Principles of Digital Communications
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Lecture – 20
ML, MAP Detectors for AWGN Channel

We have studied continuous additive white Gaussian noise channel and showed that it is equivalent to a vector additive white Gaussian noise channel. We also derive an optimum detector for this case, and we showed that it turns out to be what is known as map detector, that is maximum or posterior probability detector. This optimum detector basically minimizes the error probability that is the probability of disagreement between the transmitted message and the detected message.

So, quickly to recapitulate it let us see what we have done.

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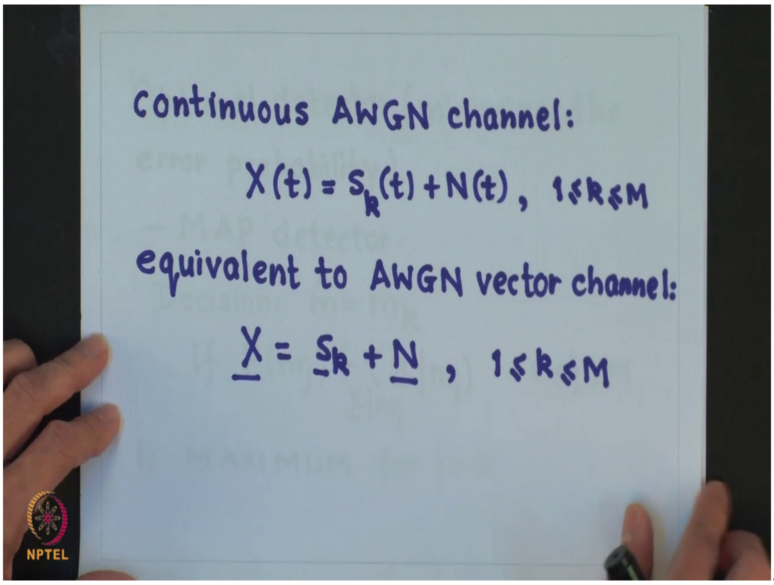

So, we have a continuous AWGN channel of this form, and we showed that it is equivalent to AWGN vector channel of this form.

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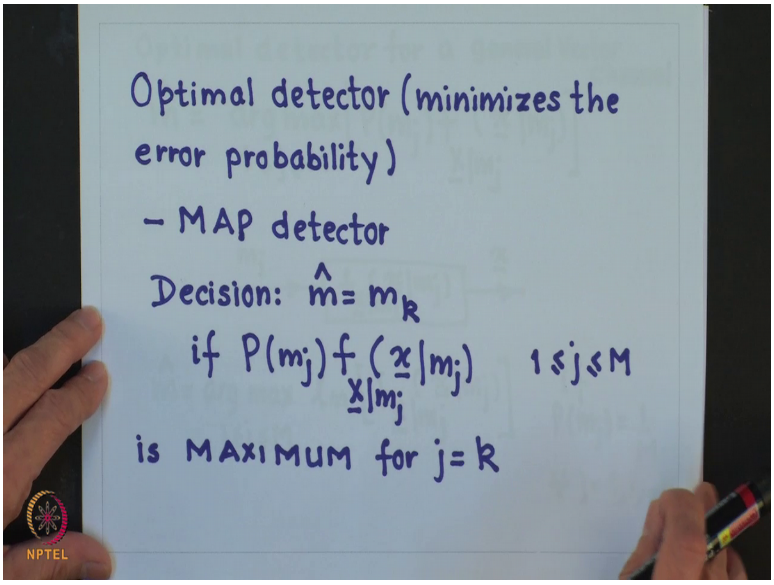

And, then the optimal detector which minimizes the error probability turns out to be the map detector and the decision rule turns out to be that decision will be equal to m k, if probability of the message m j multiplied by the conditional PDF; this expression turns out to be maximum for j equal to k. So, you evaluate this for all j s from 1 to M.

