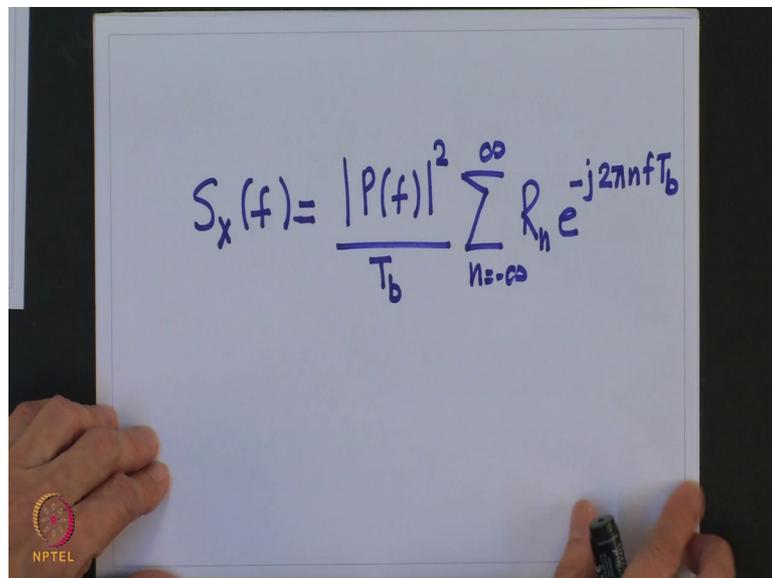


Principles of Digital Communications
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Lecture - 37
Line Coding - III

We mentioned that power spectral density is a useful tool in the analysis or understanding the characteristic of a particular line code.

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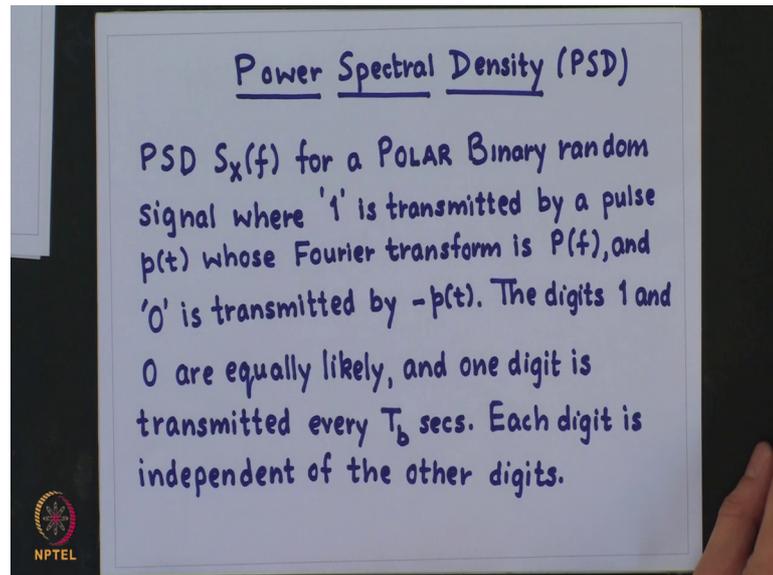
The image shows a whiteboard with a handwritten equation for the power spectral density $S_x(f)$. The equation is:

$$S_x(f) = \frac{|P(f)|^2}{T_b} \sum_{n=-\infty}^{\infty} R_n e^{-j2\pi n f T_b}$$

The whiteboard is held by a person's hands, and a small NPTEL logo is visible in the bottom left corner.

