Microwave Theory and Techniques
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Module – 04
Lecture – 19
Microwave Couplers – II: Branch Line Couplers

Hello, in the previous lecture we are talked about Coupled Line Directional Coupler, so where we had seen that there is an input from port number 1 which directly goes to the port 2.

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And if you put a another line which is close to this particular line, then because of this current flowing in this direction induced EMFs is in this direction. So this becomes coupled port and this is an isolated port.

The analysis of this was actually done by using even an odd mode concept. So, I had explained you the concept very briefly, but today I am going to talk about that same even an odd mode concept in much more detail.

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But let just have a quickly look into what we had covered in the last lecture; so then even mode excitation then odd mode excitation.

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After that we had seen that the coupling is maximum for length equal to lambda by 4 and these are the expressions for even mode and odd mode impedances in terms of characteristic impedance and coupling.

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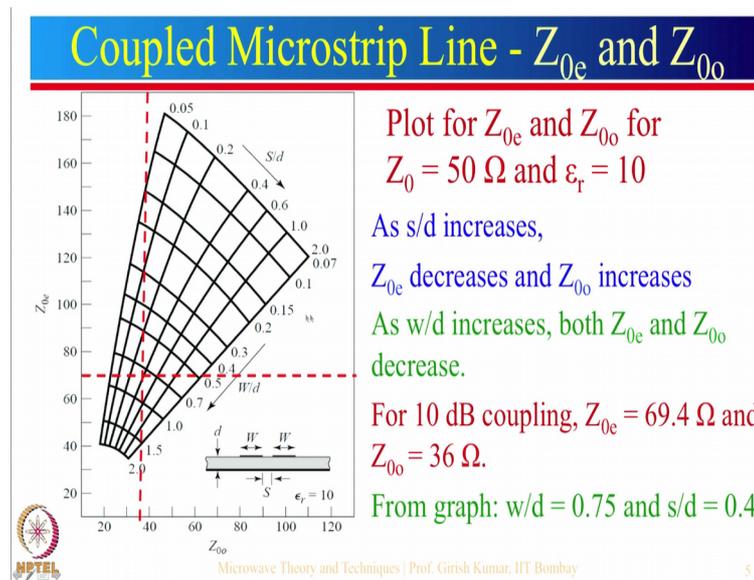
So, after that we had looked at the basic design concept that for 10 dB coupling, these are the values of even and odd mode and from the graphs we had read what are the values of w by d and S by d.

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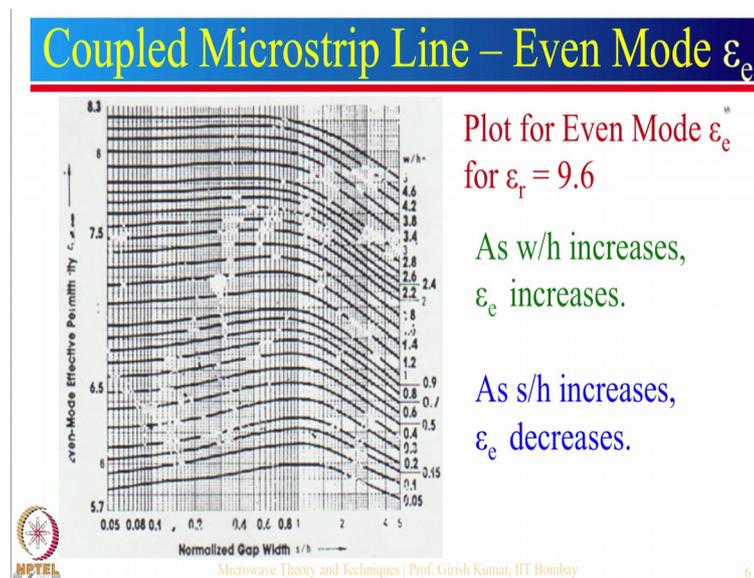
Then we looked into the even mode epsilon e as well as odd mode epsilon 0 and we had also talked about that the effective epsilon e is nothing, but the square root of epsilon e and epsilon 0.

