Microwave Theory and Techniques
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Module - 8
Lecture - 36
Low Noise Amplifiers – II: NF Circles and LNA Design

Hello and welcome to second lecture on Low Noise Amplifiers. In the last lecture, we looked at two noise sources; one was thermal noise due to resistor and that is given by this particular expression.

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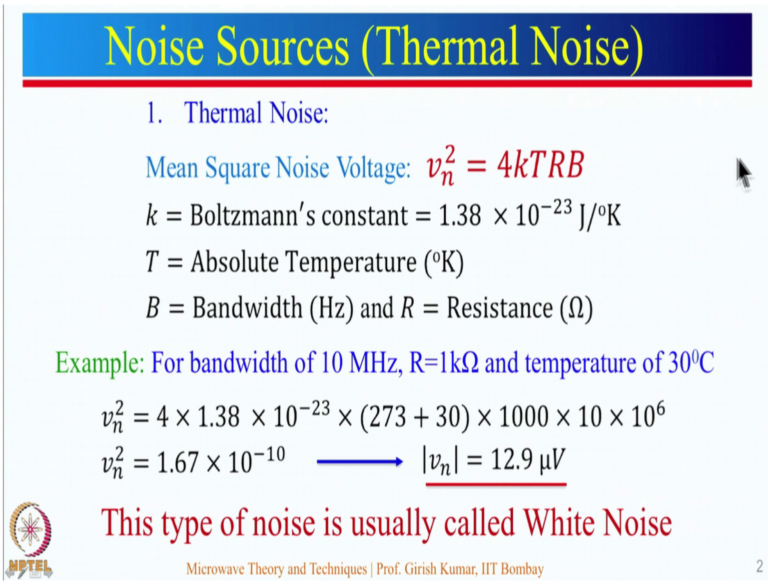

Then we found out what is the maximum available noise power from this particular resistor that is given by the expression kTB. After that we looked at the second noise source which is shot noise because of the pn junction we saw the expression for that is i n square is equal to 2 q I dc B.

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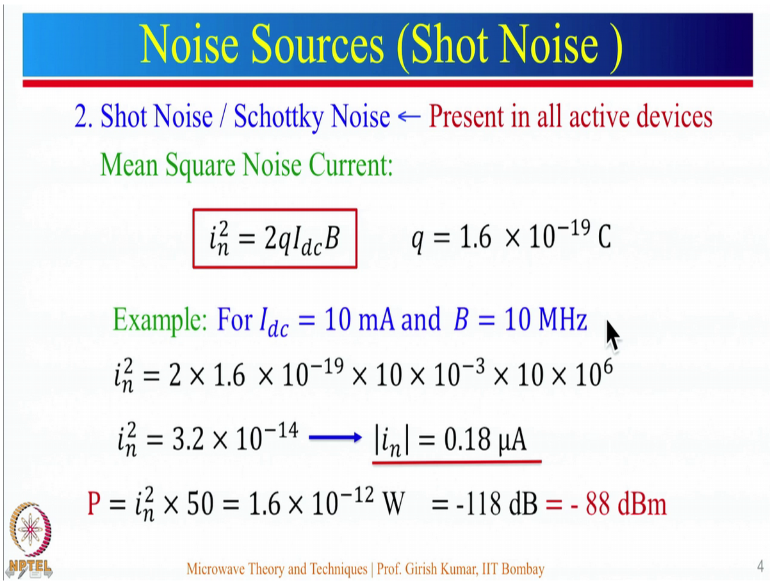

And then for this particular example we saw that the noise power is equal to minus 88 dBm that may be quite high for let us say mobile phone application. So, that is why most of the time the first stage should really have a small current as possible.

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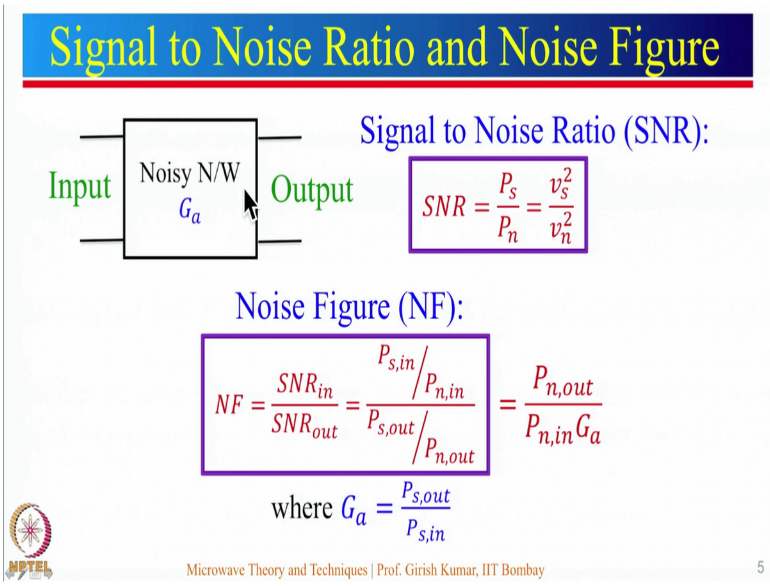

After that we defined signal to noise ratio and noise figure.

