

**Microwave Theory and Techniques**  
**Prof. Girish Kumar**  
**Department of Electrical Engineering**  
**Indian Institute of Technology, Bombay**

**Module – 03**  
**Lecture – 12**  
**Smith Chart and Impedance Matching – II: using Lumped Components**

Hello everyone. In the last lecture, we had started discussion about Smith Chart and Impedance Matching.

(Refer Slide Time: 00:27)

### Input Impedance and VSWR

<p><b>Input Impedance</b></p> $Z_A = R_A + j X_A$ <p><math>Z_A</math> represents input impedance of the antenna or amplifier or other components.</p> <p>Power Reflected = <math>P_r =  \Gamma ^2</math></p> <p>For <math>Z_A = 100 \Omega</math> and <math>Z_0 = 50 \Omega</math></p> $\Gamma = (100 - 50) / (100 + 50) = 1/3$ $\text{VSWR} = (1 + 1/3) / (1 - 1/3) = 2, P_r =  \Gamma ^2 = 1/9 = 11.1\%$	<p><b>Reflection Coefficient and VSWR</b></p> $\Gamma = \frac{Z_A - Z_0}{Z_A + Z_0}$ $\text{VSWR} = \frac{V_{\max}}{V_{\min}} = \frac{1 +  \Gamma }{1 -  \Gamma }$
------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------	--------------------------------------------------------------------------------------------------------------------------------------------------------------------

NPT © Microwave Theory and Techniques | Prof. Girish Kumar, IIT Bombay 2

So, let us have a very quick overview of what we had seen in the last lecture. So, we are actually started discussing about input impedance which is a complex number and from there we are actually also defined few other quantities like a reflection coefficient, VSWR and reflected power and we have also seen that if the load impedance does not match with the characteristic impedance then power reflector is not 0 but it has a finite value.

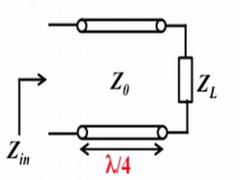
Ideally, we would like power reflector to be equal to 0 or as low as possible.

(Refer Slide Time: 01:02)

## Impedance Matching

Impedance Matching from  $Z_L = 100 \Omega$  to  $Z_{in} = 50 \Omega$

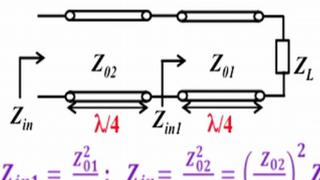
Using Single Section  
Quarter Wave Transformer



For  $\lambda/4$  transformer:  $Z_{in} = \frac{Z_0^2}{Z_L}$

$Z_0 = \sqrt{Z_{in} Z_L} = \sqrt{50 \times 100} = 70.7 \Omega$

Using Double Section  
Quarter Wave Transformer



$Z_{in1} = \frac{Z_{01}^2}{Z_L}$ ;  $Z_{in} = \frac{Z_{02}^2}{Z_{in1}} = \left(\frac{Z_{02}}{Z_{01}}\right)^2 Z_L$

