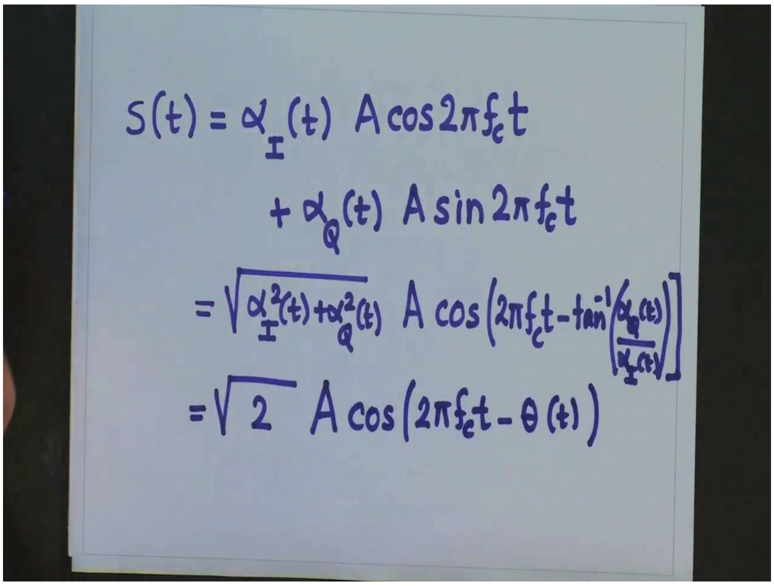


In phase bit stream or in phase waveform modulates cos 2 pi fct, quadrature waveform or quadrature bit stream modulates the carrier a sin 2 pi fct. And this can be rewritten as alpha Q t divided by alpha I t. So, this can be this values are plus minus 1. So, this I can write it as root 2 A cos of 2 pi fct minus theta t where the phase theta t is determined as follows.

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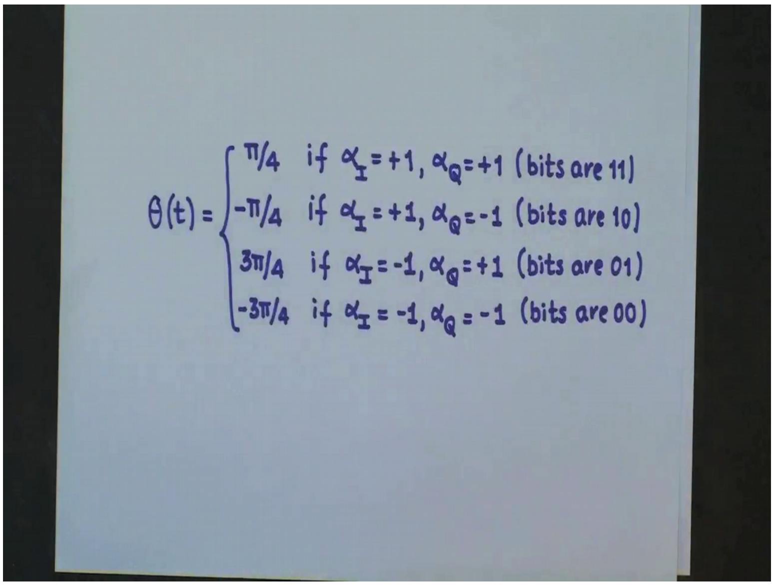


So, is equal to pi by 4 if alpha I is equal to plus 1 alpha Q is equal to plus 1 and that will corresponds to bits being 1, 1 minus pi by 4 if alpha is 1 alpha Q is minus 1 this corresponds to bits 1 0. 3 pi by 4 if alpha is equal to minus 1 alpha Q is plus 1 corresponds to bit 0 1 and finally, theta t is equal to minus 3 pi by 4 if alpha is equal to minus 1, alpha Q is equal to minus 1 and bits are 0 0.

So, using this alpha t and demultiplexing, we generate after modulation we get this waveform.