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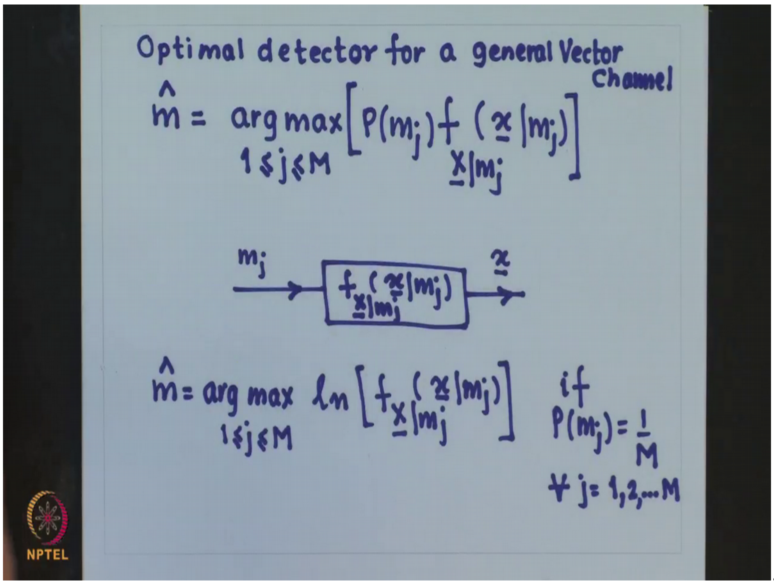

And this decision rule can be put in another console form as written here that, this is basically choose, that argument j such that this quantity is maximized. It is important to note, that in this derivation we have no we are explicitly use the statistics of the noise.

So, this derivation of map detector is basically very general and it is applicable to for any general vector channel. The specification is only in terms of the conditional pdf of the observe vector given, you have transmitted message m j. And for the maximum likelihood detector, which is a simplified version of the map detector, when the probabilities of the messages are all equiprobable, you get the decision rule to be of this form ok.

Now, let us extend this study to a specific case of additive white Gaussian noise, we show will explicitly utilize the statistics of the noise.

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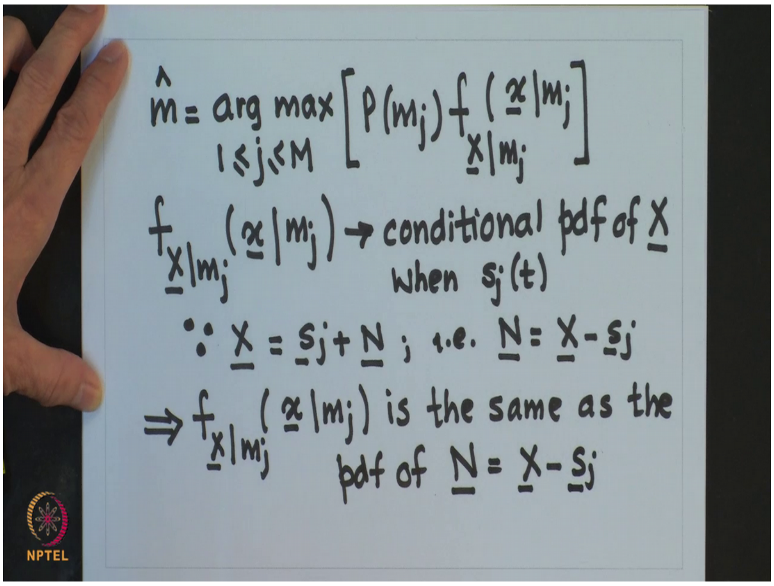

Now, let us rewrite the decision rule for map detector, which is equal to this is a probability of transmitting the message m j, this is a conditional pdf ok.

Now, for the additive white Gaussian noise, we will try to solve this decision rule. Now, we know that probability of conditional probability, when message m is m j is transmitted. So, this is equivalent to saying that, when message signal s j t is transmitted. Now, we know that our received projected vector is satisfying this relationship that is your noise vector is equal to X minus S j. So, this is a vector which we get by projecting your received signal onto the basis signal, which spanned the signal set ok.

