Principles of Digital Communications
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Lecture - 42
Partial Response Signaling - II

We learned that with the duo binary pulse we can transmit message symbols at the rate of 2 b samples per second, where b is the bandwidth of the channel, but this introduces controlled ISI.

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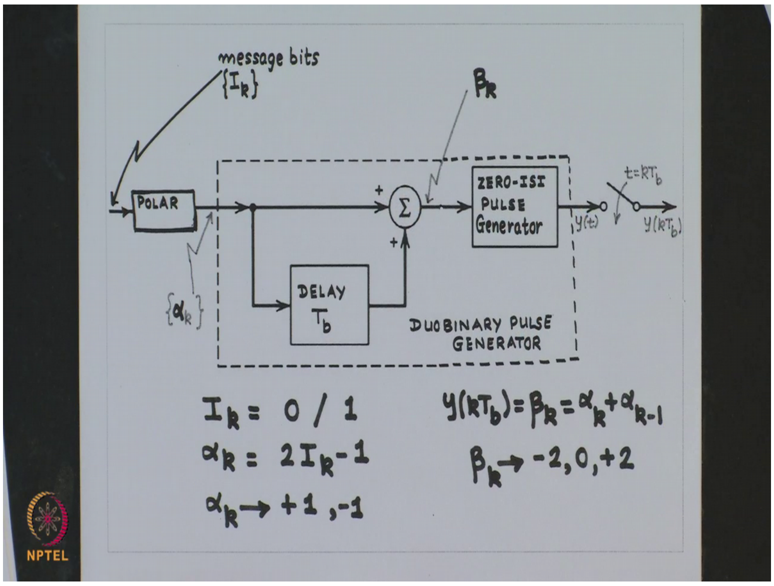

Now we saw that this is the block diagram for output of a duo binary pulse generator. The input bits are I k and output sample values here y k T b or b k is given by this relationship alpha k plus alpha k minus 1, where alpha k are the polar conversion of the message bit I k for I k equal to 0, we get value to be minus 1 and phi k equal to 1, we get equal to plus 1.

Now, we saw that for I alpha k plus 1 minus 1 this gets map to bk with for 2 minus 2 0 or plus 2. So, whenever we receive minus 2 we know that the input message bit has been 0 and whenever it is plus 2 the input message bit is plus 1. But whenever we receive 0 then the detected bit at that instant is dependent on the previous detected bit. So, if the previous detected bit was 0, then the current bit will become 1 and if the previous was 1 it will become 0.

So, what this implies that? If there is an error in the detection of the previous bit then there will be a cumulative effect on the error, till the arrival of the next correct minus 2 or plus 2 value. So, we said is it possible for us to relate this b k directly to the message bit I k and this can be done as follows.

For just presume that we do some kind of processing on this message bit and generate another sequence of message bit let us call it as b k, this b k message bit is related to the input message bit I k and the previous values of b k.

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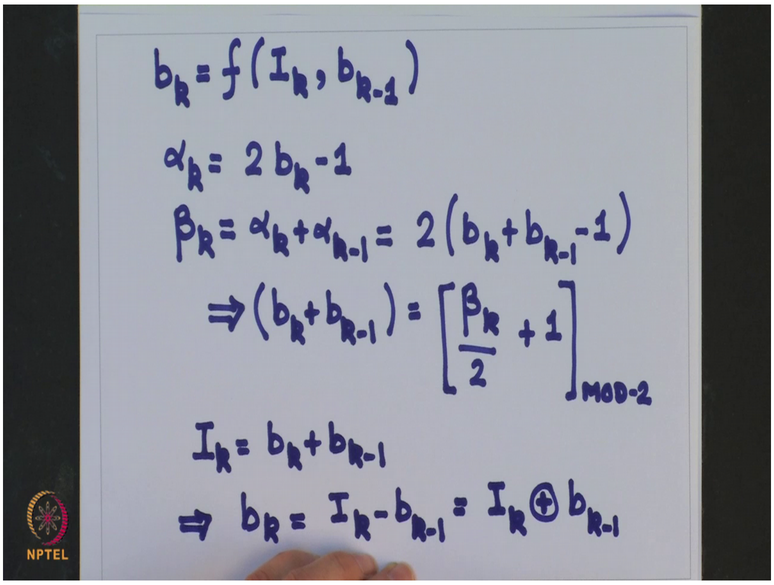

So, my b k let me assume to be a function of my message bit I k and b k minus 1. If I assume this then I can write my at this end remember my alpha k now would be equal to twice b k minus 1, because the input to this at this point will not be I k, but it will be bk. So, I get this and then your beta k would be equal to alpha k plus alpha k minus 1 correct. So, this I can rewrite it as 2 b k plus b k minus 1 minus 1.

So, what this implies that my b k plus b k minus 1 this would be equal to b k by 2 plus 1 correct, and this will be modulo 2 addition ok. So, we want that this should be b k should be directly related to I k. So, we want that your I k should be equal to b k plus b k minus 1. So, this implies that your b k should be equal to I k minus b k minus 1 which in modular to arithmetic is nothing, but I k modulo 2 addition b k minus 1 subtraction and addition would be same for modular 2 arithmetic ok.