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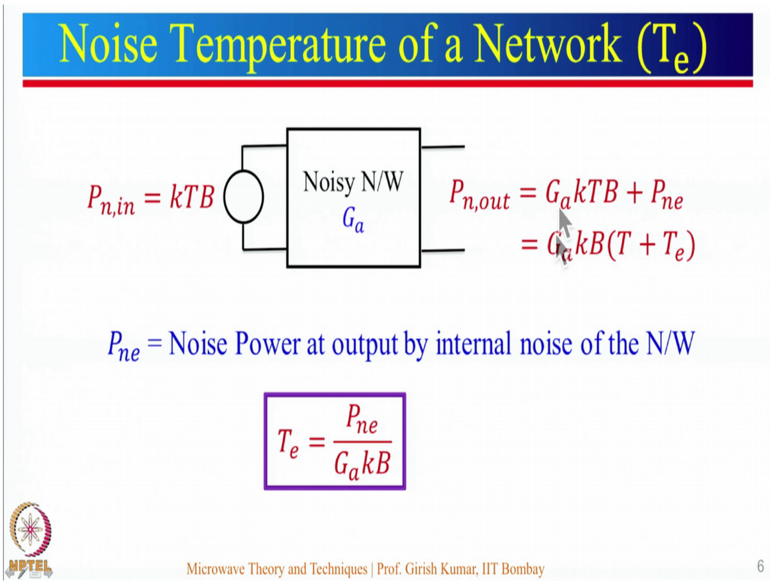

And then we defined noise temperature of the network after that we did the derivation and found out the overall noise figure for cascaded stages.

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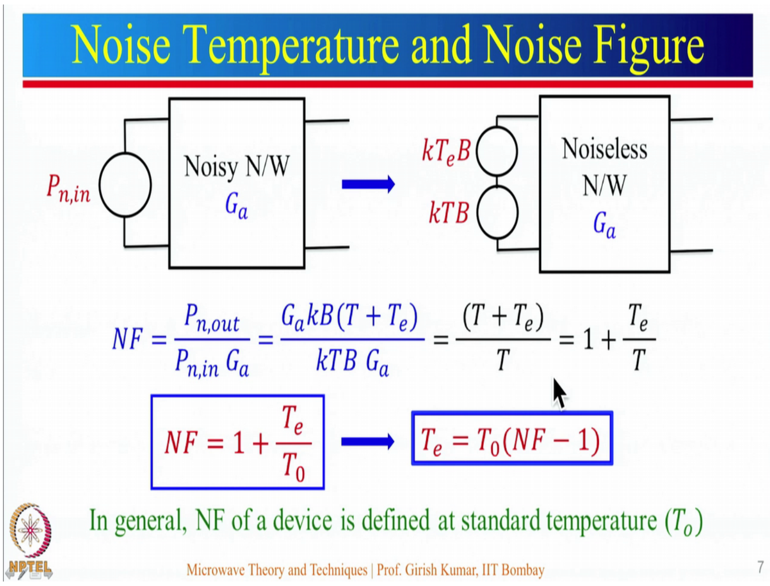

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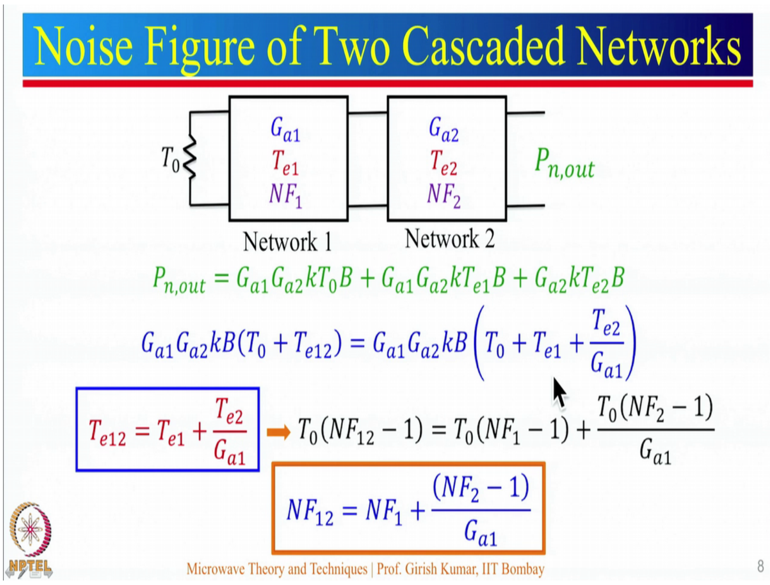

So, for 3 cascaded networks we found out NF 13 is given by this particular expression, where first stage noise figure comes as it is, second stage noise figure is divided by gain of the first stage, and for third stage noise figure is divided by gain of first as well as second networks.