So, now the point S j is constant and the noise vector is a random point so; obviously, the vector X is a random point with the same distribution has the noise vector, but centered at the point given by S j vector correct. So, what this implies is that, this conditional pdf is the same as the pdf of noise ok.

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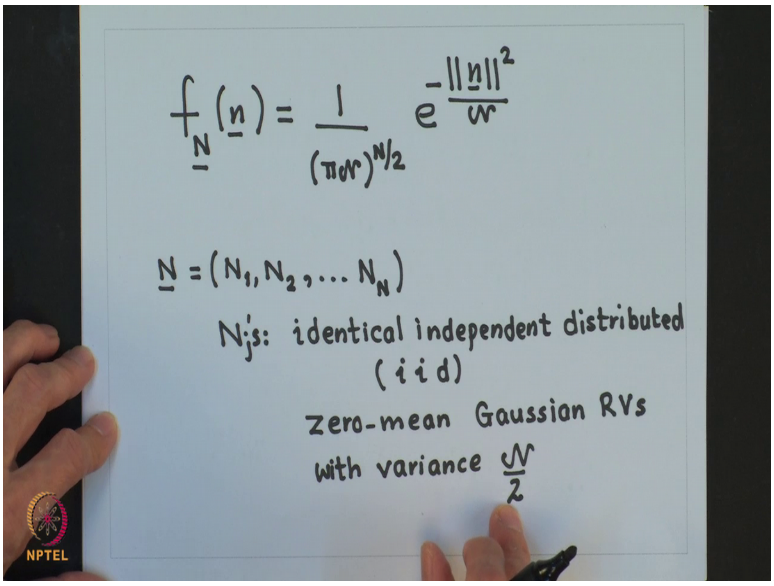

Now, we know from our early earlier study, that this noise vector consists of elements, which are identical independent distributed 0 mean Gaussian random variables, because our noise processes of Gaussian and we assumed, that this noise process has 0 mean. So, unless otherwise stated we will always assume that our noise process is of 0 mean correct. And the random variables here each of them have identical variance, which is given by italic N by 2, which is the power spectral density of the white noise Gaussian process.

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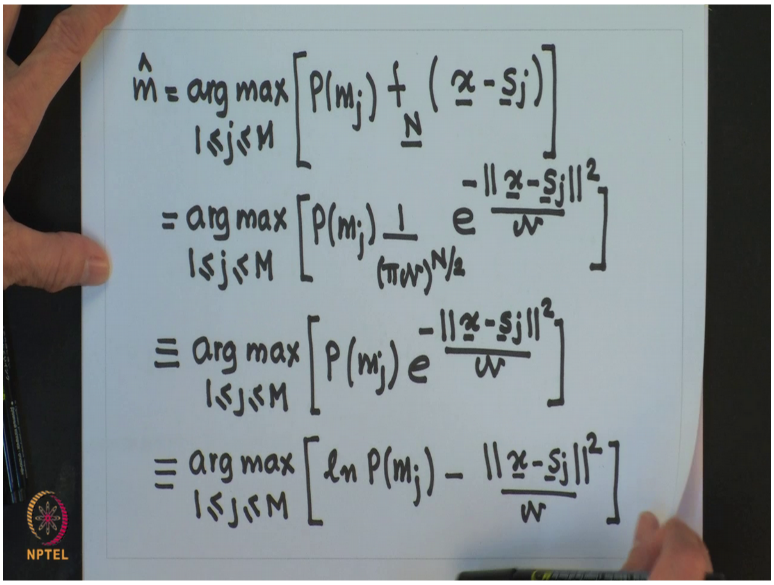

So, this is the joint pdf of the vector N. So, using this relationship we can rewrite our map decision rule as follows. Probability of transmitting the message m j and this is nothing, but the noise pdf except that it is shifted by S j vector; we just discussed this. So, this I can rewrite it as using the pdf of the noise vector.

