

**Fabrication of Silicon VLSI Circuits using the MOS technology**  
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**Lecture 08**  
**Solid State Diffusion**

Yesterday, we discussed something about the diffusion using constant source and we did derive some expression for complementary error function profiles. Today, we will continue with that. In real life the most of the time the profile which you get may be Gaussian and how it comes? So, let us see there is second case, the first case we discussed was infinite source are also called constant source diffusion.

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2<sup>nd</sup> Case :- Limited Source Diffusion  
 In this case fixed amount of Impurities reside at the Surface and then Temperature - Time cycle starts.

From Diffusion Equation we have:

$$\frac{d^2 N(x,s)}{dx^2} - \frac{N(x,s)}{D} = \frac{N(x,t=0)}{D} \quad \left( = \frac{Q}{sD} \right)$$

Let us have a Sheet charge with Dose Q (no/area) as fixed quantity and is like a  $\delta$  function at  $x=0$

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Now, let us say, we have a limited source diffusion to be performed. Since, it is a limited source diffusion what we are assuming right now is at the surface there is a sheet charge of impurities. Sheet charged has a delta function which is shown here in the figure and it contains an impurities which is, which has a concentration which corresponding to order number. But, since, it is a delta function, we say, it contains a charge density which is number per centimeter square and if I divide by also delta then it will per CC, okay.

So, this charge density which is sitting here that is the amount of charge per unit area sitting at the surface is let us say is known to me, okay. This is known to me. "How do I get it?" is the process we learn. In the figure and it contains an impurities which is, which has a concentration which corresponding to order number, but, since, it is a delta function, we say, it contains a charge density which is number per centimeter square and if I divide by also delta then it will per CC, okay.

So, this charge density which is sitting here that is the amount of charge per unit area sitting at the surface is let us say is known to me, okay. This is known to me. "How do I get it?" is the process we learn. How do I get this sheet charge, that is, fixed charge - which are fixed impurities have been put at the surface, which is very thin layer of certain amount of impurities per unit area sitting at the surface, okay.

If we come back to the diffusion equation once again and we say the  $d^2N \times s$  by  $dx$  square minus  $SN \times s$  by  $D$  equal to the initial condition by  $D$ . And in this case, where, we can show that, if the sheet charge is this and  $Q$  into delta is the volume the first, this, then we could say, it is  $Q$  the Laplace transform of this will come  $QSD$ , maybe, we can think little later, how? But, since, this equation we want to solve and we call this  $Q$  as the word which will use most often in implants, which is called dose, okay. That dose is essentially number per unit area or flux density into area or into time is actually the dose, okay.

Now, this dose as is fixed in our case because this is a limited source diffusion, okay. And say, delta function at  $x$  is equal to 0. It is called sheet charge approximation. "How do I get sheet charge?" is another simple process we will see that, if it is not exactly like sheet charge. How close it is? And if it is still not, then can I still solve an equation of second order? Okay. There are all possibilities are available, however, in most cases, we will assume that sheet charged provision is sufficiently valid. The reason, "why it is valid or not?" is the case, because, when we actually diffuse it in a silicon any impurity, I will actually monitor its profile.

And, by, at the end of the day, I know my physics, I know my numerical analysis. I will fit it anyway. So, in worst case, if I cannot get exact physics I will fit that physics to the curve I actually, monitor. Okay. And then we will say, okay, there are other effect, this - that and will justify myself that I am right. That is what all, as, I keep saying it is most important procedure.

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For this case Initial Cond<sup>n</sup> is  
 $N(x, t \leq 0^-) = Q \cdot \delta$

We have solution of Diffusion Equation as  
for earlier constant source case as  
 $N(x, s) = B(s) \exp \left\{ - \left( \frac{s}{D} \right)^{1/2} x \right\}$

$x [N(x=0, t \leq 0^-)] = \frac{Q}{s}$

$\therefore N(x, s) = \frac{Q}{\sqrt{s \cdot D}} \exp \left[ - \left( \frac{s}{D} \right)^{1/2} x \right]$

$N(x, t) = \frac{Q}{\sqrt{\pi D t}} \exp \left( - \frac{x^2}{4 D t} \right)$

This is essentially a Gaussian Profile

One can see there the solution of this right now, as, I said, please look into book. I will just go into final version. The solution of differential diffusion equation was B s exponential minus s D by 2 with linear - this condition given initial condition Laplace transform taken. I get this, you just write down this equation. One which is relevant the rest you read in the books. N x s is Q upon root s D exponential minus s by D to the power half x, essentially, this is my B s term Q by root s D, okay.

And, if I take the inverse Laplace transform then N x t is Q upon root pi Dt exponential minus x square by 4 Dt. I repeat, this is the Laplace transform solution, I take inverse Laplace transform and I get a relation which is Q upon root pi Dt exponential minus x square by 4 Dt. If you look at this expression exponential minus x square by Dt into some pre-exponent, this is Gaussian in nature. This is standard Gaussian function. Please remember this is constant, because Q has been fixed constant. I had done some diffusion at temperature T for a given time t. So, this is constant.

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For this case Initial Cond<sup>n</sup> is  
$$N(x, t \leq 0^-) = Q \cdot \delta$$

We have solution of Diffusion Equation as  
for earlier constant source case as  
$$N(x, s) = B(s) \exp \left\{ - \left( \frac{s}{D} \right)^{1/2} x \right\}$$

$\times [N(x=0, t \leq 0^-)] = \frac{Q}{s}$

$\Rightarrow \therefore N(x, s) = \frac{Q}{\sqrt{s \cdot D}} \exp \left[ - \left( \frac{s}{D} \right)^{1/2} x \right]$

$$N(x, t) = \frac{Q}{\sqrt{\pi D t}} \exp \left( - \frac{x^2}{4 D t} \right)$$

This is essentially a Gaussian Profile

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One can also see quickly from there, if I make  $x$  is equal to 0, “What is the quantity remaining  $Q$  upon  $\sqrt{\pi D t}$  if  $x$  is zero?” the pre-exponent only remains, which means at  $x$  is equal to zero what is the concentration, we define, the surface concentration at the wafer after diffusion has been performed is such that, it is only  $Q$  upon  $\sqrt{\pi D t}$  and that is called the surface concentration in a Gaussian profile. Okay. Now, this is essentially, as, I say Gaussian and, therefore, if I plot these values, these equations solution.

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For this case Initial Cond<sup>n</sup> is  
 $N(x, t \leq 0^-) = Q \cdot \delta$

We have solution of Diffusion Equation as  
for constant source case as  
 $N(x, s) = B(s) \exp \left\{ - \left( \frac{s}{D} \right)^{1/2} x \right\}$

$[N(x=0, t \leq 0^-)] = \frac{Q}{S}$

$N(x, s) = \frac{Q}{\sqrt{s \cdot D}} \exp \left[ - \left( \frac{s}{D} \right)^{1/2} x \right]$

$N(x, t) = \frac{Q}{\sqrt{\pi D t}} \exp \left( - \frac{x^2}{4 D t} \right)$

This is essentially a Gaussian Profile

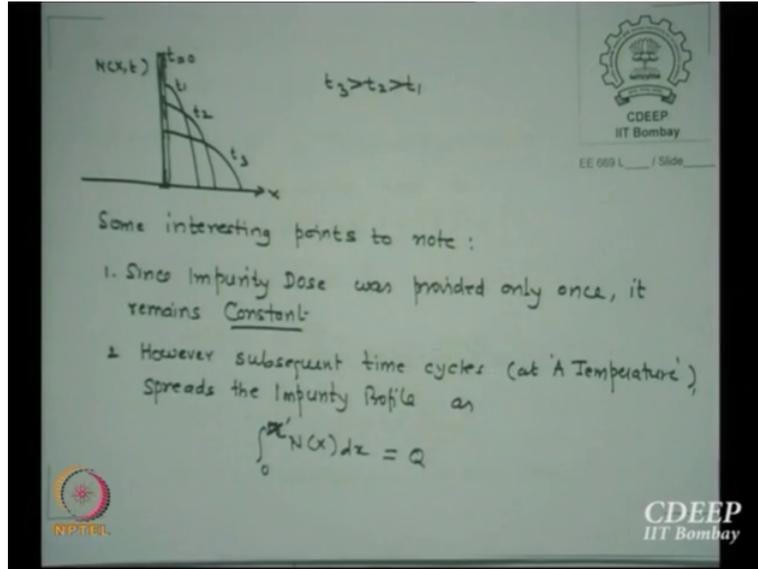
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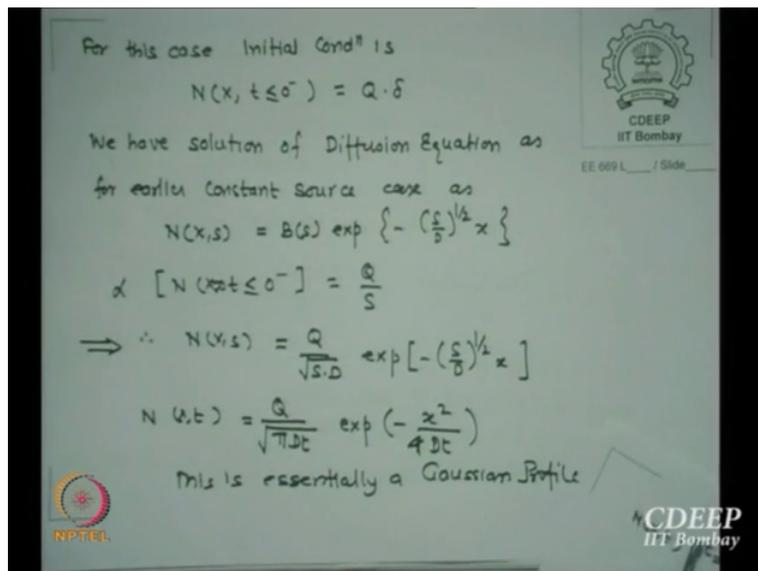
Please note down. You note down only these two. This and this and that is most important for us. Please remember, if I have a fixed charge (07:14) are sitting, the solution will be always Gaussian. This is something most important for us, because in real life there are a few things, somewhere, we may do error function profiling and, somewhere, we may do Gaussian or in some cases I did not want Gaussian, but will happen Gaussian, okay. This is what is some interesting part in this is, maybe, I have graphs to show you both profiles together. So you will learn soon.