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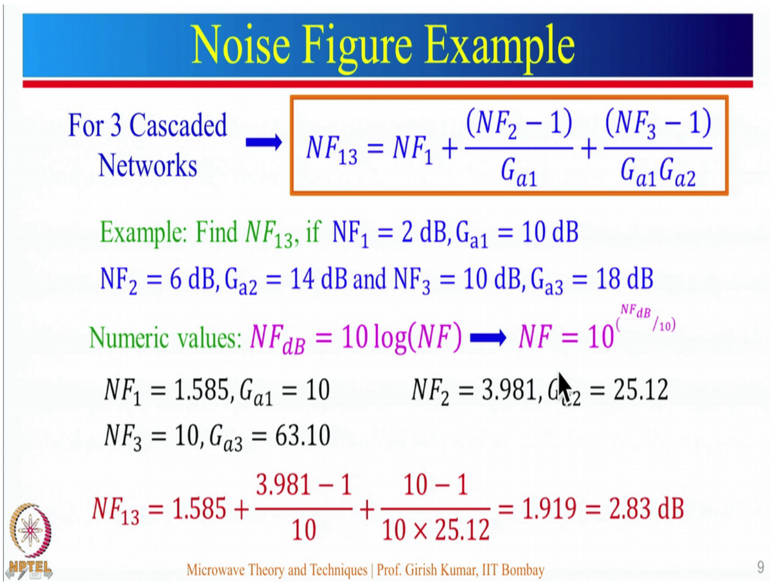

Then we took one example to find out overall noise figure and we found out that overall noise figure is 2.83 dBm where as the noise figure of individual stage was 2 dB, 6 dB and 10 dB. But the net is 2.83 dB and that is mainly because of the finite gain of the first stage and second stage. Now, we will look at how to design low noise amplifier.

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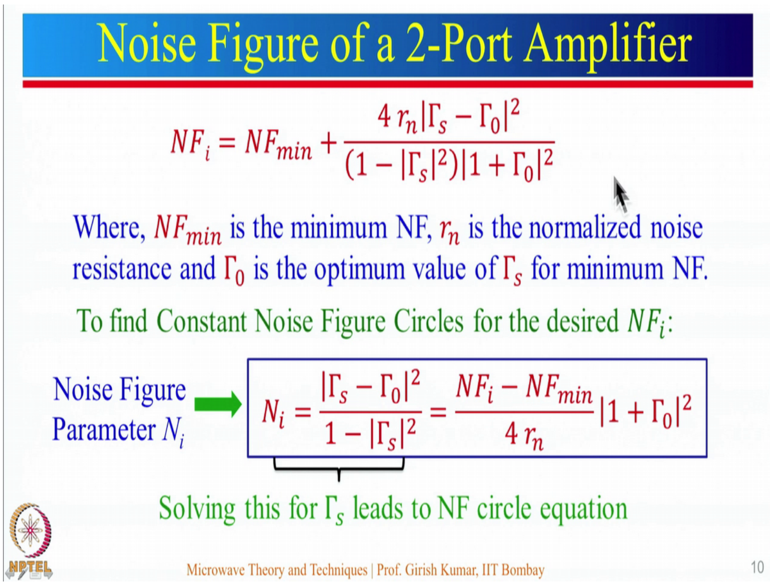

So, noise figure of a two port amplifier is defined by this particular expression. So, let me tell you what is this expression, what are the different terms over here. NF min is the minimum noise figure for that given transistor or amplifier, r n is the normalized noise resistance most of the time this normalized noise resistance is equal to noise resistance divided by z 0 and z 0 maybe majority of the time equal to 50 ohm. Gamma 0 is the optimum value of gamma s for minimum noise figure.

Now, in the previous lectures we had seen that to optimize the gain we actually draw constant gain circle for input side and output side. So, there we were only concerned about a gain, but now we are concerned about noise figure also. So, you can see here that this expression will reduce to NF min if gamma s is chosen such a way that it is equal to gamma 0. So, if gamma s is chosen as gamma 0 then this term will become 0. So, we can realize overall noise figure of this amplifier as NF min. However, if gamma s is not equal to gamma 0 then NF i will have a larger value.

Now, this is where one has to do the design aspect. So, if you have to optimize, the gain you may have to choose different value of gamma s, but if you have to optimize for low noise figure, then you have to choose different value of gamma s. We will see that later on today how to choose proper value of gamma s either to optimize gain or to optimize noise figure. In the previous lecture we had actually seen noise power due to resistor and noise power due to current i n square. So, what is this expression all about? Now, just I want to mention that a transistor may have just two pn junction, but an amplifier may have multiple transistors. So, if you try to solve all those things it will take very very long time.