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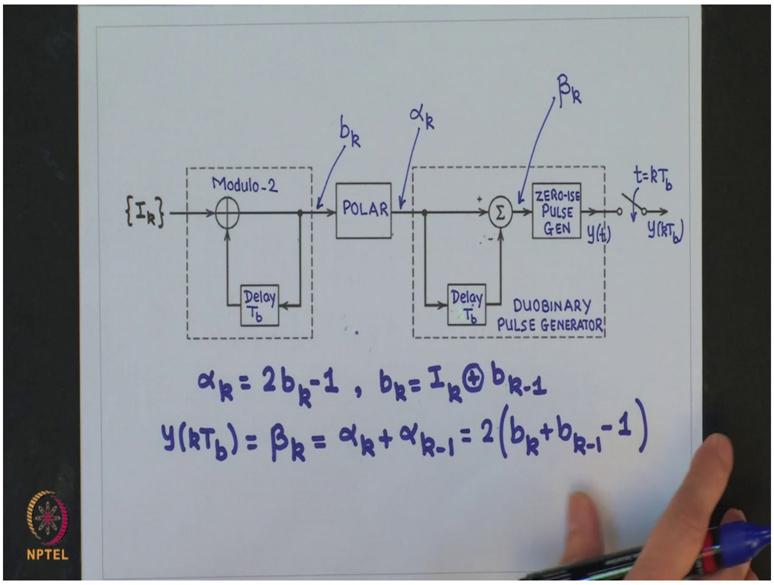

So, if we do this now then I will get my; the block diagram for my duo binary pulse transmission as follows. I have my input I k, then it passes through this block out here which is nothing, but the modular 2 addition of I k and b k minus 1 to obtain b k and then there is a polar conversion to alpha k and then this is your duo binary pulse generator, remember b k is same as y k T b correct fine. So, this relationship would be satisfied.

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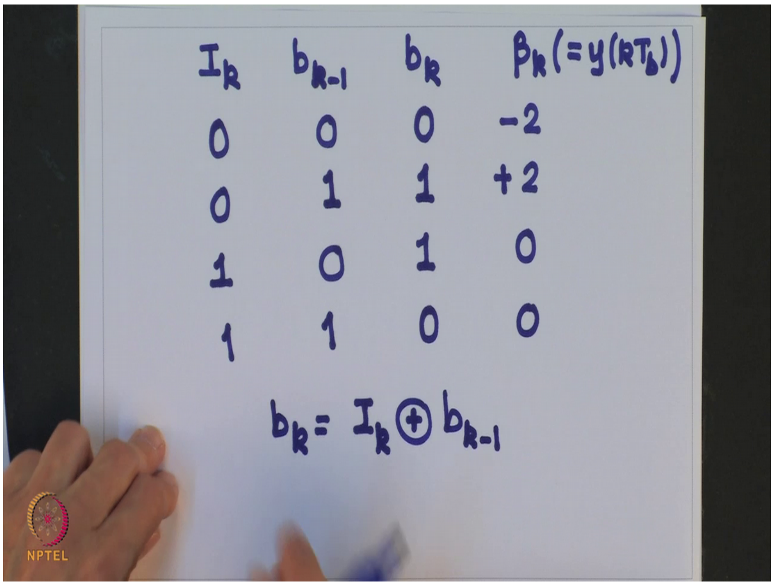

Now, if you look at the truth table for this we can write down the truth table for this very easily; I k b k minus 1 I have b k and then beta k which is nothing, but equal to y of k T

b. So, when this is 0, and this is 0 exclusive all would be equal to 0 and b k would be equal to minus 2 corresponding to the polar value of this 2. Then I will have 0, and if this is 1 this will become 1 and then this will become plus 2 polar values of this 2. So, then you have 1, 0 this would again become 1, and this become 0 polar value of these 2 bits 1 1, this is 0 again, this would be equal to 0.

So, from this we see that from here also it is clear that your b k is nothing, but I k modulo 2 addition with b k minus 1 and we see that in this case that your 0 gets mapped to minus 2 or plus 2. And one gets mapped to the output value 0 correct. So, if we find out the conditional PDF for this, remember these are all Gaussian signals correct. So, if we take a conditional PDF, the conditional PDF would look something like this.

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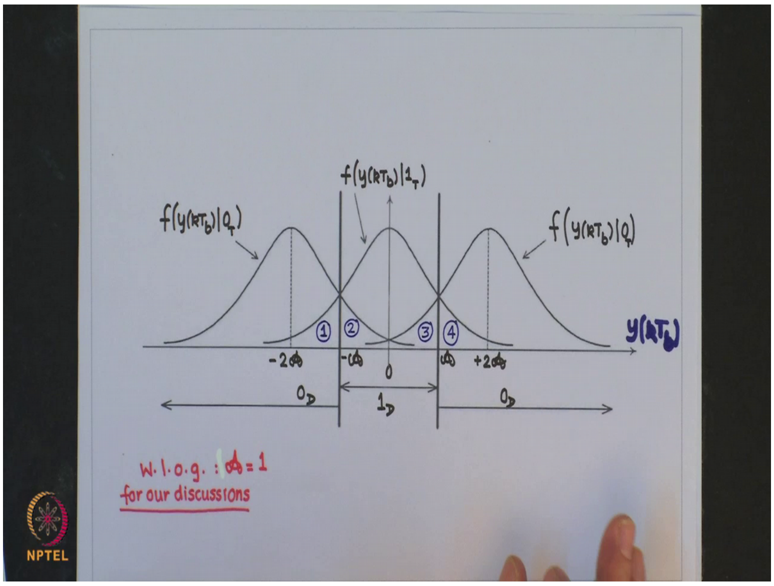

This is the conditional PDF which I will get for y k T b given that I have transmitted 0 T that is transmitted value is 0 I will get it either plus 2, or I will get it minus 2. So, this is the conditional PDF for this for the output given that 0 has been transmitted and given that 1 has been transmitted, 1 T denotes 1 transmission, this is the Gaussian PDF which I will get. So, 0 and 1 are equiprobable, so if 0 and 1 are equiprobable it is easy to see that this intersection points are going to decide your decision boundaries.