$Z_{02}/Z_{01} = \sqrt{Z_{in}/Z_L} = \sqrt{50/100} = 1/\sqrt{2}$

Take  $Z_{02} = 60 \Omega$  and  $Z_{01} = 84 \Omega$

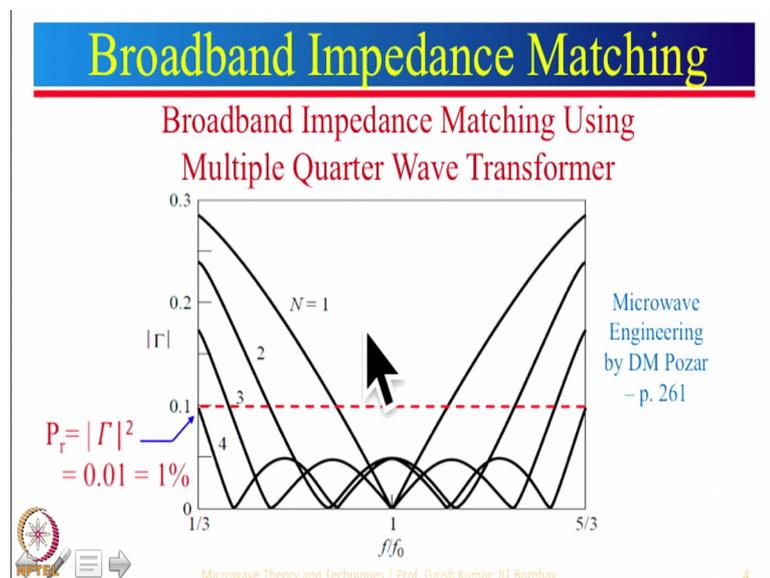


Microwave Theory and Techniques | Prof. Girish Kumar, IIT Bombay

3

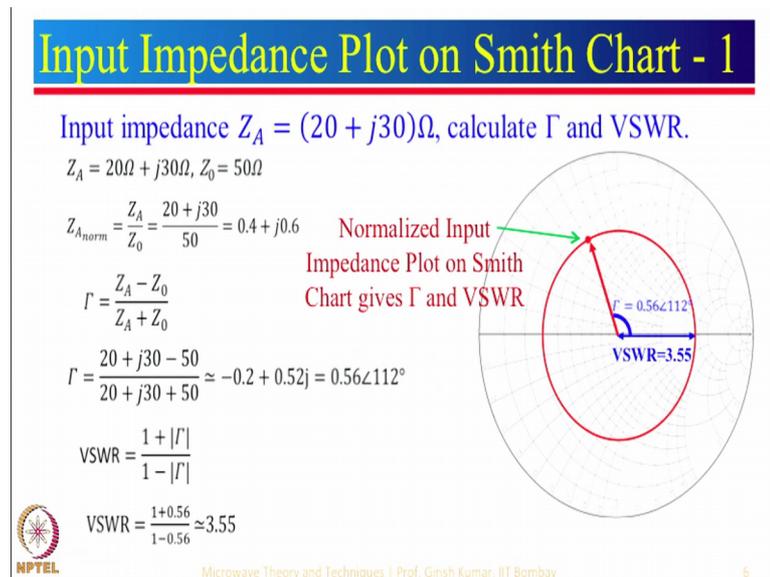
Then we had seen two cases; one was single section quarter wave transformer and then we had seen double section quarter wave transformer.

(Refer Slide Time: 01:12)



After that we saw the frequency response and we had seen that for a single quarter wave transformer, bandwidth is relatively less compared to two quarter wave transformer which has a larger bandwidth and if we use larger number of sections, we can get a much broader bandwidth and it also mention that you can use tapered line or exponentially tapered line to realize much larger bandwidth.

(Refer Slide Time: 01:42)



Then, we had looked into the Smith Chart and we took an example of the Smith Chart also. So, we had taken the example where the input impedance is given by this particular expression. So, we had done the calculation using two method; one was the calculation method. So, from this  $Z_A$  and 50 ohm of  $Z_0$ , we have calculated reflection coefficient of this value and VSWR of 3.55.

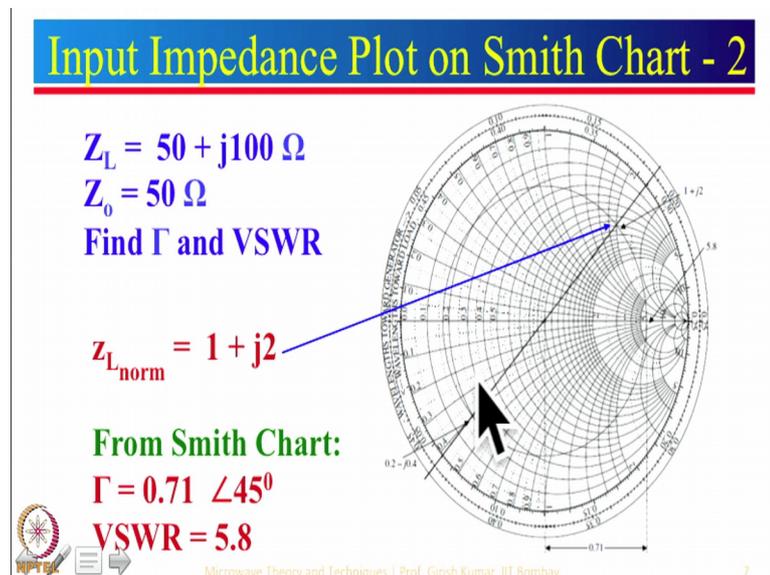
Now, the same thing can also be done using the graphical method by using Smith Chart. So, the first step in that case would be is we get the normalize value. So, you get the normalize value. So,  $Z_A$  by  $Z_0$  which is this value and how do we plot this on the smith chart, you locate 0.4 on the smith chart and then you move along the constant resistance circle and stop at a point where the value is plus  $j 0.6$  which corresponds to the reactive curve here and after stopping this point, you take this particular point as a radius value from here to here and draw the circle

So, once you draw the circle wherever the circle cuts the real axis on the right side, remember on this side also, there is a real axis point, but this value will be a less than 1 and we know VSWR is always greater than 1. So, this one gives us the value of VSWR and the reflection coefficient magnitude can be found in actually two ways; one is you actually draw this line over here and then the draw line from here and look at this horizontal axis. So, wherever it cuts here that will straightway give us the reflection coefficient value which is the amplitude and the angle can be measured.

