

Microwave Theory and Techniques
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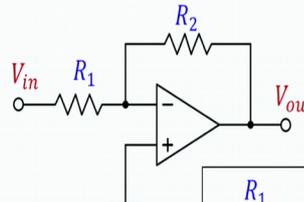
Module – 7
Lecture – 32
Microwave Amplifiers – I: Basics and Power Gain Expressions

Hello, everyone. Today, we are going to talk about Microwave Amplifiers. In the previous lecture, you had heard about different types of transistors. So, today, we will see the application of transistors for designing Microwave Amplifiers. However, we are going to start with an inverting amplifier Op-Amp, the reason for that is you are already familiar with how Op-Amps work.

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Inverting Amplifier using Op-Amp 741

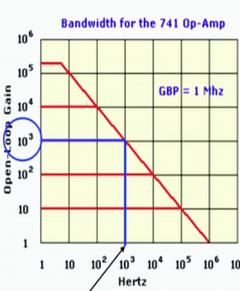
Design an inverting amplifier for a gain of -1000 (60 dB)



Gain = $-\frac{R_2}{R_1}$

$R_1 = ?$ $R_2 = ?$

R_1	R_2
1 Ω	1 k Ω
10 Ω	10 k Ω
100 Ω	100 k Ω
1 k Ω	1 M Ω



1 kHz !

Graph Image Source: <https://pe2bz.philpem.me.uk/Parts-Active/IC-Analog/OpAmps/Lm741/741.htm>

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So, let us see a simple example of an inverting amplifier and we have a design statement here that design an inverting amplifier for a gain of minus 1000. So, an inverting amplifier configuration is shown over here, where we can see that there is a feedback resistor R 2, in some books they write R F. And here the resistor is R 1 and we can say that the gain of this particular amplifier is nothing but V out divided by V in which is given by the expression minus R 2 by R 1.

The problem statement is we have to design this for a gain of minus 1000. So, we have to choose the values of R 1 and R 2. So, there are several possibilities of choosing R 1 and

R 2. So, we can choose R 1 as 1 ohm R 2 as 1 kilo ohm or 10 ohm, 10 kilo ohm; 1 kilo ohm, 1 mega ohm. So, now, which one is the best choice among these different values of resistors? All of these values will give us gain equal to minus 1000, but are these practical values. So, this is where the difference comes when you are designing an amplifier you must know, what are the parameters we should choose so that it fulfills different requirements?

So, for example, over here if we choose R 1 equal to 1 ohm, then what will happen input impedance looking at this point is equal to R 1 and that would be equal to 1 ohm? Now, for an amplifier generally desirable characteristic is that it should have a very high input impedance; so that means, if we choose the value of 1 kilo ohm, then R 2 comes out to be 1 mega ohm.

Now of course, 1 mega ohm resistors are available, but if you think about a practical 741 Op-Amp, then the practical 741 Op-Amp has an input impedance between these 2 forces equal to 2 mega ohm. So, this 2 mega ohm somehow comes in parallel with this particular resistor. So, if you choose this value of 1 mega ohm, I can tell you will not get a gain of 1000. In fact, gain will be slightly less than that.

Now, there is a another problem associated with the Op-Amp and this is the typical gain variation with frequency. So, let us see what we have over here. So, you can see that there is a peak gain of around 10 to the power 5 over here and then, the gain is reducing not typically for a 741 Op-Amp, gain bandwidth product which is what is written over here is 1 mega hertz. So, if we design an amplifier for a gain of 1000, then the withdrawal line here corresponding to gain of 1000 and we can see that the corresponding frequency will be 1 kilohertz.

So, that means, this particular amplifier will be only useful to amplify the signal up to 1 kilohertz; so, that means, it cannot be used for audio signal because audio signal bandwidth can be up to 20 kilohertz. So, if you try to use up to 20 kilohertz you will not get this particular characteristic of very high gain.

