Principles of Digital Communications
Prof. Shabbir N. Merchant
Department of Electrical Engineering
Indian Institute of Technology, Bombay

Lecture – 16
Signal Space Representation – II

Let us recapitulate our study on orthogonal expansion of signals. What was what we studied was given a set of orthonormal basis signals and any signal S t it is possible to approximate this signal S t in terms of the weighted linear combinations of this orthogonal basis signals, in such a way that the energy in the error signal is minimized.

And we saw that this is achieved by taking the first derivative of the energy with respect to the coefficients of expansions and equating this to the 0s and solving the equation and the result we got was that the error signal turns out to be orthogonal to all the basis signals which we use in the linear combination to approximate the given signal S t.

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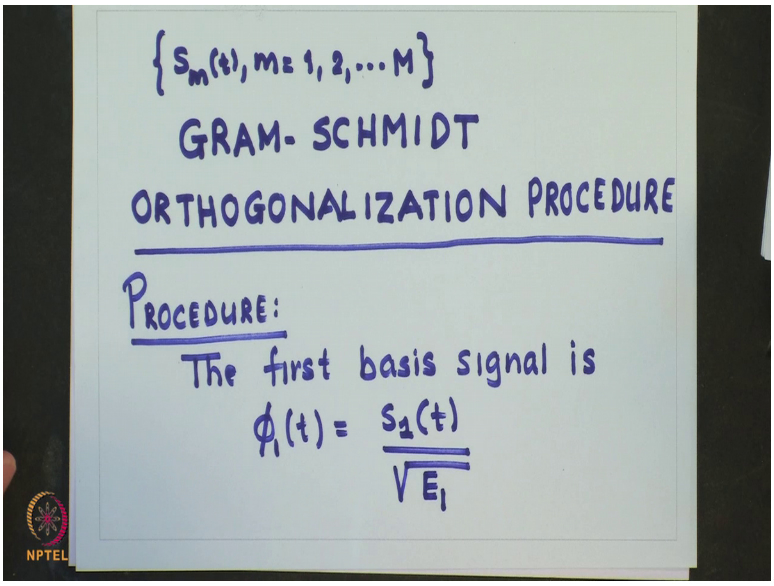

Now, suppose that we have a set of finite energy signal waveforms S m t where m goes from 1 to capital M and from this set we wish to construct a set of orthonormal waveforms. So, to achieve this we follow what is known as Gram Schmidt orthogonalization procedure. This will allow us to achieve our objective. The procedure is very similar to the one which is used for vectors ok.

So, let us look into this procedure; the first basis signal can be chosen as follows. It is the S 1 t and you normalize with the energy of this S 1 t correct, your S 1 t energy is given by this integral; without loss of generality we are assuming that all our signals are real signals ok.

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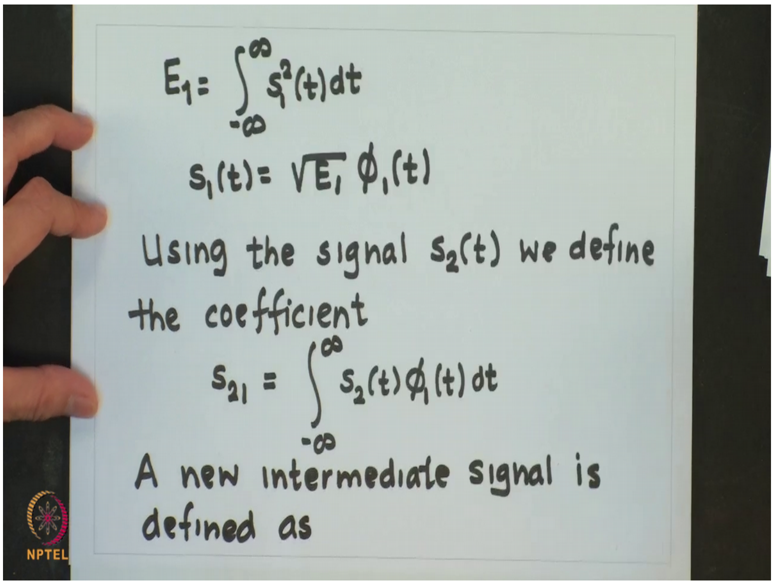