Now, when we evaluating this for all the j s remember that this quantity out here remains same. So, we could simplify our decision rule, by this following expression. So, this notation which I am using implies the simplification of the decision rule. Now, we know that the log function is a monotonic function of the positive argument.

So, what we could do is that, we could take the log of this function and use it for our decision rule; if we do this basically this gets modified as follows. And the reason for that will become very clear the expression gets a little simplified. So, this becomes log of probability. So, when I take log of this quantity this reduces to this expression.

So, this expression can be rewritten by multiplying this term by italic N by 2.

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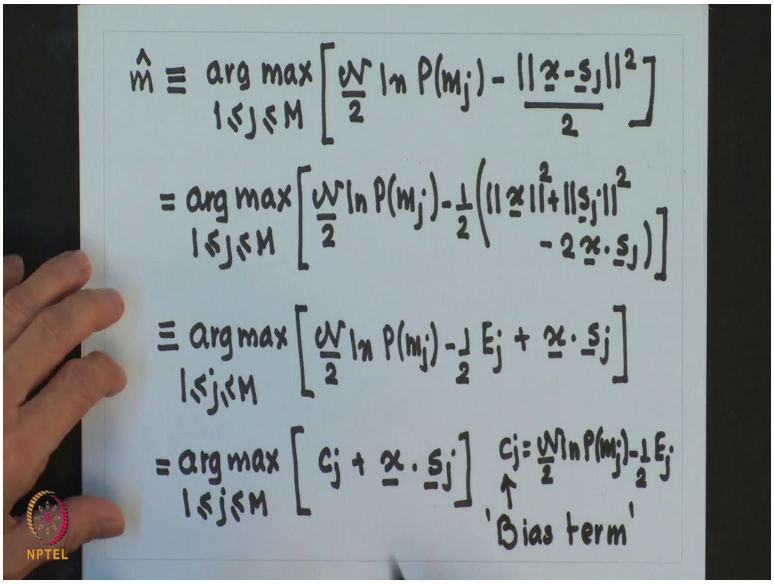

So, if I do that, my decision rule gets modified to this expression, this I can rewrite it as this I can expand it as follows.

So, now this quantity is being evaluated for all j from 1 to m, and this is the received vector which the quantity this quantity will remain the same for all the competition. So, we can drop it. So, your decision rule gets simplified to this expression, this I can rewrite it as. So, c j is basically a constant term for the jth competition and that is equal to italic N by 2 log of P m j minus half E j and this is also known as bias term. So, my map decision rule for the additive white Gaussian noise vector channel, turns out to be following, where c j is the bias term and given as follows.

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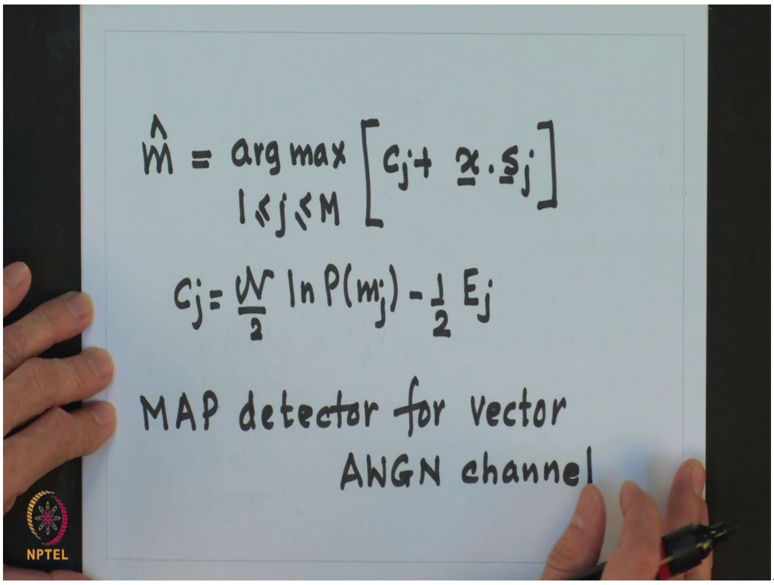

So, this is a map detector for vector additive white Gaussian noise channel.