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So, if I plot  $N \times t$  versus  $x$  and, since, remember the sheet charge is fixed. Is that correct? So, if I start, actually, giving a heating cycle at a given temperature  $t_1, t_2, t_3$ .  $t_1$  is smallest, sorry,  $t_3$ , yeah,  $t_1$  is the smallest. So, initially, after  $t$  is equal to 0, some impurities will diffuse, okay.

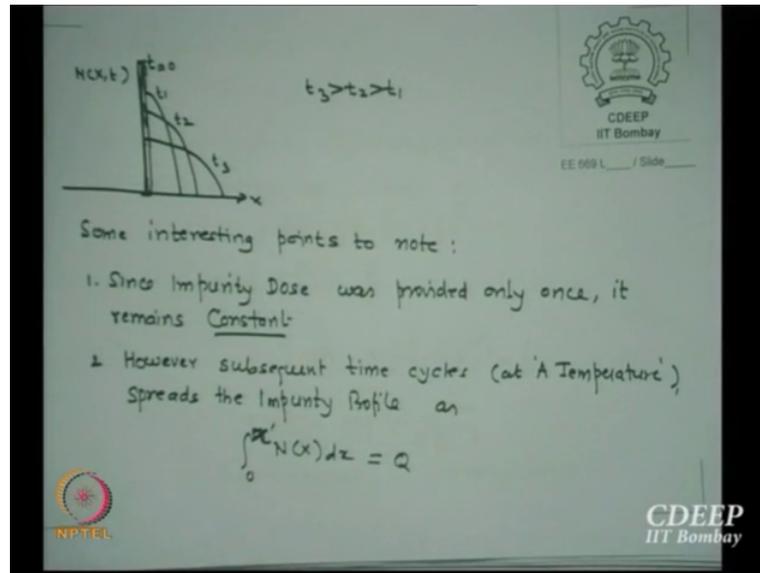
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And they will follow which profile? Gaussian profile, where  $t_1$  in that formula which I wrote in that case,  $Q$  upon  $\pi$  root  $D t_1$ ,  $D$  is the temperature at which  $t_1$  into  $t_1$  exponential  $x$  square upon  $4 D t_1$ . So, this is the profile I get, I repeat, what I say, if diffusion is performed at temperature

$t_1$  then  $D$  for that  $t_1$  we call it  $D_1$ ,  $t_1$  is the time for which diffusion must performed. It is  $4 D_1 t_1$ . So, this is the profile I get at time  $t_1$ .

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So, please remember, since, the charge is fixed, if you push inside something, what will happen? Area under the curve must remain same as  $Q$ , because that is what the assumption has total amount of impurity you pushed are constant. So, obviously, they will get redistributed. So, at first,  $t_1$ , they go inside to some depth, okay. And this concentration has decreased. Please, initial may be very high but it will first  $N_s$  has appeared, okay.  $N_s$  is what?  $Q$  upon root pi  $D_1 t_1$  is the  $N_s$ . So, the first time, I do drive in for time  $t_1$ . I see surface concentration has gone down. I keep hitting it for other time total time of  $t_2$ , same amount of impurity had to be there. Now, this surface concentration will go down and it will go deeper. But, still area under this curve should be same as the initial value.

So, I have a next profile which is at  $t_2$ , I did another driving another cycle of temperature, time cycle for time  $t_3$ . It will go  $N_s$  will further go down. Please remember, but, still this profiles will be always Gaussian in nature. They will always remain Gaussian in nature. So, as long as, your charge is fixed initially the profile will be always Gaussian and it will start flattening.

Now, one question which I have not said it very clearly here, if you are a silicon surface may be paper solve, if there is a sheet charge sitting here there is a silicon here, but on this side there is a here, okay. So, if I heat it, the impurities have no such choice of direction you know. It may get inside silicon partly may go out of this. This is called out diffusion. So, the impurities then the next time what impurities you were looking for was not  $Q$  there is that because part of the impurities I already lost in there. However, never happens in real device. The reason is the during the Drive-In this cycle of time temperature cycle. We actually do this process. We shall see technically, later, in oxygen ambient. This time cycle is pro always done in oxygen ambient.

So, what happens? That the silicon gets oxidized and we will see later of silicon dioxide is an excellent mass for impurities, okay. It does not pass impurities, unless, it is very thin. Most of the impurities cannot come out of the oxide. So the impurities which are getting in the outer surface gets oxidized and do not allow impurities to out diffuse. They all go into silicon.

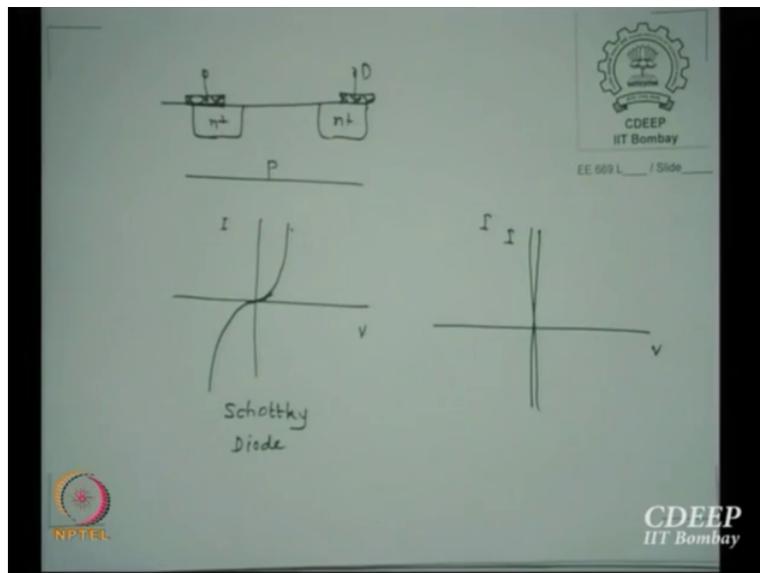
So, now, one thing you have to understand that all your processing should be says that the impurities are always in silicon and not outside. Of course, this can be debated, how some things may happen but that is some other day. So, interesting points to note again, which I said earlier, since, impurity dose were provided only once it remains constant. However, subsequent time cycle at any 'A temperature' spreads the impurity profile and given this  $0$  to  $x$  dash  $N \times dx$  always remains  $Q$ . Because, whatever, you have started that many impurities will always per unit area remain there. Integral of  $N \times D \times x$  is area per unit area. So that remains constant.

So this is something different from error function profiles or complementary function profiles. There the surface concentration was held to what value solid solubility value, because source was constant all the times. So impurities can reach to the peak of solid solubility at a given temperature. So there the surface concentration remains constant and the as you start pushing impurity the more and more deeper they may go but the  $N_s$  there remains constant to the solid solubility limit and there is this  $N_s$  is not saying it keeps on decreasing as you go for time cycles one after the other.

Is that clear? This is the difference between Gaussian profile and, so, if you are looking for surface concentration very high. “Your profile should be what kind?” complementary error function profile. Because, at the surface I want very large concentrations. This is a necessity in cases of like emitter diffusion in PNP or NPN transistors or in a mass transfer when I am doing a source diffusion or drain diffusion. I want source to be highly doped, so, it is drain and the upper surface should be very highest dope possibility.

The reason there we are looking for highest doping is from electrical properties, we know, if any other metal sits on the making a contact to this source and drain, metal semiconductor junction is a diode. It is called Schottky diode. But, we know, if metal n plus plus or metal p plus plus kind of diode is made, heavily dope p semiconductor on if the metal sits on it, the IV characteristics of such a diode is more like an ohmic very low resistance - ohmic contact, okay. What is the problem in circuits? If you see maybe, will this is what I keep saying that, I have all the game for my device.

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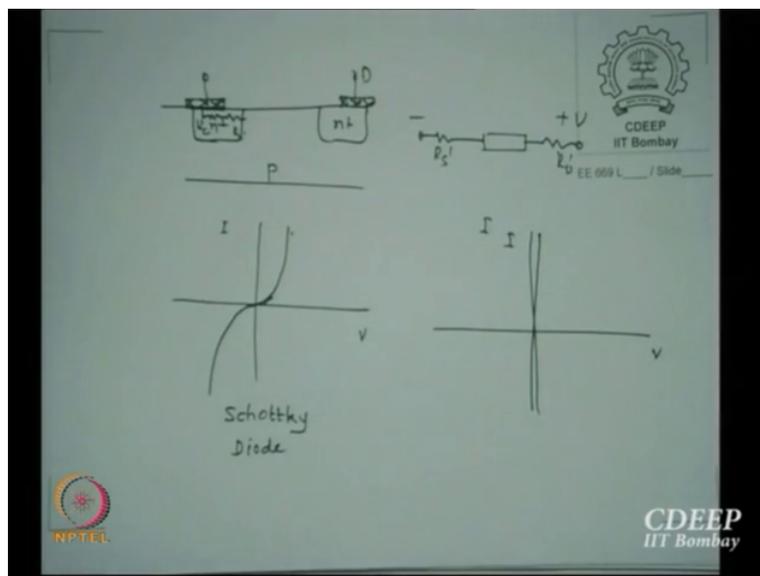
I have shown you earlier figure, but, I will show you again. Let us say, this is my n plus or p plus whichever it is. And I am making contact to shows drain. This is metal, this is semiconductor. Now, if this surface concentration is very high then normal Schottky diode with n this is more like, of course, this is not as good also, but, maybe, slightly bad slope there. Whereas, this is Schottky diode. Any metal semiconductor contact has a barrier and it always shows a

characteristics like a diode. Whereas, “What kind of contact we are really looking for here?” we are looking for a contact, which roughly gives ideally this, but, at least this.

Of course, this I may be a slope may be slightly more or less depending on the process you do, but, I want an Ohmic contact. It is called Ohmic contact.  $R$  is constant across minus  $V$  or plus  $V$ . And it should be very low, typically less than a ohm preferably or 10 ohms or less than 10 ohms or at least, a 1 ohm if possible.

Now, that is not possible because better semiconductor always here, unless, the doping there is extremely high. This is called contact resistance or specific contact resistance. Since, doping is very high, the resistance here may be smaller but this contact resistance may not be smaller. This  $R_{n+}$  and  $R_{\text{contact}}$  are in series and  $R_{\text{contact}}$  may over rule the  $R_{n+}$ . Why  $R_{n+}$  is even smaller? Because, the area there is very high. Source drain areas are much higher than the contact area. So in some sense, if rule by area increases the resistance goes down. So source resistance is smaller compared to the contact resistance same thing true here.