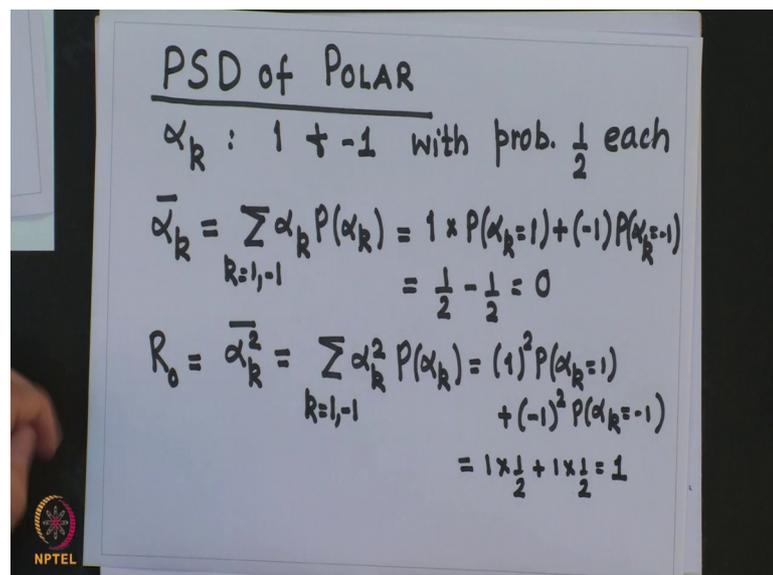
We have derived the following relationship power spectral density is equal to Fourier transform of the basic pulse which we used for transmission $P(f)$ its mod squared divided by the duration of the pulse. And this is the discrete time Fourier transform of the function R_n . Now, we will use this relationship and try to find out power spectral densities of signal like polar, bipolar, and on off format. To do that what is the important for us to evaluate this discrete autocorrelation R_n .

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So, we will start with the first example of evaluating the power spectral density for a polar binary random signal where 1 is transmitted by a pulse $p(t)$ whose Fourier transform is $P(f)$, and 0 is transmitted by $-p(t)$. We will transmit two digits 1 and 0 both are equally likely and we transmit this digit every T_b seconds. It is important to note that we assume that each digit is independent of the other digits.

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So, power spectral density of polar line code. In this case the value of alpha can be 1, and minus 1 with probability half each. First we will calculate the mean value of this random

variable α_k and we will see very soon why I need to evaluate this k can take 2 values 1 and minus 1.

This is equal to one multiplied by probability of α_k equal to 1, plus α_k equal to minus 1 and its probability this is half minus half which is equal to 0. Now we also need to evaluate R_n , so the first for n equal to 0 this is equal to this expression here. This in intern can be written as if you assume equiprobable I get this quantity r_0 to be equal to 1.

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$$R_n = \overline{\alpha_k \alpha_{k+n}} = \overline{\alpha_k} \overline{\alpha_{k+n}} = 0 \quad n \geq 1$$

$$S_x(f) = \frac{|P(f)|^2}{T_b}$$

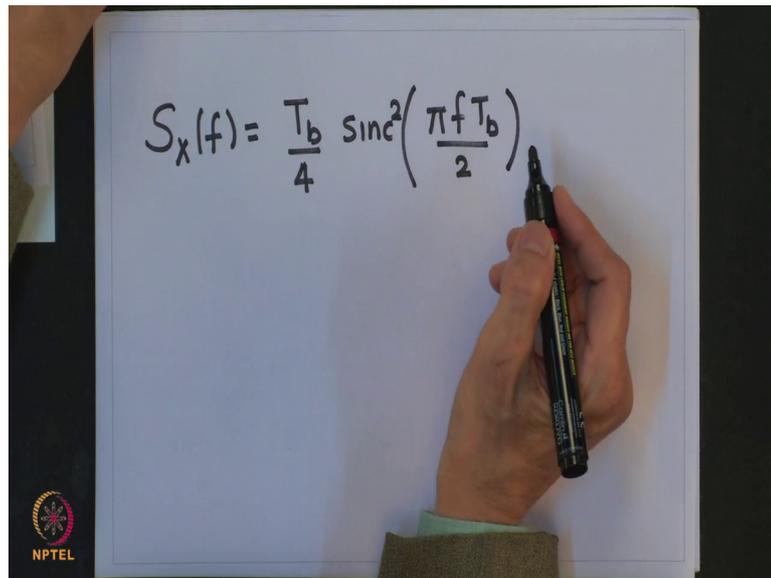
$p(t) =$

$$P(f) = T_b \operatorname{sinc}\left(\frac{\pi f T_b}{2}\right)$$

Now, because each digit is independent of the remaining digits R_n , which by definition is equal to $\alpha_k \alpha_{k+n}$ expectation of this; this would be equal to the product of two averages because any two digits are independent and we have evaluated this each of this equal to 0, so this is equal to 0.