So, to make life simpler manufacturers generally gave the values of NF min r n and gamma 0 at a given frequency. So, then you can design amplifier using these parameters for either best possible noise figure or best possible gain or maybe we may compromise between the two. So, now, we have to find out constant noise figure circle. Now, till now we talked about stability circle, then we talked about gain cycle, now we are going to talk about noise figure cycle. So, I hope that by seeing all these circles your mind does not go into circle, now I try to make things as simple as possible for you people.

So, we have to now find out the noise figure circle for that we do little simplification. What do we do it is we solve this particular expression for gamma s. So, you can see over here these terms here gamma s minus gamma 0 square is write here 1 minus gamma s square is write over here. So, this is the term corresponding to this part here. Now, this has to be represented in this form here. So, NF min will go to that side, so NF i minus NF min now 4 r n will come in the denominator. So, that comes over here and then this one plus gamma 0 whole square goes to this side. So, that is what over here. So, we can say that gamma s minus gamma 0 square divided by 1 minus gamma s square is given by this particular expression here and that particular thing is defined as noise figure parameter N i.

So, now, for a given device NF min r n gamma 0 will be known to us and then we have to design for the desired NF i. Now, you may say that we would always like NF i to be equal to NF min, but as I just mentioned if you choose gamma s equal to gamma 0 then only we will get this whole thing equal to 0 over here, but that may give rise to lower gain.

So, let us look at how we can plot the noise figure circle and then I will tell you how to choose the optimum value of gamma s. So, now, we have to solve this particular equation here N i will be known so for a given value of NF min r n gamma 0 and desired

NF i. So, this will be known that means, N i is known. So, we solve this equation for gamma s.

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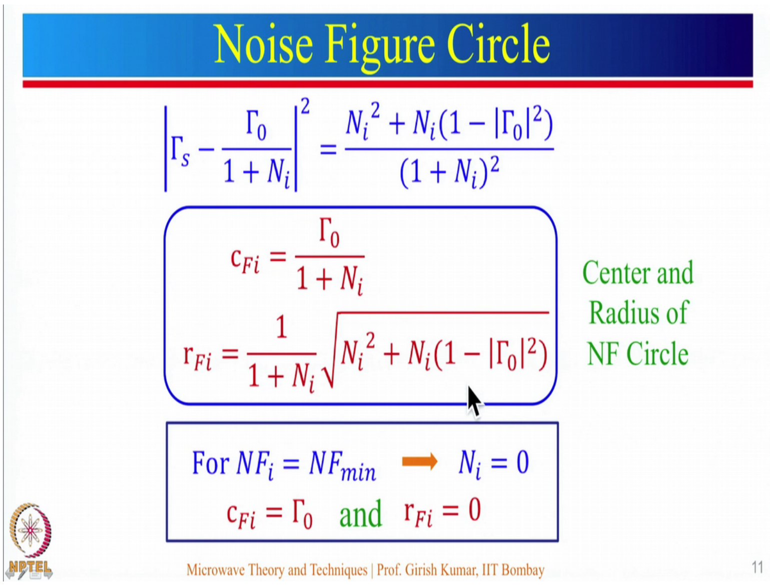

So, noise figure circle equation comes out to be gamma s minus this term which is equal to this here. So, what is this term here? This is nothing but center of the noise figure circle. So, that is given by this particular expression. And what is this here? This corresponds to the radius of the noise figure circle. So, we have to take square root of that because this is equal to r F i square. So, r F i is given by this particular expression.

So, now let us just take the special case when NF i is equal to NF min. So, if NF i is equal to NF min that will give rise to N i equal to 0 as shown in the previous slide. So, if you now substitute the value of N i equal to 0 over here. So, what this term will be? C F i will become gamma 0. What will be r F i? You can say that N i is equal to 0, this is 0, this is also 0 this will be 1 plus 0, so overall this will become 0. So, as I mentioned for absolute minimum noise figure when NF i is equal to NF min then we have to choose gamma s as equal to gamma 0, ok. So, that corresponds to the single point.

However, as I mentioned earlier this may not give rise to the optimum gain. So, let us see now how we can use this particular information and the information which we had studied in the previous lectures about the constant gain circles. Let us combine these two and then design low noise amplifier.