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So, essentially, you are having now parasitic resistance then there is a channel and then another parasitic resistance  $R_{\text{source}}$   $R_{\text{D}}$ . If I apply the voltage plus minus if on in channel, now, I see the current is flowing from here to here or electrons go from source to drain. So, if there is a resistive drop here other than the channel there will be drop in source a regions and

drain regions. So, what may, what does mean the actual VDS is not same as what you actually applied. And what is the implication the current is proportional to VDS and therefore one sees that, at a given gate voltage the current actually has reduced, okay.  $V$  by  $R$ , if  $R$  increases the current decreases.

Now, this issue looks to be trivial when we look from technology side so much, but when I go to circuit, I suddenly find my charging time has certainly increased or discharge time has increased because  $R$ 's are not as small as I expected. Of course, you cannot make it zero any day, but, whatever small possible. So, all the technology details are actually taken care from this circuit point of view. So this issue I am trying to again and again impose on you that do not think we do technology for the heck of it. Yeah, many of us do like heck of it as well and then I say 20 years I spent in lab and I enjoy doing many things. So, when we first were making a Thyristor it never fired, but, after 2-3 turns we realized that where is the why further blocking is not possible? So, we change our mass and we suddenly got firing.

That was a day when I say, okay, I made a Thyristor. It was a very small power transistor. We are just blocking some 50 volts, but, even then that was the Thyristor reverse blocking of 100 volts. Making a device is much more interesting than talking, okay. So do and go and make devices, okay. This is a life as well. Hence, creation is most important part in life and therefore you should learn this. Talking of course is easier, I always challenge all my friends even when I was in (19:41) for that if they can make a diode of a given specification on a wafer.

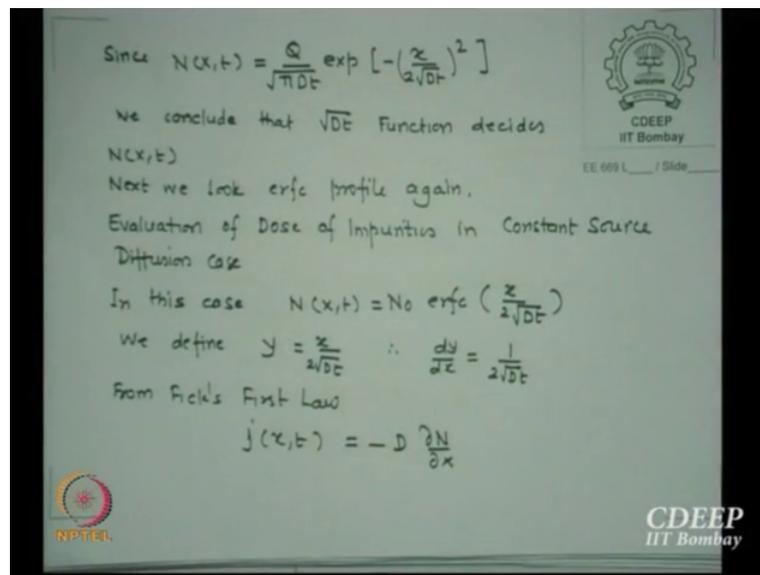
Let us say, it gives a leakage current of say 10 micro, 10 nano ampere to 100 nano. What I fixed, I do blocking voltage of 100, forward resistance of 0.2 ohm at 100 milliamps. Repeat this  $n$  times and see all them, actually, gave this characteristics. This is challenge, because, if I do one device out of 100 and in runs of 100 I get 10 devices, no one is going to pay money for him, okay. So, the problem is reliability and problem you have process standardization. So, making a device in lab is very interesting and you jump, but in real life you cannot jump, because one only is working, okay. So much money he is wasted on that, okay.

So, please take it that in real life making things are very well learned, because you have to keep practicing to know what is being done actually. So, there the theory comes to your help. If this has happened (20:43) this is the reason behind. So, I modified that term, okay. So that in next

process run, I do come closer to what I was looking and if I still do not get it then I go to another friend I say, I try both, “What do you say?” he may have his third idea and try again. So, there is a hit and trial, but based on some thinking what can go wrong, if you do not get your device with results as you thought, okay.

So, device design and process design go hand in hand and when asked to really work hard to get a process going which is what the characteristics, device characteristics you are looking for and therefore, the circuit characteristics you are looking for, okay.

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So a final version of Gaussian profile is  $N \times t \propto \frac{Q}{\sqrt{\pi Dt}} \exp\left[-\frac{x^2}{4Dt}\right]$ , this is  $x^2$  square by  $4Dt$  has been written in square form  $\frac{x^2}{4Dt}$ , which is same as  $\frac{x^2}{4Dt}$ . So, obviously, one can see from here the profile has a direct relationship with  $\sqrt{Dt}$  term, okay. Through exponential and through pre-exponent  $\sqrt{Dt}$  term is appearing both ways and therefore  $\sqrt{Dt}$  will decide the surface concentration and  $\sqrt{Dt}$  will also decide the Gaussian slopes or the depth up to which this - Please, yesterday, I talked about  $D$  has a unit of centimeter squared per second,  $Dt$  as a units of centimeter square. So,  $\sqrt{Dt}$  has this unit of length or centimeters.

So, essentially, now, I am saying for a given temperature where I fix  $D$ , which I given  $D$ , I will now and if I know what is  $Q$  then I would say I will be able to figure out how many impurities are at the surface after a time cycle of  $t_1$  and also up to how deep they will actually move in,

okay. Larger the time they will start moving deeper and deeper whereas by doing deeper diffusions, the penalty I will pay is reduction in surface concentration, because profile is flattening, is pressing as if okay. Next, we look error function profile once again. Let us see, why we were so enamored by only Gaussian.

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Since  $N(x,t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left[-\left(\frac{x}{2\sqrt{Dt}}\right)^2\right]$

We conclude that  $\sqrt{Dt}$  Function decides  $N(x,t)$

Next we look  $\text{erfc}$  profile again.

Evaluation of Dose of Impurities in Constant Source Diffusion case

In this case  $N(x,t) = N_0 \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$

We define  $y = \frac{x}{2\sqrt{Dt}} \therefore \frac{dy}{dx} = \frac{1}{2\sqrt{Dt}}$

From Fick's First Law

$$j(x,t) = -D \frac{\partial N}{\partial x}$$

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So, let us come back and say okay let us look error function. Next we look into error function complement function profile again. What they say, if I introduce at a given temperature time cycle using constant source diffusion some impurity which is error function. So, I want to know, “How many impurities are actually per unit area gone in after that time cycle?” that is the dose. Because, as many impurities per unit area into that distance, if per unit integral of that is area, is the dose. So, I like to know, what is the dose, if I do only error function profiles? Okay. In the case of error function profile,  $N \times t$  is  $N_0 \text{erfc}\left(\frac{x}{2\sqrt{Dt}}\right) \times 2\sqrt{Dt}$ . Remember this term, I brought  $2\sqrt{Dt}$ , because, I wanted to have similar term both sides.

So that, I can write  $y$  is equal to  $X$  upon  $2\sqrt{Dt}$  anywhere, okay, to make integral solution easier, okay. We define  $y$  is  $x$  upon  $2\sqrt{Dt}$  and therefore,  $dy$  by  $dx$ , one of the assumption I am making all through in this that  $D$  is constant at a temperature and we had last time said  $D$  is a function of  $N$  and therefore  $x$ . Our assumption is  $D$  is constant in real life one may have to do more complicated analysis to get the dependence, okay. From fixed first law, we know, the  $j$  is the dose there we defined. This is not  $j$ , is the flux per unit area per unit time.

So,  $j \times t$  is minus  $DN$  by  $dx$ , this is fixed first law, okay. That is what we started with fixed first law. Is that okay? I started with Gaussian, already, I said, okay, this profile I can get. But I want to “Can you think now something?” I want to see this. Can this  $Q$  be created out of the error function profile? That is my ultimate aim I am looking for. Can I create  $Q$  out of that? So in error function, how many impurities per unit area I am can push? Let us see at given time what how many they go in, okay, at a given temperature, because, what is a temperature is going to decide  $Dt$  product. Is that clear? And it also going to decide  $N_0$  value. This is most important there the surface concentration is also decided by this, but, here, also, the pre-exponent is temperature dependent.

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Since  $N(x,t) = \frac{Q}{\sqrt{\pi Dt}} \exp\left[-\left(\frac{x}{2\sqrt{Dt}}\right)^2\right]$

We conclude that  $\sqrt{Dt}$  Function decides  $N(x,t)$

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Evaluation of Dose of Impurities in Constant Source Diffusion Case

In this case  $N(x,t) = N_0 \operatorname{erfc}\left(\frac{x}{2\sqrt{Dt}}\right)$

We define  $y = \frac{x}{2\sqrt{Dt}} \quad \therefore \frac{dy}{dx} = \frac{1}{2\sqrt{Dt}}$

From Fick's First Law

$$j(x,t) = -D \frac{\partial N}{\partial x}$$

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So, for a given temperature  $N_0$  may be constant, but  $t$  may vary and therefore impurities per unit area can be varied by just pushing, either, at higher temperature or longer time. Which will you prefer? I want a sheet charge. Higher temperature lower times because then it will be very shallow this diffusion, but highest of concentrations ahead on the top. So, we say, okay, here is sheet charged, okay. That is what we are trying to do it, okay. So, from the fixed first law  $j \times t$  is minus  $D \frac{dN}{dx}$ .