So, whenever the output this is your output y k T b, whenever this quantity lies between minus 1 to plus 1 decision will be in the favor of 1, so 1 will be decided, 1 D means the detected bit is 1. And similarly 0 D means the detected bit is zero. So, whenever the mod of y k T b is greater than 1 I decide in favor of transmission 0 d, so I will get this fact fine.

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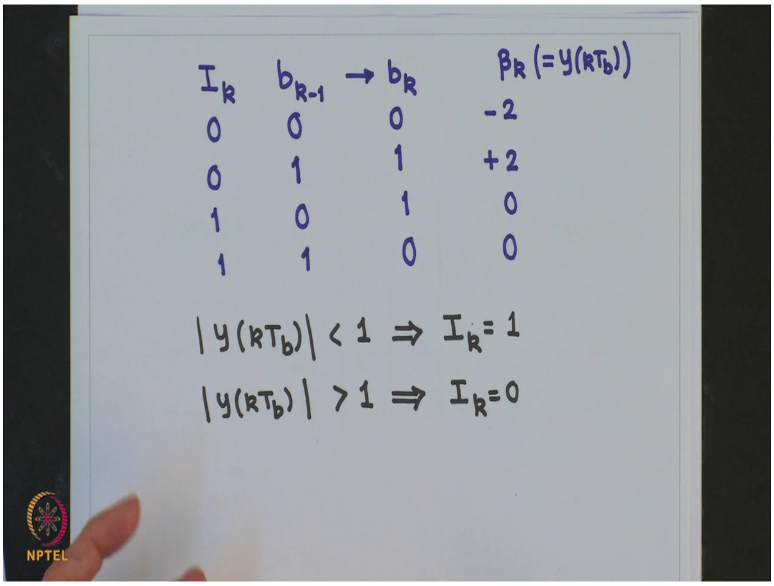

Now, given this let us try to evaluate the performance of this duo binary pulse and compare it with the zero ISI pulse. So, here in this case please remember without loss of generality we have considered A to be equal to 1 because your alpha k for 0 bit I will get minus 1, and for 1 bit I get plus 1, but it could have been for 0 bit I could have chosen minus A and for 1 bit I could have chosen plus a. And this figure has been drawn considering that, but for our discussion we can assume that A is equal to 1.

In this case please observe that probability of y k T b equal to minus 2 and probability of y k T b equal to plus 2 there will be totally is half because probability of 0 being transmitted is half. So, the probability of this occurrence is one-fourth and the probability of this happening probability of y k T b equal to plus 2 is one-fourth and probability of y k T b equal to minus 2 is going to be one-fourth. And probability of y k T b equal to 0 correct will be equal to half because probability of transmitting one is half.

So, now we have to calculate the probability of errors now this probability of errors can be easily calculated. Look at this areas which I have indicated area one denotes everything below this curve from this point onwards right up to infinity. Similarly 2 denotes the area under this curve right up to the infinity, 3 denotes from this point right up to in minus infinity, this will be plus infinity, this will be minus infinity. And similarly here for area 4 from this point onwards the area under this curve right up to the plus infinity.

Now, as far as the probability of error is concerned approximately as far as 1 and 4 are concerned correct that is the error which will occur when I have transmitted 1. So, any anything lying here, anything lying here would go in error. So, these probabilities I can find out this would be equal to sum of this plus this multiplied by half would be the probability area under this curve.

As far as probability of wrong detection that is pertaining to 0 being transmitted is the area under this curve from here up to this point and for this point it is from here up to this point correct. But we will approximate it correct will assume that this area from here up to this point is same as going from here up to minus infinity and here also from area under the curve going from this point to this point is same as going from here up to plus infinity correct.

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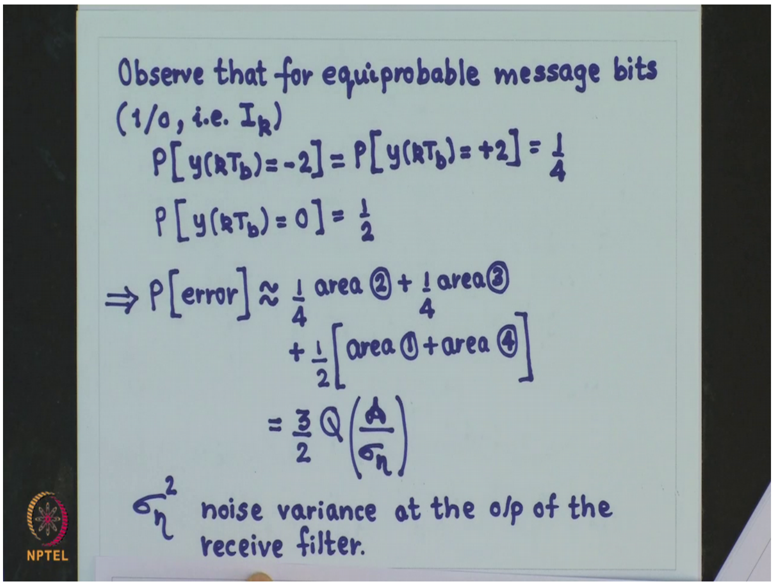

So, if I take that then I can write down my error as follows. So, this is we mention equal to one-fourth because total is half, and the probability of y k T b is equal to equal to 0 is equal to half, and probability of error is approximately equal to remember there is an approximation involved here, and there is approximation involved here for the area 2 and 3 correct. Therefore, this is one-fourth, and this is also one-fourth correct whereas, this is this area, so 1 and 4, so I get this correct.