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Then we looked at the design example of a microstrip line for a 20 dB coupler and we had noticed that the directivity of this particular thing was very poor and that is not acceptable for many of the applications.
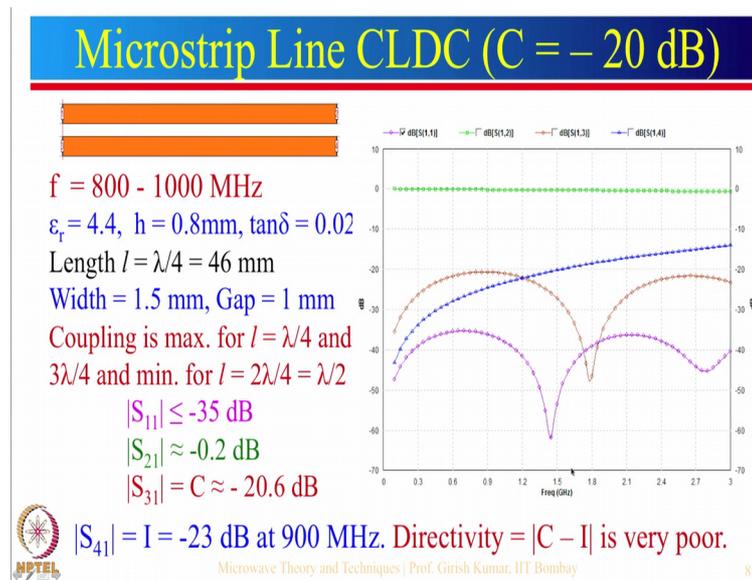
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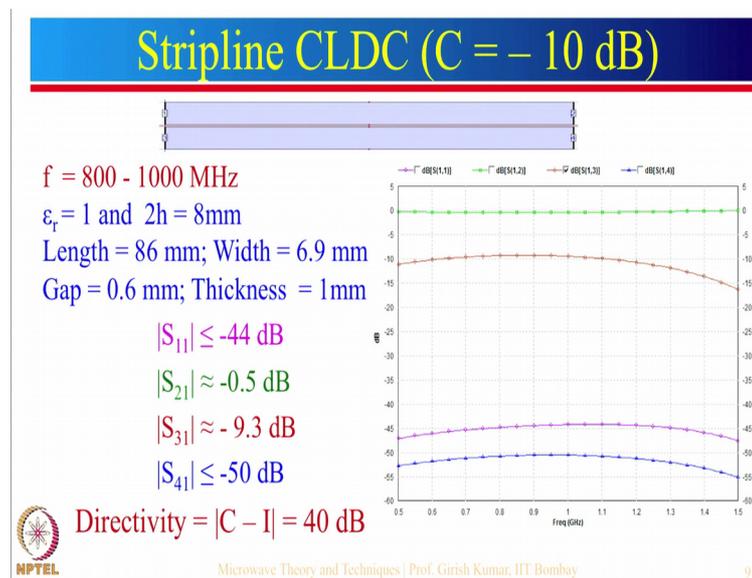
So, we looked at the concept of the stripline and we had used air as a substrate or you can say that we had used two metallic plate which were suspended in the air and we had a ground plane on the top as well as at the bottom. Even for 30 dB coupling we got more than 40 dB directivity.

Now, these couplers are good for generally speaking coupling of minus 10 dB to minus 20 or minus 30 dB, but many a times there is a requirement that we want the coupled power to be high or we can call it a tight coupling also known as strong coupling. So, what can be done to increase the coupling between these two coupled lines? So, of course, we can keep on reducing the gap which increases the coupling. Another way is that we actually connect one line over here and let us say another line over here. So, what will happen there will be now, direct coupling here direct coupling here. So, this is the concept which is the topic of today and that is a two branch line coupler. So, let us see what it is.

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So, here we have a port 1 you can say port 2, port 3, port 4. Now, here we know already that maximum coupling takes place when the length is equal to lambda by 4. So, this length is lambda by 4 this length is lambda by 4 and now, we have put two branches in between these two lines and these lengths are also equal to lambda by 4. Let us just see first the concept part and then we will try to complete the S matrix for this 3 dB coupler.

So, let us start with let us say we are giving the input at port 1. So, when we give the input at port 1 part of the power will go here and part of the power will go here. So, this power when it reaches here it experiences a phase delay of minus 90 degree. Why? Because length is lambda by 4 and phase which is theta equal to beta l. So, beta is 2 pi by

lambda into lambda by 4 will be 90 degree since there is a delay angle will be minus 90 degree.