So, what it this specify that if I choose my basis signal phi 1 t to be equal to S 1 t divided by the root of E 1, then I able to express my S 1 t as root E 1 phi 1 t fine. Now let us use the signal S 2 t in the set. We define the coefficient S 2 1 as projection of S 2 t on phi 1 t right. So, what we are saying is that we are trying to find the linear combination of S 2 t in terms of phi 1 t such that energy in the error is minimized correct. So, then let me I will get a new intermediate signal which we defined as g 2 t.

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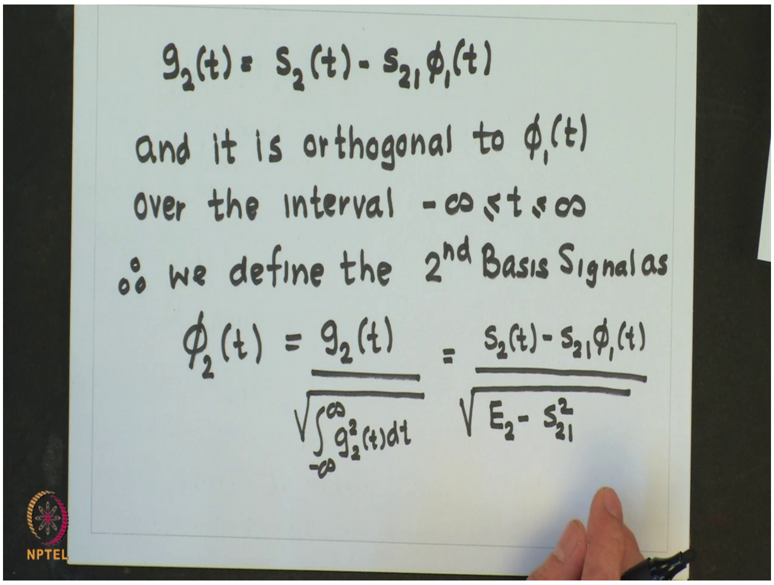

This is basically nothing, but the error signal between S 2 t and its approximation using phi 1 t correct. Now we know from our study on orthogonal expansion of signals that this error signal is going to be orthogonal to phi 1 t over the interval, we are using this interval without loss of generality ok.

So, if you do this therefore, we define the second basis signal as that is this error signal. And we normalized by its energy correct and this can be rewritten as follows. We have seen this earlier that this quantity would be equal to E 2 minus S 2 1 square ok. What we could do is basically take S 3 this signal and try to express it as a linear combination of phi 1 t and phi 2 t correct and then get the error signal of approximation between S 3 t and this linear combination of phi 1 t and phi 2 t.

So, we will get an intermediate signal that is g 3 t; now g 3 t will be; obviously, orthogonal to phi 1 t and phi 2 t correct. So, that could be used as another basis signal after normalizing it with the energy of g 3 signal square root of that fine.