Now, this is simple to show that this quantity area under these curves we have done it earlier. So, it is going to be 3 by 2 Q A by square root of the variance of the output noise correct, now this is the noise variance at the output of the receive filter. Now, please recollect that we are using the same model at which we had used for 0 ISI.

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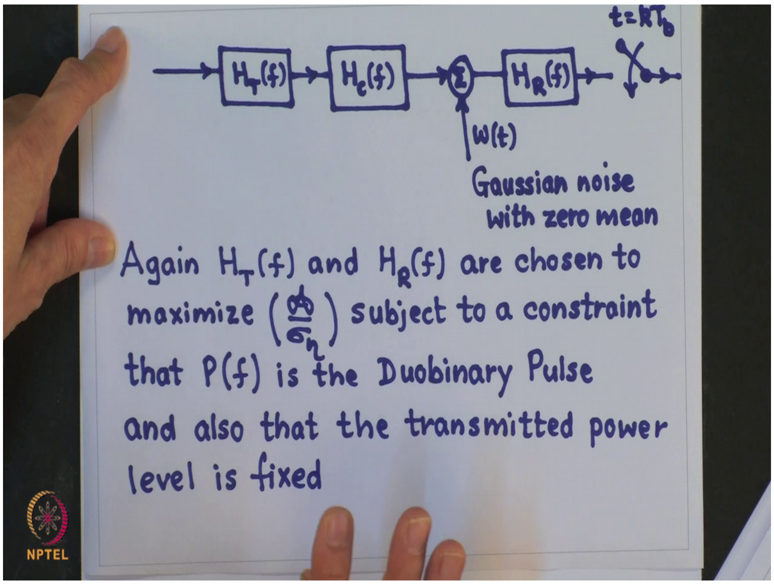

So, we have a input stream here and it passes through transmitting filter channel on the channel we have Gaussian noise with 0 mean and we have a receiving filter. The only difference is now here that product of H T f, H C f and H R f should be such that we get P f to be duo binary pulse rather than zero ISI pulse that is only difference. And so again we will see that we want to choose H C f is assumed to be fixed.

So, again the transmitting filter and the receiving filter which are also known as terminal filters are chosen to maximize this quantity. Because the error turns out to be proportional to this quantity correct. So, Q is a decreasing function. So, I have to maximize this quantity and constrain is that P f should be the now duo binary pulse and the transmitted power level is fixed. So, if we assume this let us try to compare the performance of the duo binary system with the zero ISI system.

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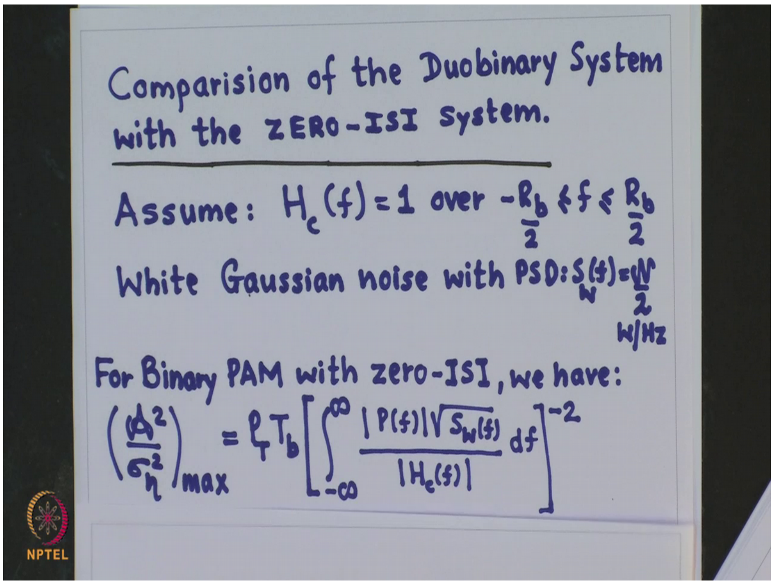

So, the we will assume that over the frequency band of interest H C f is equal to 1, White Gaussian noise we assume with the power spectral density given by italic N by 2 watts per hertz ok. So, for the binary pulse amplitude modulation with zero ISI we have derived this in the previous class that this ratio maximum will turn out to be this quantity correct.

And P f would satisfy the condition for zero ISI correct that is important. So, now, in this case we will just try to let us find out what is this ratio for the case when we have the white noise and this to be equal to 1.