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At the surface ( $x=0$ ) Flux density is  
$$j(0,t) = -D \left. \frac{\partial N}{\partial x} \right|_{x=0}$$
  
From erfc profile we get  
$$\left. \frac{dN}{dx} \right|_{x=0} = N_0 \frac{d}{dx} \left[ \text{erfc} \left( \frac{x}{2\sqrt{Dt}} \right) \right]$$
$$= N_0 \cdot \frac{d}{dy} [\text{erfc}(y)] \cdot \frac{1}{2\sqrt{Dt}}$$
$$= \frac{N_0}{2\sqrt{Dt}} \frac{d}{dy} (1 - \text{erf}(y))$$
$$= - \frac{N_0}{2\sqrt{Dt}} \frac{d}{dy} [\text{erf}(y)]$$
$$= - \frac{N_0}{2\sqrt{Dt}} \cdot \frac{2}{\sqrt{\pi}} \cdot \exp(-y^2)$$

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At the surface, the flux density  $j(0,t)$  is minus  $D$  and  $dN/dx$  at  $x$  is equal to  $0$ . At  $x$  is equal to  $0$  term we can evaluate  $dN/dx$  at  $x$  is equal to  $0$  and then say, okay, this is the flux available at  $x$  is equal to  $0$  plus density. So, using the  $N(x,t)$  profile for a complementary error function, I write  $dN/dx$  at  $x$  is equal to  $0$  and  $0 \cdot d/dx$  of error function profiles. I already made the  $y$  term as  $x$  upon  $2\sqrt{Dt}$ . So, I can write  $N_0 \cdot d/dx$  into error function, and, why this term is appearing?  $1$  upon  $2\sqrt{Dt}$  additional term product term appeared. What for?  $y$  is equal to  $x$  upon  $2\sqrt{Dt}$   $dy/dx$  is  $1$  upon  $2\sqrt{Dt}$ . So, if you make  $d/dx$  then I can write  $d/dy$  into  $dy/dx$ . Is that correct?

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At the Surface ( $x=0$ ) Flux density is

$$j(0,t) = -D \left. \frac{dN}{dx} \right|_{x=0}$$

From erfc profile we get

$$\left. \frac{dN}{dx} \right|_{x=0} = N_0 \frac{d}{dx} \left[ \text{erfc} \left( \frac{x}{\sqrt{4Dt}} \right) \right]$$

$$= N_0 \cdot \frac{d}{dy} [\text{erfc}(y)] \cdot \frac{1}{2\sqrt{Dt}} \quad \frac{d}{dx} = \frac{d}{dy} \cdot \frac{d}{dx}$$

$$= \frac{N_0}{2\sqrt{Dt}} \frac{d}{dy} (1 - \text{erf}(y))$$

$$= -\frac{N_0}{2\sqrt{Dt}} \frac{d}{dy} [\text{erf}(y)]$$

$$= -\frac{N_0}{2\sqrt{Dt}} \cdot \frac{2}{\sqrt{\pi}} \cdot \exp(-y^2)$$

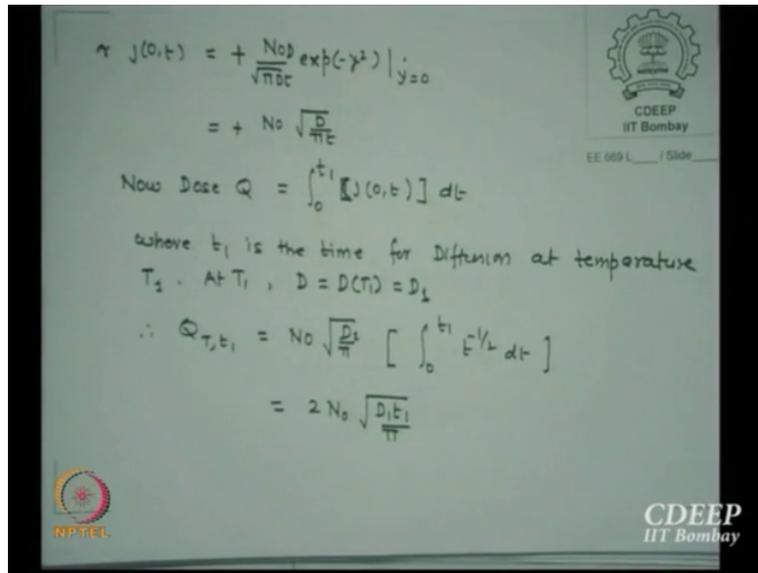
Anything, I can write  $d$  by  $dx$  can be written as  $d$  by  $dy$  into  $dy$  by  $dx$ , okay. So, since, I have simple algebra here calculus part here. This is that  $dy$  by  $dx$ , this is your  $d$  by  $dy$  error function  $y$ , this is  $N_0 / (2\sqrt{Dt})$   $d$  by  $dy$   $1 - \text{complementary function}$  is  $1 - \text{error function } y$ . So, I write  $d$  by  $dy$  minus error function  $y$ . And, well constant the differential of  $1$  is  $0$  and differential of this is still, so, minus sign appears minus  $N_0$  upon  $2\sqrt{Dt}$   $d$  by  $dy$  of error function  $y$ . Is that clear? The idea is to get a term which is algebraically I can find, okay. Error function  $y$ , if I differentiate, yesterday, I gave you a table for all of them.

So, differential of error function  $y$  is  $2$  upon  $\sqrt{\pi}$  exponential minus  $y$  square. Now, this term is  $d$  by  $dy$  of this error function  $y$ . This  $N_0 / (2\sqrt{Dt})$  was pre-part of that. So this is, now,  $dN$  by  $dx$  at  $x$  is equal to  $0$ . Why we are interested in? Because, I want to know the flux density at  $x$  is equal to  $0$ . How much is impinging? Okay. I repeat, I am just trying to find the flux density at  $x$  is equal to  $0$ , I use Fick's law, I figure out if I take  $dN$  by  $dx$  is equal to  $0$  multiplied by diffusion coefficient at given temperatures.

Then I can differentiate, I use  $N$  is equal to  $N_0 e^{-x^2 / (4Dt)}$  complimentary function  $x$  by  $\sqrt{4Dt}$ . Try to rewrite that  $dN$  by  $dx$  by differentiating this replace  $x$  by  $2\sqrt{Dt}$  to  $y$  and therefore,  $dy$  by  $dx$  term appears. And, when I expand a complimentary to  $1 - \text{error function}$  I differentiate that. So,  $1$  differential goes  $0$  but the other one is minus  $d$  by  $dy$  error function  $y$ . But, I know, the differential of error function  $y$  is  $2$  upon  $\sqrt{\pi}$  exponential minus  $y$  square. This

is from the error function theory. Since, I know this, I know this, I know  $dN$  by  $dx$  and, since, I know my  $dN$  by  $dx$ , if I multiply by minus  $d$ , "What do I get?"  $j_0 t$ .

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$$j(x,t) = + \frac{N_0 D}{\sqrt{\pi D t}} \exp(-x^2) \Big|_{y=0}$$

$$= + N_0 \sqrt{\frac{D}{\pi t}}$$

Now Dose  $Q = \int_0^{t_1} [j(x,t)] dt$

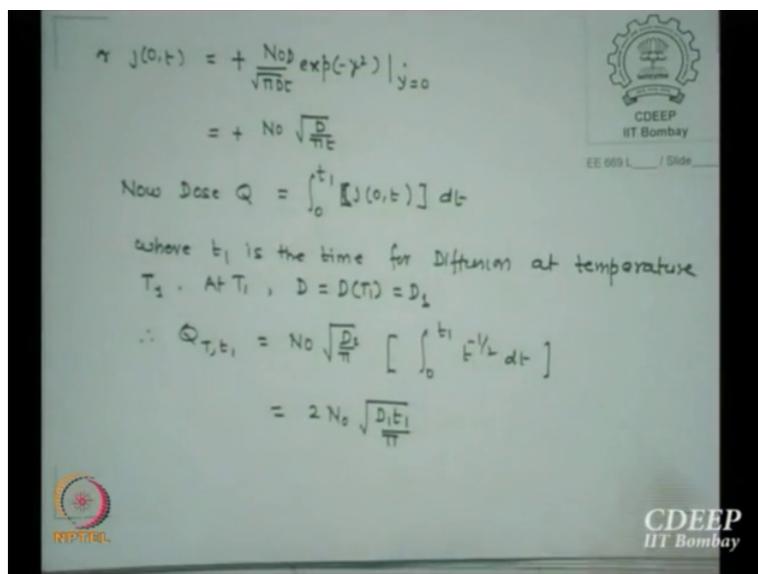
where  $t_1$  is the time for Diffusion at temperature  $T_2$ . At  $T_1$ ,  $D = D(T_1) = D_1$

$$\therefore Q_{T_1, t_1} = N_0 \sqrt{\frac{D_1}{\pi}} \left[ \int_0^{t_1} t^{-1/2} dt \right]$$

$$= 2 N_0 \sqrt{\frac{D_1 t_1}{\pi}}$$

So,  $x$  is equal to 0 is same as  $y$  is equal to 0. Is that okay?  $y$  is equal to  $x$  by  $2 \sqrt{Dt}$  means, when  $x$  is 0,  $y$  is also 0.

(Refer Slide Time: 30:47)



$$j(x,t) = + \frac{N_0 D}{\sqrt{\pi D t}} \exp(-x^2) \Big|_{y=0}$$

$$= + N_0 \sqrt{\frac{D}{\pi t}}$$

Now Dose  $Q = \int_0^{t_1} [j(x,t)] dt$

where  $t_1$  is the time for Diffusion at temperature  $T_2$ . At  $T_1$ ,  $D = D(T_1) = D_1$

$$\therefore Q_{T_1, t_1} = N_0 \sqrt{\frac{D_1}{\pi}} \left[ \int_0^{t_1} t^{-1/2} dt \right]$$

$$= 2 N_0 \sqrt{\frac{D_1 t_1}{\pi}}$$

So, at  $j_0 t$  at  $y$  is equal to 0, which means  $x$  is equal to 0. If I take the value of this, the so-called flux density at  $x$  is equal to 0 is  $N_0 D$  upon  $\pi t$  under root of  $D$  by  $\pi t$ .

And, this is very important this is  $j_0 t$ . Please, remember, this is only  $j_0 t$ . How do I, why are interested in? I will, now, say, over the time this impurities are getting in but  $N_0$  remaining constant. So, I am now going to integrate for any time and figure out after a time  $t_1$ , "How many impurities have actually per unit area has reached by  $t_1$ ?" okay, which is called area under the curve, okay. So, I say, the dose is now 0 to  $t_1$  for which diffusion was performed  $j_0 t dt$ , is that okay? Once, I get surface flux density I integrate that for a given time  $t_1$  and find the actual dose in a given time  $t_1$  which silicon has received, okay.

So, if I different integrate  $j_0 t$ , I substitute this term here. Please, remember the convention which I am going to use at any temperature  $T_1$ . The  $D$  at that  $T_1$ , I will call it  $D_1$ . At any other temperature  $T_2$ , I will call it  $D_2$ . Third temperature I may call it  $D_3$ . And, so, on so for so, I do not have to write  $D T_1 D T$  capital  $T$  every time, okay. So, then the  $Q$  at the given temperature for a given time  $t_1$ , you integrate this and if you integrate this, this is very simple  $t$  to the power minus half plus half upon half. So, that gets  $2$  and  $0 D_1 D_1 T_1$  upon root  $\pi$ .