So, what you do? You simply extend this line over here and measure this angle, whatever is that angle will be the actually reflection coefficient angle. Now, magnitude can also be measured slightly different way, you can do the calculation. So, what you do? You measure from here to here, what is the value of this particular thing and you measure this value, take the ratio of this divided by this value over here and that will give the value of the reflection coefficient.

So, either you can draw the vertical line, read it from here or you measure this value and measure, this value take the ratio. So, both ways you can find the reflection coefficient.

(Refer Slide Time: 04:15)



Let us take one more example. So, here is an example where the load impedance is given as 50 plus j 100 ohm and desire is 50 ohm. Now, in this particular case, what we can do that we can locate this particular point over here. So, how do we locate? We actually do the normalization. So, you divide this whole thing by 50 which is the characteristic impedance that will give me 50 divided by 50, 1 and 100 divided by 50 will be 2.

So, 1 plus j 2, how do we do that? I have shown you different different types of Smith Charts so that you know depending upon what is available to you, but this one here is relatively better to you know look at because locate the points because it has a much larger number of circles. So, let us see here what you do? You locate 1. So, 1 will be the center point and then we have a plus. So, what you do? You move in the direction above and there you stop at a point where this curve corresponds to j 2.

So, this will be the point where you stop and then you take this point with this particular radius draw the circle. And once you draw the circle, wherever this point is cut, you can see that I have put this thing here vertical line from the reflection coefficient if you see that will give me the reflection coefficient value which is 0.71. So, you can see here reflection coefficient is 0.71 but the point at which it is cut, you can see this value corresponds to 5.8. So, that is the value of VSWR which is 5.8.

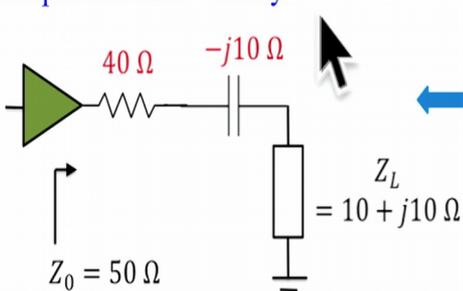
Now, for this particular thing to measure the angle what you do; you measure this particular angle ok. So, this one and this one, so this angle is nothing but angle of 45 degree. So, corresponding to this  $Z_L$ , these are the values of reflection coefficient and VSWR. Of course, you can do this calculation using the equations also, reflection coefficient is  $Z_L$  minus  $Z_0$  divided by  $Z_L$  plus  $Z_0$  and this will be the same answer you will get by using that equation.

(Refer Slide Time: 06:39)

### Lossy Impedance Matching

Design an impedance matching network(IMN) using lumped components to match  $Z_L = 10 + j10 \Omega$  to  $Z_0 = 50 \Omega$ .

Simple Solution - Lossy



Bad solution, as resistor will consume power


Microwave Theory and Techniques | Prof. Girish Kumar, IIT Bombay
8

So, let us now look at the impedance matching. Why impedance matching is important? What will really happen if you do not do impedance matching or what will happen if you do lossy impedance matching? So, let us just take a case where  $Z_L$  is given by 10 plus j 10 ohm and what we want? We want the impedance to be 50 ohm. So, let us see what we have, let us say this is the amplifier. So, that amplifier actually has an output impedance of 50 ohm.

So, for maximum transfer to the load, it will be better that the load impedance is equal to 50 ohm well, but the load impedance is not 50 ohm but we would prefer that impedance looking from here should be 50 ohm. Now, one of the solution, this is I have already written here it is a bad solution, but let us see one of the solution. This is  $10 + j 10$  ohm. So, we add something in series which is  $-j 10$  ohm. So,  $10 + j 10 - j 10$  will become 0. So, the reactive part will become 0 and this is 10 ohm, we add a 40 ohm. So, that will become 50 ohm over here.

Now, this is definitely a definitely a not a good solution because the power will be absorbed in this particular resistor. So, this is not at all a good option, ok. So, what we generally do? We would like to design a lossless impedance matching network and what are the lossless impedance matching networks? Well, we can use inductors and capacitor. Now of course, inductors and capacitors also have small losses. In fact, you might be knowing that an inductor is generally represented by series resistor and series inductor. Of course, series resistors value is generally very small that can be maybe 0.1 ohm to maybe 0.5 ohm but that can be negligible.