So, does that mean that we cannot use 741 Op-Amp for a very high gain? In fact, I would generally recommend donot try to design these Op-Amps for very high gain of 1000. I generally recommend that you design this particular value of gain in 2 or 3 stages. So, for example, if you use 3 stages, then each stage can give us a gain of 10, 10, 10. The

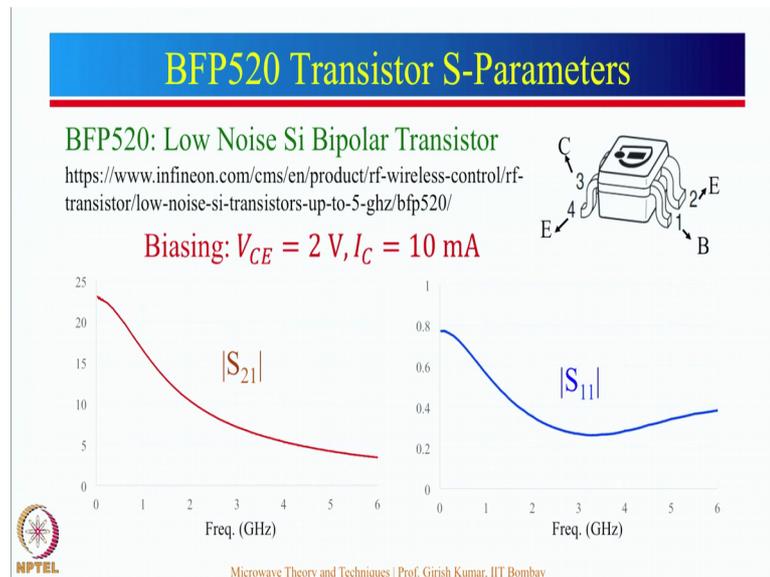
another thing is if you want to use only 2 stages, then you can use something like maybe a gain of 10 and 100; maybe gain of 31.6 and 31.6. So, that you can get overall gain of 1000.

Another thing which I would generally recommend that for most of these application is better to design non inverting amplifier. We would be giving input at this particular point and this would be grounded in that particular case gain will be given by $1 + R_2/R_1$. So, here the advantages, if we give input here input impedance is very high. So, one can realize a high input impedance amplifier. There are many applications where we have to amplify a very low signal and that signal maybe at a very low frequency. It can be even a DC voltage. So, for example, if let us say a sensor is connected at this particular port over here and this sensor can be a temperature sensor pressure sensor thermocouple and so on.

And these sensors generally give very small voltage that could be of the order of 1 microvolt or 1 millivolt. So, we have to amplify that particular signal with a very high gain. So, you can use sometimes these kind of a configuration, but be careful you should know what is the application for which application, you have to design the amplifier. Now, from here we will shift to the very high frequency and that is microwave amplifier, but I just want to bring to your attention one more time the gain decreases as the frequency increases.

Now, this is 1 megahertz, if we go to 1 gigahertz gain response will be extremely poor for 741 Op-Amp. In fact, it will not even work, but I want to mention there are several Op-Amps; they have a gain bandwidth product of 1 gigahertz to even a few gigahertz. So, if you want to use Op-Amp at very high frequency, it is better to use those Op-Amp then to use 741 Op-Amp.

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So, let us go to the next part how to design microwave amplifier using transistor. So, here we have taken an example of BFP520 transistor. This is actually known as Low Noise Silicon Bipolar Transistor. It is available for Infineon Company and you can actually click on this particular thing and see the data sheet of this particular transistor. So, I just want to mention a few things over here. So, this transistor is to be biased for V_{CE} equal to 2 volt, I_C equal to 10 milliampere, the transistor looks like this over here. So, we have a terminal 1 which is base; terminal 2 and terminal 4 both are emitter. 2 and 4 should be connected with each other so that we have a emitter which is common and this is collector.