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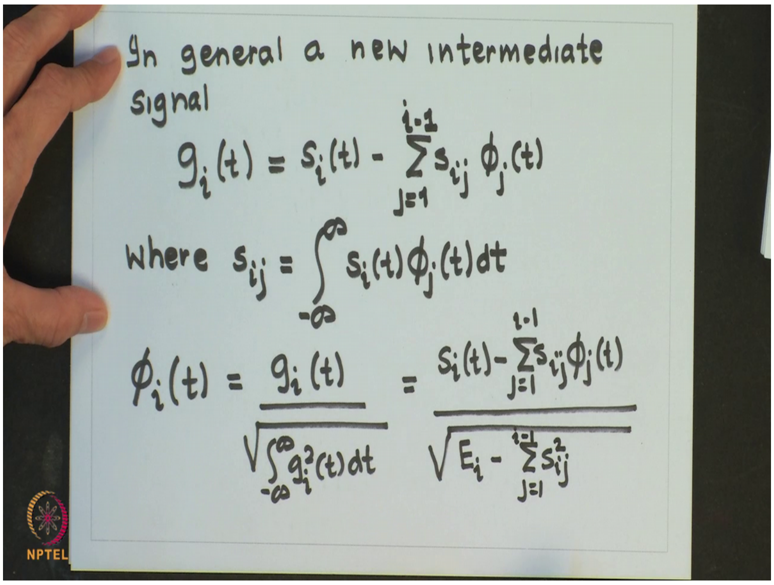

So, in general I can always form a new intermediate signal as follows g i t is equal to s i t minus linear combination of basis functions which have been computed earlier. So, there will be i minus 1 basis functions and your coefficients of expansion that is s i j would be equal to projection of s i t on the basis function phi j t correct. And then we can get my basis signal phi i t to be equal to g i t normalized by the square root of its energy which can be rewritten as this is E i; the energy in s i t signal minus fine.

So, the orthogonality process is continued until all the M signals waveforms have been exhausted and N less than or equal to M orthonormal waveforms have been constructed.

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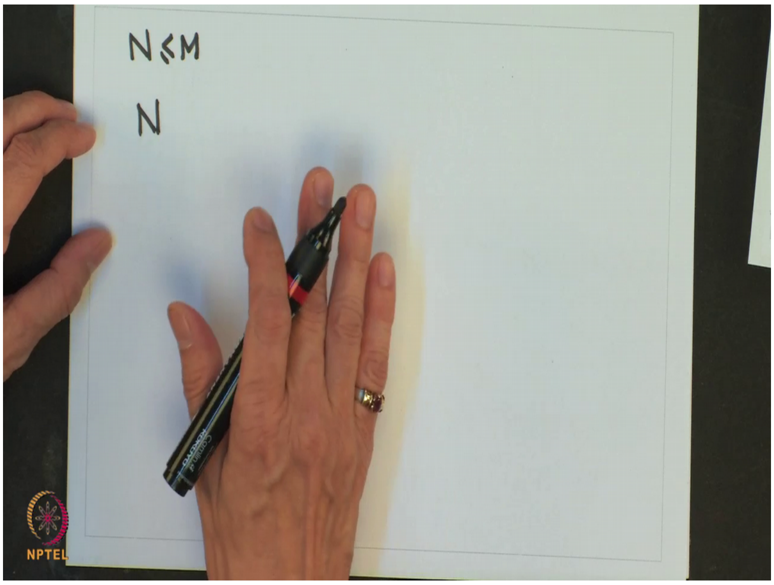

So, the dimensionality N of those signal space will be equal to M if all the signal waveforms are linearly independent; what it means that none of the signal waveform is a linear combination of other signal waveforms ok. So, let us take an example to appreciate what we have done. So, let us say that I have 4 waveforms given as shown here S 1 t, S 2 t, S 3 t, S 4 t ok.

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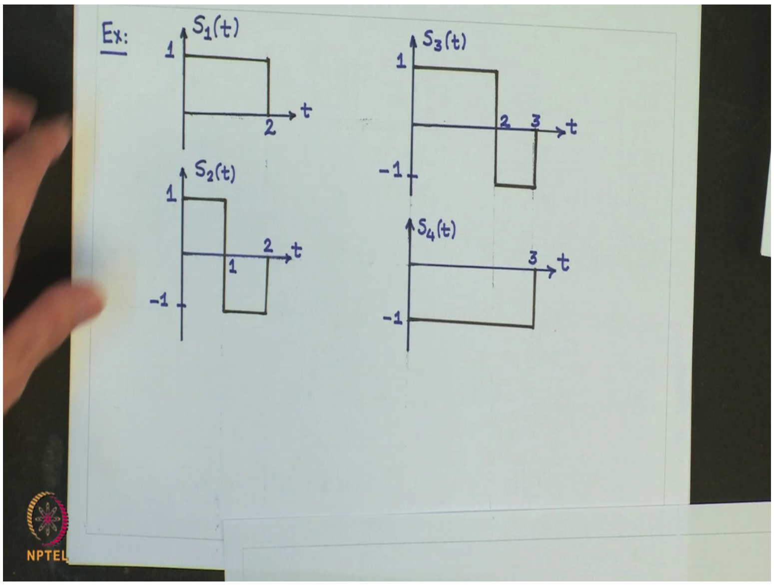

