Principles of Digital Communications
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Lecture – 40
Pulse Shaping for Zero ISI – III

The overall frequency response which we denoted by P f is product of the three frequency responses one due to the transmitting filter, other due to the communication channel filter and the third is the receiving filter, which learned at this overall response should satisfy zero ISI criterion.

Now, usually the communication channel frequency response is fixed, but we still have the flexibility in the design of the transmitting and the receiving filters. So, the question is how do we choose this filters? These filters are also known as terminal filters. A natural approach to design of this transmitting and receiving filters which we will call as terminal filters is to minimize the probability of error. So, let us look at this design.

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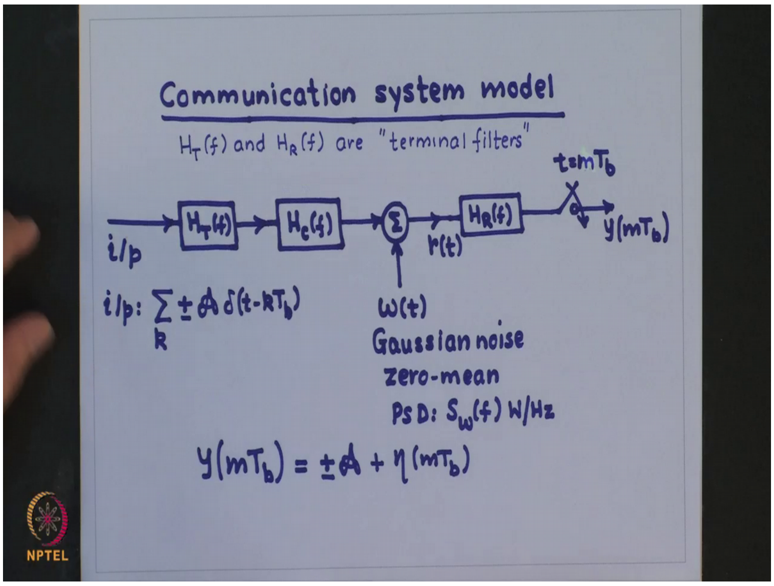

Let us revisit the communication channel which we had considered for our study in pulse shaping, the communication channel is given as follows. We have the input which is impulse train we will assume that we have used the polar line code, so the amplitude of the impulses are plus minus A, this passes through a transmitting filter this translating filter could also include a modulator. But for our study at present we are not really concerned with that, then the output of the transmitting filter passes through a communication channel which we have modeled as H c f frequency response.

On the channel there is a noise W T we assume that it is a Gaussian noise with zero-mean, we will assume the power spectral density is given as S W watts per Hertz to start with we do not assume it to be a white, it could be a non white. At the receiver input to the receiver R T passes through a receiving filter and then it is sampled at sampling instances given by t equal to m T b where T b is equal to 1 by R b, where R b is a rate of transmitting the message samples per second.

Now, we have shown that if it is a zero ISI then sampler output out here would be given by plus minus A plus the noise component eta m T b. This eta out here is the sampled version of the noise which passes through this filter H R f. So, let us calculate the variance of the output noise.

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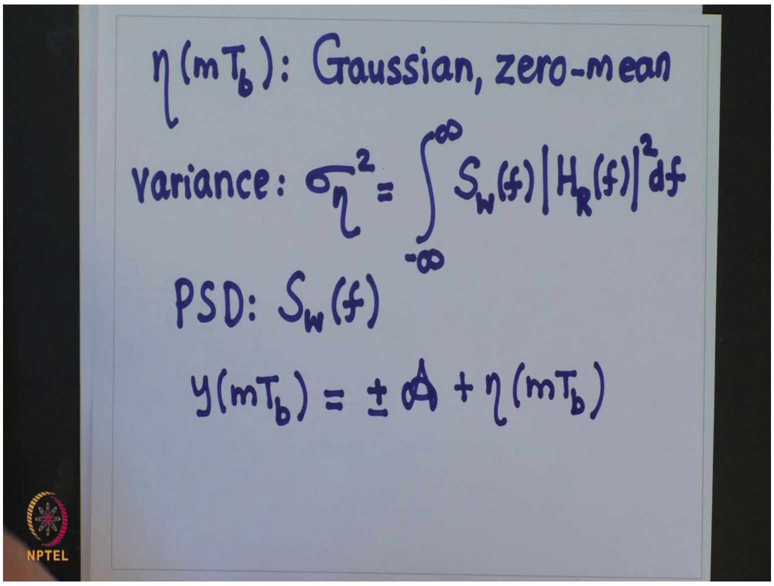

That would be given by as follows this is Gaussian with zero-mean. So, the variance of this sample would be equal to the integration of the output power spectral density which is the input power spectral density of the noise process multiplied by the filter frequency response squared where the input power spectral density is S W.

Now, because the noise is Gaussian we can find out the conditional pdfs of the sampled value. The sampled value is y m T b and this as we said earlier is equal to plus minus A plus eta m T b. The conditional pdf would be as shown in this figure.