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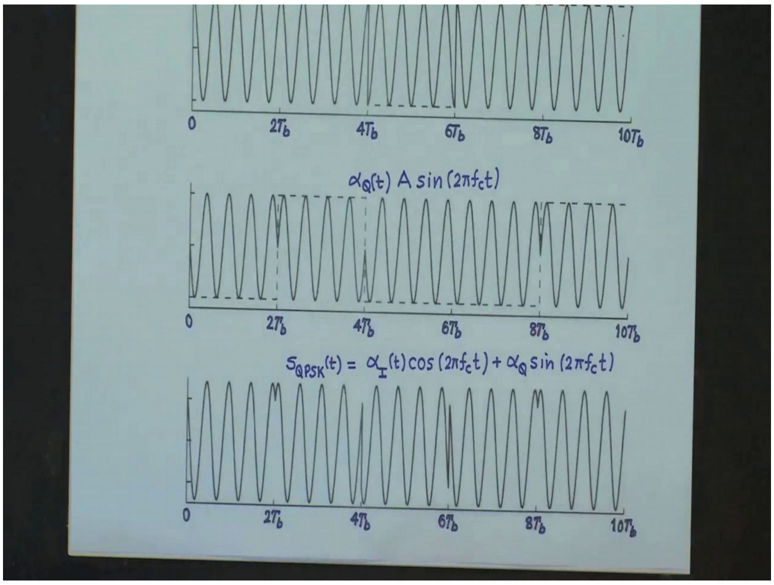


So, alpha I t, a cos 2 pi fct alpha Q t a sin 2 pi fct, and then if the summation of this 2 is your QPSK signal which we have generated correct. So, now from this equation it is clear that the transmitted signal is a QPSK signal, and depends upon the specific even and odd quadrature bits, which select the phase theta t of the sinusoidal carrier root 2 a cos 2 pi fct minus theta t.

So, you get 4 signals out of this for different values of alpha Q t and alpha I t and this can be represented by the following 2 orthonormal basis signals.

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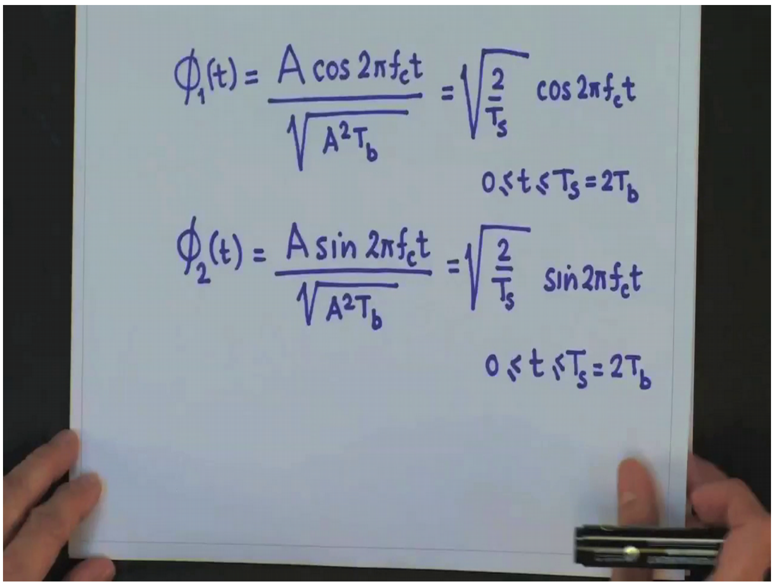

Phi 1 t is this signal and phi 2 t is given by this signal and using this 2 signals basically for representation, we can get the signal consolation for QPS signals as shown in this figure.

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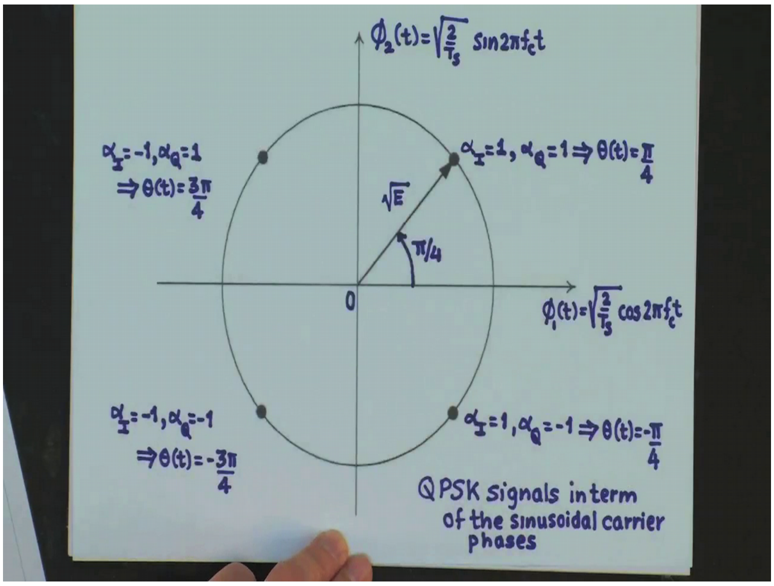