So, what we will do is basically we will start with this and let us try to obtain the orthonormal basis signal in terms of which we can represent any of this given 4 signals ok.

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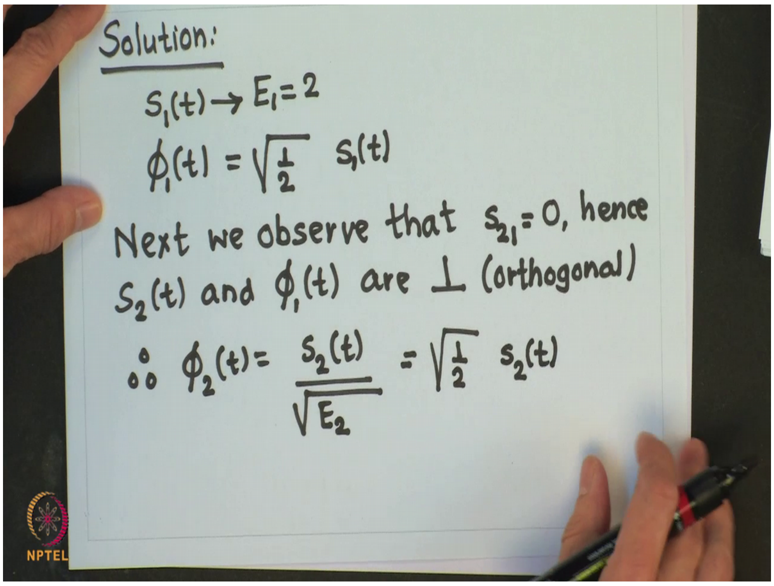

So, we will choose from this the solution would be as follows. So, S 1 t its energy is equal to 2 square it multiply by t. So, 2 correct. So, my phi 1 t is equal to 1 by root 2 S 1 t. Next we observe that S 2 1 that is the projection of signal S 2 t on S 1 t is equal to 0, hence this implies that S 2 t and phi 1 t are orthogonal; this means orthogonal correct. Therefore, I can choose my phi 2 t to be equal to S 2 t normalized by the square root of its energy. And again in this case if you see the energy in the signal S 2 t is equal to 2; so, root of 1 by 2 S 2 t fine.

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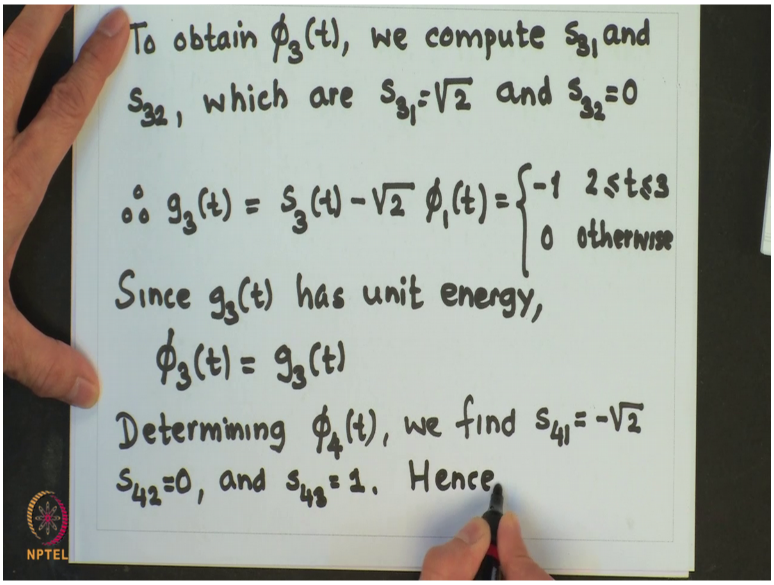