And all the capacitors are generally represented by a parallel resistor but generally speaking that parallel resistor value is in general greater than 10 Kiloohm or even a 100 Kiloohm so, again a parallel resistor can be ignored. So, we can say that most of their time inductors and capacitors are relatively lossless. So, let us now see how we can do lossless impedance matching for this particular case so, that maximum power transfer takes place from here to here.

(Refer Slide Time: 09:18)

## Lossless Impedance Matching

Design 1: IMN for  $Z_L = 10 + j10 \Omega$  to  $Z_0 = 50 \Omega$

Normalization with  $50 \Omega$ :  $z_L = 0.2 + j0.2$  and  $z_0 = 1$

1. Locate  $z_L$  on Smith chart
2. Move along constant resistance circle to reach point  $z_1$  intersecting  $g=1$  circle (dotted line)
3. Locate  $y_1$  at radially opposite point from  $z_1$  on  $r=1$  circle
4. Move along  $r=1$  circle to reach  $z_0 = 1$  point

Microwave Theory and Techniques | Prof. Girish Kumar | IIT Bombay

So, we are going to take the same problem. So,  $Z_L$  was given as  $10 + j10 \Omega$  and we would like to have  $Z_0$  equal to  $50 \Omega$  or you can say we want input impedance to be  $50 \Omega$ . So, let us see how this Smith chart can be used for proper lossless impedance matching. So, let us go again step by step so, the first step is we normalize with  $50 \Omega$  which is  $Z_0$ . Now, all the time, we do not have to normalize with  $50 \Omega$ . So, please read what is there suppose if  $Z_0$  is  $100 \Omega$ , then you have to normalize with  $100$ ; if  $Z_0$  is suppose  $70 \Omega$ , then you normalize with  $70 \Omega$ .

So, you have to see what has been defined or given. So, in this case now,  $10$  divided by  $50$ ,  $10$  divided by  $50$ , so  $0.2 + j0.2$  and  $Z_0$  is now equal to this is also normalize  $50$  by  $50$ . So now, let us locate this particular point on the Smith chart and let us see how we can use impedance matching concept using Smith chart. So,  $Z_L = 0.2 + j0.2$ , so how do we do? First locate the real part  $0.2$  on the Smith chart. So, this is  $0$ , you know this is  $1$ , infinity.

So, from here you locate  $0.2$ . So, that is the first type, then since it is plus, we will go plus, we will go up here and this is that reactance  $0.2$  circle. So, we stop at this particular point and we gave a nomenclature here which is nothing but  $Z_L$  as you can see here. So, that is a step one, locate  $Z_L$  on Smith chart. Now, what we do? We actually draw another circle. So, you can see there is a this circle here. This is known as  $r=1$  circle. So, you draw another circle like this which has been shown in the dotted line.

Many smith charts do not have this dotted line. So, you have to draw it yourself and many a times people ask how do you draw this, well it is simple. This is the you can say the diameter so you take the center point of this and then go from here, take the center point and draw the circle here. So, this is the circle which we name as  $g$  equal to 1 circle, right. Now, remember everything is in terms of impedance right now because we are plotted impedance. We will actually show you also how impedance and admittance can be interchanged.

So, first locate  $Z_L$ . Now, we move along the constant  $r$  circle, it is very very important you move along constant  $r$  circle. So, what happens if you are moving along constant  $r$  circle; that means resistive value has not changed; only thing which is changing is the reactive value. So, you move from here, go up to a point where this dotted line is there. So, you stop at this particular point, designate this point as  $Z_1$ . So, for this  $Z_1$ , the real part has to be exactly same as the previous real value if you are getting a different value; that means, you are made a mistake, you have not moved along the constant resistance circle.

So, you have to move from here constant resistance circle, stop at a point where this dotted line is there. I will tell you the reason also after few minutes, why we stop here. So, read the value so, this value is  $0.2 + j 0.4$ . So, this is the step move along constant resistance circle to reach point  $z_1$  intersecting  $g$  equal to 1 circle which is a dotted line. Now, at this particular point, what you do? You take from here you take go to a diagonally opposite point.

So, you have to move, take a scale, draw the line from here passing through the center and you draw the line here. This line will definitely cut this particular circle here. If it is not cutting at this particular point and either you have not drawn the circle properly or you have not drawn the line properly through the center ok. So, from here basically what it is  $z_1$  now becomes  $y_1$ . So, it is in fact a one of the very very simple way that you can get a  $1$  by  $z$  simply by moving in the opposite thing.