So, then at this point again the power gets divided part of that goes here part of that comes over here. So, phase difference at this point will be from here to here 90 degree, another 90 degree so this will be minus 180 degree; or you can say 180 degree. Let us see now, what is happening at port 2. So, one path comes from here which is 90 degree; another path which is there from here to here 90 degree another 90 degree, another 90 degree. So, the phase delay from here is minus 270 degree from here it is minus 90 degree. Now, minus 270 can also be represented as plus 90. So, we have a one path which is giving minus 90 degree another path which is giving plus 90 degree, and if we design this particular coupler such a way that the power coming from here and power coming from here these two things are of equal magnitude. Then what will happen? Path from this and path from this will cancel each other. So, there will be no power which is going to port 2.

And we can also design now, that we can send half power here half power here. So, if we send half power here half power here this is known as a 3 dB coupler. We will also talk about other values, but a 3 dB coupler is a very very commonly use thing and it has many application. And here one thing I want to emphasize again from here to here it is minus 90 degree phase difference, from here to here minus 90, minus 90 minus, 180 degree. So, between these two ports phase difference will be 90 degree. So, now, let us try to build the S matrix here. So, 1 by square root 2 is taken outside which is corresponding to half power here half power here remember as I mentioned this is 3 dB coupler.

So, what we have designed for that there should be no reflection. So, no reflection means S 11 is equal to 0. So, from here to here no power goes here. So, that is also 0. So, in this particular case port 2 is known as isolated port, this is direct coupled port and this is coupled port. Then from 1 to 3, so the path is 90 and 90, 180 degree. So, there is a minus 1 1 by square root 2 has been taken outside path from here to here is minus 90 degree phase delay. So, there is a term which is minus j. So, similarly you can complete the rest of the terms you can use the symmetry of this particular network and complete the rest of the matrix.

So, now, let us see how do we do the analysis of this? Now, this is nothing, but a 4 port problems. So, this 4 port problem is simplified by using a symmetry. So, we can see that there is symmetry along this particular axis. So, by using the concept of the symmetry and as I mentioned for the coupled line what do we do instead of feeding only port 1 which is value let us say 1, this is 0, 0, 0. Now, we will think about that we are feeding let us say plus 1 here plus 1 here and then that will be a even mode symmetric second time we think about feeding here plus 1 and minus 1 this will be odd mode symmetry and then take the average of the two, so plus 1 plus 1 divided by 2 will be 1 and plus 1 minus 1 will be equal to 0 over here. So, that is the basic concept for the even and odd mode analysis.

So, let us see what we have here now. So, that is a normalized circuit. So, we have here instead of writing Z 0s, Z 0 we have written here 1 and these impedances are normalized. You can see that these are Z a, Z a and these are Z b, Z b.

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So, now, just to tell the concept what we do? So, 4 port networks is decomposed into set of two decoupled two port network because of the symmetry along this particular axis here. And then what we do? We use even and odd mode circuits using symmetry and anti symmetry, ok. And then actual responses obtained by using some of the responses of even and odd mode excitation.

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So, now, let us go step by step. So, here is a even mode excitation odd mode excitation. So, here this is the line of symmetry. So, for even mode its written here plus half plus half or you can say 1 1 and then you later on divide by 2.

So, here when we give the input plus, so along this particular axis you can say that there will be if this voltage is same here there will be no current over here. So, we can say that this will be resulting into open circuit at this particular point for odd mode excitation. Now, this is plus half this is minus half the voltage you can say that at this particular point will be equal to 0 which is replaced by a short circuit.

So, what do we do? We actually first of all find the ABCD parameters of this particular network here and ABCD parameters for this network and from ABCD parameters we can find the reflection coefficient. If you recall earlier it is written as A plus B by Z 0 minus C Z 0 minus D but here we are actually writing these thing as normalized with respect to Z 0. It is easy to remember then A B C D, ABCD here sum of A plus B plus C plus D and this is A plus B minus C minus D and this is the transmission coefficient or you can say this is S 11 and this is S 21, ok. So, this is how we can do the analysis. Again now, let us move on.