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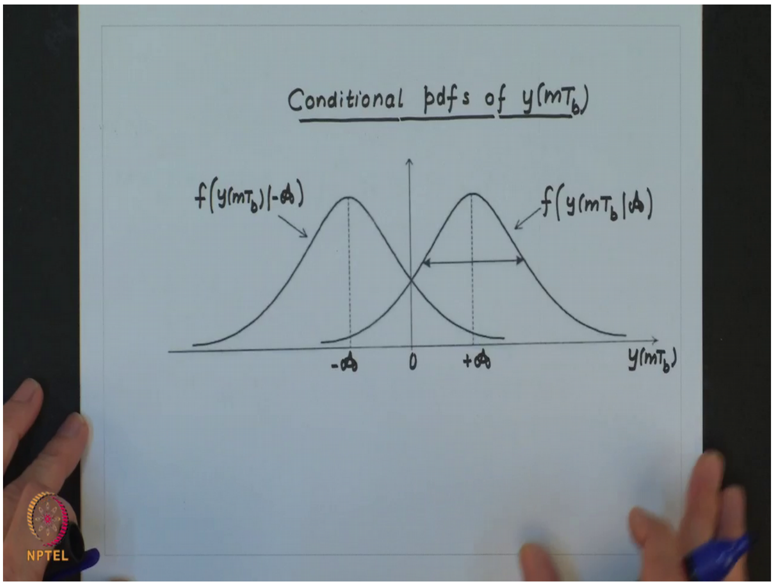

This is the conditional pdf which will have given I have a transmitted plus A, and this is the conditional pdf which I will get given that I have transmitted minus A, correct. And we will assume that this symbols are equiprobable in that case the threshold for detection would be equal to 0 and the probability of error in this case would be equal to Q function of A by square root of the variance of the output power.

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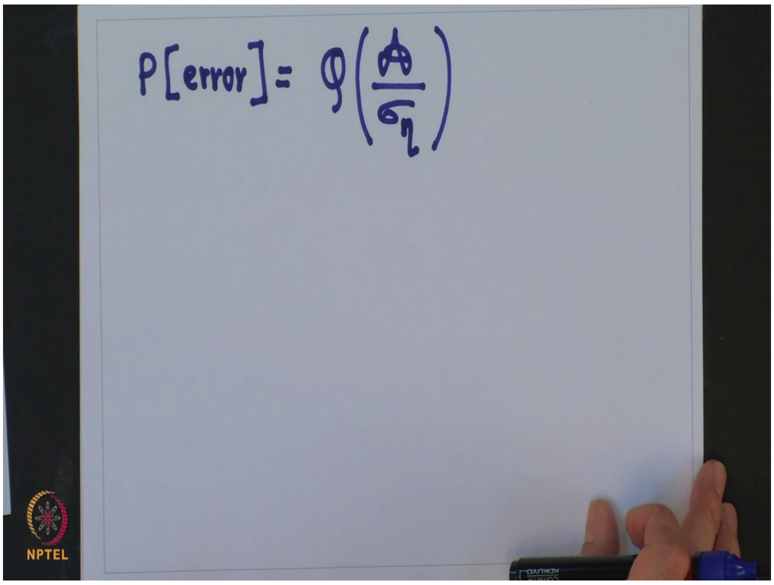

Now, if you want to make this probability of error small we would like to see that this quantity out here becomes large because Q is a decreasing function of its argument. So, we have to maximize this quantity if we want to minimize the probability of error.

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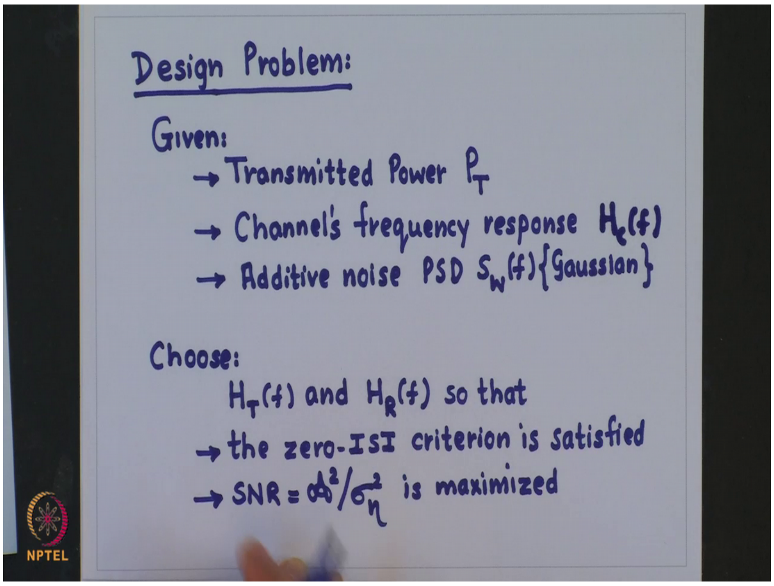

So, given this now our design problem reduces to the following. I have been given transmitted power P T. I know the channels frequency response that is H c f and the input power spectral density at the receiver is Gaussian with zero-mean.

Now, a problem is to choose the terminal filters that is the transmitting filter and the receiver filter so that it satisfies the zero ISI criterion and the signal to noise ratio at the sampling instance which is given by this quantity is maximized because this will minimize the probability of error.