So, these are the 4 signals which we get. The phase of each signal is relative to this pi 1 t axis. So, this signal out here is at pi by 4 this signal is minus pi by 4 this is 3 pi by 4 and this is minus 3 pi by 4.

So, now we will calculate the minimum error probability receiver for the implementation of the QPSK which we have discussed.

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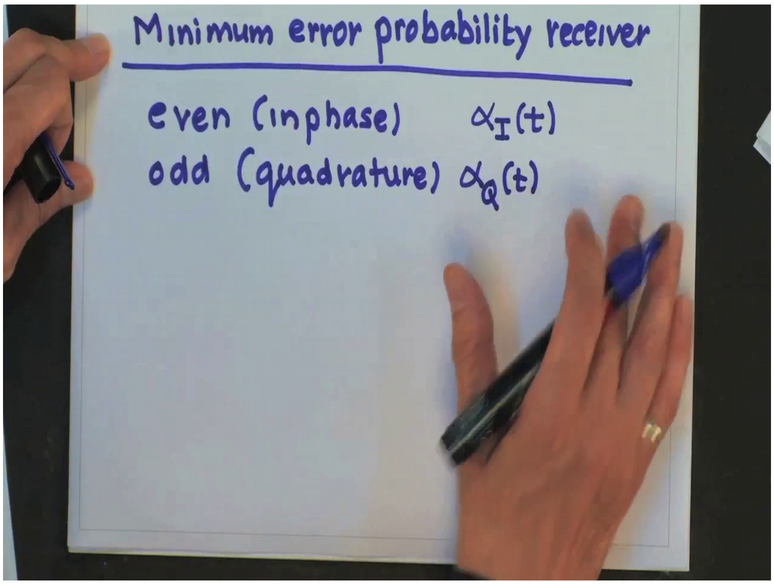


Now, note that the even bit stream. that is your in phase and the odd bit stream which is your quadrature, both this can be treated separately since the alpha I t bit stream of waveform does not have any component along pi 2 t basis signal, and similarly alpha qt bit stream of waveform does not have any component along the orthonormal basis signal phi 1 t.

So, what this implies that QPSK signal can be considered to consist of 2 separate non interfering DPSK signals. So, the signals phase diagram would be as shown here.

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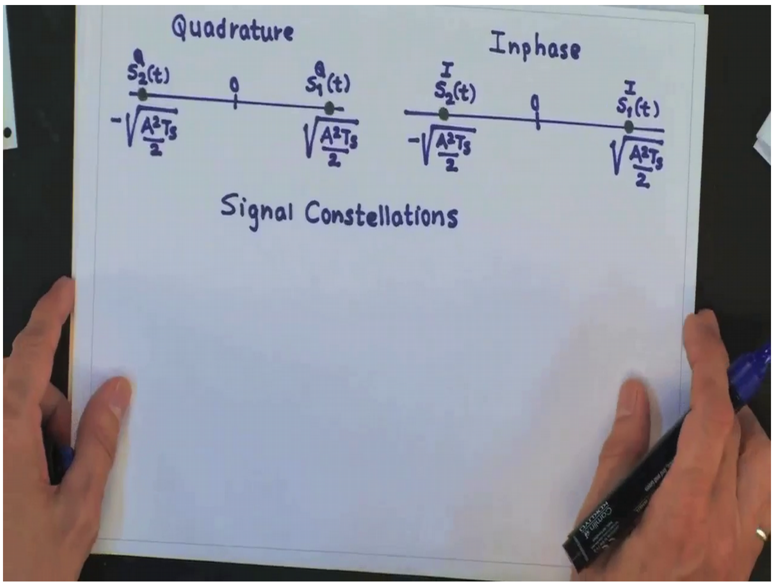

This is for the in phase part and this is for the quadrature part. Where this signal S I t S 2 t and here S 1 Q t and S 2 Q t are as defined on this slide.