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So, even mode analysis how do we do it now? So, this is the open circuit think about, now this does not exist at all. So, from here you can see that there is a one branch over here and then there is a transmission line and then there is a another branch over here. So, now, recall ABCD parameters. So, we can actually speaking divide this problem into 3 separate segments. So, one segment will consist of this another segment will consist of this line and the third segment will consist of this line here, as I mentioned divided and solve the problem.

So, over here now, we can look from the input impedance this is an open circuit. So, we can find out what is the input impedance from here which can then be replaced by a shunt admittance. So, for shunt admittance we had seen the formula is 1, 0, y, 1. Now, this one here is a transmission line. So, for transmission line we had seen that the impedance is represented here as Z a which is the characteristic impedance of the line but the ABCD parameter of this will be 0, j Z a, j y a. Remember these are normalized values, and then 0. For this again what we see there is a shunt element so that will be 1, 0, y and 1.

So, these j terms are coming because you are looking from here equivalent thing. So, remember open circuit line if you look from here and since this length will be now, equal to what? It will be half of lambda by 4. So, this length is lambda by 8 lambda by 8 is less than lambda by 4 and open transmission line will act like a capacitive. So, we can say

that j y b is nothing but capacitive term in the form of y, ok. In y we write capacitance as j omega c, ok, so that is why there is a j term over here.

So, now, all we need to do multiply these 3 matrices simplify which leads to this particular expression here and this is for even mode ABCD parameter.

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Now, let us do for odd mode. In case of odd mode what we have here? There is a short circuit here short circuit here. So, you can actually neglect this particular part, ok.

So, now, we looking from here if we look at the input impedance from here since this is a shorted input impedance of this will be inductive. So, in terms of y will have a negative sign ok. So, this is something similar to minus j by omega l, ok. So, that is what is the capacitive admittance. So, again this is a shunt y, so 1 0 y 1 then this is a transmission line and then there is a shunt element here. Multiply the 3 matrices we get ABCD for odd mode. Once we have done that, now we have to find the S-parameters from these ABCD parameters. So, how do we do that now? We have to apply the principle of superposition. So, let us see S 11.

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S-Parameters of 2-BLC

To find S-parameters, apply superposition of even and odd mode reflection and transmission coefficients.

$$S_{11} = \frac{1}{2}(\Gamma_e + \Gamma_o), S_{21} = \frac{1}{2}(\Gamma_e - \Gamma_o)$$

$$S_{31} = \frac{1}{2}(T_e - T_o), S_{41} = \frac{1}{2}(T_e + T_o)$$

$$\text{where,} \quad \Gamma = \frac{A + B - C - D}{A + B + C + D}$$

$$T = \frac{2}{A + B + C + D}$$

What will be S 11, now? So, that will be half of why we are taking half, because it was 1 and then minus 1, ok. So, when we did the analysis for even mode it was plus 1 plus 1, so that corresponds to gamma e. For odd mode it is plus 1 minus 1. So, for S 11 we have to add the two and take the average. For S 21 port 1 was here port 2 was down below. So, for that now, again we are looking at a reflection, but plus half here minus half here. So, we have to take the difference that gives me S 21. For the transmission coefficient now, S 31, 1 was here, 2 was here, 3 was here. So, this is not transmission coefficient and this is the transmission coefficient for port 4. So, above it was plus so we have a plus term and down below when the other port it was a minus, so this is where the minus term is.

And where just to rewrite again reflection coefficient is given by this, transmission coefficient is given by this particular expression. So, we already know ABCD parameters for even mode odd mode. So, from even and odd mode ABCD parameters we can find out reflection coefficient even mode odd mode and transmission coefficient even mode and odd mode and from that we can calculate all the S parameters.

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So, after that now, we apply various design condition. So, the first design condition is we do not want any reflection at port 1 so that means, S 11 is equal to 0. The second design condition is we would like to have a complete isolation that means, S 21 is equal to S 12 is equal to 0. Now, of course, these conditions will not happen at all the frequencies, ok. So, we are putting this particular condition only at the center frequency, ok. So, away from the center frequency things will be different, ok.

Now, by putting these two conditions S 11 equal to 0 and S 21 equal to 0 in the previous equation we can simplify that and that leads to y a square minus y b square equal to 1, of course, this whole thing can be written in form of the impedance also. So, simply change this y a is 1 by Z a, y b is 1 by Z b simplify this you will get this particular condition.