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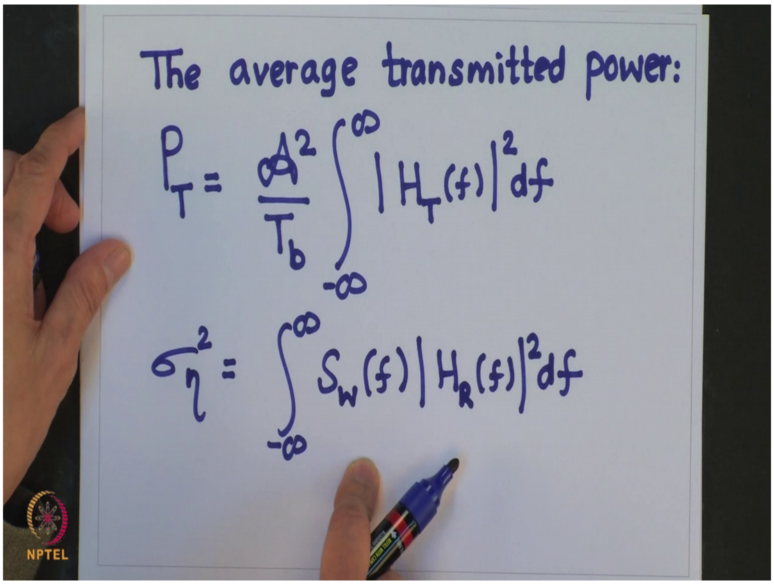

So, let us first calculate the average transmitted power that we are integrating it by P T that would be equal to this quantity out here remember the average power of a train of impulses is given by this.

We know the variance of the noise, I repeat here I am interested in finding out the ratio A squared by square root of this variance. So, what I will do is basically I will write A in terms of the given power, correct. So, I will get A squared and variance I have this term.

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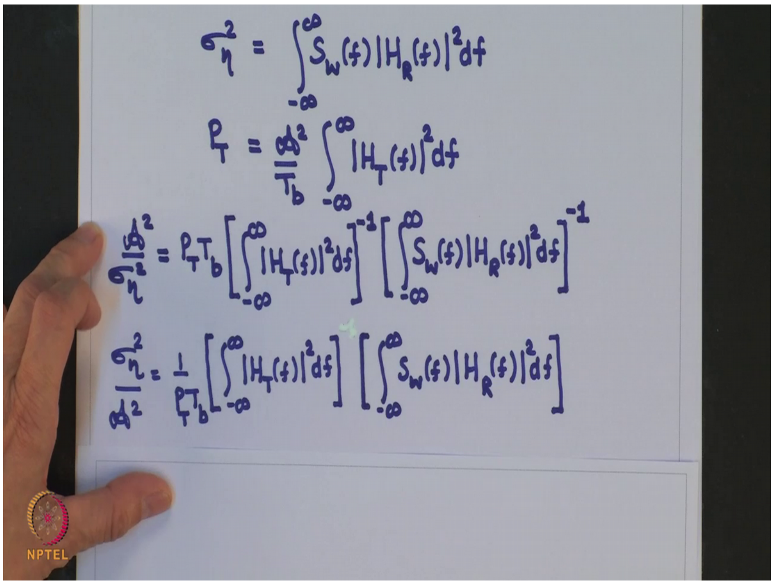

So, if I write that expression I would get the expression as shown here, correct. So, this quantity out here is my A squared and this quantity out here is variance.

Now, maximizing this is equivalent to minimizing the inverse of this quantity which I have written here. So, just this quantity is the inverse of whatever I have written above.

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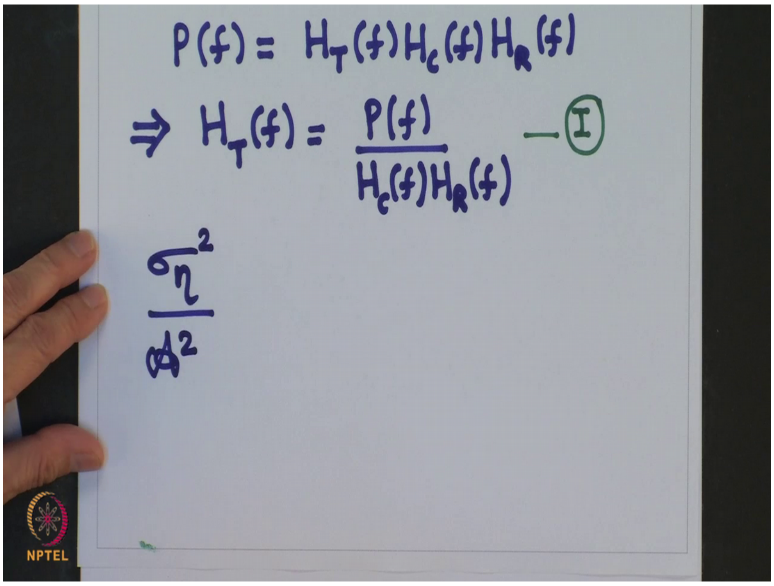

Now, what we do is basically we know that the overall frequency response is given by this expression, from this I eliminate the transmitting filter frequency response let me say this is equation 1. Then knowing this I can write down my H c f in terms of this quantity and plug it into the earlier ratio which we have written out here. So, this quantity out here I am going to replace it by this quantity out here.