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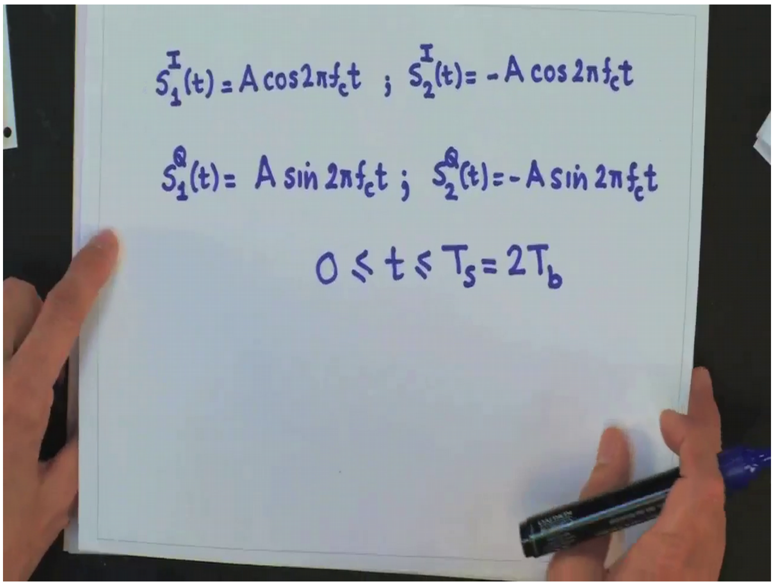


This is easy to realize because remember that your alpha I t is a polar waveform, which modulates cos 2 pi fct and x qt is a polar waveform which modulates a sin 2 pi fct. So, for this the basis signal in the in phase branch would be this and the basis signals for the quadrature branch would be as written on this slide correct.

So, for this case now, the receiver would look as shown on in this slide.

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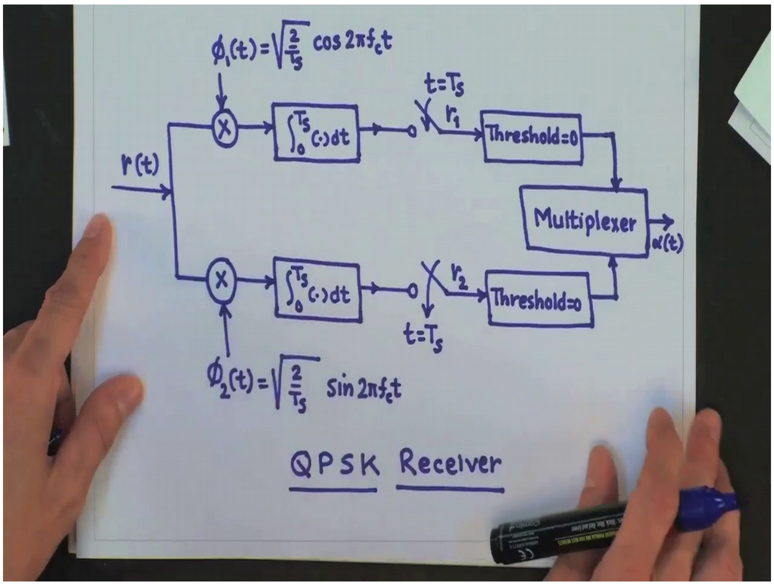

We get the received signal and you project them on phi 1 t and phi 2 t, you get the projections r 1 and r 2, you take the independent decisions. So, this will give the even stream and this will give the odd bit stream and you multiplex them to get their final waveform alpha t.

So, let us calculate the probability of error for any one of this because you are going to be the same. So, if we know the distance here.