So, now we proceed to obtain phi 3 t. So, for this we need to compute the projection of S 3 t on phi 1 and phi 2. So, these are the coefficients which are S 3 1 is equal to root 2 and S 3 2 is equal to 0 very straightforward form correct. Looking at this itself figure we can see that the projection of this onto the phi 1 t; phi 1 t is similar to this correct except for the normalizing factor of root 2 correct.

So, you will have the projection of this on phi 1 t, but a projection of this on the phi to 2 which is similar to this except for the normalization will be equal to 0 fine. So, this will get therefore, my g 3 t will be equal to S 3 t minus root 2 phi 1 t. And this will be equal to minus 1 for this interval and it is 0 otherwise correct. So, since g t 3 t has unit energy, it follows that your phi 3 t is equal to g 3 t correct.

Now, we have to still proceed; so, determine phi 4 t we find S 4 1 is equal to minus root 2 correct. So, projection of this on phi 1 t will be equal to minus root 2, S 4 2 would be equal to 0 correct fine. I am showing this because phi 2 t is similar to this and S 4 3 would be equal to 1 ok.

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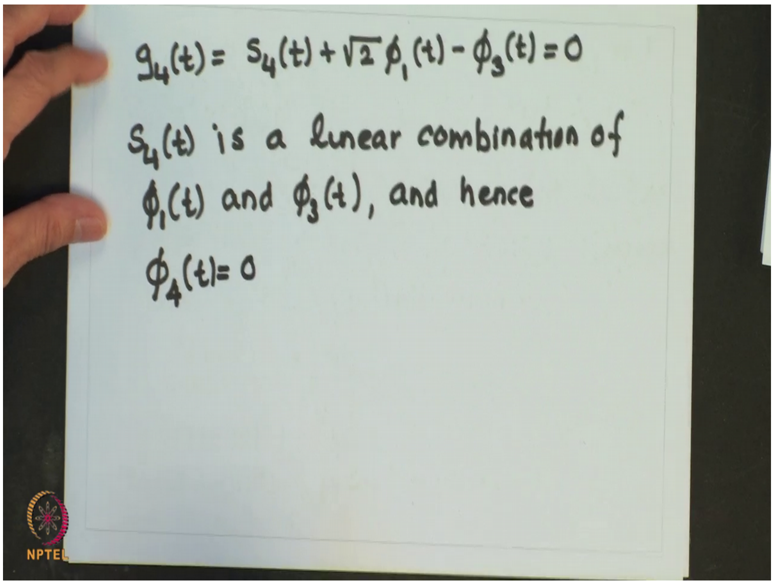

So, hence my g 4 t would be equal to S 4 t and this turns out to be 0. So, what it means that S 4 t is a linear combination of phi 1 t and phi t 3 and hence phi 4 t is equal to 0 fine.