And you must be familiar with let us say common emitter amplifier, common base amplifier, common collector amplifier. So, some of those can be used, but with little modification. Now generally speaking a manufacturer at microwave frequency does not give h parameters, but it actually gives S parameter. So, this S parameters are measured using network analyzer. So, one of the port of the network analyzer which is actually generating different frequencies is connected to the input port of the transistor and the output port of the transistor is connected to the other port of the network analyzer which does the measurement. Of course, the network analyzer can do measurement in both the directions also.

So, by using this network analyzer one can actually measure S_{11} , S_{21} , S_{12} as well as S_{22} . So, here the plots of S_{21} and S_{11} are given. So, let us see what we have here? So, this is the frequency from 0 to 6 gigahertz and this is the gain value you can see that it varies from 0 to up to 25. These are numeric values. So, let us see how magnitude of S_{21} varies. You can see that as frequency increases S_{21} value is decreasing. So, let just look at some typical numbers here. At 2 gigahertz frequency, S_{21} is equal to 10. So, gain is equal to S_{21}^2 . So, that will be equal to 100 and if we now take 10 log of gain that comes out to be 20 dB; so, gain at this frequency is 20 dB.

So, at this particular frequency, you can say which is approximately 0.6 gigahertz, gain is going to be equal to S_{21}^2 which is 20 square which is equal to 400 and 400 is equivalent to 26 dB. But you can see that as frequency increases, gain keeps on decreasing and one can see that somewhere at 4 gigahertz now, gain is just about 5 square which is equal to 25 and as frequency increases further, gain is reducing further. Now corresponding to this now let us see we have S_{11} plot. One can actually see here that at lower frequencies S_{11} is very poor. If you look at a value of 0.84 S_{11} and recall reflection coefficient is equal to S_{11} which is equal to 0.8. So, reflected power will be square of this which is 0.64.

So that means, 64 percent power will reflect back. So, one should really do something at this particular frequencies. So, if you want to use this particular transistor at lower frequencies, we must try to optimize the input impedance of this particular transistor. You can see that at around 3 to 4 gigahertz, reflection coefficient is relatively less than 0.3 which corresponds to reflected power of about 10 percent which may be acceptable. But if we know that, we are going to operate at a frequency of 3 gigahertz or let us say a Wi-Fi frequency of 2.5 gigahertz. Then, we know what is the value of S_{11} . So, this can be improved by providing impedance matching network.

Now, the similar thing needs to be done at the output port also S_{22} also again may not be equal to 0; at all these frequencies, it may have a finite value. So, we need to provide output impedance matching network. So, for a given device generally speaking S parameters are specified for given biasing condition by the manufacturer. So, now, what we are going to do? We are going to look at the expression for how to find out Γ_{in} input of a device.

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Derivation of Γ_{in} of a Device (Amplifier)

$\Gamma_{in} = ?$

$\Gamma_L = \frac{a_2}{b_2} \Rightarrow a_2 = \Gamma_L b_2$ — 3

From 1, using 3 & 4

$$\Gamma_{in} = \frac{b_1}{a_1} = S_{11} + \frac{S_{12}S_{21}\Gamma_L}{1 - S_{22}\Gamma_L}$$

$$\Gamma_{in} = \frac{S_{11} - \Delta\Gamma_L}{1 - S_{22}\Gamma_L}$$

where $\Delta = S_{11}S_{22} - S_{12}S_{21}$

S-Parameters:

$b_1 = S_{11}a_1 + S_{12}a_2$ — 1

$b_2 = S_{21}a_1 + S_{22}a_2$ — 2

$b_2 = S_{21}a_1 + S_{22}\Gamma_L b_2$

$b_2 = \frac{S_{21}a_1}{1 - S_{22}\Gamma_L}$ — 4

Now, this concept is common to any general device, but since this topic is microwave amplifier. So, I have written here in the bracket amplifier.

So, let us see what we want to do over here and why we want to do it? So, first thing is we want to find out gamma input. So, let us look at how to find out gamma input. So, let us see over here. So, here a given device is specified by its S parameters at the desired frequency and given biasing conditions. Now you can see here this is the incoming wave a 1; here is a incoming wave a 2.