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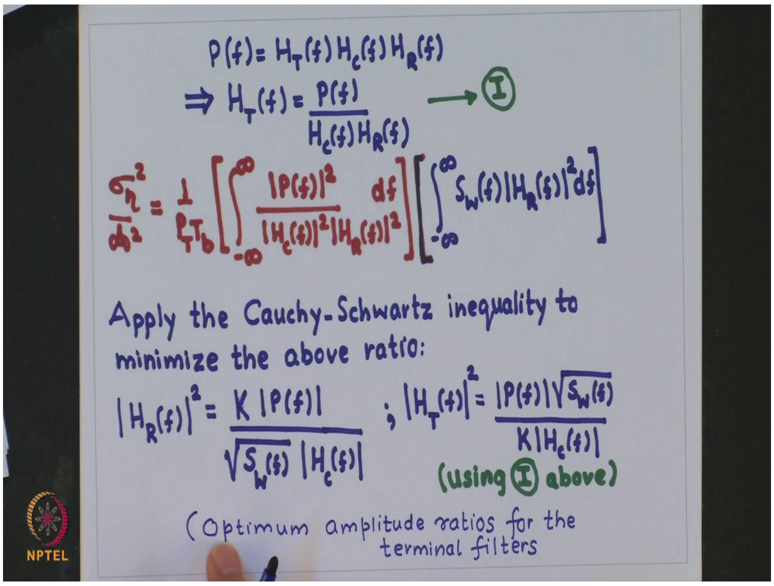

If I do this I get this quantity out here. So, H T f squared I have replaced by this from here. Now, I want to minimize this ratio.

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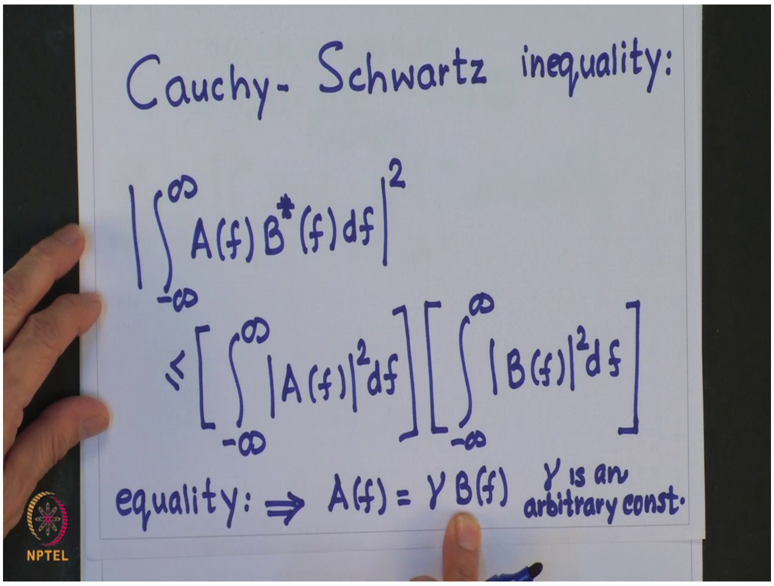

We will use Cauchy Schwartz inequality here which states that if I have A f and B f as shown on the left hand side of this expression, then the left hand side is always less than equal to the right hand side and the inequality holds only if A f is proportional to B f. Using this Cauchy Schwartz inequality an identifying A f and B f in our case.

So, your B f squared is this, A f squared is this. Using this I identify this I have to just say that A f is proportional to B f. If I do this I get this expression out here, correct, simple. The root of this will be equal to the sum constant root of this correct I get this expression.

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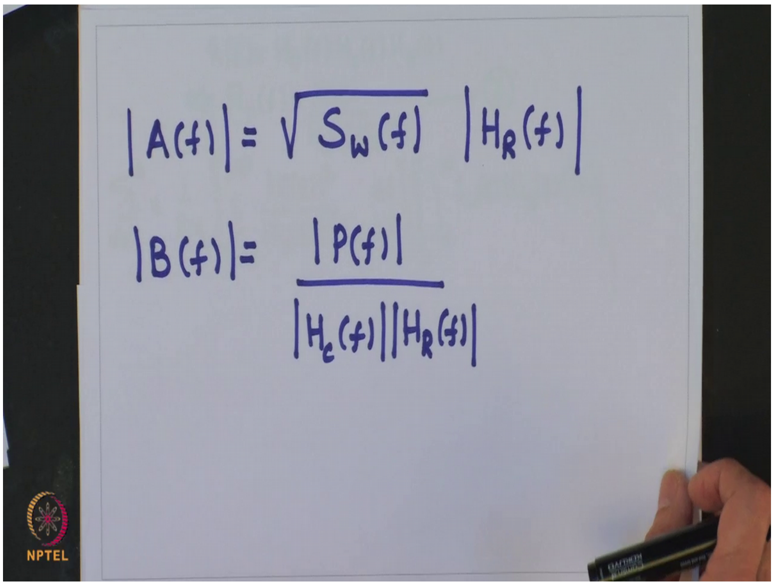

Now, this expression shows that at the receiving filter your noise gets deemphasized by the root of the power spectral density of the noise on the channel. So, this will affect your P f also. So, your transmitting filter takes care of that it pre emphasizes P f by root of the power spectral density. This are the optimum amplitude ratios for the terminal filters which we have designed.

Let us take a case for a white noise. For white noise this will become a constant this will become a constant. So, it will be just dependent on P f and H c f and you will have a constant. So, I can immediately write this as follows.