Now, we are adding a third design condition. Now, this is the first time we are putting a condition that we want to design 3 dB coupler. This condition is valid for any other coupling value also. So, you can design it for 1 dB or 2 dB or 3 dB or 5 dB up to 10 dB. So, what happens beyond 10 dB? Actually speaking beyond 10 dB the line width of the branches line will become very very thin which are not practically realizable. So, if you want a coupling of minus 10 dB or minus 20 dB it is better to use coupled line directional coupler. And if you want a tight coupling do not use coupled line directional coupling because there the gaps become very small you use branch line coupler.

So, here is the condition for 3 dB coupler that means, the power will be given to the two output ports which are equal. So, S 31 square is equal to S 41 square equal to half and if you put this condition and simplify what do we get; Z b is equal to 1 and Z a is equal to 1 by square root 2.

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Design of 2-BLC

After simplification:

$$z_b = 1, \ z_a = \frac{1}{\sqrt{2}}$$

So, characteristic impedances of the main line and branch lines are:

$$Z_a = Z_0/\sqrt{2} \ \ and \ Z_b = Z_0$$

For characteristic impedance of the I/O ports, $Z_0 = 50\Omega$, the main line and branch line impedances for equal power division are **35.35Ω** and **50Ω**, respectively.

Now, these are normalized value when we denormalize we can say that Z a is equal to Z 0 by square root 2 and Z b is equal to Z 0. So, if we take Z 0 as 50 ohm then Z b becomes 50 ohm and Z a becomes 50 divided by square root 2 which is 35.35 ohm. So, let us see the realization of this with the design example.
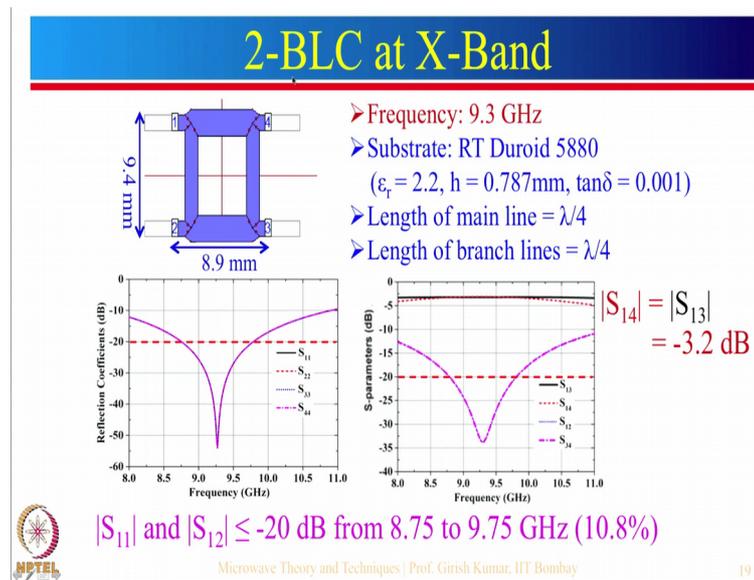
So, here is a two branch line coupler at x band we have taken center frequency as 9.3 gigahertz which is within the x band of 8 to 12 gigahertz. Now, since the frequency is very high please do not take a glass epoxy substrate because losses will be very high. So, here we have taken now, I would say an expensive substrate, so which has an epsilon r 2.2 thicknesses you can say 0.787 mm or close to 0.8 mm which actually corresponds to 1 by 32 inches tan delta is relatively small 0.001, ok.

So, since the frequency is 9.3 gigahertz we have to take these length as lambda by 4, ok. So, one can do the calculation corresponding to this find out first lambda 0 by 4 and then you have to first calculate corresponding to 50 ohm what is the line width ok, from the line width calculate what is epsilon effective and corresponding to this 35 ohm calculate what is the width. And then from that calculate epsilon effective and these things will then give you the exact length of the main line and branch line.

So, please remember this length will be slightly different than this length here. Even though we say these things are lambda by 4, but please remember for this epsilon effective is different compared to this particular branch line, ok. So, the lengths will be slightly different, ok. You can see that at this particular thing the entire size is you can say less than 10 mm by 10 mm, so 1 centimeter by 1 centimeter.