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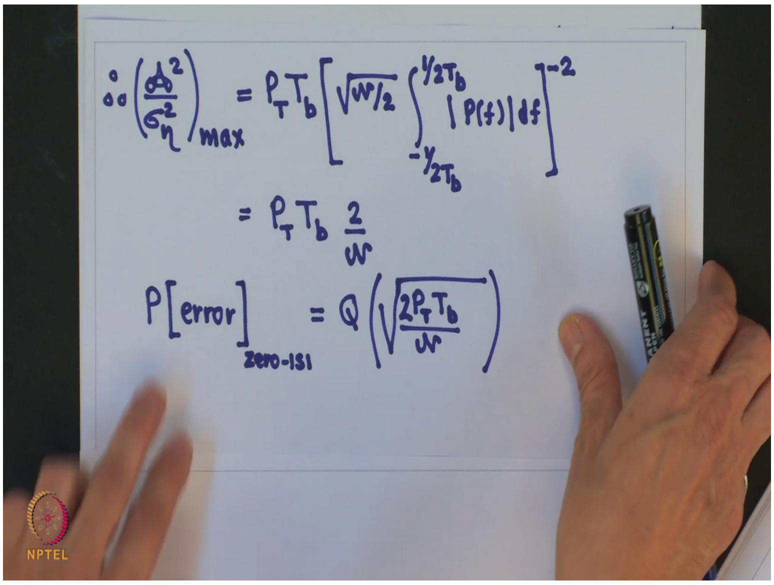

So, if I just plug in that values it is straightforward I get this quantity H C f is equal to 1. So, this is equal to this I get this quantity. Now, remember that integration of this quantity would be equal to 1, we have seen this because p f is equal to H T f multiply H C f multiplied by H R f correct and P f being real. So, mod is also real and the integration of P f over the frequency range will give you pt at T equal to 0, and that we have chosen equal to 1 for zero ISI pulse.

Therefore, this quantity will turn out to be 1. So, this reduces to this expression out here correct. So, probability of error now would be given by Q of this quantity we have seen this earlier correct. So, this is equal to this is for zero ISI ok. Now let us see the same thing what happens when we have duo binary pulse.

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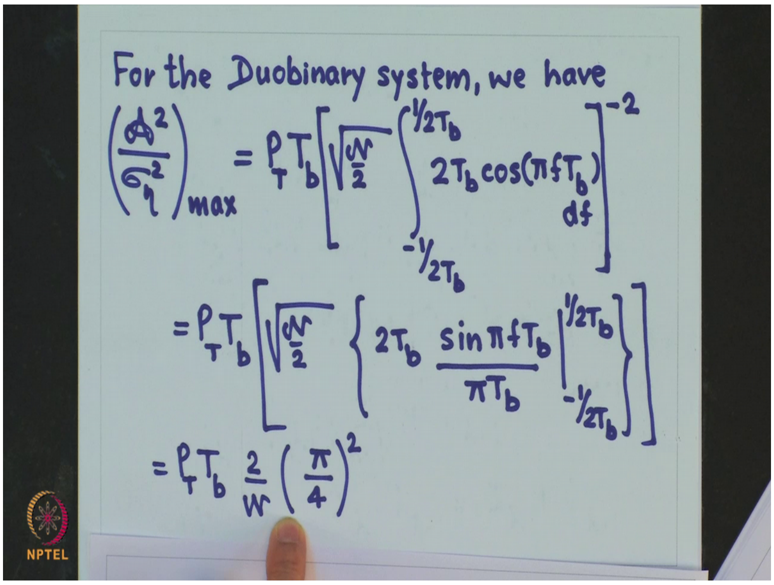

For the duo binary pulse again we will be required to evaluate this quantity is A squared by variance of the noise at the output of the receive filter. The maximum value will the expression will remain the same, the expression is the same one without any changes. Here also now again we will assume to be the white noise constant 1 P f will be chosen to be a duo binary pulse that is the only difference correct, otherwise this expression remains the same ok.

So, here we integrate so for the duo binary pulse this is the expression for the P f, this we have done in the earlier class. And if I integrate this quantity out here I will get sin of this quantity divided by pi T b, evaluate over lower limit and upper limit straight forward. I calculate it I get equal to this quantity correct.

And now we plug in this quantity into the probability of error for the duo binary case which we have shown to be 3 by 2 multiplied by Q of the square root of this quantity fine. And this quantity square root of this quantity the square root of this quantity so fine.

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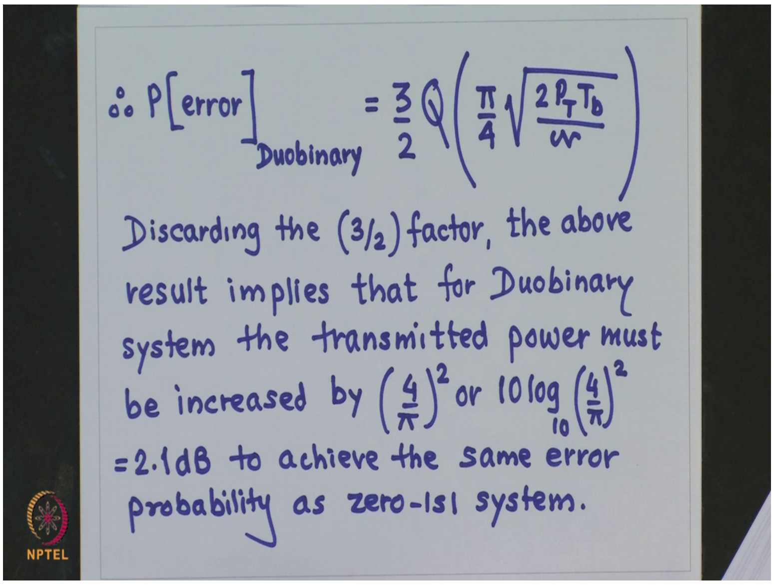

So, if you do that I get the probability of error for the duo binary case to be equal to this value fine. Now, we can discard this factor of 3 by 2 then the above result implies that for a duo binary system the transmitted power must be increased by 4 by pi squared or in terms of log this turns out to be 2.1 dB to achieve the same probability as zero ISI system. Because I want this term to be the same as I get from my zero ISI system, I want this term to be the same as the term here. So, I have to multiply this by power I have to multiplied by 4 by pi square correct, fine.