Now, if your probabilities of message transmissions are equiprobable; that means, each of this is equal to 1 by n, then this relation can be further simplified and in that case what I will get it is as follows.

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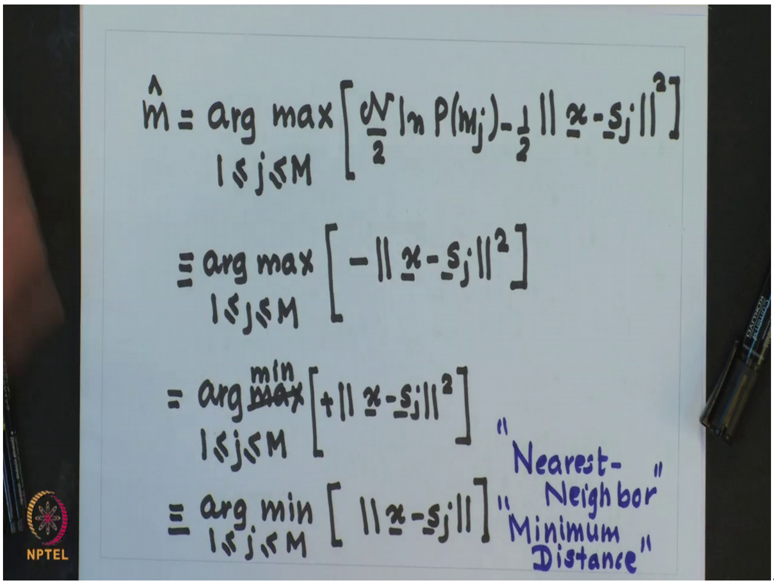

So, I will rewrite my map detection rule for vector AWGN channel, this my rule and if this quantity is constant for all j, this when equiprobable transmission case, this will get modified to the following decision rule, which is equivalent to this expression.

So, I want to make this plus. So, this will get changed to minimum correct. So, this will be equal to argument minimum equivalent. So, let us provide a geometric interpretation of the results derived. So, the receiver basically receives the vector x, and looks among all vector S j to find that is closest to the received vector x, using standard Euclidean distance.

Now, such a detector is called the nearest neighbor or minimum distance detector. So, note in this case when the signals are equiprobable, the map and the ml detector coincide and both are equivalent to the minimum distance detector or the nearest neighbor detector.

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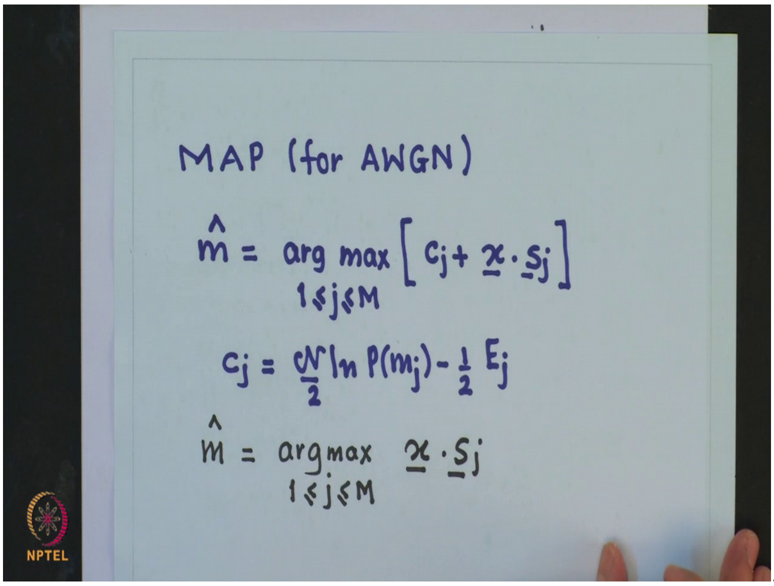

Let us look at the map detector for the AWGN channel; I repeat here that expression here this is your decision rule, where c j is given to this.