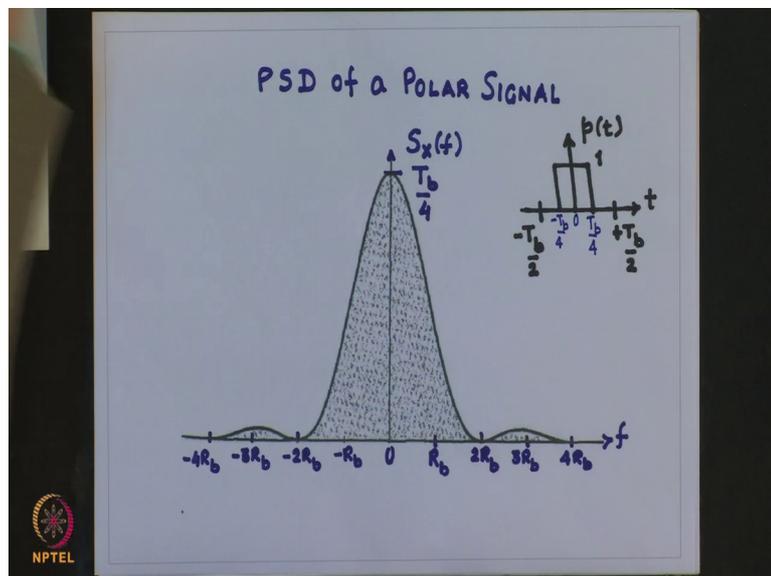
So, this is a reason why we started with the evaluation of expectation of α_k for n greater than equal to 1 this will fall. So, from for this case we get the power spectral density to be equal to just this quantity. And if we assume that our pulse $p(t)$ is half width return to 0 rectangle pulse, as shown here. Then we know the Fourier transform of this pulse to be equal to sinc function of this quantity, sinc x is defined as $\sin x$ by πx .

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$$S_x(f) = \frac{T_b}{4} \text{sinc}^2\left(\frac{\pi f T_b}{2}\right)$$

Given this we can write the power spectral density for the polar case to be equal to this expression here. If you plot it, the power spectral density would look something like this.

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Now, if you see this spectrum basically it goes right up to the frequency infinity. So, for such a spectrum it would be difficult to define really bandwidth. So, there are different definitions of bandwidth one definition of bandwidth which is known as essential bandwidth is the first non DC null frequency which is here $2 R b$. So, the essential

bandwidth for the polar signal for using this kind of a pulse $p(t)$ that is written to 0 polar half width is twice R_b .

And this is 4 times the theoretical bandwidth which is also known as Nyquist bandwidth, we will learn this concept very soon. We will show that when you transmit the pulses at the rate of R_b , the bandwidth required the minimum bandwidth required for x transmission is $R_b/2$. So, in this case we require 4 times $R_b/2$ then the only way to reduce this bandwidth is increasing the pulse width for full width pulse that is maximum possible pulse width. The essential bandwidth would become half of this it will become $R_b/2$, but still it will be twice the theoretical bandwidth which is $R_b/2$, so this polar signaling is not the most bandwidth efficient.

We also see that it has nonzero power spectral density at DC that is f equal to 0, this will rule out the use of AC coupling during transmission. And we also see that there is no discrete clock frequency component in the spectrum of the polar signal, but on rectification of the return to 0 polar signal. However, it will yield a periodic signal of clock frequency R_b and this can be readily used to extract timing.