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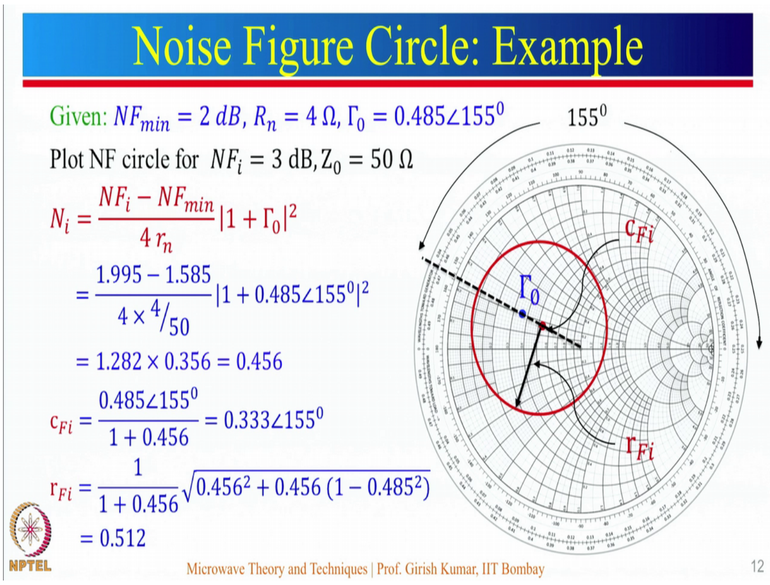

But before that, let us take a noise figure circle example. So, here is NF min equal to 2 dB, r n is 4 ohm gamma 0 is 0.485 angle 155 degree. These are specified so we want to plot noise figure cycle for NF i equal to 3 dB z 0 is 50 ohm. So, by using this particular expression we can find the value of N i. So, substitute the various values.

Again please remember do not put the values directly in terms of dB otherwise this would become 3 minus 2. Please do not do that you have to convert these dB values into corresponding numeric values. So, the numeric value for 3 dB is 1.995, numeric value of 2 dB is given by 1.585 divided by 4 into r n, r n is capital R n divided by 50. Then this term will be 1 plus gamma 0 is given by this particular expression and we have to take magnitude of that.

So, N i comes out to be the real number it is not a complex number because here we are taking magnitude of this particular term. So, from here now we can calculate the value of C F i, C F i is nothing but gamma 0 divided by 1 plus n i. So, C F i comes out to be this one here. Now, you can note here that C F i has same angle as that of gamma 0, ok. And what is the value of r F i? We substitute the value of N i, r F i comes out to be 0.512. Now, let us plot noise figure circle on the smith chart.

So, first thing what you should do locate gamma 0 on the smith chart. So, gamma 0 is 0.485 angle 155 degree. So, you draw a line which is at an angle of 155 degree then locate gamma 0 which is 0.485. So, you know that now that this is normalized equal to 1.

So, 0.485 of this normalized value to be shown over here. Then locate C F i, C F i is 0.333 locate that on the smith chart and then draw the circle over here. You can choose any value of gamma s on this particular noise figure circle and that will give constant noise figure that is why it is known as constant noise figure circle.

I just want to mention where will be the location of the other noise figure circles for different values of N F i. So, just recall this point here corresponds to NF min which is equal to 2 dB. Suppose we are interested in NF i equal to 2.5 dB. So, 2.5 dB noise figure circle will be somewhere here this is 3 dB noise figure circle 4 dB noise figure circle will be much larger like this over here. So, with this particular information now let us see how we can design a low noise amplifier.

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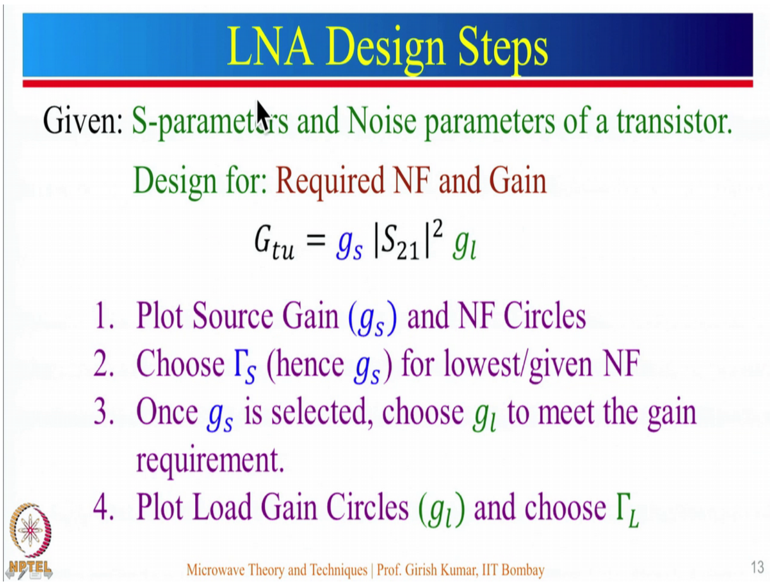

So, these are the design steps for designing a low noise amplifier. So, S-parameters and noise parameters of a transistor will be generally given for given biasing condition and frequency. So, let us say now we have to design for required noise figure and gain. We have already seen how to draw the noise figure circles now let us see how do we put gain circles over noise figure circle.