So, let us see the results for this particular case over here, ok. So, let us first see this is the input here port 1, port 3, port 4. So, at port 3 and 4 since we have designed it as a 3

dB coupler you can see that S 14 or you can say S 41 they are symmetrical S 41 and this S 31 or S 13, these two you can say here that the power is almost equally divided and this value is about minus 3.2 dB. Ideal would have been minus 3 dB there is a additional loss the main reason is that there are some radiations from these microstrip lines, ok. And of course, there will be small conductor loss and very small dielectric loss.

Now, let us see what is S 11. So, this is here is S 11 and this is the plot you can say for S 21. So, we can see from here they are fairly symmetrical to each other and we can say that S 11 as well as S 21 means this is the reflection coefficient this is the isolation they are less than minus 20 dB from 8.75 to 9.75 gigahertz. You can see that it has a bandwidth of approximately 1 gigahertz which is about 10.8 percent.

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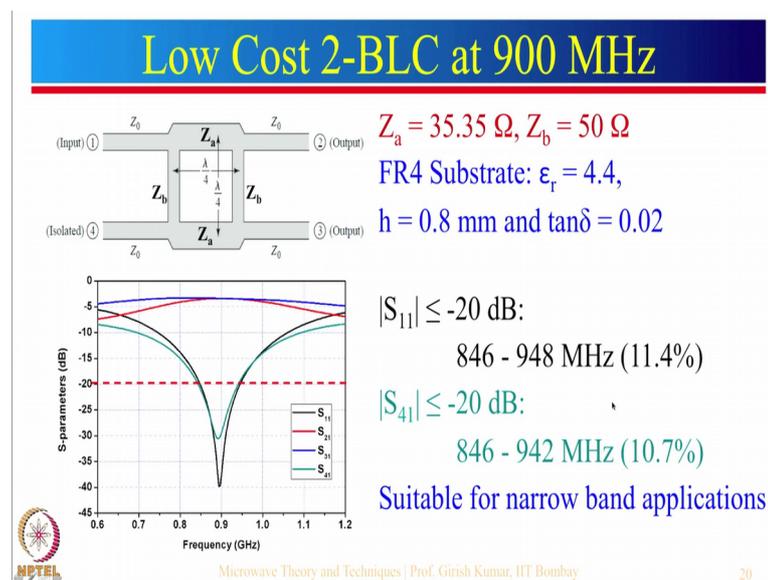
Now, the same thing let us see a low cost realization at 900 megahertz. So, I just want to tell you know you should use a expensive substrate mainly for applications at very high frequency or applications which are mainly for either defense or space because their cost is not that much important as much as performance is important, ok. And for commercial application cost becomes always very very important and a minor sacrifice can be done to the performance, ok, but that does not mean you do over compromise on the performance, ok.
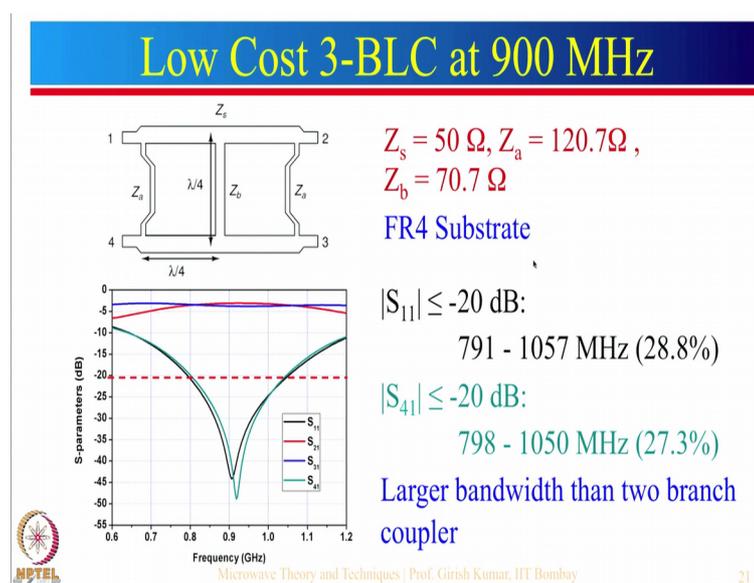
So, let us see over here. Now, this is the again same two branch coupler. So, we know that this is 35.35 ohm, this is 50 ohm. So, for 35.35 ohm you can see that the width is

large here, for 50 ohm this width is relatively small. We have used a low cost substrate which is FR 4 substrate also known as glass epoxy substrate. So, these are the parameters and these are the plots. So, you can see here that the from here to here just notice that the port numbers are defined differently, ok. So, it is not always that we should say 1 2 3 4, ok. So, here it is 1 2, you can say 3 4. So, look at the port numbers, whatever the numbers are there the response will be corresponding to that.