So, in this sense this polar signal is transparent because there is always some pulse positive, or negative regardless of the bit sequence. And we know that a line code in which the bit pattern does not affect the accuracy of the timing information is said to be a transparent line code. So, now let us take another line code, we will try to evaluate the power spectral density for a bipolar line code.

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PSD of BIPOLAR LINE CODE

$$\alpha_k: \begin{matrix} 0, 1, -1 \\ \frac{1}{2} \quad \frac{1}{4} \quad \frac{1}{4} \end{matrix}$$
$$\bar{\alpha}_k = 0 \times P(\alpha_k=0) + 1 \times P(\alpha_k=1) + (-1) \times P(\alpha_k=-1)$$
$$= 0$$
$$R_0 = \bar{\alpha_k^2} = 0^2 \times \frac{1}{2} + 1^2 \times \frac{1}{4} + (-1)^2 \times \frac{1}{4} = \frac{1}{2}$$

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Now in this case your alpha k can take values 0, when we have symbol 0 we do not transmit no pulse, so the alpha k is 0. And whenever we have symbol 1 it could take either plus 1 or it could depend it could take minus 1, it will happen this in alternate fashion. We assume that probability of symbols 0, and 1 are equiprobable. So, in this case the probability of occurrence of this alpha k would be equal to half for 0, this would be equal to one-fourth, and this would be equal to one-fourth respectively.

Let us again evaluate the average value for this, this would be equal to 0 multiplied by probability of alpha k equal to 0 plus 1. Probability of alpha k equal to 1 and minus 1; probability of alpha k equal to minus 1, this turns out to be 0, your R 0 which is nothing, but alpha k squared average of it. Here it is easy to see that this would be equal to 0 multiplied by half plus 1 multiplied by one-fourth plus this is square and this would be equal to half.

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$$\begin{aligned} R_1 &= \overline{\alpha_k \alpha_{k+1}} = \sum_k \sum_{k+1} \alpha_k \alpha_{k+1} P(\alpha_k \alpha_{k+1}) \\ &= (1)(1)P(1,1) + (-1)(1)P(-1,1) \\ &\quad + (1)(-1)P(1,-1) \\ &\quad + (-1)(-1)P(-1,-1) \\ P(1,1) &= P(-1,-1) = 0 \end{aligned}$$

Next let us calculate R_1 . R_1 is by definition $\alpha_k \alpha_{k+1}$ expectation. This is equal to summation over all k and summation over all $k+1$, this is 1 followed by 1 probability of getting 1 1 plus. So these are different combinations which we can occur for the two adjacent α_k 's. Probability of joint occurrence of plus 1 and minus 1, now please note that probability of 1 followed by 1 and probability of minus 1 followed by minus 1 is 0, because the polarities alternate.

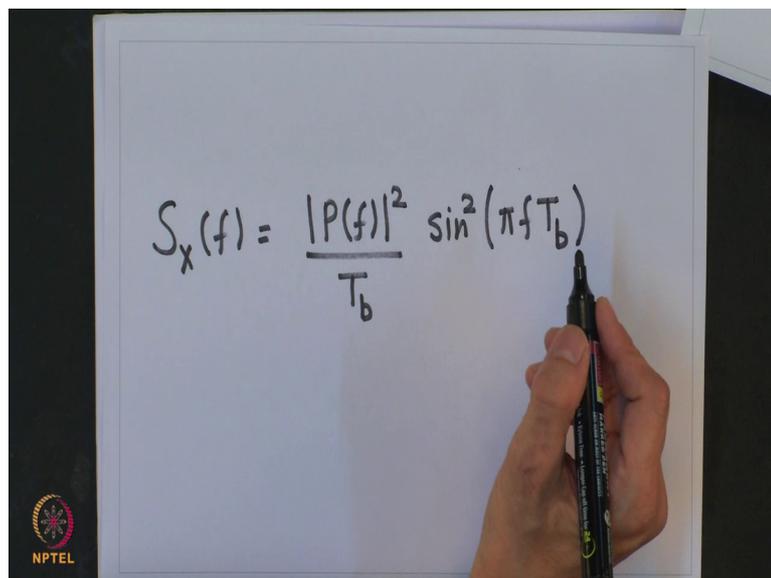
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$$\begin{aligned} P(-1,1) &= P(-1)P(1|-1) = \frac{1}{4} \times \frac{1}{2} = \frac{1}{8} \\ \text{||| } P(1,-1) &= \frac{1}{8} \\ \therefore R_1 &= \frac{1}{4} \\ R_n &= \overline{\alpha_k \alpha_{k+n}} = \overline{\alpha_k \alpha_{k+n}} = 0 \text{ for } n \geq 2 \\ S_x(f) &= \frac{|P(f)|^2}{T_b} [1 - \cos 2\pi f T_b] \end{aligned}$$