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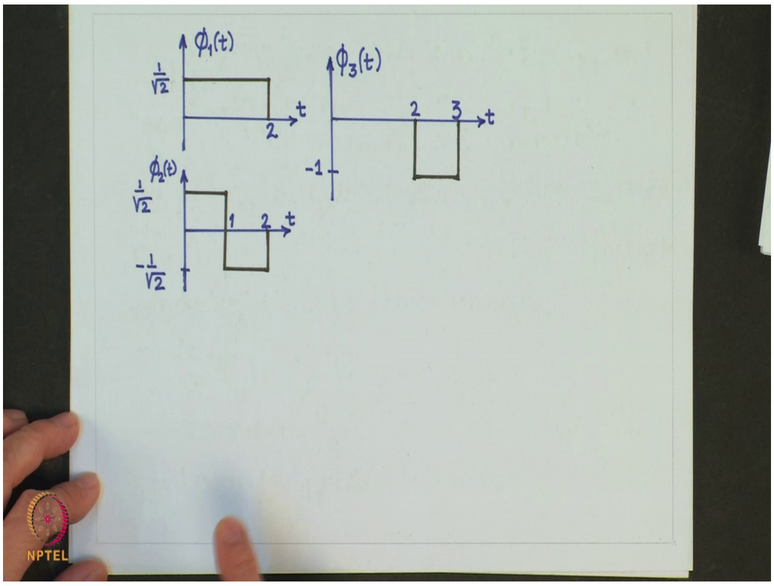

So, based on this the 3 orthonormal basis signals which we have computed are shown here phi 1, phi 2, phi 3. So, this resembles S 1 t except for the normal in factor, this also represent resemble the S 2 t except for the normalizing factor and this is really different form S 3 t correct.

So, once we have constructed the set of orthonormal waveforms; in this case we have got 3 orthonormal waveform, we can express the 4 signals which were given as linear combinations of this orthonormal waveform correct?

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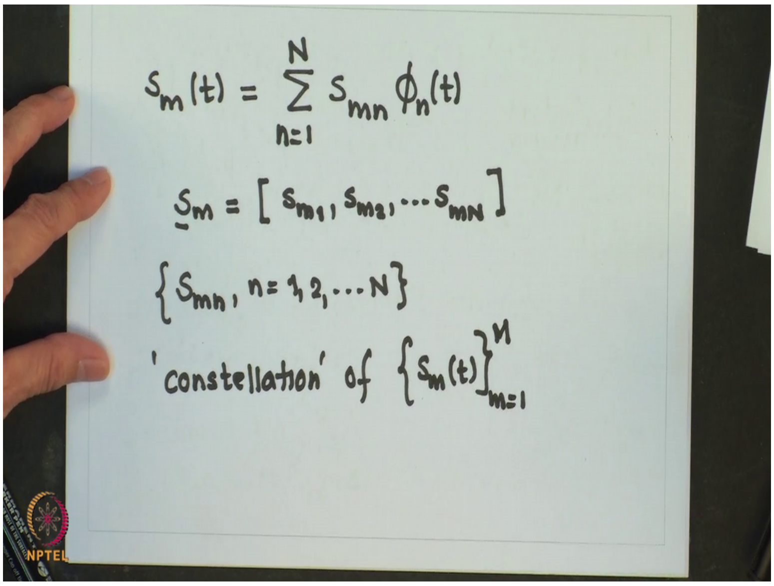

So, in general once I have my orthonormal basis signal; I can express my signal S m t as linear combination of basis signal correct. So, in our case this m would go from 1 to 4 this capital N would go from 1 to 3 sorry this capital N would be 3 correct this n would go from 1 to 3.

So, based on the above expression; this signal may be represented by the vector. So, I can represent it as a vector like this in general this will be N or equivalently as a point in the N dimensional signal space correct with coordinates; therefore, what we have shown that a set of m signals can be represented by a set of m vectors in the N dimensional space where N is less than or equal to m correct. So, the corresponding set of vectors is called the signal space representation are also known as constellation of the given signal set.