Now, we have seen that duo binary pulse gives us sufficient amount of power around DC it is a cost function. So, many a times we have communication channels which are AC coupled to prevent the dc from being transmitted or prevent the slow drift of DC. In such cases we require the P f also to be of the same characteristic in the sense that it should have a null around 0.

Now, this kind of channels example would be telephone channel or magnetic recording channel. So, how do you generate the pulse which satisfies this requirement of providing null around DC. So, this kind of thing is known as what is known as modified duo binary signal.

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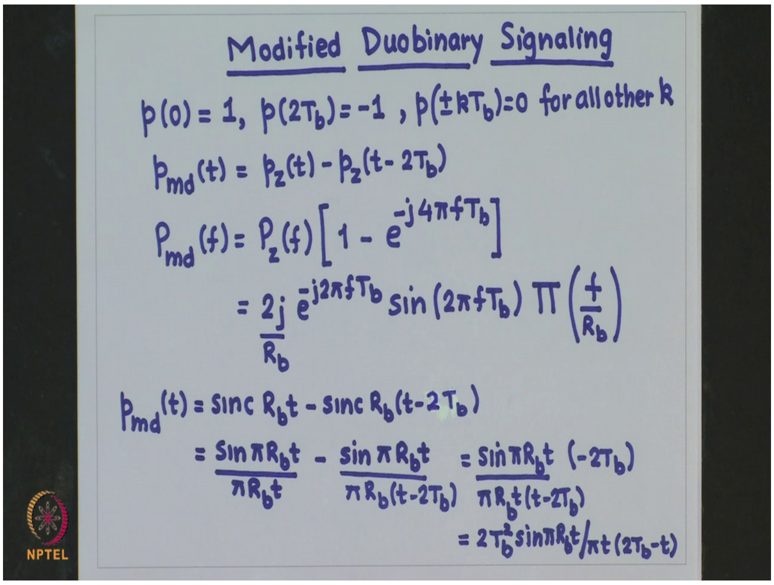

So, if we take the sampled values of the p t to be equal to this p 0 equal to 1, and p 2 T b equal to minus 1 and this is equal to 0 for all other k. Then I know that I can this is known as the pulse which we will get from this I will call it as modified duo binary pulse and it is indicated by md. So, it is easy to see that the difference between that pulse and the pulse corresponding to zero ISI would be only at for this value correct.

So, we can see that this can be generated from my pm dt can be generated from my zero ISI pulse using this relationship it is similar to what we did earlier for the duo binary pulse. So, if I do this basically it is straightforward to show that the frequency response of this is equal to this quantity simple using the Fourier transform properties and this can be simplified by removing e raised to minus j two pi f T b outside the bracket and you can show that this expression turns out to be this.

And from this we can immediately even write down the time characteristic of the modified duo binary pulse and this would be equal to this quantity out here which if you simplify turns out to be this correct. And if you look at the Fourier spectrum and the time domain characteristic of this modified duo binary pulse they would look something like this.

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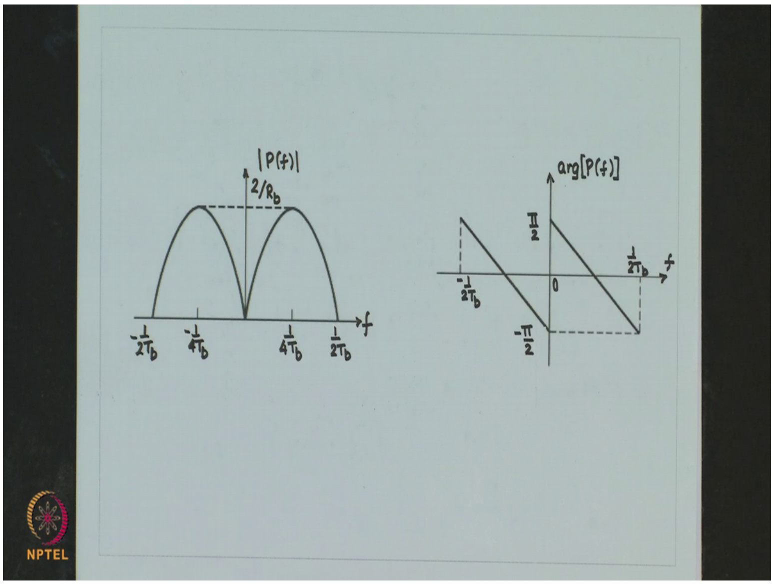

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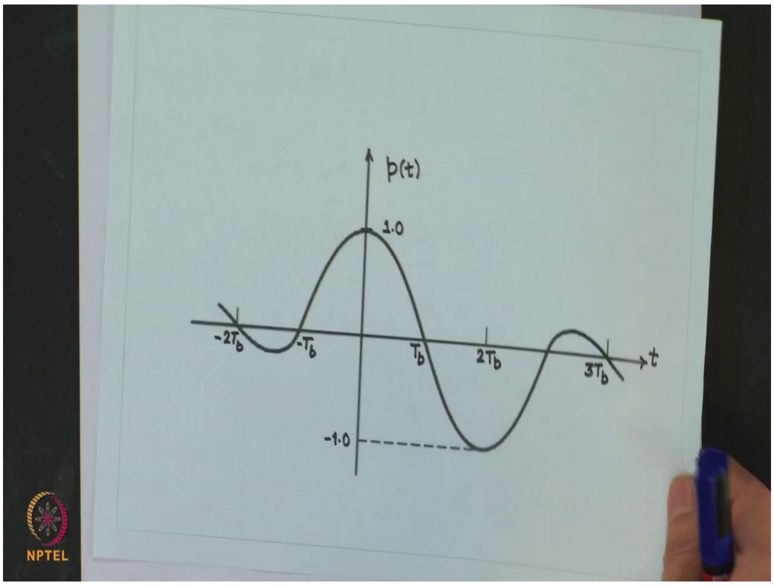

So, we have a null at 0 this is the modi the amplitude response of the P f and this is the phase response and this is the time domain thing for this.