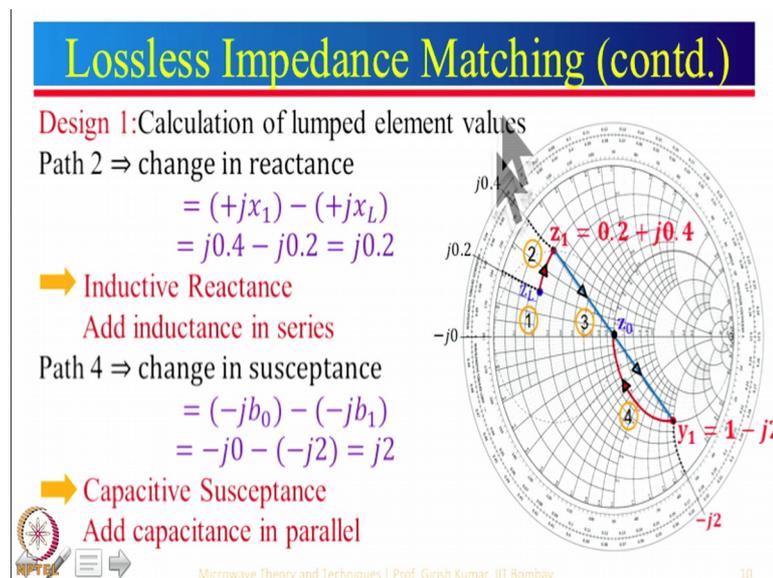
So, whatever is the impedance value, you can get the admittance value. Now, since this particular point lies on this circle, remember this point will always be a real value be  $1$  a few things you have to remember. Now, this is  $y$  ok, this was  $z$ , we have move opposite

this is y. So now, read first real part which is equal to 1, imaginary part you can read from here this particular thing and that value comes out to be 1 minus j 2.

So now, locate y 1 at a radially opposite point from z 1 on r equal to 1 circle. So now, at this point we know what is y. So, when we know y this is 1 minus j 2. Now, just think about if I add plus j 2, what will be the value then; if I add plus j 2 total y will become equal to 1. So, from here you move along this particular constant r circle or here it is r equal to 1. So, again you have not added any resistor value, you are simply moved over here and move along r equal to 1 circle to reach z 0 equal to 1 point.

So, you have reached at this particular point and at this point, what is input impedance? Now, equal to 1; what is the reflection coefficient here, equal to 0; what is VSWR, equal to 1. So, what we have done, let us go step by step again from here we have to go to point here we took the reflection and then we move along this.

(Refer Slide Time: 15:47)



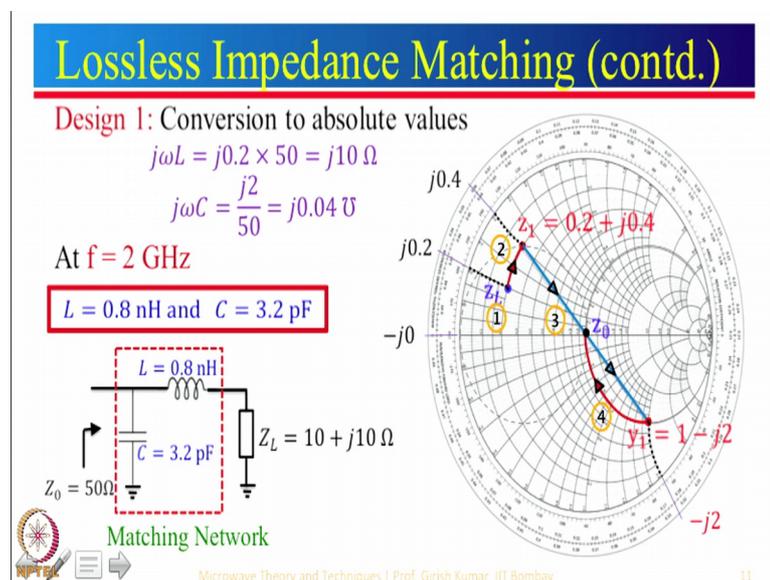
So now, let us see how these things happen. Now, first thing we are doing here is we are adding basically a lumped element values we need to calculate that. So, what we did, from 0.2 we went to 0.4 so; that means, what is the difference of these two, the difference is plus j 0.2.

So, at this point, we have to add plus j 0.2 and these are the impedance values. So, if you want to add something in the impedance, then what you do, you add the element in

series. So, and this is a plus value; that means, you have to add inductance in series to reach from here to here ok. So, step 1; then from here we take a reflection, we had seen the reflection and this we had seen as 1 minus j 2, from here we need to add plus j 2. So, remember now, this is y.

And in y we have to add plus j 2. So, in y what is plus it is capacitance because y is equal to j omega c. So, we have to add a capacitance in parallel because anything you want to add in y, in y we have to put the elements in shunt, then they get added.

(Refer Slide Time: 17:10)



So now, let us see the next step. Now so, the next step is conversion to the absolute values. So, we know that the first thing which was j 0.2, but that was normalize value.