Now, in this decision rule, if we have an additional constraint, that beside the symbols being equiprobable your message signals each of them have equal energy. In that case basically this quantity c j can be drop and I can simplify my map detection rule to be equal to this expression correct fine. So, this is a simplified form of map detector, which I will get for equiprobable symbols and equal energy message signals.

Please note, that this vector x is not directly accessible. This is obtained by taking the projection of the received vector x t onto the basis signal phi j t where j goes from one to capital N, is it possible for me to write this expression in terms of the received signal x t and we will try to do this.

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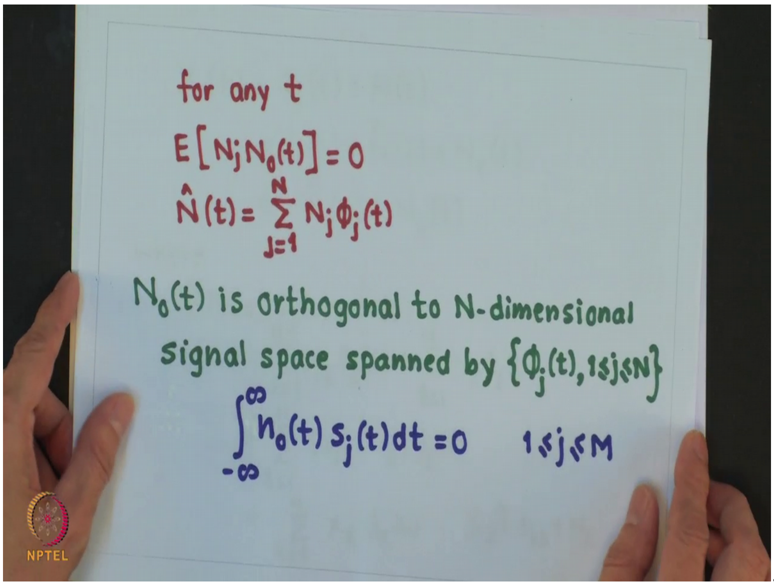

To to do that basically let us recollect quickly we are shown earlier that, your noise your signal basically X t is composed of s j t plus N t and N t we are broken up into 2 parts.

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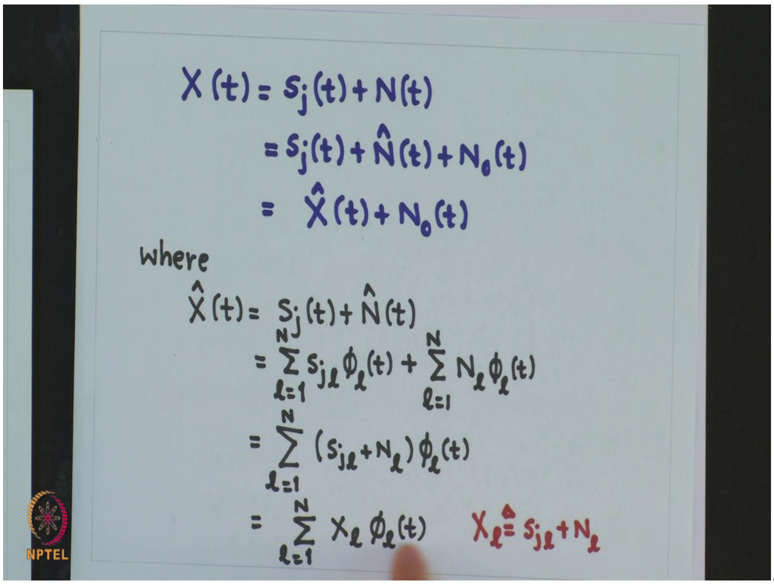

One component is basically in the same plane or in the same signal space as spanned by the message signal set and there is another component N 0 t, which you cannot represent by the basis signals used to represent the given signal set. So, we got this equal to X hat t plus N 0 t correct and from X hat t can be represented in the terms of basis signal phi l t by using this expansion coefficient except this we have done earlier ok.