So, now, I know, if I do a error function profile that means I do impurities entering for a given time  $t_1$  at temperature  $T_1$  then so many impurities per unit area can be deposited close to the surface. Of course, there is a burst there, this is not, this is the total area under the curve is what we are received, okay. Now, this fact that this charge has something to do with  $t_1$ . So larger the time I do impurity will get inside more and more. So, the area under the curve will keep on increasing. Higher the temperature I do  $D_1$  will increase, so again, but both are under root functions. So, root functions, so, they do not go proportionately.

So, if I just increase time root of that time only the corresponding charge density increases. Is that point clear? This is relevant, because, suddenly, you made 10 minutes to 20 minutes, do not say, they double, okay. It will not double. So, root 2 times it will go, okay. So, this fact is also used in our analysis or in our actual process, okay. If I double it how much I will get? Okay. So, I know, roughly, how much I should be able to get, okay. So, this is a very important term which I will use it again, this I will use it in actual junction formations. Please remember the charge density in a complementary error function profile.

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$$j(0,t) = + \frac{N_0}{\sqrt{\pi b t}} \exp(-\gamma^2) \Big|_{y=0}$$

$$= + N_0 \sqrt{\frac{D}{\pi t}}$$

Now Dose  $Q = \int_0^{t_1} j(0,t) dt$

where  $t_1$  is the time for Diffusion at temperature  $T_1$ . At  $T_1$ ,  $D = D(T_1) = D_1$

$\therefore Q_{T, t_1} = N_0 \sqrt{\frac{D_1}{\pi}} \left[ \int_0^{t_1} t^{-1/2} dt \right]^{N_0(t)}$ 

$$= 2 N_0 \sqrt{\frac{D_1 t_1}{\pi}}$$

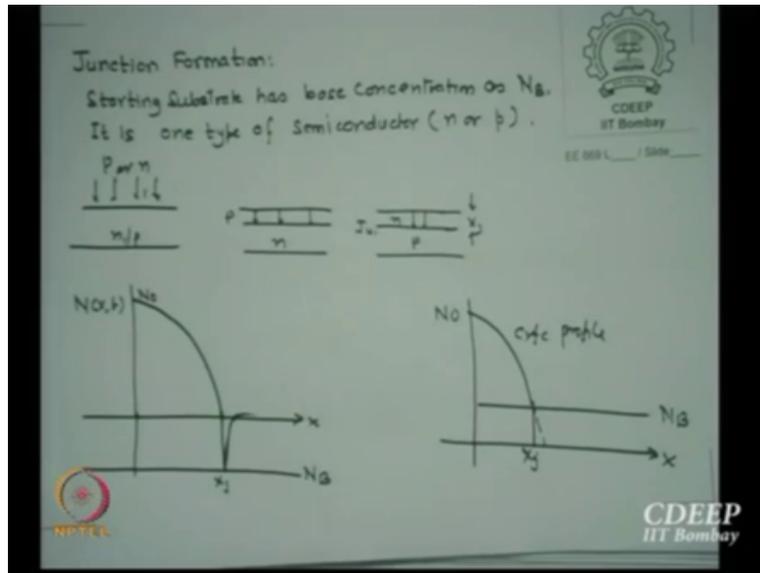
Let us look at a complementary error function profile, this is  $n_0$ , this is  $N \times t$  and this is any given time  $t_1$ , okay. Now, if I do much smaller time say  $t_1$ . I can make even shallower, okay. If I do a smaller time, I can make shallower or I can also make both small temperature higher but this small. So, I - sorry, it is not coming well. So, I can also make slightly higher and 0 and sharply following because time cycle is even reduced. In both cases I may maintain queue, okay. I increase the temperature and decrease time proportionately. So, the details change, so, I may have same charges. But, I may have more closer to sheet charge. Is that clear?

So to create the sheet charge is much easier, if I do my first diffusion as complementary error function. Use this as the available charge density in the case of, available density in the case of Gaussian, there that Q term was appearing. Is that clear? In Gaussian profile, this Q you can use now. Is that clear to you? This Q we can use there because first profile we did with complementary and then start actually heating cycles without source, okay. So that will become Gaussian profiles with initial charge of this much, okay.

So this is the trick we will come back to this little later. So point I was saying, I know how many impurities per unit area are just at the close to the surface after my so-called error function diffusions, okay. The given time and temperature of my choice, I can decide what profile I get

and therefore what queue I will get. I can always evaluate this. All this analysis has been done for what purpose we want to see junctions or devices meet.

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The first important thing for us is the junction formation for which all these diffusion process has been introduced. You can have a starting substrate either n kind or p kind, okay. And we call the concentration as the base concentration or background concentration, either, it is called base concentrations or it is also called background concentration. So a wafer can be which is a background to you, can be either n type or p type. All that, I will do is, if it is a n type wafer to make a junction p n junction. I must put impurities of p type. If it is a background is p type, you will do n type impurity diffusion to make p n junction. Is that point clear? If I do p n and it is a p n junction, if I do n and p it is a n p Junction. But, it is still a diode, okay.

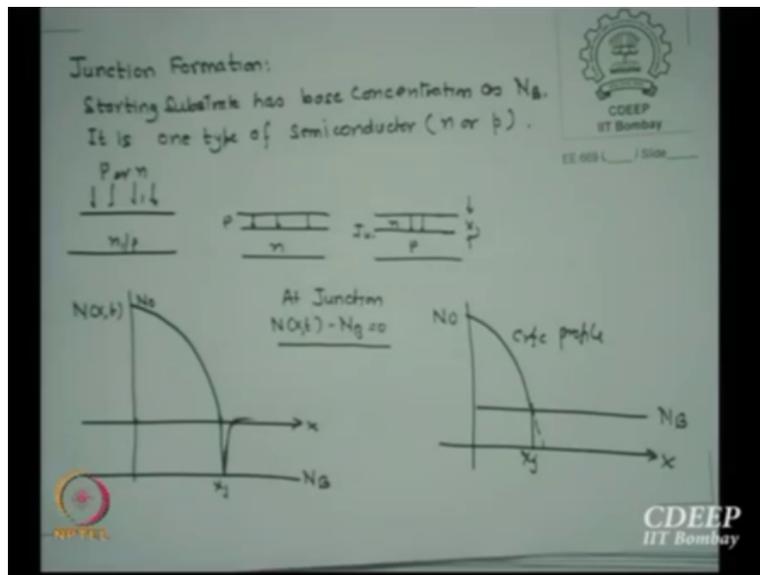
Now, or maybe you can see from here separately shown, if it is n wafer, I have a p area on the top, if I have p wafer. So, impurities do go inside of the other kind and they form a junction, okay. We will see how with that junction. So, if you see a profile, normally, and NB will be opposite that of the impurities you are introduced to make a junction, okay. So, if this is one may say plus and minus sign. So, if this n is positive, the background is the opposite minus of that.

You can also show this way, which is how most books show. So, if I have a background concentration of minus NB or whatever it is. And, I introduce this impurities inside and they start

getting inside the wafer for a given temperature cycle, okay, for at a given, a given temperature and given time. So,  $D \sqrt{t}$  something I fixed, so the impurity profile is something like this. As the impurities start from the surface their concentration starts reducing.

Let us say, the background concentration was some number, which is smaller than the surface concentration we start with. So, as this concentration start decreasing at certain point, the net acceptor impurities are equal to net donor impurities. Assuming all are ionized which may or may not be true every time. All electrons and whole concentrations are equal or they neutralized recombine themselves. It is called compensations, is that call, is called compensation. So, a p type impurity will compensate the n type impurity and then that compensation becomes 0 or concentration both sides are we call that say junction, at the so at actual junction, what is the net concentration? Zero.

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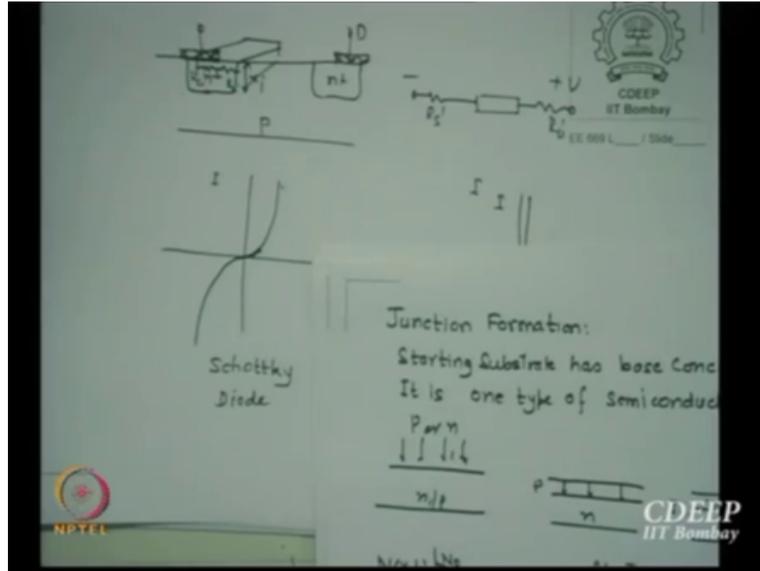
At junction the net concentration  $n$  minus  $p$  or in this profile minus  $N_B$  is always - so for a junction  $N(x) - N_B$  at junction or  $N(x) = N_B$ . Is that clear? Alternately, saying, whenever the profile concentration goes to the background concentration junction actually is formed at that point. You can see I intentionally made this at this point, whenever they will become this, the actual concentration will be 0 and we say there is a junction sitting there.

Now, can you tell me how do I find an  $x$ ? Similarly, many books and many simplest way. Do not look at for the signs, just put NB on the same scale as that of  $n$   $x$ . And wherever, this  $n$   $x$  profile intersects NB that is your junction. Is that clear? Whenever  $n$   $x$  profile in, actually, intersects with  $n$  B, you say, you have a junction. In reality, this is happening, but you can also represent in this fashion it is equally because we are anywhere not showing you what is internal to device. What we are saying calculations. So, for this program method is also equally good.