This is the reflected wave b 1; this is reflected b 2. This device is now connected with the source which has a source impedance and there is a load over here. So, the objective here is to find out gamma input ok. So, how do we start? Let us start with S parameters. We have already discussed about S parameters in our previous lecture. So, S parameters that defined in terms of b 1, b 2 and these are $S_{11} S_{12} a_1 a_2 S_{21} a_1$ plus $S_{22} a_2$.

Now, let just look at the another thing here which is gamma L. So, how do we define reflection coefficient? Generally, speaking reflection coefficient is defined by you can say reflected wave divided by incident wave. So, now, if you look at this side here what is the reflected wave looking in this side that will be a 2 and what is the incident wave that will be b 2. So, gamma L is given by reflected wave which is a 2, incident wave which is b 2. So, from here we can find out a 2 is nothing but equal to gamma L times b

2. So, we substitute this particular value over. We substitute this value of a_2 in this particular equation here. So, b_2 is equal to now $S_{21} a_1 + S_{22} a_2$ is $\Gamma_L b_2$.

So, now we simplify this particular thing for b_2 . So, this term will come to this side and then we divided. So, b_2 expression is given by the equation number 4. So, now, what we do? We actually put this value of b_2 in this particular equation over here, but after we write a_2 equal to $\Gamma_L b_2$. Why we are doing that? Because, our objective is to find Γ_L which is equal to b_1 by a_1 . So, we want to get b_1 by a_1 , we have already removed a_2 from here as you can see from this equation, now we will remove a_2 from here. So, let us see how we proceed. So, this is how we proceed. b_1 is $S_{11} a_1 + S_{12} a_2$. a_2 is $\Gamma_L b_2$ and b_2 is given by this particular expression.

So, now we solve that equation and we get this particular expression over here. So, what is this expression? Γ_{in} is equal to $S_{11} + S_{12} S_{21} \Gamma_L$ divided by $1 - S_{22} \Gamma_L$. Now generally for a device we are familiar with S_{11} . Now some additional term has come over here. Why this additional term has come over here? The reason for that is the S parameters of a device are defined when you put match load at the input or output side, over here it is not terminated with the match load; but it is actually terminated with some unknown load impedance Z_L .

So, if Z_L is unknown, if it is not equal to 50 ohm; then, Γ_L will not be equal to 0, but suppose if it is terminated with 50 ohm in that case Γ_L will be equal to 0 and if Γ_L is equal to 0, we put 0 over here Γ_{in} will be equal to S_{11} . So, that is how S parameters are defined.

Let us look at another thing also here and that is S_{21} , we know is a forward gain from here to here and amplifier should actually send the signal from the input side to the output side. What is S_{12} ? S_{12} is if we give input at the output side. What is the output? Generally, speaking we would like S_{12} be close to 0. So, in this case also if you put S_{12} equal to 0, then what we will get Γ_{in} will be equal to S_{11} and in that particular case it does not matter what is Γ_L .

So, what it really means is that if amplifier is only I am sending signal in this direction and not sending any signal over here. This is also known as a unilateral case. In fact, we would like an amplifier to be perfectly unilateral case; that means, it only send signals in

this direction. It does not send signal in this particular direction. If it does send some signal in this particular direction, it is known as bilateral case.

Now, this equation can be further simplified. So, let us see what we have here. So, from here you can see that denominator is $1 - S_{22} \Gamma_L$ which remain same here. This particular thing is multiplied. So, you will get $S_{11} - S_{11} S_{22} \Gamma_L$ and plus this term over here. So, that can be simplified in this particular form over here so where delta is given by this expression here. What is delta over here? Actually, speaking it is nothing but determinant of the S matrix; determinant of S matrix will be $S_{11} S_{22} - S_{12} S_{21}$. We will use this particular equation later on.

So now, what is the purpose of finding Γ_{in} ? Actually speaking the purpose of finding Γ_{in} is that, we find what is the value of Γ_{in} after considering all these things here. Then, we design impedance matching network over here. So, that maximum power transfer can take place from the source to this particular device.