So, let us see here what are S 21 and S 31. You can see here these are the two curves and you can see that at the design center frequency they are approximately same and instead of minus 3 dB these are about close to minus 3.2 dB or so, which is not so bad. Now, let us see S 11 plot which is shown in the black color and this is S 41 plot which is shown in this particular color over here. So, one can see that for minus 20 dB or below S 11 or S 41 less than minus 20 dB the bandwidth obtained is from 846 to 948 or 946 to 942 because we had designed at 900 megahertz.

Now, of course, of this can be scaled slightly let us say if you want a gsm band which is 890 to 960, so bandwidth required is 70 but here we are getting a bandwidth of approximately you can say 100 megahertz which is more than 70. So, all you need to do it is that if you want the design specification from 890 to 960 all you do it is you reduce these branches slightly, ok.

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Low Cost 3-BLC at 900 MHz

$Z_s = 50\ \Omega$, $Z_a = 120.7\Omega$, $Z_b = 70.7\ \Omega$

FR4 Substrate

$|S_{11}| \leq$ -20 dB:
    791 - 1057 MHz (28.8%)

$|S_{41}| \leq$ -20 dB:
    798 - 1050 MHz (27.3%)

Larger bandwidth than two branch coupler

Now, we notice that the two branch couplers do not give too much bandwidth typical bandwidth obtained is of the order of 10 to 11 percent and many cases you may require larger bandwidth. So, here is an example of 3 branch coupler and for this 3 branch coupler let me first show the concept point of view. So, here is a input 1, there is a output 2 3 4. So, first concept point. So, from here to here 90 degree, another 90 degree, so phase will be minus 180.

So, from here to here there are 3 paths, so path one 90, 180, then 90, 90, 90, then 90, 90, 90. So, all the 3 paths basically give me the phase difference of minus 270 degree. So, this is minus 180 this is minus 270. So, the output will have a phase difference of 90 degree.

Now, let us see what is happening over here. You can see that this is relatively thin line this is thicker this is relatively thin line; why we have chosen that I will explain that now. So, from here to here the path is minus 90 degree, so from here to here to here 90 90, 180; 180 90, 270; 270 90, 360. So, we can say 360is equivalent to 0. So, then the path is minus 90. So, one path here minus 90 this path here minus 90, so these two paths will add up.

But now, let us see what is happening here this is 90, another 90, another 90. So, that is minus 270 degree which is equal to plus 90 degree. So, now, the way these impedances are designed that the path from here to here will add up, but path from here will subtract. So, what we want is that these two impedances should be large that means, the width should be small. So, less power comes from here less power comes from here, but larger power comes from here. So, that the output at port 4 is equal to 0.

So, again analysis is very similar to two branch analysis what you do would draw a dotted line over here. So, this is a symmetrical configuration. So, now, we will have a one branch, one line, one branch, one transmission line, one branch. So, now, we actually have to multiply 5 matrices. So, one for this, second, third, fourth, fifth, you multiply all those 5 matrices, do the solution for even mode odd mode and then put the boundary condition, solve the values and this is what you will get. So, what we have here? Z s is 50 ohm which is series impedance that is over here.

Now, Z a is 120.7 which is over here and here and Z b corresponds to 70.7 ohm. Now, again all the lengths are lambda by 4. So, let us see the response here, you can see that

these two are outputs at port 2 and 3 you can say that they are almost identical within very small error. And let us see what are the bandwidths corresponding to minus 20 dB and minus 20 dB for S 11 and S 41 and we can see that the bandwidth is much larger than two branch coupler here the bandwidth is of the order of 27 to 28 percent so which is much larger than two branch coupler.