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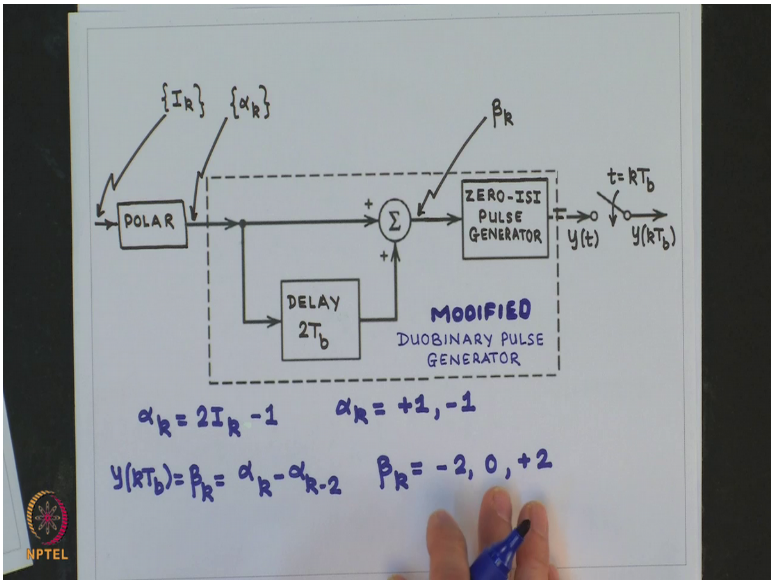

And then we can easy to say again that I can my duo binary pulse generator. This would be modified duo binary pulse generator would be given as shown on this slide.

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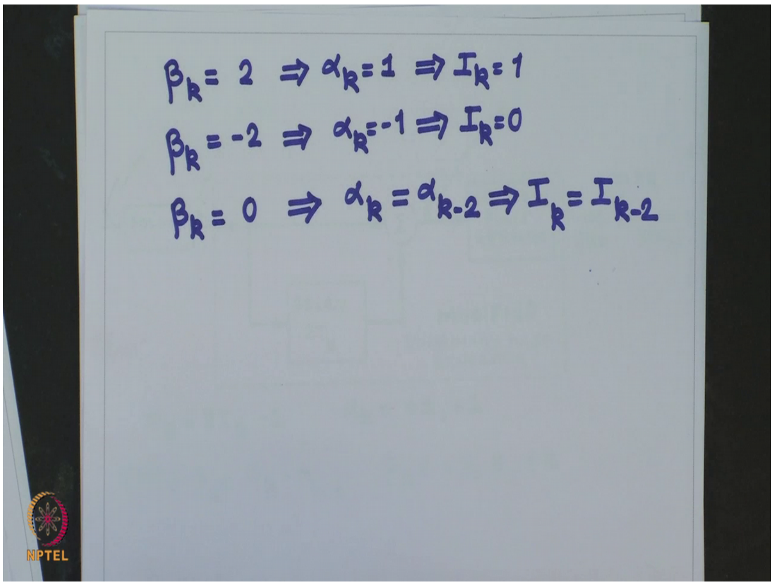

Again you will get this values from here we see like what with the same argument which we followed for the duo binary pulse we can see that but whenever b k is equal to 2, my the present transmitted bit has to be 1 and if it is minus 2 it has to be 0. And when it is equal to 0 I have that the current bit is the same as the bit detected 2 bits previous to this. Because now we are involved with 2 T b, correct that is the only difference correct, so I k would be equal to I k minus 2. Now, again the same problem if you make a decision error there will be a cumulative effect on the error.

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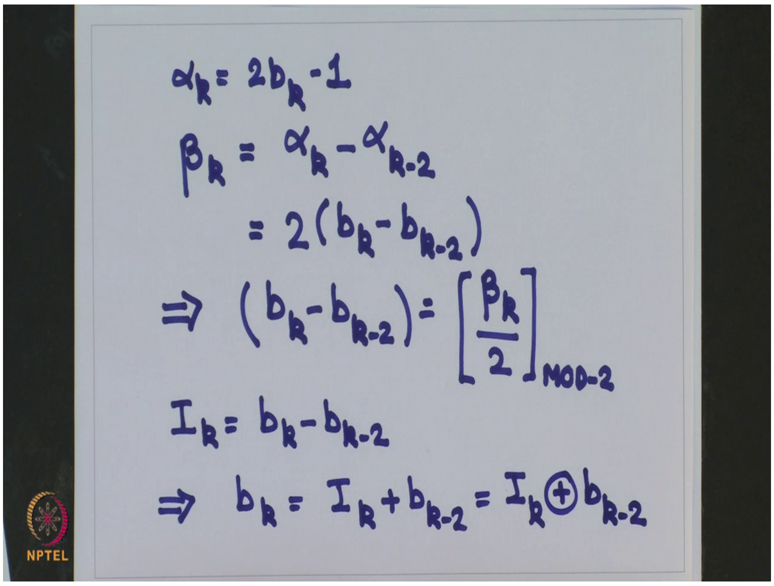

And so if you want to get rid of this it is simple again to see that I can do some kind of preprocessing on this and get my b k such that. So, my alpha k is again same as 2 b k minus 1, my beta k in this case is going to be alpha k minus alpha k minus 2 correct. So, this you can show if I use this is nothing, but b k minus b k minus 2.