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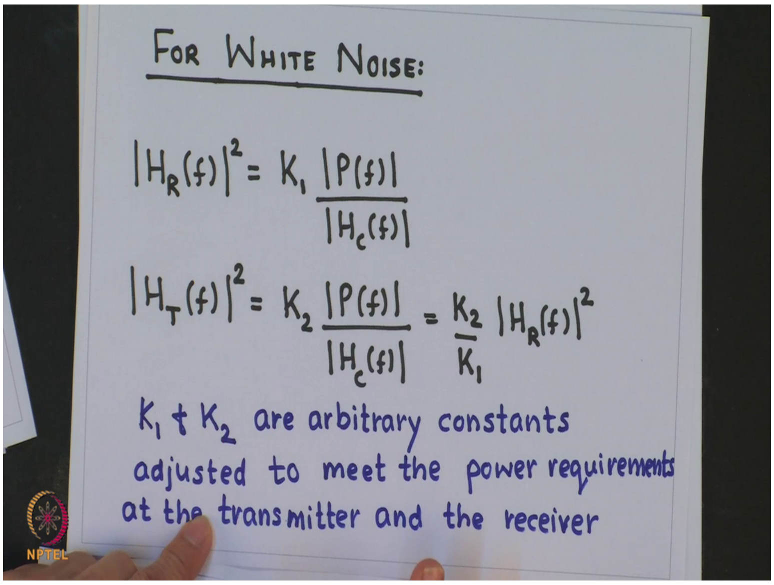

This quantity will reduce to this, this quantity will reduce to this where K 1 and K 2 are appropriate constants we are which are adjusted to meet the power requirements at the transmitter and the receiver. Please note that this K out here is also some arbitrary constant ok.

Now, the phases of this filter H T f and H R f are arbitrary, but the phase functions must cancel each other. It is important to note that because the overall product of the three filters should give us P f.

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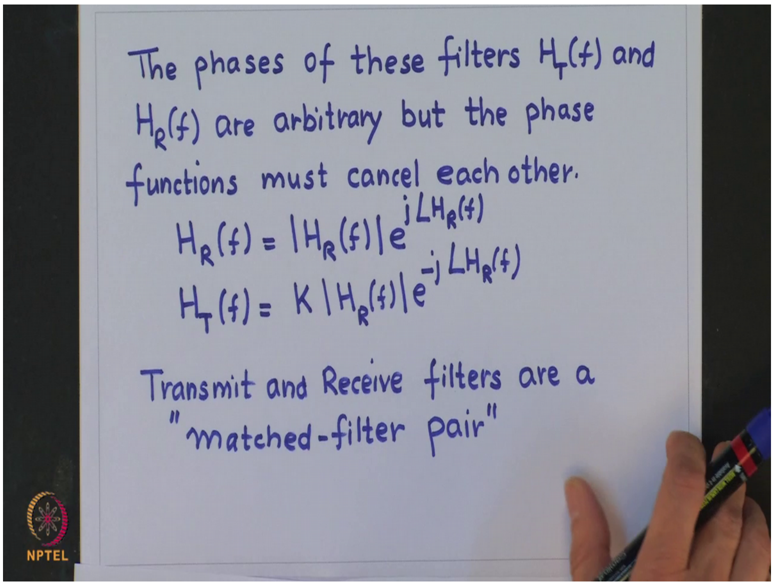

So, in that case if I have my H R f of this form, where my H T f is going to be some proportionality constant multiplied by mod of H R f amplitude response will be the same and the phase will be just the opposite. And this both transmit and receive of filters are max filter pairs.

Now, having done this let us try to evaluate what is the maximum A squared by variance value which we will get from for this optimal filters.

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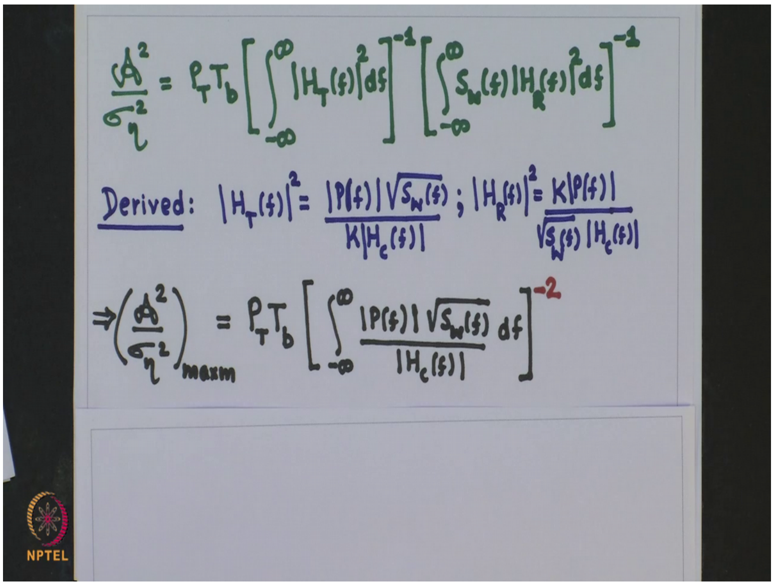

So, that is very straightforward. We know that this quantity out here is given by this expression fine. Now, we have also found out what are the optimum values for H T f and H R f. I repeat it here in a generic case this is my H T f optimum filter and this is my receiving optimum filter. So, if I take these two filters and plug it into this equation here which I have shown here, it is not very difficult to see that I will get this expression. If you take this quantity and put here and if you take this quantity and put here you will get the same expression for the both and that is why it becomes minus 2, correct.