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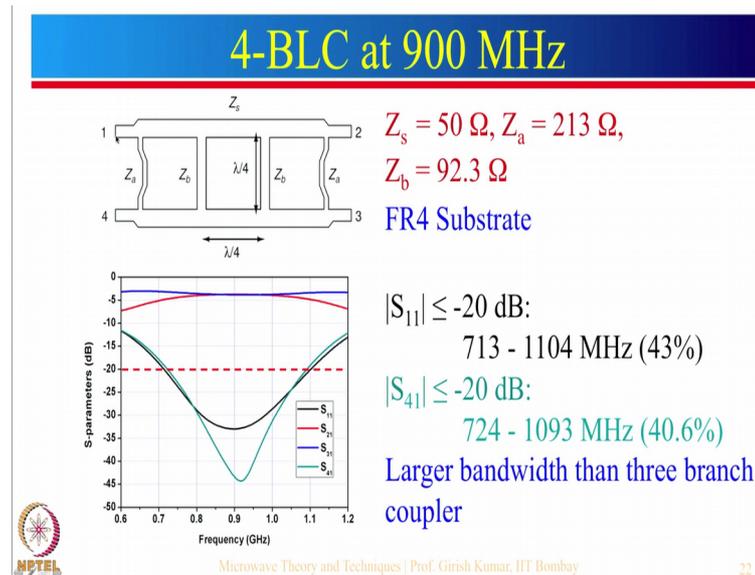
Now, instead of using two branch coupler one can get even a larger bandwidth by using 4 branch line coupler, ok. So, here again let us see the concept point of view. So, we have input at port 1 going to port 2, port 3 and port 4 here. So, let us see from here 90, another 90, another 90, so phase delay here is minus 270 degree, but to this point that is 90, 90, 90, another 90, minus 360 degree, but the phase difference at these two port actually remains 90 degree so which is same as port 2 or same as port 3.

Now, over here we want the output power to be close to 0 at this particular point here, ok. So, now, again we have multiple paths here. So, these are one path, then another path, then another path, then another path. So, again what we have to ensure that the paths are leading to the 0 value here. So, here just to mention again now, so this path is 90 degree which is minus 90 degree we can say 90 degree delay. So, this is minus 90, so from here to here, to here, to here, to this here. So, this path here again leads to minus 90 degree. And the path from here to here to here lead to plus 90 and path from here to here leads to plus 90.

So, now, one signal is coming from here one signal is coming from here which are getting added up and one signal coming from here one signal coming from here they are getting added up. So, you can see that this impedance is same as this. So, these two will cancel each other and this impedance is same as this these two will cancel over here and the net resultant is that S 41 will be equal to 0.

So, let us see the response. So, we can again see that power going to 2 and 3 you can say that it is fairly flat, and let us see now what is the bandwidth for S 11 and S 41 we have kept the same criteria as for two branch coupler or 3 branch couplers which is minus 20 dB minus 20 dB. Now, you can see here in this particular case bandwidth obtained is of the order of 40 percent and 43 percent. So, this bandwidth is much larger than 3 branch coupler.

Now, as far as design is concerned it looks very nice, however, when we try to do the implementation there is a serious problem and what is that serious, problem the serious problem is this value, ok. So, Z a is equal to 213 ohm. Now, that is a very high impedance, and this very high impedance will lead to this very thin line. In fact, this line may be of the order of 0.1 mm to 0.2 mm depending upon the substrate specifications ok. So, the realization of this particular thing becomes very very poor, ok.

So, in the next lecture we will see how this particular problem can be overcome and what we did to solve this particular problem, ok. So, just to summarize in the last two lectures we have talked about coupled line directional coupler. So, we had seen that coupled line directional couplers are good for coupling of minus 10 dB to minus 20 or minus 30 dB whereas, branch line couplers are good for minus 2 dB or minus 3 dB or up to minus 9 dB or so, ok. And then we had seen 2 branch line coupler, 3 branch line couplers and 4 branch line coupler. And we had noticed that from two branches if you go to 3 and then when we go to 4 bandwidth increases. However, for 4 branch line coupler we notice that the impedance is very high so the line width becomes very small. So, in the next lecture we will see how to solve this particular problem.

Thank you very much. We will see you next time. Bye.