And, we have also shown that this random variable N j and N 0 t for any t turns out to be 0 and this N hat t is nothing, but expansion of the basis signal phi jt. So, what it implies from this that N 0 t is orthogonal to N dimensional signal space spanned by phi j t correct. So, I can write for a specific sample function n 0 t this expression, which implies that n 0 t is orthogonal to S j t. Remember S j t lies in the same space has N hat t. So, this relationship is valid.

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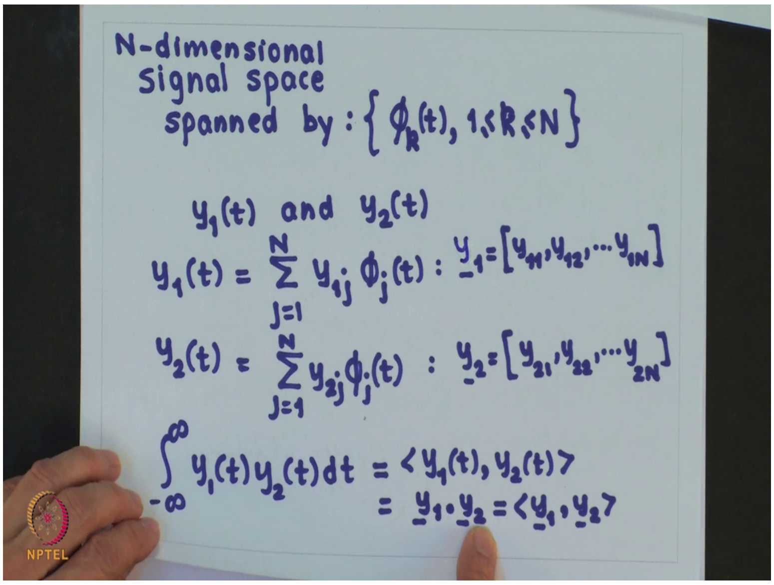

So, now we have also studied that, if I have an N dimensional signal space spanned by phi k t, where y 1 t and y 2 t are any 2 signals in this signal space, then I can rewrite y 1 t and y 2 t in vector form as shown here. And, then we know that cross correlation between y 1 t and y 2 t can be obtained by the dot product of the 2 vectors.

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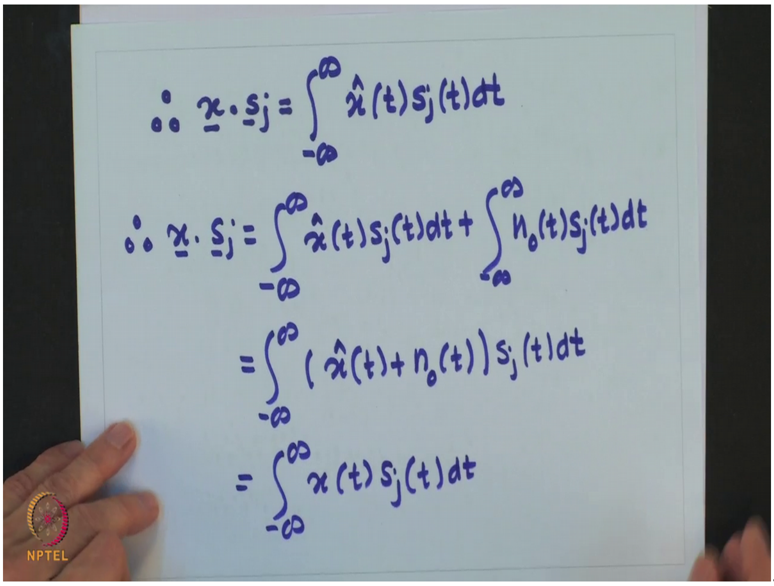

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So, based on this result we could rewrite our this dot product between x dot and S j also as follows. This is equal to therefore, this quantity, which I am writing here second term is 0 we have just shown it. So, this is equivalent to writing it remember that this are the sample function of the respective processes. And this is equivalent to cross correlation between the vector received vector x t and S j t the signal message signal.