So, for the case where the channel is ideal in that case I can assume that my S c f is equal to 1 for the frequency less than the bandwidth of the channel, and if I assume that K 1 and K 2 are equal and we have a white noise then in that case the filters which I will get is as simple as this, ok.

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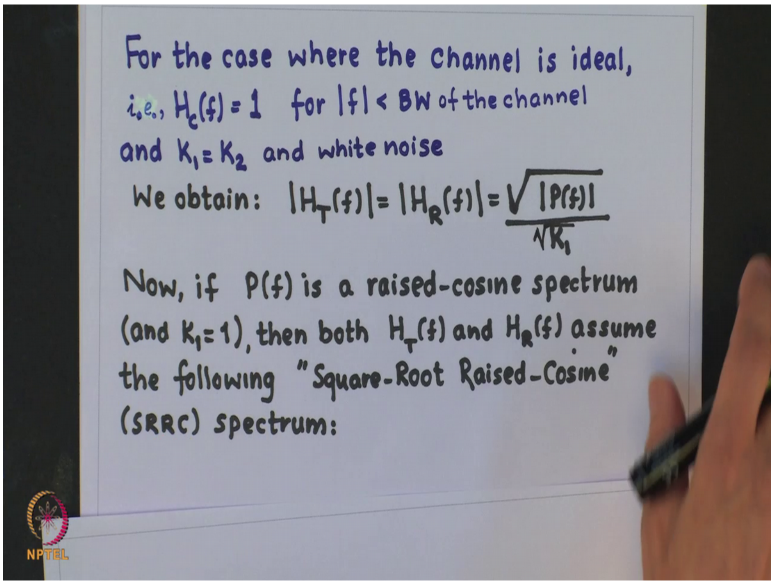

This factor root K 1 is a constant K 1 is some constant. I have divided between the transmitting filter and the receiving filter without loss of generality correct, because H T f is proportional to H R f, correct.

So, now if you look at this and if I assume here my P f to be a race cosine spectrum because it has to satisfy zero ISI criterion and the; that means, the P f will satisfy a vestigial spectrum characteristic requirement, and one of the popular vestigial spectrum characteristic we have seen is the raised cosine spectrum which is used in practical applications. So, if I assume that and if I assume K 1 equal to 1 then both H T f and H R f will assume the following square root raised cosine spectrum its clear from this expression correct and we call this square root raised cosine spectrum as SRRC.

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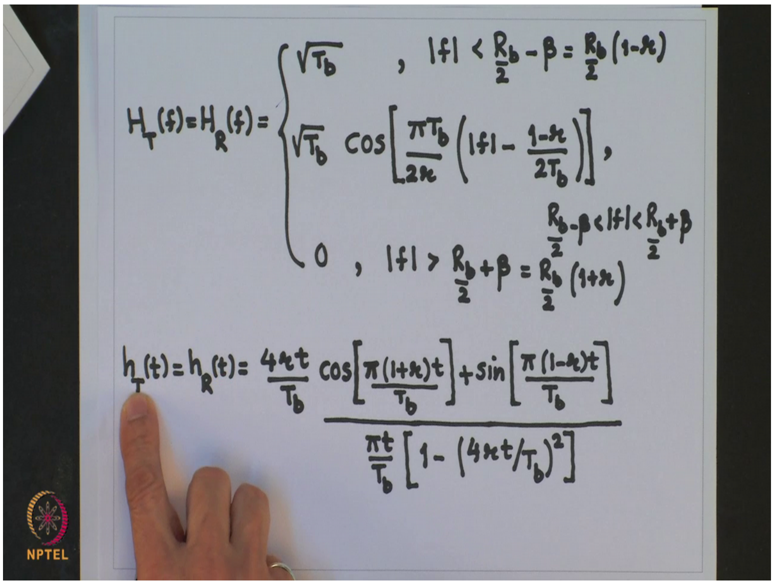

So, we have seen earlier what is the raised cosine spectrum, and if you take the square root of that that expression which I will get is I have written here. We have done this earlier, so just I am putting down the same expression, but the square root of that and now if you find out the inverse of this which will give me the impulse responses for the transmitting filter and the receiving filter which I show here. So, these are the impulse responses for the transmitting filter and the receiving filter.

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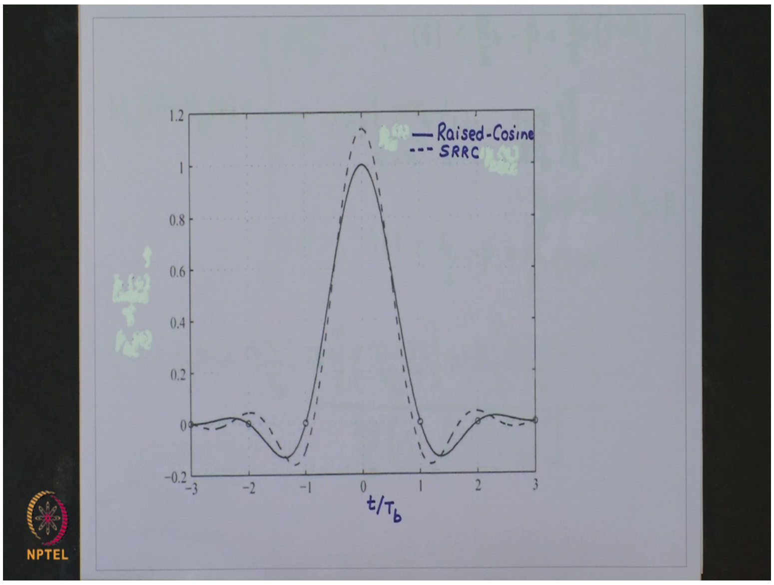

In this slide I show the impulse responses for the case when we use the raised cosine spectrum and when we use the SRRC spectrum. So, the solid curve out here represents your P T which is the inverse of the raised cosine spectrum and the dotted one shows the impulse responses of h T t and h R t that is the transmitting filter and the receiving filter responses. Please note that at the sampling instances SRRC does not pass through zeros.