So, again to design an amplifier you must follow the steps which we had discussed when we were talking about microwave amplifier design. So, first you check whether the amplifier is stable or not. So, for that you calculate the value of delta, calculate the value of k, if delta is less than 1 and k is greater than 1 then it is unconditionally stable. So, you do not have to draw the stability circles. If that condition is not satisfied that means, amplifier is conditionally stable you have to draw input and output stability circles and then choose the value of gamma s and gamma l which is far away from those stability circle.

Then comes the next part which is G tu. Now, before again you look at G tu what do you do? First you calculate G tu max, so that desired gain has to be less than G tu max then you also find out the value of m check that m is less than 0.05. So, that you know that the gain error is relatively small. Then comes the next part now for the desired value of the gain, what should do? S 21 square is known. So, we have to now choose the value of g s and g l as we did earlier. But now there is slight change now we do not give much focus to g l now we focus more on g s value. Why we focus more on g s value? Because this g s corresponds to the constant gain circle for the input side that will give us the value of gamma s and noise figure circle also decides the value of gamma s. So, first we focus on how to find the value of gamma s or you can say the value of g s then we look at the g l value.

So, the first step is choose the value. So, the first step is choose the value of g s which is less than g s max, then plot; the constant g s circle and also plot NF circle. Choose the value of gamma s and hence you can say g s either for the lowest noise figure or for given noise figure and then you will know what is the value of g s. Once you know the g s then you calculate the value of g l to meet the gain requirement. After that only you draw the load gain circle and then choose the value of gamma l.

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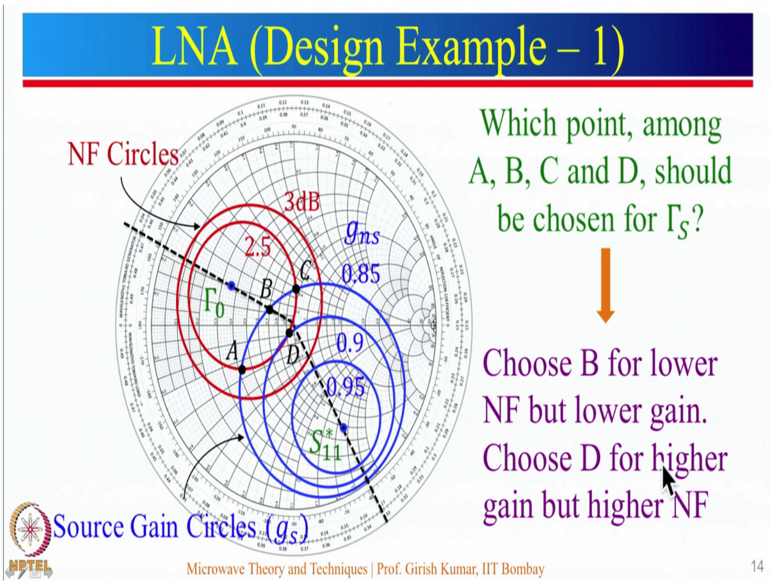

So, now, we will take two different example. So, here is a design example 1. So, you can see here this is similar to what I had shown for the noise figure example. So, this is gamma 0 and this is the noise figure circle for 3 dB this is the noise figure circle for 2.5 dB. I just want to mention this 2.5 dB noise circle is just a representation I am not telling that this is the exact 2.5 dB noise figure circle.

Now, let us see different source gain circles. So, here if you choose this particular point which is S 11 conjugate then we would get maximum gain, but these are the other constant gain circles for different value of g ns, I am actually showing normalized value of g ns which can be let us say 0.85, 0.9, 0.95 here of course, it will be equal to 1. The reason to show these g ns is because g s may have different values for different amplifier or transistor.

If one now tries to design this particular amplifier for maximum gain which will correspond to this particular point you can see that noise figure will be very poor over here because this itself is 3, dB if we look at another circle that may be 4 dB, here noise figure maybe 4 dB or even 5 dB. So, it is not a good idea to choose this particular point for maximum gain. So, we have to sacrifice gain now.

So, let us see if we look at this particular circle you can see that gain is reduced slightly, but it is becoming closer to the noise figure circle. So, let us look at this particular circle here which is normalize value of 0.9, you can see that this is intersecting noise figure 3 dB circle at this point and this particular point, ok. But it is intersecting 2.5 dB noise circle at this particular point and then there is another one. So, you can see that gain is reducing, but then we are getting closer to the lowest noise figure.

So, let me ask a very very simple question which point among A, B, C and D should be chosen for gamma s. So, let us say we have point A, B, C, D. Let us first look at points A B C, you can see that all the 3 points A B C are on the constant gain circle which corresponds to normalize value of g ns which is 0.85. Now, if we choose point A and C you can see that the noise figure will be about 2.5 dB, but if we choose point B you can see this point B is closer to gamma 0 point so that means, noise figure here will be lower than noise figure corresponding to point C and point A.