So, what this implies is that b k minus b k minus 2 would be equal to b k by 2 mod 2 modular 2. So, this we want to be equal to I k, so this implies that my b k is going to be equal to I k plus b k minus 2 which will be equal to this is modular 2 arithmetic ok.

So, similar thing correct, so if I do this I will get my block diagram for the pre coding of I k this is also known as differential coding. So, we instead of transmitting I k directly you do this kind of pre coding which is known as a differential encoding or differential coding and then transmit correct.

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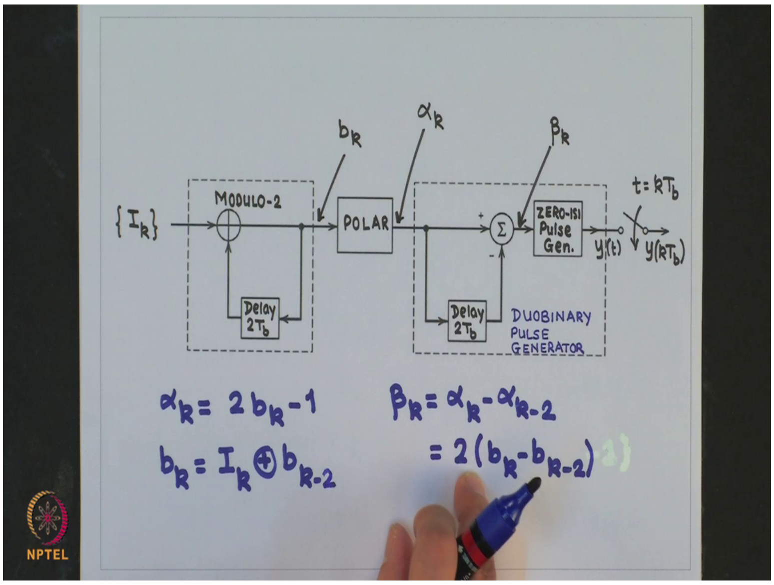

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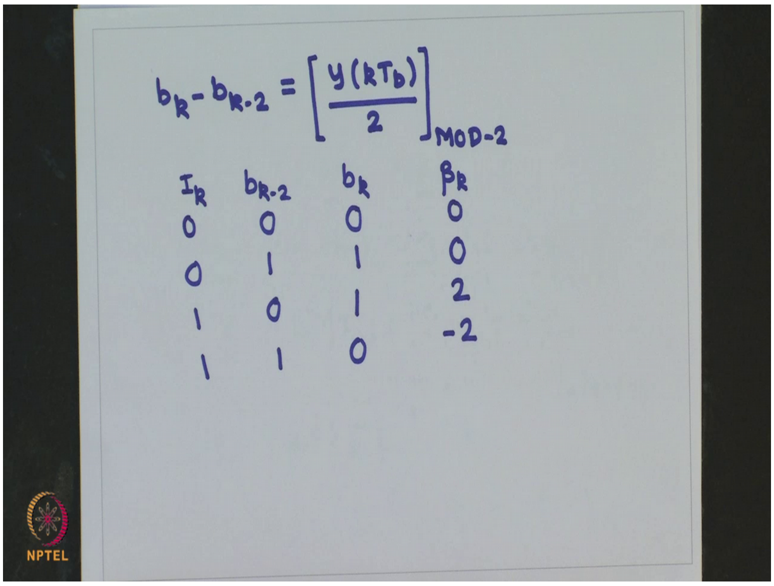

If you look at the truth table for this we can easily generate. So, you see that for both 1 can get mapped to either plus 2 or minus 2 and 0 gets mapped to 0 correct. So, in this case also you see that the performance of this system would be inferior to zero ISI by approximately 2.1 Db. Because more or less the PDF, conditional PDF figure what we saw earlier for the case of a duo binary case will remain the same. So, you will again ignoring that factor before Q the thing would be almost more or less same correct ok.

Now, this binary pulse amplitude modulation can also be extended to M-ary pulse and modulation using the duo binary pulses ok, we will not go into those details at the moment, but it is possible to do that. Now remember that we have designed all this pulses and the transmitting and receivers filters based on the assumption that my channel filter remains fixed.

But in a practical scenario this channel filter H C f could change, or the receiver is not aware of the channel filter. If the receiver and the transmitter aware of the channel filter then it is possible for us to incorporate in the design of duo binary pulse or zero ISI pulse. This we have seen earlier correct that is design of optimum terminal filters, but in the case when H C f is not known then in that case if you take even if you have designed your filter for zero ISI you will get ISI, because the filters will not be match you know P f value will not satisfy the Nyquist criterion for zero ISI.

So, in such a case you will have to do some type of a signal processing at the receiver end and try to remove this uncontrolled ISI duo binary pulse and modified duo binary pulse gives rise to controller ISI, which we can take care of at the receiver. But this because of the unknown information about H C f this will give rise to uncontrolled ISI. And, we will see how to take care of this uncontrolled ISI in practical scenarios and this will do it next time.

Thank you.