So, we have to multiply with this value as 50 so that it becomes now de normalized value and this is equivalent to j omega L. So, let just calculate this is j 10 ohm and what was y, y was j omega c which is equal to j 2 and in y we have to divide with 50. Remember in impedance, we have to multiply with 50 and in y we have to divided by 50 and again this 50 is only because our Z 0 was 50.

If our Z 0 was suppose 100, then these would have been 100; say from here we can calculate this value is 0.04 over here the unit is ohm, here the unit is mho. Now, till now frequency has not come into picture. So, we can now, substitute the value of the desired frequency. So, till now, you can actually see that this design is valid whether you do at 1

megahertz or 10 megahertz or 100 megahertz or a 1 gigahertz; do not try to do this at 10 gigahertz or beyond because beyond 10 gigahertz or so, these lumped element values are very very small.

So, let us see even at 2 gigahertz, what are these values ok. So, we now put frequency as 2 gigahertz. So,  $\omega$  is equal to  $2\pi f$  ok. So, do not put here  $f = 2$  gigahertz, this is not correct,  $\omega$  is  $2\pi f$ . So,  $2\pi$  into 2 gigahertz will be  $10^9$ , you substitute that value and then calculate the value of  $L$  from here,  $C$  from here and that comes out to be 0.8 nanohenry and  $C$  equal to 3.2 pikofarad. You can see that these are very small values of inductor and capacitor at low frequencies, you are probably used to of using inductors of a microhenry or even millihenry and capacitance, you are probably used to of using microfarad or nanofarad.

But over here, the inductor and capacitor values are very very small. Now, just to tell you suppose your frequency was instead of 2 gigahertz, suppose if it was 0.2 gigahertz; that means, 200 megahertz, then this value will become 8 nanohenry and this will become 32 pikofarad. Now, let us try to complete the design. So, from here we are added something in series and that was impedance. So, impedance addition will be series. So, here is the load impedance, we added something in series which is inductor. So, that will be the inductor value of 0.8 nanohenry.

Then from here, we are taken the reflection and in  $y$  we are adding something. So, remember in  $y$  whenever we want to add, this has to be in shunt. So, from here we have added in shunt and what we added to added plus  $j 2$  which is equivalent to the capacitance which we have calculated here so, that is 3.2 picofarad. So, you can see now  $10 + j 10$  ohm has been transferred to 50 ohm.

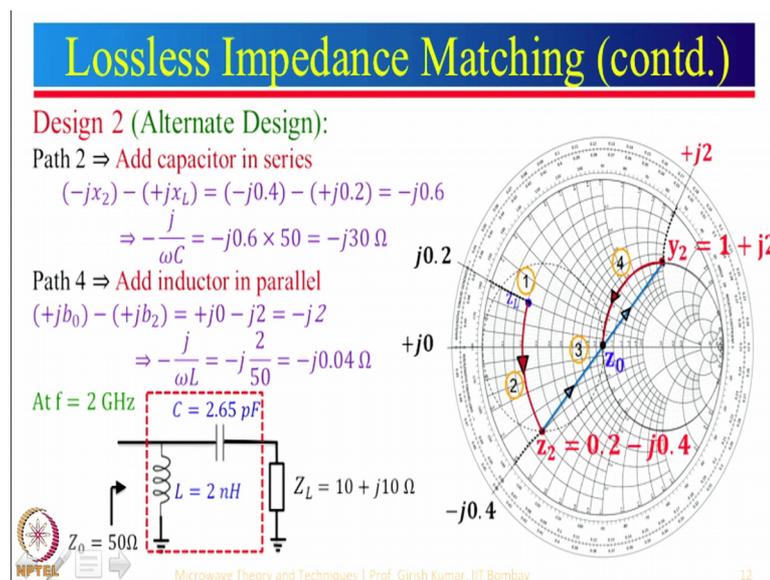
So now, this is the another concept which lot of times students ask me. This was the amplifier here. So, amplifier now see is a 50 ohm impedance and this is 50 ohm. So, maximum power transfer takes place from here to this particular point, but the load impedance is  $10 + j 10$  ohm. So, how maximum power transfer gets over here, well you have to actually think of this as concept point of view.

So, from this point maximum power got transferred from amplifier or any other particular device. So, from here maximum power got transferred. So, what will be the power loss in this? If we assume that these are the ideal components, then there will be a

no power dissipation here, there will be a no power dissipation here but maximum power transfer has taken place. So, from here all the power will go to this particular load impedance which is not 50 ohm, yet because of this matching technique, we have transferring of course, as I mentioned earlier inductors and capacitors are not going to be ideal but I did mention that these inductors and capacitors have generally low losses or at least chose inductor and capacitor which have a low loss values.