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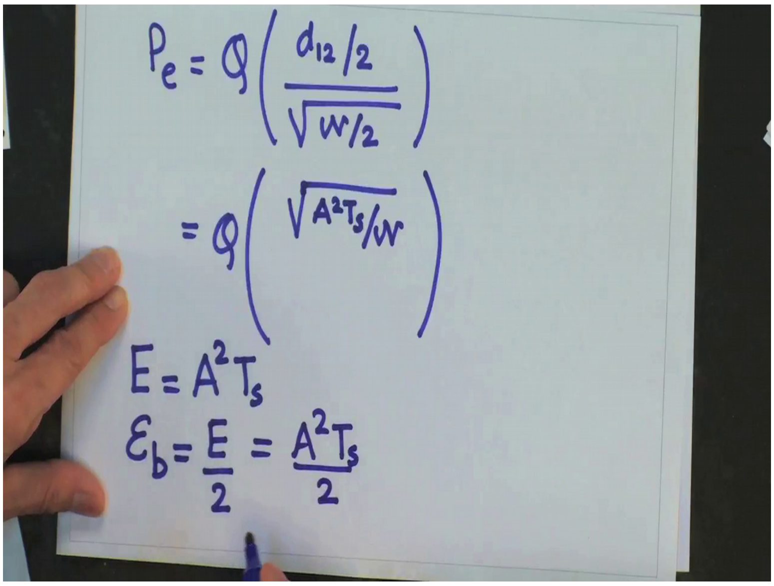


So, the probability of error would be given by the following expression d 1 2 by 2 divided by root of n italic n by 2, this is equal to look at this distance this is twice of this. So, substituting there, we will get root of A square T s by italic N.

Now energy of the QPSK signal we know is equal to a squared T s. So, the energy per bit is equal to E by 2 which is A square T s by 2. So, from this we get the probability of error to be equal to Q times root of 2 E b by italic N right substitute here, and this you get this quantity. So, this is indeed the same as derived earlier.

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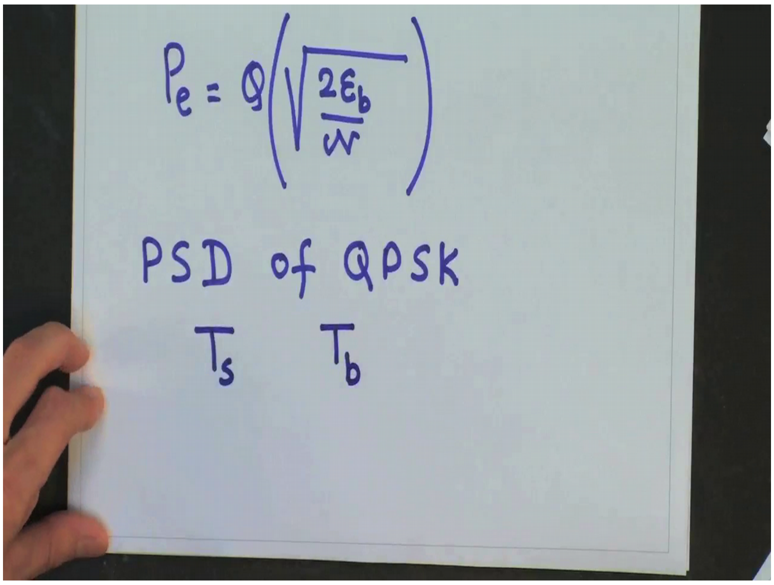


. So, finally, we have to calculate the power spectral density of the QPSK signal, now realize that QPSK signal is the sum of 2 BPSK signal and there is a statistical independence of the even and the odd bit streams, which are the 2 BPSK signal that make up the QPSK signal and since they are uncorrelated, it follows that the power spectral density of the QPS signal is twice the power spectral density of each of the BPSK signal. It is just scaled by 2 and with Ts substituted a for T b ok. QPSK signals phase could change by plus minus 90 degrees or plus minus 180 degrees.

Now, this can be of particular concern when QPSK signal is subjected to filtering before detection. Specifically the 180 degree and 90 degree shifts in the carrier phase can result in changes in the carriers amplitude, that is QPSK signals envelope and consequently affecting the probability of symbol errors on detection. There is a form of QPSK known as off set of staggered QPSK, which mitigates this problem of 180 degrees phase shift and this will study next time.

Thank you.