So, please do not choose point A and C you can choose point B. So, point B should be chosen for lower noise figure, but that will have a lower gain also. Now, if we choose point D you can see that at point d noise figure is of the order of 2.5 dB and gain is corresponding to 0.9. So, you can choose d for higher gain, but higher noise figure also. You can also choose another point over here, but then that would mean noise figure is deteriorating further, but gain will increase, ok. So, one has to really decide whether noise figure is more important or whether gain is more important. If gain is more important choose a point which is closer to other and if noise figure is more important then choose a point closer to this particular gamma 0.

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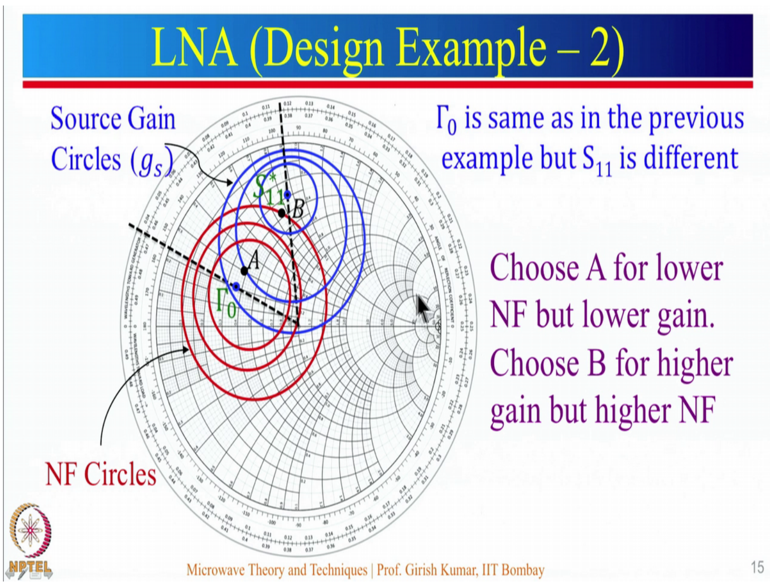

Let us take another example. So, in this example we have kept the noise figures as before, but now S 11 is different. So, here S 11 conjugate is located at this particular point and these are the constant gain circles, ok. So, again I have just shown two points A and B. So, if you look at point A, this point A is very very close gamma 0 so that means, this will give us very low noise figure, but then you can see that it will give relatively less gain. If we choose a point B you can see that gain will be higher, but noise figure will be poor at this particular point over here. So, you can choose A for lower noise figure, but lower gain choose B for higher gain but higher noise figure.

But I just want to mention here how to make a proper choice. So, draw a line between gamma 0 and S 11 conjugate, ok. So, that line will basically be corresponding to the two circles which are intersecting each other. So, it is better that choose any point along this particular line and you can then decide to choose if you choose this point that will be the lowest noise figure, but lower gain also and if you choose this particular point then it will be maximum gain and poorer noise figure.

So, any point over here can be chosen depending upon the desired gain value or desired noise figure. So, let us say since we are designing a low noise amplifier we choose point A which is very very close to gamma 0. So, corresponding to this value of A we know now what is the value of g ns or you can say g s, then you find out what is the corresponding value of g l check that g l has to be less than g l max then you draw output stability circle then choose the value of gamma l. The last step then left is you have to design impedance matching network since I have already discussed impedance matching network in detail when we talked about amplifier design and also impedance matching network has been discussed in much more detail when we talked about transmission line. So, I am not going to discuss that again now. So, please refer to those earlier slides, ok.

Now, we will take a real life example. So, here is an LNA which we fabricated using this particular IC.

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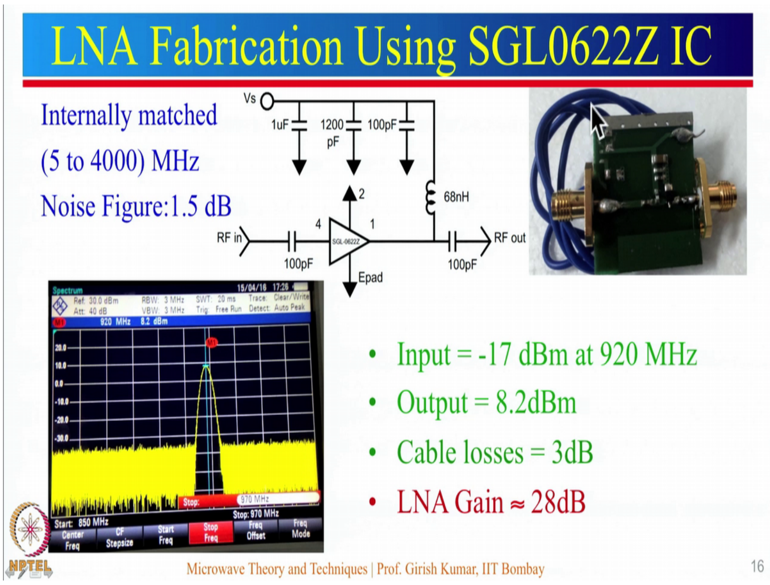

So, this particular IC has the specification of noise figure equal to 1.5 dB and it is internally matched from 5 to 4000 megahertz and this is one of the typical circuits given by the manufacturer. But I want to mention that please use this particular circuit little carefully because this particular circuit is not very good if you want to design at 5 megahertz. So, why I am saying that? So, in this particular circuit you can see the coupling capacitors are given as 100 picofarad and 100 picofarad.