So, you have to be careful when you are choosing these matching circuit and by doing this particular thing, you have actually brought the load impedance from here to this particular thing not directly from here to here, but you have use the zigzag path from here, then here and then coming over here. Let us take a one more example so that you know that our confidence gets built up. So, the one thing here is that instead of a moving up, let us move downward. So, what will happen?

(Refer Slide Time: 22:24)



So, you can use either this approach or you can use this particular approach, both the approaches are correct. Later on we will tell you what are the differences and what are the pluses and minuses. So, in this particular case what we did? So, located again the same thing 0.2 plus 0.2, so this is the point which has been located, so from here instead of a going up, we are going down, we stop at a point where it cuts this g equal to 1 circle up. Now, instead of a writing z 1, I have written z 2 just to be different, read the value real part should be exactly same as the real value which was here which is 0.2.

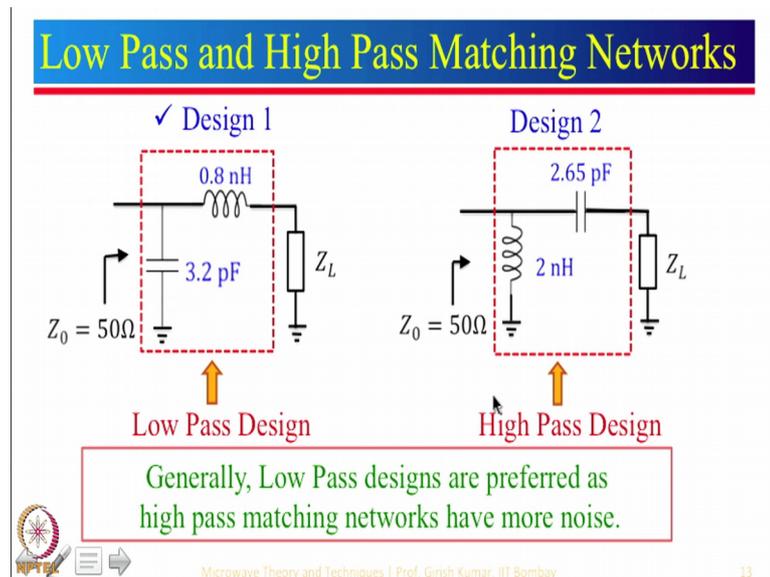
The imaginary part is now you can see this is minus  $j 0.4$ . So, we add something in series and that difference will be now equal to minus  $j 0.6$  because this was plus  $0.2$  we have to reach to a minus  $0.4$ . So, we have to do minus  $0.6$  and minus  $j 0.6$  can be realized using their capacitor, why because minus  $z$  is equal to minus  $j$  by  $\omega C$ . This is the normalize value, you multiplied with  $50$  because it is impedance value.

So, from here we can calculate their capacitance value, then from here what you do, you go through the center point go over here and you can see that real value of the  $y^2$  will be equal to  $1$ , it has to be if otherwise you have made a mistake and read the corresponding value of the imaginary value. So, you can see here that imaginary value is plus  $j 2$ . Now, we have to reach to the central point here.

So, plus  $j 2$  if we add minus  $j 2$ , then what will be the actual value, that will be  $1$ . So, we need to add a minus  $j 2$ . Now, in  $y$  minus implies inductance so,  $y$  is equal to minus  $j$  by  $\omega L$  which is minus  $2$  and why we divided by  $50$ . So, this is the value corresponding to this here. So, at  $f$  equal to  $2$  gigahertz, again you can see till. Now, frequency has not come into picture. So, at  $f$  equal to  $2$  gigahertz you can now calculate the value of the inductor, the value of the capacitor.

So, again now, what we did? So, from here, we went to this, we added a capacitance in series, took a reflection. Now, it is  $y$ , so in  $y$  we have to add something which will be in shunt. So, this is now the solution ok, so now, the question is between the first solution and this solution which one is better. So, let us look at a comparison of these two.

(Refer Slide Time: 25:24)



So, here is the design one where we are added a inductor and then we had a added a capacitor in shunt. Here capacitor was added in series, inductor was added in shunt.

Now, this is a actually known as a low pass design. Actually we are going to cover low pass filter, high pass filter later on. But just to check quickly, you can do a very quick check also for a low pass. Generally, what is a low pass design? A design which passes the low frequency and blocks the high frequency; what is a high pass design? It passes high frequencies and blocks the low frequency. So, you can quickly check here what happens at low frequency. So, you can take the lowest frequency as omega equal to 0 or DC.