Whatever I said, I may repeat again by writing, if you have drawn already, please, may I?  $N$   $x$   $t$  profile intersecting with the background concentration, you have the junction. And  $x$   $j$  word was used, sorry, I forgot that. This distance from  $x$  is equal to 0 or this distance from  $x$  is equal to 0 is called junction depth. So in this case, what is junction depth? Whatever the impurities, p type impurities or n type impurities touches the background at that point is the junction depth. So, if I want to make larger junction depth, what should I do? I should put more time for impurities to get in, so that the junction depth is larger or if you want smaller I should not put lot of temperature time cycles. So that junction depth are smaller.

In the case of contact resistance maybe we will come back to that figure. So this is what one of the values for a junction I want to calculate it is  $x$   $j$  and what is the  $N$  0? You will be actually knowing complementary error function and in the Gaussian what will happen? This value will be  $N$  s  $N$  0 upon  $\pi$ , whatever, function I wrote. That will be the  $N$  0 for the case of Gaussian. But, just a minute that figure which I had drawn for you.

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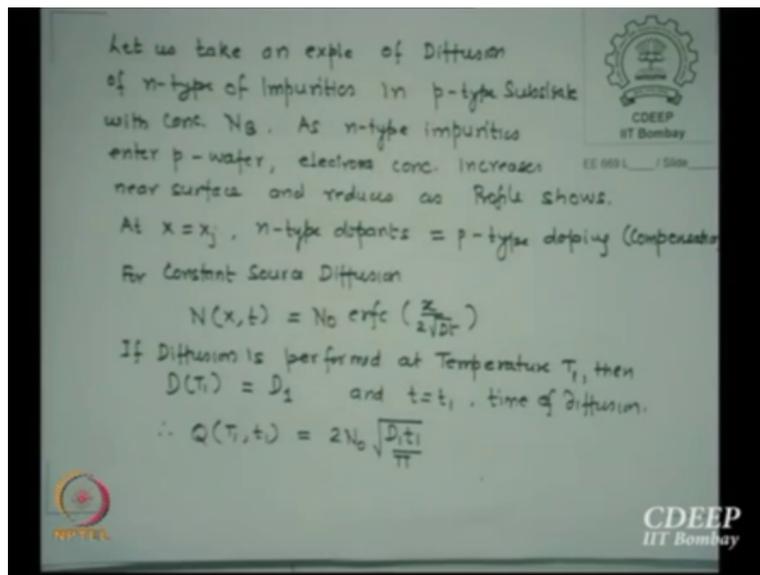


You can see from here, the junction depth in a mass transistor for source and drain is how much? Impurities, we are coming from top source and going down. So, this is my  $x_j$  and if I calculate, this is the three-dimensional area actually, if you see it. So, if I see here semiconductor bar there of  $n$  plus diffusion,  $n$  plus area. Larger the  $x_j$ , larger is the cross sectional area, area is this. So, larger the junction depth resistance is smaller. But, when I scaled the device now, I am looking for 5 micron down to 14 nanometer, 11 nanometer, 10 nanometers. I do not know, 0 nanometers. This source drain areas will become smaller and smaller.

So, what increases? Resistance. Contact resistance is increasing, because area is decreasing. The resistance have source drain increasing. So what is the implication? The actual speed goes down, okay. All the currents are your volt is scaling down. So, you are not allowing larger current to flow because of excess voltage. You are reducing voltage, you are decreasing junction depth scale down. So, one of the major hits a speed, okay, major hit a speed. Of course, how do we recover some other day, but this is one possible. So, scaling looks to be very good in every sense for everyone said it.

Here is a problem, the first time we hit is the source drain resistance and their contact resistance should also, will also start increasing as scaling technology goes down. As, 90 nanometer it was safer, 14, 16 will be even difficult and 9, 11, 7 may be even more difficult. But, I told you once which is my friend statement, we all understand physics or at least think we understand physics, but silicon does not. So, it behaves as it wants and we keep, we are left think - why it is thinking that way, okay. That is the fun part in that. Okay.

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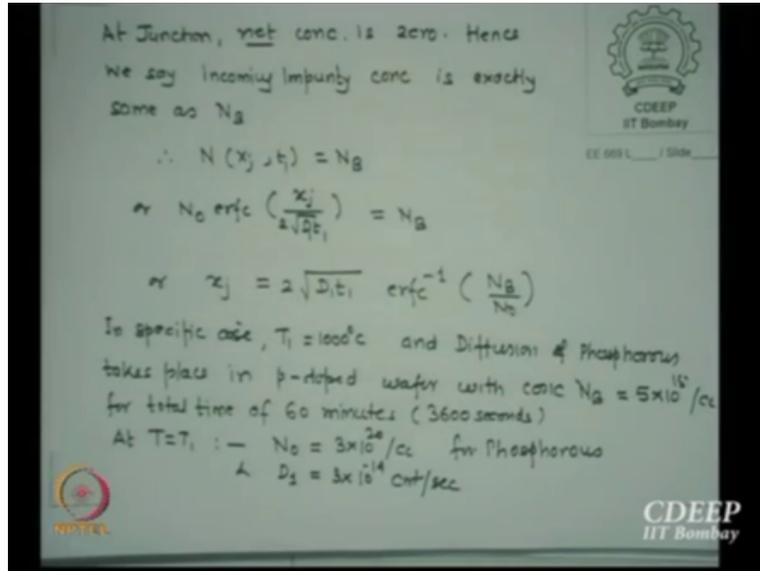


Let us take an example of diffusion of n type impurity p-type substrate, n-type impurities enter p – wafer electron concentration increases near the surface and reduces as profile shows at x is equal to  $x_j$ , n-type dopants is equal to p-type doping and this is called compensation, okay. If they do not compensate there will no junction, okay. If they do not compensate there is no point anywhere, this will - that will happen p and n will actually merge everywhere and there will uniform  $N_i$ . But it does compensate and therefore you do get junctions.

Now, here is we already said, if I do pre-constant source diffusion this is the profile  $D(T_1)$  is  $D_1$   $t$  is equal to  $t_1$  is time of diffusion and the impurities which I pushed at the surface, because the depth is very small let us say  $2 N_0 \sqrt{D_1 t_1}$  by  $\pi$ , okay. Please remember once again at junction the net concentration is 0. Net means, background the doping profile minus background is always 0 at junction. That is what the why the definition is of the junction, okay. Sushant say, I do, I write

again now, so, whatever, I said please note down. This is relevant because, I am going to create queue out of error function profile.

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So, again, I said at junction net concentration is 0. Hence, we say, incoming impurity concentrations is exactly same as  $N_B$  and therefore we say  $N_0$  error complementary function  $x_j$  upon root  $D_1 t_1$  must be equal to  $N_B$  at what point  $x$  is equal to  $x_j$ . Now, this is known function.  $N_B$  is known to me. What does it, what is known to me?  $N_B$  is known to me starting substrate.  $N_0$  is known to me. Why? I fix the temperature.

So  $D_1 t_1$  is known to me. Is that clear? Everything is known, what is not known is  $x_j$ . So, if I expand this  $x_j$  is  $2 \text{ root } D_1 t_1 \text{ complementary error function inverse } N_B \text{ by } N_0$ , okay,  $x_j$  is  $2 \text{ root } D_1 t_1 \text{ complementary error function inverse of } N_B \text{ by } N_0$ , so, give up. So, since, error function is a function I need the value which I will get it from its table and I will show you soon what those tables are, okay. So knowing  $N_B/N_0$  because I know  $t_1$ , I know  $D_1 t_1$ . So, I can evaluate the junction depth. And, I can therefore, you can now quickly see larger the temperature.

This term will increase, this is complementary  $1 - \text{something}$ . So, even if this increases this complementary will also increase. Please remember complementary is  $1 - \text{something}$ , if do not increases  $1 - \text{that term}$ , actually, enhances. So,  $x_j$  is larger, larger the time you diffuse or larger the, at larger temperature you diffuse. This is how controls are there. But, this is only one control. That is I have one fixed temperature and one fixed time. But, I want little more number of variables for me, okay. Maybe one example before we start the other part.

Let us take I am doing a diffusion of phosphorus in a p-type wafer which is doped  $5 \times 10^{15}$  per CC. This numbers are typical numbers not necessarily for every device. Most devices mass transistor starts with  $5 \times 10^{15}$  when we are working below about 100 nanometers. Now, this concentration may be  $10^{18}$  or  $2 \times 10^{18}$ , as the scaling down is happening. Why are actually increasing has anyone taught you this process (49:07) given you some idea. As you scaled down to keep that scale factor everything has to be readjusted by scale factors. So, concentration in the substrate has to be increased to get  $VT$  of what you are looking for.

So, initially, let us give some example  $5 \times 10^{15}$  per CC substrate. I am diffusing impurities of phosphorus at 1000 degree. And let us say, I do 60 minutes of diffusion. Please take it diffusion coefficient is centimeter square per second. So time must be declared in seconds. So it is 3600 seconds. So, please take when you do calculations do not make, do not put 60, put 3600, 60 time things may change, of course, since, mostly under root. So, at least eight times of seven and I eight times it will be. So, which may be a large variation in actual values.

Now, we can, I will show you graph soon. So at  $T$  is equal to  $T_1$   $N_0$  is  $3 \times 10^{20}$  for phosphorus. This is data is available to us. At  $T$  is equal to  $T_1$  which is thousand degrees  $N_0$  is  $3 \times 10^{20}$  per CC for the phosphorus. At that temperature of 1000 degree the diffusion coefficient of phosphorus since silicon is  $3 \times 10^{-14}$  centimeter square per second. This data is made available to you either by me or by book or by the model P, modeling in the software, okay.

Since, I know  $D_1$ , I know  $t_1$ , I can calculate, please note down. This is a problem I am solving. I want to know  $x_j$  for this case. I have a substrate concentration  $5 \times 10^{15}$  at 1000 degree for 60 minutes. I introduced n-type phosphorus impurities and then try to see what is the

junction depth? Up to which phosphorus has gone in? Okay, to form a junction that is my ultimate aim. So, I chose these values of randomly. You can in real life I may give you some other values and you will have to solve for that, okay. Is that okay?