When maximum power transfer will take place from the source to this particular device, that will be the case when Γ_S looking from this side is equal to Γ_{in} in conjugate. So, you all know that for maximum power transfer, load impedance should be equal to source impedance if it is real impedance. But for complex impedance, load impedance should be equal to complex conjugate of the source impedance. So, that is what we need to do. So, first we find out the value of Γ_{in} by using this expression and then, we design a impedance matching network. So, that Γ_S of that particular thing will be equal to Γ_{in} conjugate.

Now, by using the same concept, we can find out Γ_{out} of the device also, but before we look into that I will just tell you intuitively what you should expect? So, if instead of in if it is out, this will b_2 divided by a_2 . S_{11} will become S_{22} ; S_{22} will become S_{11} , S_{12} , S_{21} will swap each other. So, it will remain same and Γ_L will become Γ_S . So, let us see.

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Derivation of Γ_{out} of a Device

S-Parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$$\Gamma_s = \frac{a_1}{b_1} \Rightarrow a_1 = \Gamma_s b_1$$

$$b_1 = S_{11}\Gamma_s b_1 + S_{12}a_2 \rightarrow b_1 = \frac{S_{12}a_2}{1 - S_{11}\Gamma_s}$$

$$\Gamma_{out} = \frac{b_2}{a_2} = S_{22} + \frac{S_{12}S_{21}\Gamma_s}{1 - S_{11}\Gamma_s}$$

→

$$\Gamma_{out} = \frac{S_{22} - \Delta\Gamma_s}{1 - S_{11}\Gamma_s}$$

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So, this is the derivation of gamma out of the device. So, again what we want to find out we want to find out gamma out is equal to b 2 by a 2. Now in order to find out the output impedance or gamma out, we must make source equal to 0. So, you can see that source is now made 0, since we had taken a voltage source. So, 0 voltage source would mean short circuit. Had there been a current source, then we would have made current source equal to 0 that would imply open circuit.

So, now objective is to find gamma out equal to b 2 by a 2. If you look at this expression here, I had just mentioned to you that this can be derived just by looking at the gamma in expression. But over here, let us quickly see how we can do the derivation. Again, we start with the S parameters which are given over here. In this case now let us define gamma S. What is gamma S looking at this particular side over here? And for gamma S, what we have to again say? It is a reflected wave divided by incident wave.

So, if you are looking from this side, what is reflected wave? a 1. What is the incident wave? b 1. So, from here we can write a 1 is gamma S times b 1, substitute this value of a 1 and simplify you will get this particular expression and this expression can be written in this particular form where delta is nothing but S 1 1 S 2 2 minus S 1 2 S 2 1.

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Gain using Mason's Signal Flow Rules

S-Parameters:

$$b_1 = S_{11}a_1 + S_{12}a_2$$

$$b_2 = S_{21}a_1 + S_{22}a_2$$

$\frac{b_2}{b_s} = ?$

$$a_2 = \Gamma_L b_2$$

$$a_1 = \Gamma_s b_1 + b_s$$

$$\Gamma_s = \frac{a_1}{b_1}, \text{ if } b_s = 0$$

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So, now we will find the expression for the gain and here, we are going to use Mason's Signal Flow Rule or generally speaking this Mason's Signal Flow Rule is taught in the control theory, but we can use the same concept over here also. So, let us see we have S parameters of the device, we have a load here and there is a source impedance. Remember this is not 50 ohm. This Z S will be chosen properly so that we can provide impedance matching network later on. So, here this source is supplying b s waveform. So, now, let us see how we can actually speaking built this signal flow graph. So, we start again with the S parameters over here. We also know what is a 2? a 2 is gamma L b 2. You can just quickly check here, what is gamma L? Gamma L is reflected wave which is a 2 divided by incident wave. So, that gives us a 2 equal to gamma L b 2.