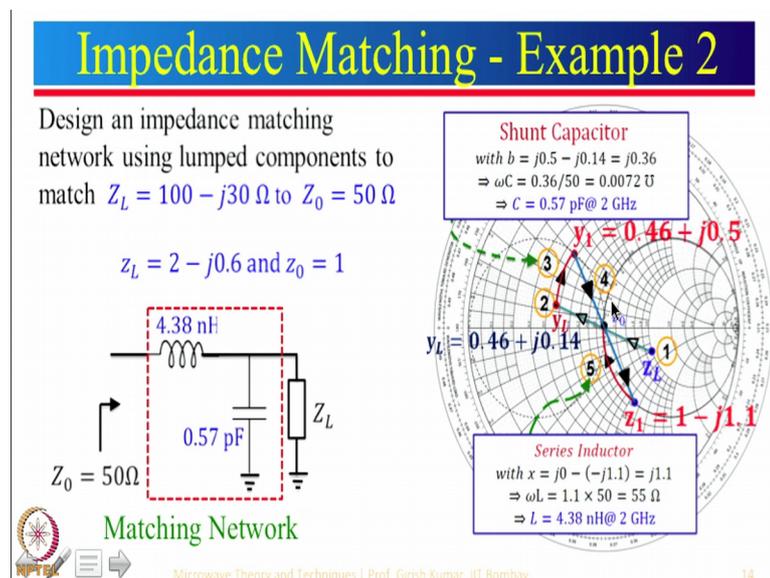
So, at DC inductor will act as a short circuit. So, whatever we gave here comes here; at DC, capacitance will be open circuit. So, nothing will go here; everything will come over here ok, but at infinity what will happen? This will act as a short circuit the power will try to go over here and if anything which is going through this inductor will act as a open circuit because infinity multiplied by inductor will act as an open circuit. So, nothing will go here, that is how it is low pass design. Let us look at a high pass; at high pass again test at two extreme frequencies DC which is omega equal to 0.

So, at omega equal to 0, this will act as an open circuit so, nothing will go there. At omega equal to 0, this will act short circuit. So, most of the power will go through here and at omega equal to infinity, this particular thing will act as an open circuit because Z

is equal to  $j\omega L$ . So, if  $\omega$  is, this will be open circuit and this particular thing  $Z$  is  $1$  by  $j\omega C$  so, this will act as a short circuit. So, at high pass everything will go through. By the way, this low pass design is also known as an integrator and high pass is also known as a differentiator.

Now, generally low pass designs are preferred as high pass matching networks have more noise because all these things are acting as a differentiator and we know that differentiators are always adding noise whereas, integrators are always smoothing out the noise. So, generally speaking, low pass design is preferable compared to high pass design.

(Refer Slide Time: 28:01)



Let us take a one more example. We just introduce this thing quickly and this example here is  $Z_L$  is equal to  $100 - j30 \text{ ohm}$  and that is to be matched to  $50 \text{ ohm}$ .

So, what is the first step, you normalize  $Z_L$ . So,  $Z_L$  is now  $100$  by  $50$  is  $2 - j0.6$  and  $Z_0$  is equal to  $1$ . So, what you do, you locate this particular thing on the smith chart. So,  $Z_L$  you locate over here. Now, in this particular case it is different now, you cannot just move along this particular circle because it will never ever cut this particular circle here. So, what you do? From here you take the reflection. So, you have to ensure that you keep writing the thing. So, this is  $Z_L$ , take the reflection, now you write this as  $y_1$ . So, this is  $Z_L$ , this will become  $y_1$ . So, read the value, so you can read the value as this here. So, if at this particular point you move along the constant conductor circle and that

particular thing over here will cross at this particular point where it is this here. So, this value you read it so, that is now plus  $j 0.5$ .

So that means, from here to here, you are going to move and add this particular value. So, what is the difference between the two that is  $0.36$ . So now, from here you take the reflection, so this was  $y 1$  now, you write this as  $Z 1$ . So, at this point, you add this value here which is plus  $j 1.1$  and you will reach the central point. In fact, many a times I call this as Bull's eye also. So now, let us look at the solution so, from here we went to  $y$ , in  $y$  we added something. So, if you are adding something, in  $y$  it has to be in shunt and then from here you went to  $Z$ , we have to add something in  $z$ . So, that will be in series. So, this is what the design which is in series ok.

So, please look into this particular thing. In the next lecture, I will go through this particular design one more time, but till then do little more practice so that you have a more confidence. A please remember impedance matching is very very important in most of the amplifier design or in most of the system design ok. So, you cannot ignore impedance matching and in fact, in the next lecture, I will talk about some different impedance matching technique. So, till then please read it and do some practice with arbitrary load impedance and see if you can do the final design.

Thank you very much, bye.