So, now using this we can quickly modify our optimal map detection rule, we have seen that optimal map detection rule in a AWGN channel is given by this.

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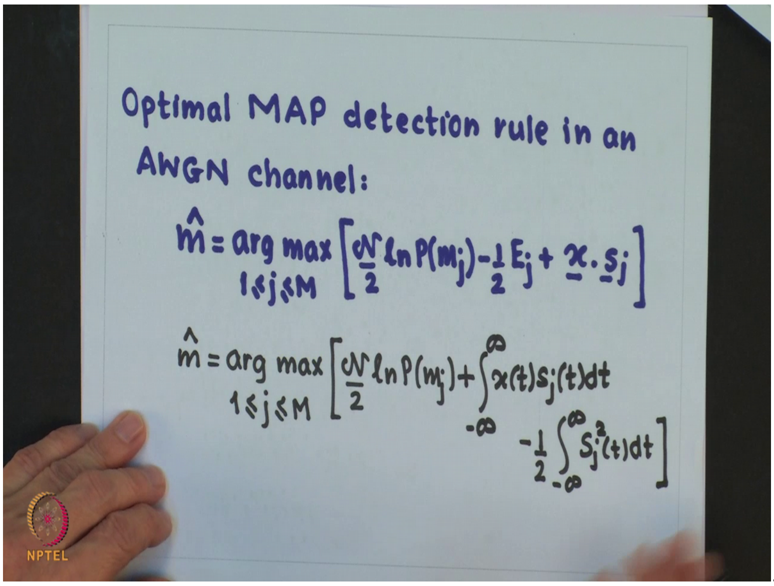

So, this quantity I can rewrite it as this quantity I can rewrite it, this quantity is equal to this quantity and this basically is nothing, but the energy in the signal. So, this also is rewritten in this format fine. So, this becomes your optimal map detection rule for the your continuous AWGN channel now, and we can similarly rewrite the ml detection rule as follows.

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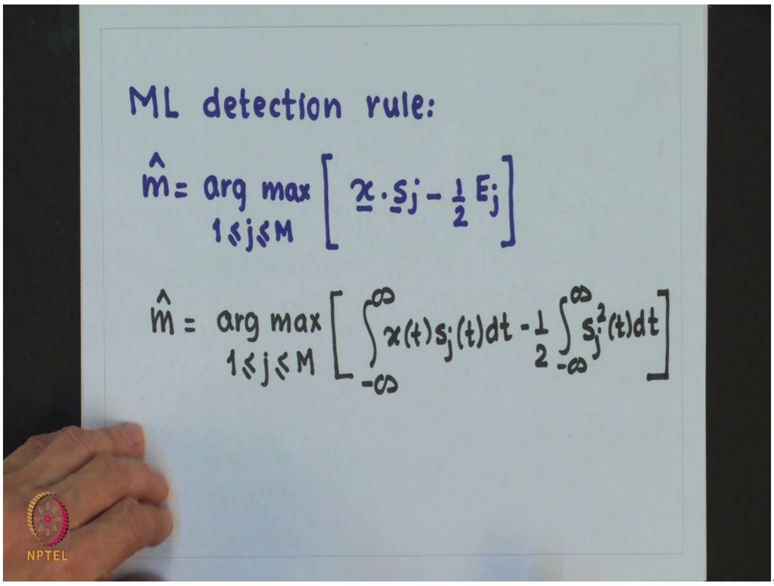

This is your ml detection rule for the additive white Gaussian noise vector channel correct. So, this I can rewrite for the continuous AWGN channel in this format, based on the result derived for this dot product. This is equivalent to writing and this quantity is equivalent to writing this expression correct.

So, this becomes your ml detection rule for the continuous AWIGN channel. So, having done this basically we will see basically how to implement this rule in a practical scenario, and this will do in the next class.

Thank you.