What is gamma S? So, gamma S, we had seen previously it was a 1 by b 1, but that was the case when b s was equal to 0. If b s is present in that particular case you can see b s is also coming. So, a 1 will be now equal to you can see from here gamma S b 1 plus b s. So, now, these 4 equations are represented in this graphical form over here. So, let us see how we can build this particular signal flow. So, let us start with let us say b 2. So, b 2 is given by S 2 1 a 1. So, b 2 we locate a 1. So, a 1 multiplied by S 2 1 plus S 2 2 a 2. So, we locate here a 2. This is S 2 2. So, these are known as path gain. Now let us look at the other equation which is b 1. So, b 1 is S 1 1 a 1. So, b 1 is S 1 1 a 1 plus S 1 2 a 2; so, plus S 1 2 a 2.

Now, we need to look at this equation here. So, a 2 is gamma L b 2. So, a 2 is gamma L times b 2 and a 1 is given by this expression here. So, let us see if this is a 1. So, b 1 times gamma S and plus b s is coming from here. So, now, let us see what is the expression for Mason Signal Flow?

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Gain using Mason's Signal Flow Rules (contd.)

$$Transfer\ Fun. = \frac{P_1[1 - \sum L(1)^1 + \sum L(2)^1 - \sum L(3)^1 - \dots] + P_2[1 - \sum L(1)^2 + \sum L(2)^2 - \sum L(3)^2 - \dots]}{1 - \sum L(1) + \sum L(2) - \sum L(3)}$$

$\sum L(1), \sum L(2) \dots$ = sum of all 1st order, 2nd order loops

$\sum L(1)^1, \sum L(2)^1 \dots$ = sum of all 1st order, 2nd order loops that do not touch path P₁

$\sum L(1)^2, \sum L(2)^2 \dots$ = sum of all 1st order, 2nd order loops that do not touch path P₂



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So, this is the transfer function which can be used to find out the gain of the amplifier. So, what it shows over here. This transfer function can be used to find out the transfer function from let us say 0.1 to 0.2 ok. So, what this expression has here. Let us start with the denominator you can see here it is 1 minus summation L 1 plus summation of L 2 minus summation of L 3. So, let us see what are these things?

So, we can see here L1 L2 L3 these are nothing but sum of all first order second order and third order loops. I will define in the next slide, what are these things here. Now here, we have in the numerator you can see here there is a P 1. This is the gain of path number 1 this is the gain of the path number 2 and what are these terms here? These are the terms, these are the different loops which do not touch the path 1 and these are the loops which do not touch the path P 2.

Now let us see how we can find out the various parameters for this particular case here.

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Gain using Mason's Signal Flow Rules (contd.)

Path from b_s to b_2

$P_1 = S_{21}$

$P_2 = 0$

- 1 Path: No node is touched more than once $\longrightarrow P_1 = S_{21}$
- 2 First order loop: Three first order loops $\longrightarrow (S_{11}\Gamma_s), (S_{22}\Gamma_L), (S_{21}\Gamma_L S_{12}\Gamma_s)$
- 3 Second order loop: Product of any two non-touching loops $\longrightarrow (S_{11}\Gamma_s), (S_{22}\Gamma_L)$
- 4 Third order loop: Product of any three non-touching loops \longrightarrow (none)

$$\frac{b_2}{b_s} = \frac{S_{21}}{1 - (S_{11}\Gamma_s + S_{22}\Gamma_L + S_{21}S_{12}\Gamma_s\Gamma_L) + S_{11}S_{22}\Gamma_s\Gamma_L}$$

$$= \frac{S_{21}}{(1 - S_{11}\Gamma_s)(1 - S_{22}\Gamma_L) - S_{21}S_{12}\Gamma_s\Gamma_L}$$

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We want to find out the gain which is given by b_2 divided by b_s ; b_2 is the output, b_s is the input. So, let us see what is the path we have? There is a only 1 path from b_s to b_2 and we can write that path gain is nothing but equal to 1 multiplied by 1 into S_{21} .