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$\therefore \sqrt{Dt} = \sqrt{3 \times 10^{-14} \times 3600} = 1.04 \times 10^{-5} \text{ cm}$   
 Further  $\text{erfc}^{-1}\left(\frac{N_B}{N_0}\right) = \text{erfc}^{-1}\left(\frac{5 \times 10^{15}}{3 \times 10^{20}}\right)$   
 $= \text{erfc}^{-1}(1.66 \times 10^{-5})$   
 As  $\text{erfc}(x) = 1 - \text{erf}(x)$  . let us define  $\text{erfc } x = 1.66 \times 10^{-5}$   
 $\therefore x_j = 2\sqrt{Dt} \cdot 3.06$  or  $\text{erf } x = 1 - 1.66 \times 10^{-5}$   
 $x_j = 2 \times 1.04 \times 10^{-5} \times 3.06$   $= 0.999984$   
 or  $x_j = 0.636 \mu\text{m}$   $\therefore x = \text{erf}^{-1}(0.999984)$   
 $= 3.06$

So, I calculate, you know, always, since, I solved myself, I write all steps you need not, okay, right now. You may your side do anyway, “Aap hath se kuchh calculate Karte nahin. Aap calculator par jaiye” that is much easier. I normally, do not use calculators. So, I just do some quick analysis. Hopefully, they are okay.

Maybe, you know, most times I can assure you my second decimal may not be same but otherwise I am very accurate in numerical calculations otherwise. Okay. Of course, here it was so trivial but otherwise also. So, I get error function inverse NB by N 0 is this. So which is 1.66 into 10 to the power minus 5. So we will recall back complementary error function x is 1 minus error function x. Is that clear? Okay.

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$\therefore \sqrt{Dt} = \sqrt{3 \times 10^{-14} \times 3600} = 1.04 \times 10^{-5} \text{ cm}$   
 Further  $\text{erfc}^{-1}\left(\frac{N_B}{N_0}\right) = \text{erfc}^{-1}\left(\frac{5 \times 10^6}{3 \times 10^{20}}\right)$   
 $= \text{erfc}^{-1}(1.66 \times 10^{-5}) = x$   
 As  $\text{erfc}(x) = 1 - \text{erf}(x)$  . ~~Let us say  $x = 1.66 \times 10^{-5}$~~   
 Let us define  $\text{erfc } x = 1.66 \times 10^{-5}$   
 or  $\text{erf } x = 1 - 1.66 \times 10^{-5}$   
 $= 0.999984$   
 $\therefore x = \text{erf}^{-1}(0.999984)$   
 $= 3.06$   
 $\therefore x_j = 2\sqrt{Dt} \cdot 3.06$   
 $x_j = 2 \times 1.04 \times 10^{-5} \times 3.06$   
 or  $x_j = 0.636 \text{ } \mu\text{m}$

Let us define error function  $x$ . Let us say, this is  $x$ . So, what will happen complementary error function  $x$  is this value. If I re-find complementary error function inverse  $1.66 \times 10^{-5}$  to power  $x$ . Then I say complementary function of  $x$  is this value. Is that okay? Yes, inverse. Okay. So, I bring inverse on this side, so, error function complementary function  $x$  is essentially  $1.66 \times 10^{-5}$ . Therefore, error function  $x$  is  $1 - 1.66 \times 10^{-5}$ . Achchha, now from where this I will show you  $0.999984$  this is the value after subtraction. Please remember this  $1.66 \times 10^{-5}$  should not be thought smaller and neglected. This number is very crucial  $0.999984$  by subtraction. Therefore,  $x$  is now this is error function  $x$ .

So, therefore,  $x$  is error function inverse. Is that correct? I want  $x$  to be multiplied with this for that term. So  $x$ , I am now say is how much? The reason why I did this because the table which I have available is only error function table. For a given  $x$  the value of error function  $x$  is known to me. So, what do I do? How do I get  $x$  then? I start looking from the error function table this value  $0.9999$  and to the left of it, what is the  $x$  value? I actually find out from there. Is that point clear?

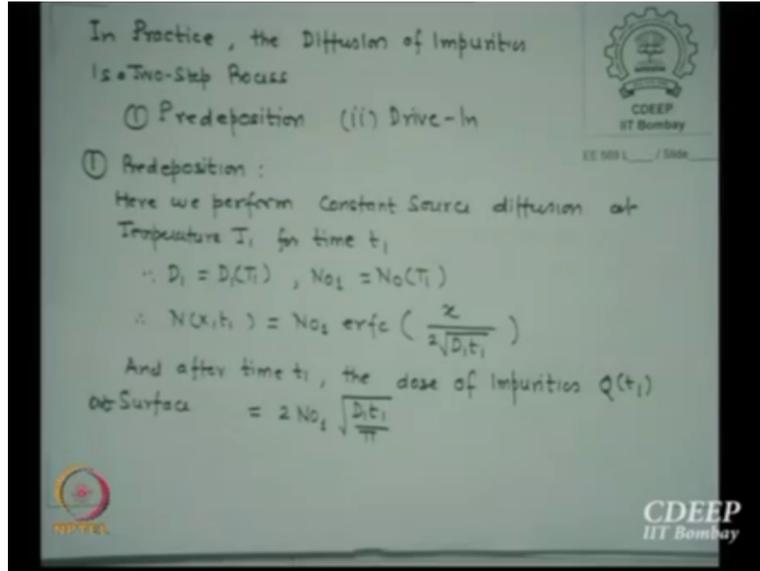
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$\therefore \sqrt{Dt_1} = \sqrt{3 \times 10^{-14} \times 3600} = 1.04 \times 10^{-5} \text{ cm}$   
 Further  $\text{erfc}^{-1}\left(\frac{N_B}{N_0}\right) = \text{erfc}^{-1}\left(\frac{5 \times 10^{12}}{3 \times 10^{20}}\right)$   
 $= \text{erfc}^{-1}(1.66 \times 10^{-5}) = x$   
 As  $\text{erfc}(x) = 1 - \text{erf}(x)$  . ~~Let us say  $x = 1.66 \times 10^{-5}$~~   
 Let us define  $\text{erfc } x = 1.66 \times 10^{-5}$   
 or  $\text{erf } x = 1 - 1.66 \times 10^{-5}$   
 $= 0.999984$   
 $\therefore x = \text{erf}^{-1}(0.999984)$   
 $= 3.06$   
 $\therefore x_j = 2\sqrt{Dt_1} \cdot 3.06$   
 $x_j = 2 \times 1.04 \times 10^{-5} \times 3.06$   
 or  $x_j = 0.636 \mu\text{m}$

So I have a table which he says, x an error function x. So, I have so many x point to point 0, 1, 2, 3, 4, 10, 4 up at least given these are, so, I says this value somewhere here and corresponding to this I get value from the table which is my x. Once, I know my x, I know the pre-exponent is 2 No 1 by D1 t1 that multiplied by this term. I will get junction depth. So x j is root 2, 2 root to D1 t1 minus into 3.06 x j is multiplied up this which is 0.636 microns. So, if I substrate of 5 into 10 to the power 15 per CC boron or boron doped p-type and I introduce phosphorus impurity that 1000 degree centigrade for 60 minutes, the impurities will go as deep as point 0.6 microns or 0.63 microns or 0.64 microns which your numbers you feel correct. This value is some way measure how deep impurities have gone in.

Please remember this earlier we used to say less than a micron is a shallow junction, okay. Now 0.6 is the deep junction, okay. The shallow junctions will be of the order of 100 nanometers, okay. So it is something numbers are change. So earlier books myself would keep saying, "This is very shallow" actually, this is extremely deep junction, okay. Compared to the earlier time, so this, scale down technologies all terminology also had changed. The micro-world has suddenly become nano, okay. If you really see the nanotechnology only one dimension is going into nano, okay. If I reduce width and lengths too small, length I may but width I small. Then you take from me there is no current, okay. W by L i is proportional W by L.

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So, if I do not put large widths, no current is possible. So, “Idhar toh bahut chhota - wota kiya aur udhar aisa lamba kardiya. Toh yeh sab game hi hain” All that we are done is since it did not fit to the actual values, “Toh usspe polynomially  $A_0$  plus  $A_1 x$  plus  $A_2 x^2$  by  $A_3 A_99 x$  to the power 99, “fit toh baithne wala hain koi bhi ho.” In practice, therefore, as I already now said that, I can create by a complementary error function fixed impurities, surface charge  $Q$  and that step. So we do two step process for any diffusion. So one is called pre-deposition and the other is Drive-In.

Now, there is a joke many years ago, some students kept on telling me driving. I say “Ki woh car nahin hain” but still he kept on saying the word driving. So, I said, no it is Drive-In but that is what I am saying, driving okay. M is M and M, so, if is you ask any Tamilian, do regards, if say M say M. I say it is M. Yeah, that is M. So that is the way life is interesting part. So, if I do pre-depositions, basically, pre deposition is a complementary error function diffusion with constant source or infinite source or whatever word you want to use. I do it at temperature  $T_1$  and time  $t_1$ .

So, I know  $d_1$ . I know  $N_0$ , I know therefore  $N_0$  times complementary error function  $x$  upon  $\sqrt{D_1 t_1}$ . This is the profile I got and for this I have calculated the dose which is  $2 N_0 \sqrt{D_1 t_1}$ , why I put 1? Because, this is first step. So, I said 1 next maybe 2, okay.  $N_0$ , because, Gaussian will have another value. So here I call it you know,  $1/\sqrt{D_1 t_1}$  by  $Q$  is essentially the charge per unit area at the surface, okay. So the first step in diffusion is creating the sheet equivalence of sheet charge which is called Predepositions, okay. So I fixed those. This is all that we did, just now I did. I am just rewriting that. I just said that a given Predeposition is essentially a constant source diffusion for a given time  $t_1$  at temperature  $T_1$ . The after, which  $N_0$  is known this and therefore I know the surface concentration.

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(ii) After Predeposition step, source of impurities are stopped. Then the wafers are reintroduced in the furnace at Temperature  $T_2$  for time  $t_2$  this process is continued. This process is called Drive-In.