Having done this let us try to take one design problem. So, we can appreciate what we have studied in a better way. Let me take the design problem as follows. I want to design a binary non return to zero system with the following specs.

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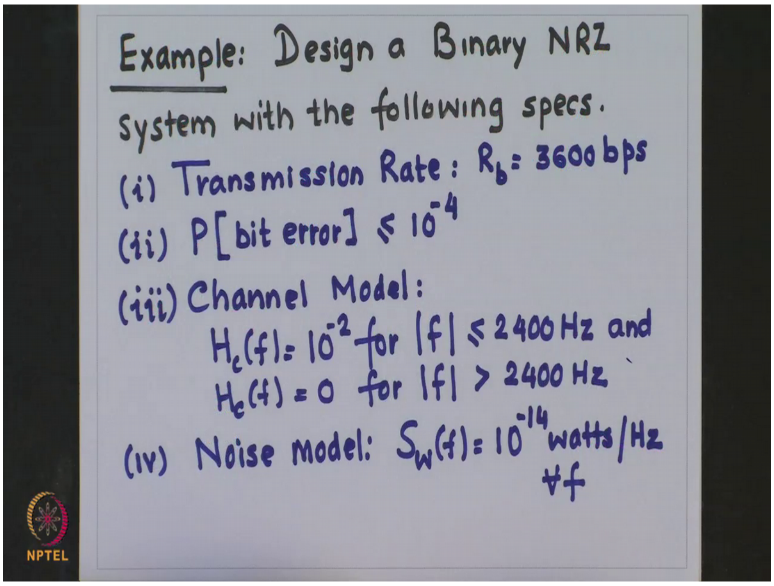

I want to use the transmission rate to be R b 3600 bits per second, I am using the binary symbols. I want the probability of bit error to be less than 10 raised to minus 4 the channel model has been given to us it is 10 raised to minus 2 for frequencies less than 2400 Hertz and H c f is 0 beyond that and the noise model is given here, which essentially means that is a white noise. Given this let us try to find out what is the transmitting power and what is the raised cosine spectrum which we should use for this case.

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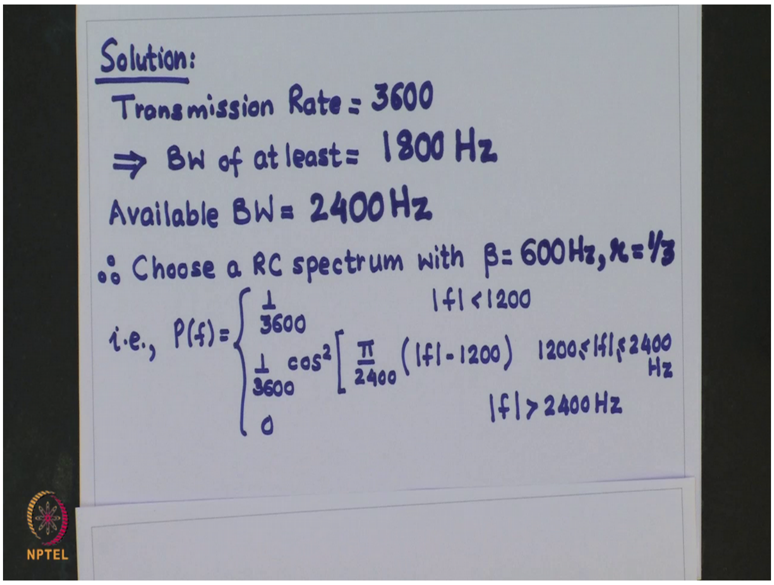

Note that our transmission rate has been given to be equal to 3600 bps. So, what this implies that the bandwidth should be at least 1800 Hertz, but we have been given that available bandwidth is 2400 Hertz. So, in this case since we have extra bandwidth we could make our implementation simplified by using raised cosine spectrum. So, we have beta equal to 600 Hertz which implies that the rolling factor is equal to 600 by 1800 that is one-third.

And from this I can get my raised cosign spectrum as given by this expression on this slide.

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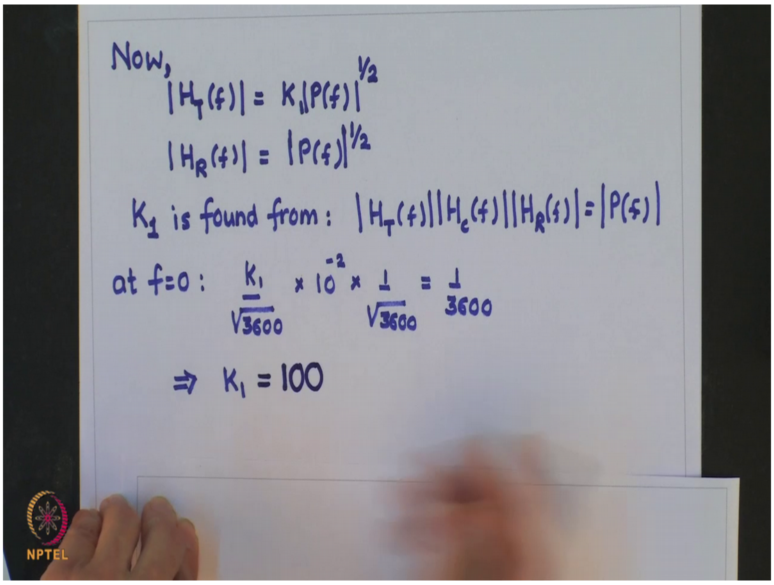