So, this quantity is equal to 0 and probability of minus 1 followed by 1 is equal to probability of minus 1 and probability of 1 given minus 1 this would be equal to one-fourth multiplied by half this probability, so this is equal to one-eighth. And similarly probability of 1 followed by minus 1 would be equal to one-eighth. Therefore, your R_1 is one-eighth plus one-eighth is equal to one-fourth.

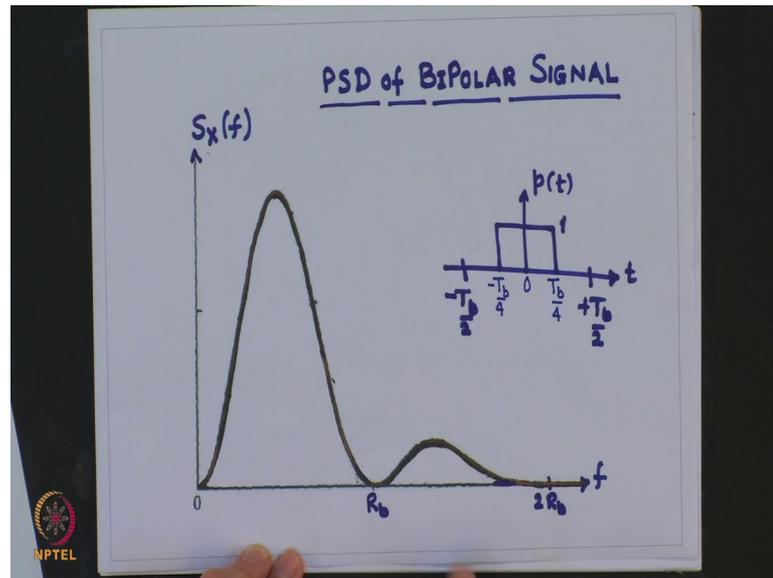
Let us calculate the value of R_n for n greater than or equal to 2. Now this can be will be independent, so I write it separate amount and this would be equal to 0 for n greater than or equal to 2 this quantities are going to be independent correct. So, from this you can calculate the power spectral density to be of this form. Now, this has interesting interpretation.

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$$S_x(f) = \frac{|P(f)|^2}{T_b} \sin^2(\pi f T_b)$$

Now we will see that if we plot this power spectral density for the bipolar signal I have drawn this specifically for half width rectangle pulse $p(t)$.

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This spectrum has a DC null irrespective of the choice of $p(t)$, the bandwidth requirement is R_b . So, still twice the theoretical bandwidth, so bandwidth is not excessive, bipolar signal has single error detection capability. So, if there is an error the polarity will be violated and we can immediately find there is a error, but the bipolar signal compared to the polar signal is not power efficient because bipolar still requires 3 DB more power for the same performance, error detection performance. And this bipolar signal is also not transparent because the long sequence of 0 will disturb the timing extraction and therefore, this is not transparent.