So, this is the path gain. There is no other path to go from this point to this particular point. So that means, P_2 is equal to 0. So, these are the different steps down. So, step 1 path. So, we can actually see that there is only 1 path which is equal to S_{21} . I just want to mention here that in to define the path, no node should be touched more than once ok.

So, now, let us see what are the first order loops ok? So, there are 3 first order loops in this particular case and the loop gain for these 3 loops given by these expressions here. So, this is the first one, second and third one. Now, what is this second order loop? Second order loop is product of any 2 non touching loops. These 2 loops are not touching each other. So, hence this term comes over here you can see that is a product of these 2 terms over here.

There is no third order loop because there are no 3 non touching loops and hence, this particular thing will be equal to 0. So, now, we can write b_2 divided by b_s . So, let us look at the numerator it is nothing but P_1 . There is no loop which is not touching this particular path hence other terms are 0. So, we are left with only S_{21} . Let us see now the denominator. Denominator is 1 minus summation of the first order loop. So, you can

see here S_{11} , then S_{22} ; then, this term over here. So, that is the summation of the first order loop and then, plus summation of second order loop and there is only 1 term. So, that comes over here.

Now, this expression can be simplified in this particular form, we will see later on how this particular form will be useful to design the gain of this particular amplifier. Now, we are going to define 3 different types of gain.

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Power Gain of an Amplifier		
Power Gain	Symbol	Formula
Transducer Power Gain	G_t	$\frac{P_l}{P_{avs}}$
Available Power Gain	G_a	$\frac{P_{avn}}{P_{avs}}$
Operating Power Gain	G_p	$\frac{P_l}{P_{in}}$

P_{in} = Input power
 P_l = Power delivered to the load
 P_{avs} = Power available from source
 $= P_{in}$, when $\Gamma_{in} = \Gamma_s^*$
 P_{avn} = Power available from network
 $= P_l$, when $\Gamma_L = \Gamma_{out}^*$


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Now, till now you might be more familiar with only 1 gain; you know output voltage divided by input voltage or output power divided by input power. So, why we are defining 3 different types of gain over here? In fact, I just want to tell you actually all these 3 gains are related to each other in a very simple manner. We are going to first start with Operating Power Gain which is given by P_l divided by P_{in} . What is P_l ? Power delivered to the load. What is P_{in} ? That is input power.

So, from here let us go to this expression here which is Transducer Power Gain. Transducer Power Gain, if you see the difference is only that P_{in} is equal to $P_{available}$ from source. So, what will be the maximum value of the power available from the source when P_{input} will be maximum? So, P_{input} will be maximum, when Γ_{in} is equal to Γ_s conjugate. So, what we have here that input impedance of the device should be complex conjugate of the source reflection coefficient. Now, let us see what is the maximum available power from a given device? That is defined by this expression over

here. So, compare to G_t , what is the difference here? Now, P_l is equal to P available from the network.

So, this is the maximum value of the power which can be delivered to the load and maximum power will be available when Γ_L is equal to Γ_{out} conjugate. So that means, maximum available power from a device would be when input as well as output impedances are complex conjugate of the source and load impedances. In that particular case, we actually say that this is the maximum available power from an amplifier. So, now, let just quickly look at the expressions ok.

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Power Gain of an Amplifier (contd.)

Transducer Power Gain:

$$G_t = \frac{P_l}{P_{avs}}$$

$$P_l = \frac{1}{2} (|b_2|^2 - |a_2|^2)$$

$$= \frac{1}{2} |b_2|^2 (1 - |\Gamma_L|^2)$$

$$P_{avs} = \frac{1}{2} \frac{|b_s|^2}{1 - |\Gamma_s|^2}$$

$$P_{avs} = \frac{1}{2} |b_s|^2, \text{ if } |\Gamma_s| = 0$$

$$G_t = \frac{1 - |\Gamma_s|^2}{|1 - \Gamma_{in} \Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - S_{22} \Gamma_L|^2}$$

$$G_t = \frac{1 - |\Gamma_s|^2}{|1 - S_{11} \Gamma_s|^2} |S_{21}|^2 \frac{1 - |\Gamma_L|^2}{|1 - \Gamma_{out} \Gamma_L|^2}$$



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We will start with the Transducer Power Gain and Transducer Power Gain expression is given by this term over here.