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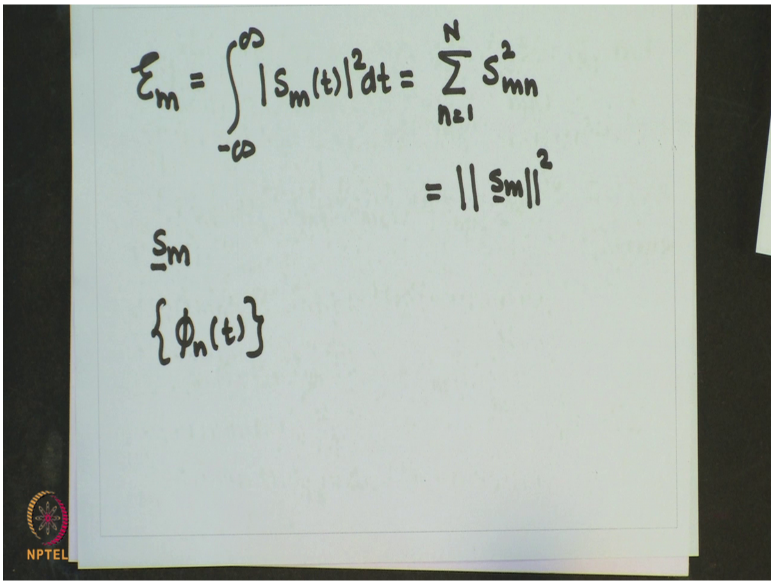

So, correct; now from the orthonormality of the basis phi nt it follows that the energy in each of this given signals can be computed as follows correct. So, the energy of the mth signal is simply the square of the length of the vector or equivalently the square of the equilibrium distance from the origin to the point S m in the N dimensional space. So, thus any signal can be represented geometrically as a point in the signal space spanned by the orthonormal functions phi nt correct.

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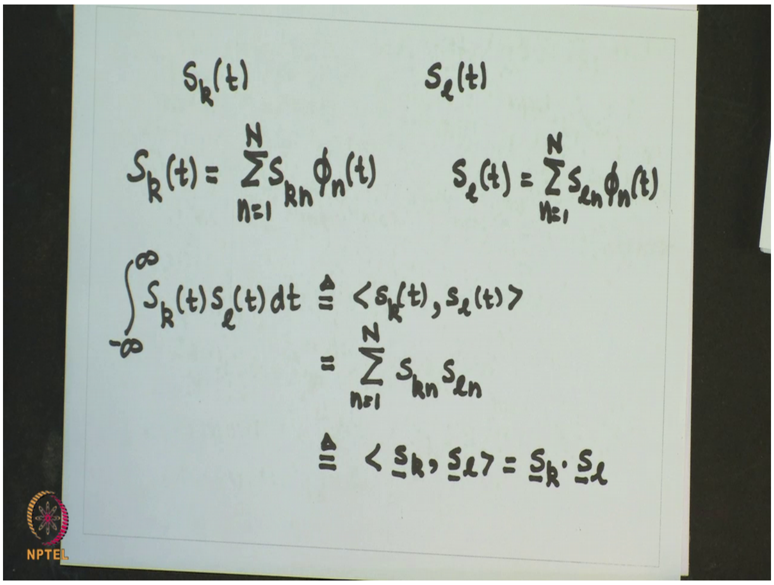

Now, for such a case if you have 2 signals in this space let us say S k t and S l t correct now both of these signals can be represented in terms of orthonormal functions as follows correct and now if you try to evaluate the this integral.

So, this is nothing, but the correlation between the 2 signal S k t and S l t, which we can write by definition as just a notation correct. So, each of this signal can be written in terms of vector and then it is easy to see that basically this will be equal to and this will indicate by the dot product of the 2 vectors S k and S l correct.

So, what this shows that the inner product of 2 signals is equal to the inner product of the corresponding vector ok.