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PSD of UNIPOLAR (ON-OFF)

$\alpha_k: 0, 1 \quad \frac{1}{2} \text{ each}$

$$\bar{\alpha}_k = 1 P(\alpha_k=1) + 0 P(\alpha_k=0) = \frac{1}{2} \cdot 1 + \frac{1}{2} \cdot 0 = \frac{1}{2}$$

$$R_0 = \overline{\alpha_k^2} = (1)^2 P(\alpha_k=1) + (0)^2 P(\alpha_k=0) = \frac{1}{2}$$

$$R_n = \overline{\alpha_k \alpha_{k+n}} = \bar{\alpha}_k \bar{\alpha}_{k+n} = \frac{1}{4} \quad n \geq 1$$

So, now we evaluate the power spectral density of unipolar on and off line code. So, in this case we will have alpha k to be taking two values 0, and 1. For symbol 0 no transmission for symbol 1 we transmit positive 1, and again the probability of occurrence of each is half. Let us calculate the average value of it this would be equal to 1 multiplied by this is equal to half your R 0 can be evaluated as follows this is also equal to half.

Now R n in general for n greater than equal to 1 would be equal to product of these two averages. Because they are independent and each of this is half. So, this is equal to one-fourth for n greater than or equal to 1.

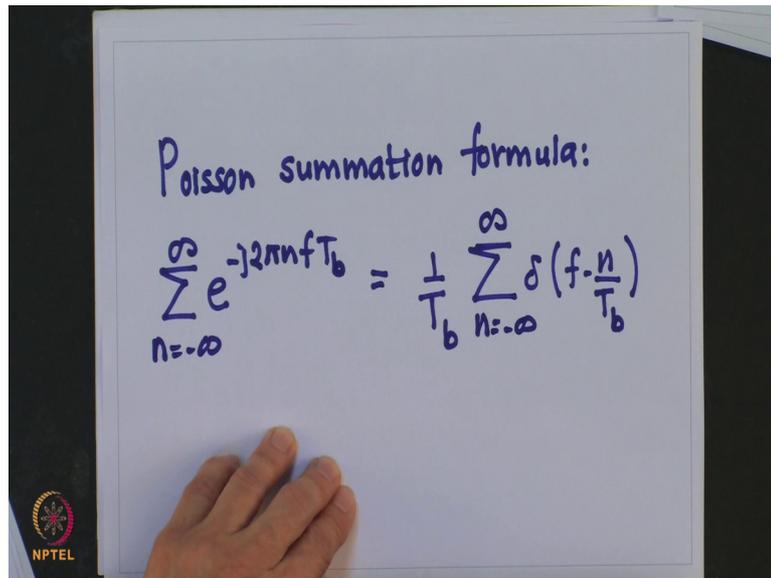
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$$S_x(f) = \frac{|P(f)|^2}{T_b} \left[\frac{1}{2} + \frac{1}{4} \sum_{\substack{n=-\infty \\ n \neq 0}}^{\infty} e^{-j2\pi n f T_b} \right]$$

$$= \frac{|P(f)|^2}{T_b} \left[\frac{1}{4} + \frac{1}{4} \sum_{n=-\infty}^{\infty} e^{-j2\pi n f T_b} \right]$$

So, now we can write our power spectral density equal to n is equal to minus infinity to plus infinity, but n not equal to 0. This I can rewrite it as follows I take one fourth term inside here.

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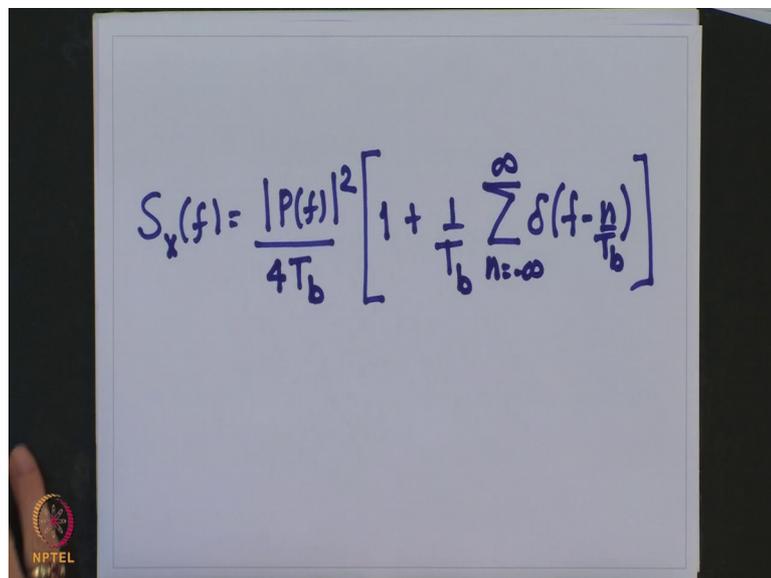
Poisson summation formula:

$$\sum_{n=-\infty}^{\infty} e^{-j2\pi n f T_b} = \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right)$$

The image shows a hand pointing to the equation on a whiteboard. An NPTEL logo is visible in the bottom left corner.

Now we will use what is known as Poisson summation follow formula, using this formula I can rewrite my power spectral density.

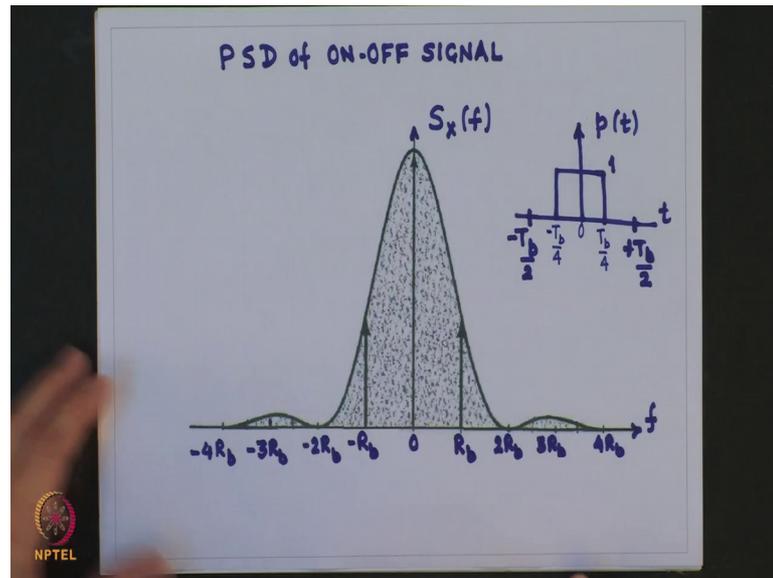
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$$S_x(f) = \frac{|P(f)|^2}{4T_b} \left[1 + \frac{1}{T_b} \sum_{n=-\infty}^{\infty} \delta\left(f - \frac{n}{T_b}\right) \right]$$

The image shows a hand pointing to the equation on a whiteboard. An NPTEL logo is visible in the bottom left corner.

Important to note that this spectra has both continuous part and a discrete part; and now if you plot this for a specific case of a half width rectangle pulse as shown here. We will get the power spectral density to be given by this figure.

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So, here it is again important to know that for the transmitted power this is less immune to noise interference than the polar scheme, average signal power is twice that required for the polar signal for the same noise immunity like what happened in the bipolar case? Unlike the polar signal this is not transparent, so it is similar to what happened in the bipolar case, because if you have a long string of 1 or 0, it will disturb the timing extraction.

And this spectrum shows that the bandwidth requirement is quite excessive twice R_b there is non 0 power spectrum at DC therefore, AC coupling will affect the transmission of this signal, this also does not have any error detection capability. But there is a clock frequency available at the rate R_b and its harmonic. So, this could be used for synchronization purpose at the receiver or at the regenerative repeaters. And another thing we see that there is a discrete component at DC that is f equal to 0.

In summary we see that the power spectral density of a line code is dependent on two factors; one is on the choice of $P(f)$ that is influenced by the choice of your $p(t)$ and the second is the kind of coding which we carry out that gets reflected in R_b . So, the choice of the pulse also is an important factor and we will see basically how this choice affects the transmission in the next class.

Thank you.