Now, we know that our transmitting filter and receiving filters are related by this expression we have just derived it. And we have to find out K 1. K 1 is found from this relationship product of this three filter should be equal to P f, now at f equal to 0 we know that P f should be 1 by 3600. So, I substitute for H T f is going to be this value, H c f is given to be this value and H R f is this. So, I get this flow from this I get my K 1 to be equal to 100.

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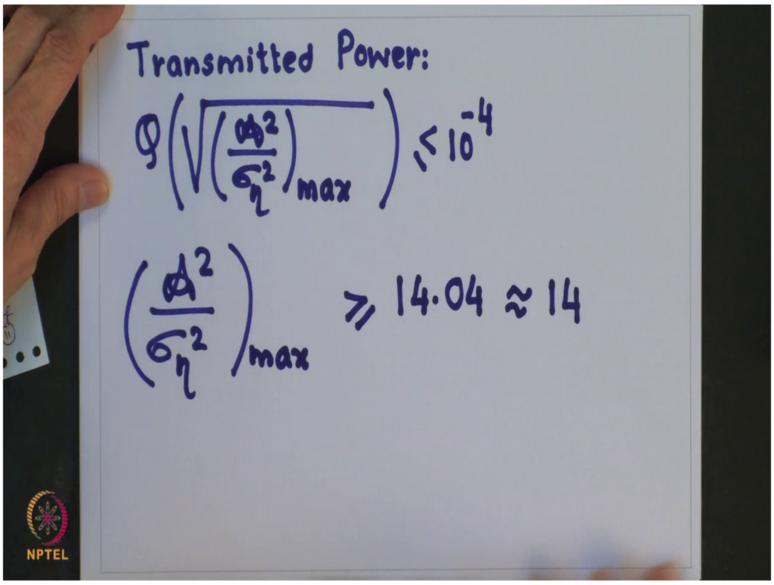

Now, let us determine the transmitted power. So, to determine the transmitted power we will use the specification which has been given for the required bit error probability. We have been told that the bit error probability which is given by this expression out here we will choose the maximum value. This should be less than or equal to 10 raised to minus 4 which implies that this quantity should be greater than equal to 14.04 which is approximately equal to 40.

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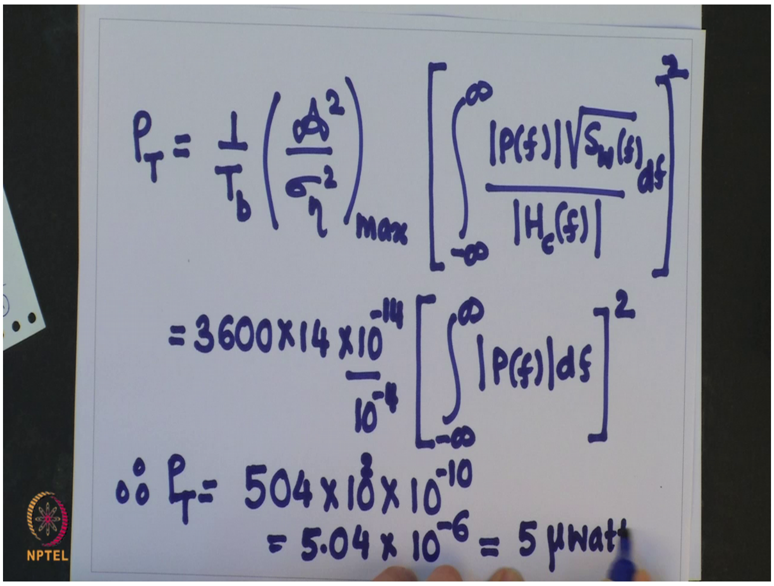

So, from this we can evaluate the transmitted power as follows. We know that our transmitted power is given by this expression, and just you have to substitute the values out here. Remember this quantity is equal to 1 because P T at T equal to 0 is equal to 1 therefore, we get the transmitted power to be equal to which is approximately 5 micro watts.

Now, we saw that when we have extra bandwidth available then we can transmit our message samples at the rate less than twice b and satisfy the constraint of practical realizability of the transmitting and receiving filters. But if you do not have the bandwidth if the bandwidth is scarce and we still want to have the bandwidth for the transmission rate of R b samples per second to be R b by 2.

Now, in that case we know that the only way to do is to use the Nyquist pulse that is use the brick wall structure, but this poses a problem in practical implementation. Is it possible to still do this? Have the rate to be R b and use the bandwidth to be R b by 2, and without using the brick wall structure. Now, we will show that this is possible using what is known as partial response signaling technique, but this will create some kind of controlled ISI which will take care of and show that we can do this kind of transmission of R b with the bandwidth of R b by 2, and this we will study next time.

Thank you.