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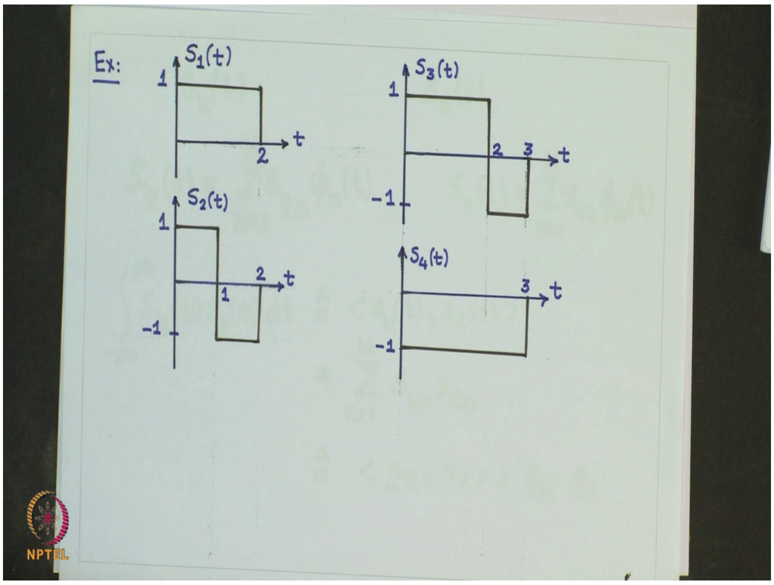

It is also important to realize that there are infinite number of possible sets of basis signal for a given signal space. For example, for this case when we started with S 1 2, S 2 3, S t 3 S 4 t this way we generated this set of orthonormal basis signals, but it is important to note that this orthogonal procedure does not provide you unique orthonormal basis signals correct because it all depends on the way we have arranged S 1 t, S 2 t, S 3 t, S 4 t.

If I had arranged it little differently correct for example, if this was S 1 t and this was S 2 t; then I would get a different set of orthonormal basis signals correct. So, if I get a different orthonormal basis signals the vector representation will also change, but it is important to note that the dimensionality will not change correct. So, in this case the dimensionality will still remain 3 correct. So, and the representation of this signals as a vector will; obviously, change because the basis signals are changing correct.

But the configurational vectors will not change correct. So, the length of the vector will remain the same correct the dot product between the 2 vectors will remain the same as you would get the answer in the earlier basis signal set correct. So, that is important to know fine.

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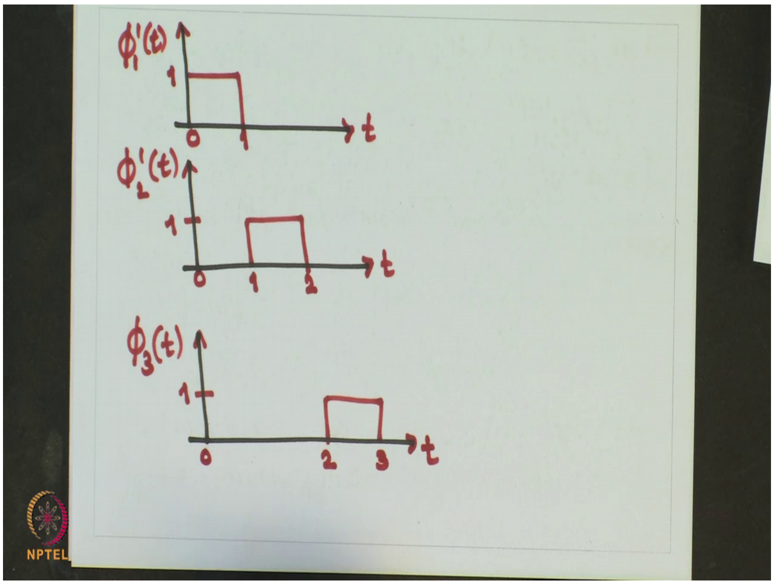

So, just for the sake of completion I can just provide you another basis signal for the example on hand; for example, I could have uses use this 3 basis signals correct. So, I just call them; so this is from 0 to 1, this is time, this is 1 to t and this is from 2 to 3 correct. So, this 3 basis signals also will be able to represent the signal space; spanned by this 4 signals. So, this proves my point that the orthonormal basis signal set is not unique ok. So, with this we end our module on the signal space representations of signals.

Thank you.