So, why are we starting with this particular expression? The reason for that is the load may not be in our hand, the load may change depending upon the requirement. But power available from the source can be optimized. What we can do here that we can actually find out what is the gamma input of the device and then, we design impedance matching network such a way that maximum power transfer takes place from the source to the input of the active device.

So, that is why we are starting with the G_t ok. So, what is G_t now? P_l divided by P available from the source. So, this is given by half b_2 square minus a_2 square. Why? b_2

square is the wave which is going to the load and a^2 is the reflected back signal from the load. So, hence we deliver to the load will be given by the expression here we take b^2 square outside, then this will be 1 and this will be a^2 divided by b^2 . So, that will be Γ^2 . So, that is the expression for P_L . Let us see how we can find out expression for P available from the source and that is equal to half of b^2 . So, this part is kind of obvious, but we have another term over here.

Now, this expression will be equivalent to this particular expression if Γ is equal to 0. So, if Γ is equal 0, this will be 1 minus 0. So, this is kind of obvious that P available from the sources half of b^2 . Now, why this term comes into picture? The reason for that is if it is not properly matched, then what will happen part of the wave will get reflected back. So, let just take the worst case assuming that Γ is equal to 1. So, if it is 1, what it would mean 1 minus 1 will be equal to 0 and that would imply that power available from the source is infinity. What is that really mean? Well, you can think differently. See, if everything is reflected back; then, we can conceptually say well infinite power is available. Eventhough actually speaking it is not available, you are not supplying anything.

So, in general we always try that Γ should be done properly so that power available from the source can be maximized. Now, let us look at the expression for G . So, P_L divided by P available from the source. So, half and half will get cancel. So, b^2 divided by b^2 that expression we have derived in the previous slide. So, that will have several terms. Let just look at other terms also in the numerator. We have 1 minus Γ^2 that is written right over here. This term was in the denominator since it is divided. So, this will go up over here. Now, all these terms over here they are coming because of this expression of b^2 divided by b^2 .

So, you can actually speaking see here this term is related to everything in the source side; this term is related everything to the load side. So, basically what you can see over here, this is the gain term corresponding to the source side and that can be optimized by properly choosing impedance matching network at the input side. Now this is the term which corresponds to the load side, this can be optimized properly by designing a proper output impedance matching network. But I still want to mention over here, you can see here this is Γ . This is not the way it was written as b^2 by b^2 . I just want to mention here you have to do little bit of a; you have to do little bit of a simplification,

γ_{in} is given by $S_{11} + S_{12} S_{21} \gamma_L$ divided by $1 - S_{22} \gamma_L$.

Substitute that expression, you will get what we had derived in the previous slide. We can see over here γ_{out} is there. So, here γ_{out} ; whereas, this is S_{22} over here. So, again for γ_{out} you have to write the expression of γ_{out} which is $S_{22} + \text{the other term}$, simplify it, you will get the same expression as in the previous slide. In fact, these 2 expressions are exactly identical just that the representation is slightly different. So, here you can see these are the input side, these are the output side. So, in the next lecture, we will start from here; we will start with the expression G_t and then, we derive other expression.

So, just to summarize, today, we started with a very simple inverting operational amplifier where we did look at the design problem of gain equal to minus 1000 and how resistors should be chosen appropriately. Then we look at S parameters of a transistor and we noticed that S_{21} of the transistor keeps on decreasing as frequency increases. So, at higher frequency really speaking, we have to optimize the gain properly so that we can get a higher gain and to obtain the higher gain, what we need to do? We have to find out γ_{in} , we have fine γ_{out} and then, we have to design impedance matching networks at both input and output sides. So, that maximum transfer of the power takes place from input to the output side.

So, thank you very much. We will see you next time.