Now, these capacitors should act as short circuit at the desire frequency, but at 5 megahertz these capacitors will provide very high impedance because we know that impedance of the capacitor is given by z equal to 1 by j omega c. So, if omega is small overall impedance will be large. So, there will be a lot of attenuation because of this capacitor. So, if you want an amplifier at lower frequencies then you must use higher value of capacitance here we have kept these values because we wanted to design this particular amplifier in the GSM band you can see here we have tested at 920 megahertz.

So, let us see now here is an input this is the coupling capacitor and output there is a coupling capacitor. You can see there is a no impedance matching network required the reason for that is this particular amplifier is internally matched. So, for this particular amplifier you do not have to draw noise figure circle you do not have to draw constant gain circle and other thing, all those things have been done by the manufacturer. So, it is a very very simplified configuration. However, I just want to mention noise figure is equal to 1.5 dB because when they try to do impedance matching internally for this broadband region, somewhere lossy network does come into picture hence noise figure is poor.

I just want to mention there are several transistors which have noise figure equal to even as low as 0.2 or 0.3dB and if you use those transistor you can realize that low noise amplifier with noise figure of 0.5 dB to even 1 dB, ok. So, here yes things are simple, but we are paying the penalty of noise figure of 1.5 dB. However, we fabricated this because for one of the application desired noise figure was 2 dB and this gives lower than 2 dB, and nobody will of course, object if you give a lower noise figure.

Let us see the other thing now the biasing condition. You can see that the output here is connected through this inductor; you can see this value is 68 nanohenry which will act as an open circuit at the desired frequency. However, again if you want to use at 5 megahertz, then this inductor is small you have to choose larger value of inductor. Then this is now connected to the supply voltage, you can see that there are 3 parallel capacitors connected to the ground.

What are these values? So, one microfarad is basically to reduce the ripple in this particular power supply, these two capacitors are mainly to reduce that transients at higher frequency and this is for mid frequency range. So, this is the fabricated pcb based on this particular design. You can get the idea of the size of this particular amplifier just by looking at these two connectors. So, typical is this dimension is of the order of 1 centimeter. So, you can get an idea that size of this amplifier is of the order of 1 inch by 1 inch which is about 25 mm by 25 mm.

So, let us see what are the test results we got. So, input was given as minus 17 dBm at 920 megahertz through a microwave generator, and the output of this particular amplifier was connected to the spectrum analyzer the response of the spectrum analyzer is shown over here. The output was measured as 8.2 dBm. So, if you just look at output minus input in terms of dB then it comes out to be 25.2 dB. However, we had connected coaxial cable and if you recall when we discuss about coaxial cables all the coaxial cables have certain losses at the given frequency.

So, we actually tested the cable losses at this particular frequency we hardly had used little longer cable. So, cable losses turn out to be 3 dB. So, you have to add all these numbers together so the gain of this LNA comes out to be 28 dB. So, you can see that gain is fairly good. If you recall the previous examples I was talking about gain of 10 dB or 14 dB or 18 dB, but this particular amplifier gives a gain of 28 dB noise figure is also relatively low which is of the order of 1.5 dB. So, this particular low noise amplifier can be used as the first amplifier block in the receiver chain.

I just want to mention that in a receiver chain there are several cascaded stages of the amplifier. The reason for that is the signal which is received by the antenna, maybe very very weak. So, typically that amplifier chain may have gain of the order of 60 dB. So, one may have to use cascaded stages of these low noise amplifier. Of course, the next stage one can optimize for maximum gain because that stage will be divided by this very high gain. So, even poor noise figure of the second stage will not add to the overall noise figure because the gain of this particular stage is very very high.

So, just to summarize today we talked about how to design low noise amplifier. So, we first started with a very simple expression for noise figure for a given amplifier and that expression basically uses 3 important terms NF min, r n and gamma 0. So, by using those terms we first find out what is N i, and then we plot noise figure circles, then along with that we plot constant gain circle for the source side. Then we choose appropriate value of gamma s depending upon what is more important. Gain is more important or noise figure is more important.

And then we took this practical example and you can see that in this particular example we do not have to do many of the things which I discussed earlier today, but as I mentioned if you have to design a much lower noise figure amplifier, then you have to draw all those constant noise figure circle and constant gain circles to design an optimum amplifier. In the next lecture I will talk about power amplifiers. So, Bye.

Thank you very much. See you next time.