At  $T_2$ ,  $D_2(T_2) = D_2$ ,  $\therefore Dt = D_2 t_2$

In Drive-In, no new impurities are introduced, but available ones from Predeposition are re-distributed (Gauss). This is the case of Limited Source Diffusion giving Gaussian Profile:

$$N(x, t_1, t_2) = \frac{Q}{\sqrt{\pi D_2 t_2}} \exp\left(-\frac{x^2}{4 D_2 t_2}\right)$$

$$N(x, t_1, t_2) = \frac{2 N_0 \sqrt{D_1 t_1}}{\sqrt{\pi}} \exp\left(-\frac{x^2}{4 D_2 t_2}\right)$$

After the Predeposition step the second step is, since, the source of impurity that I now stopped. So, what I do it? I have put a wafer inside furnace will show the technology and for a while I allow impurities to get in, okay, for a given temperature and time cycle. Is that clear? Once, I have that impurity deposited, I will take the wafer out and will see that it has the gas has to be edged and then that means certain amount of impurities on the silicon have been deposited by Predeposition step, okay. Take the wafer out clean it again and that process to be safe we will come back and then put again in the furnace at any other temperature. It can be same or any other temperature.

Let us say,  $T_2$  which for which  $D_2$  is known as such we know that. So we do it this second cycle is called Drive-In. Why word was used in? You have certain amount of impurities which you are Drive-In, Drive-In inside, they are pushed in, okay. So they are driven inside the wafer and therefore the cycle was called Drive-In cycle, okay. This can be a time  $t_2$  at temperature  $T_2$ . Please remember you can still have  $T_1$ ,  $D_1$ ,  $T_1$ ,  $T_1$  same as but that is not done. We will show you why, okay. So, we have at temperature  $T_2$   $D_2$  is the diffusion coefficient,  $Dt$  is  $D_2 t_2$ . In Drive-In, no new impurities are introduced whatever earlier only will keep inside go inside as time elapses. But, otherwise once Predeposition are Re-distributed.

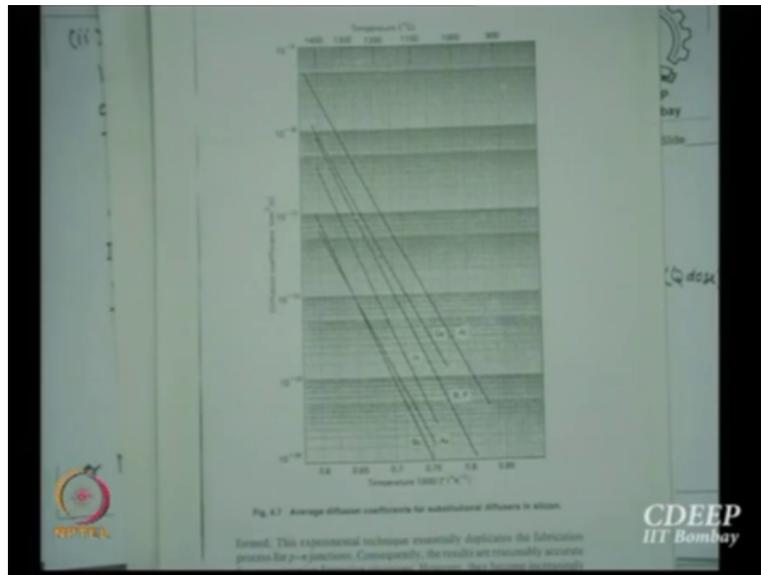
This is the case of limited source diffusion and therefore we write the new profile now. What were the profile Gaussian profile?  $Q$  upon  $\sqrt{\pi D_1 t_1}$  exponential minus  $x^2$  upon  $4 D_1 t_1$ . That was the Gaussian profile. In this case what is the time temperature cycle we have?  $D_2 T_2$  for the Drive-In cycle  $D_2 T_2$ . However, this  $Q$  is coming from where from the first Predeposition cycle, which is essentially what we just now derived, which was the number which we get  $2 N_0 \sqrt{D_1 t_1}$  by  $\pi$ .

So, if I substitute this  $Q$  here. So I get  $2 N_0 \sqrt{D_1 t_1}$  by  $\pi$  under root of  $D_1 t_1$  by  $D_2 t_2$  exponential minus  $x^2$  upon  $4 D_2 t_2$ . So how many variables now I have? I can still get some  $x_j$  by this process, I have second temperature, I have second time, I have first temperature, my first time. So, now, I have two temperatures and two time cycles to adjust my  $x_j$  and what else I had to adjust the surface concentration. So two are not identical and therefore I can adjust is the point. Did you get that? Did you get my point?

I want larger surface concentrations and I want a given junction depth, okay. If there too much junction depth that will be larger but surface will go down. So adjust your  $D_1 t_1$  such that literally some number is available at the surface by  $D_2 t_2$  in such a way junction depth is achieved by you. Is that clear? So, now, you have two temperature cycles at two different times and therefore much more control and final junction depth much easier compared to only one of them. Is that clear? Only one of them.

So this all diffusion cycles are always two state diffusion and here is a, of course, I am going to scan this some sheets now soon and put it on the this may be one or two I will give it to your class representative to you take your own Xerox. There are too many process graphs which you will use in your exams, okay.

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This is a graph, okay, which shows diffusion coefficient for all impurities possible in silicon and even in partially in some gallium arsenide - gallium phosphide devices. Please remember this is plotted against  $1000/T$ .  $T$  is in kelvins. Is that clear? So  $T$  has to be say 1000 degree means 1273 degree Kelvin. So take a ratio of 1000 by 1273 to get this number 0.6, 0.7, 0.8 kind of thing. Then whichever impurity you are looking. Let us say phosphorous boron and phosphorus has same diffusion coefficient.

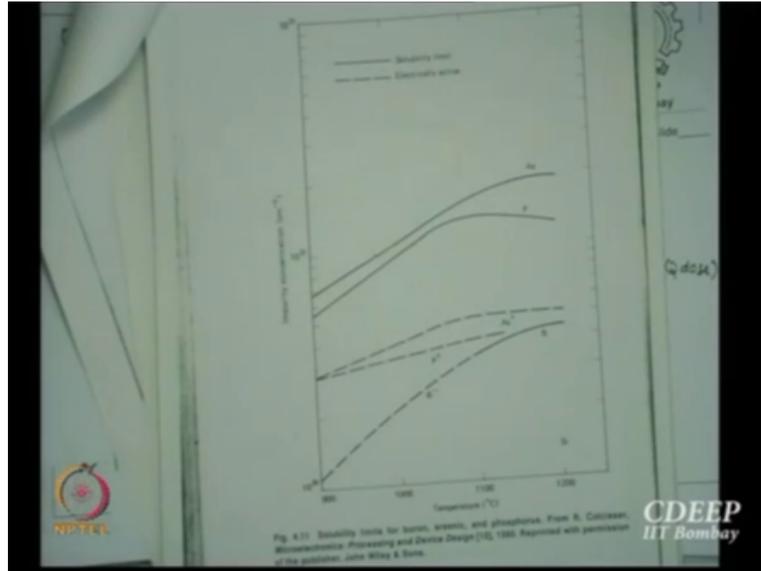
So, let us say it is 0.7. So you go on this graph, okay. And somewhere here it hits and measure the surface, diffusion coefficient for this. Now, one feature which I do not want to say it but over the years I am perturbed and maybe I am, I hope may this year I will not be. You should learn to read graphs, log graphs which is very funny to say 30 percent of the students cannot read the log graphs correctly. So most of the time their values goes by orders absurd, okay.

And, because that already is changing so many things change, okay. Now, this fact has to be understood that this is increasing function of scale minus 14 10 to the power minus 14 minus, 13 minus, 12, 11, 10. The first one, please remember this is 0.2, 0.3, 0.4, 0.5 is very clear more than half because a log scale. 0.6, 0.7, 0.8, 0.9 this one thing is this. Let us say, your  $x$  by, I mean, 1000 by  $T$  becomes more than point 0.8. You are doing temperature, which is more than such that it becomes. So this graph has to be extrapolated down. Is that clear? Extrapolated down like this. However, and this scale the values are 10 to power minus 15 into this and not minus 14 into this. I repeat, if you are gone down the next scale here it will be minus 15 into 10 will make it 10 to the power minus 14.

So this value down here essentially is order of 10 to power minus 15 into 2 or 4 or 8, whatever the, if the scale is here it may be 8, here it may be 6 4 2 into 10 to power minus 15. So, as soon as, you extrapolate roughly calculate from the scale where you are - this is a log scale. This is 0.9, 0.8 and this is 0.2. This is roughly 0.5. So, you roughly see from your extrapolation how much distance in this scale and this and evaluate the new diffusion coefficient. In case, for a case I give which I might not have thought properly and you suddenly find 1000 by  $T$  is more than 0.8 - 82 or 8. Then you will have to extrapolate and get correct value of diffusion coefficients. Is that clear?

So reading a log graph, similarly, it is between these this to this values is 6 2 into minus 13 4 5 into minus 13, 10 into minus 13 is 10 to power minus 12. So it is increasing function in a log and therefore accurately monitored from the graph values. This does not happen root  $Dt$  product goes wrong then the junction depth goes wrong and then say, "Sir, humne toh sab theekh kiya par graph read nahin theekh kiya, haina?" which is even worse, okay. So this is the first curve, I will show you.

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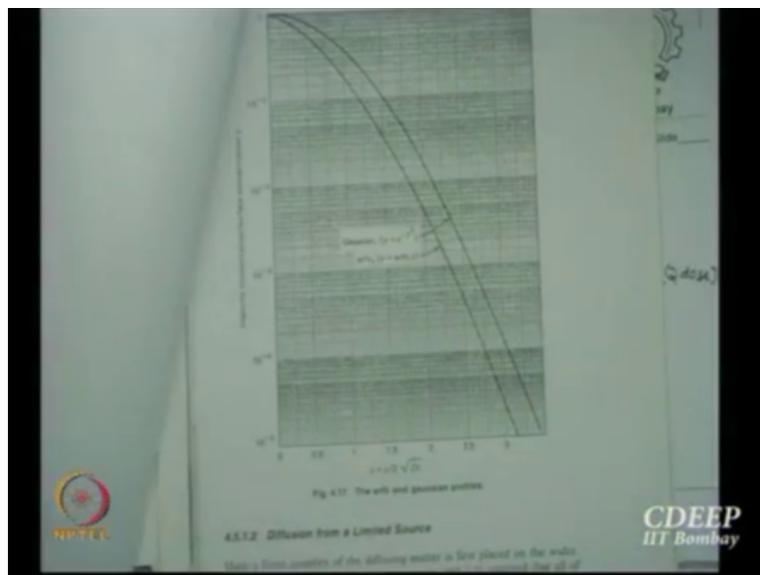


This second is this solubility curve, you can see this dotted ones are, what are those dotted ones? Activated ones. These are the atomic ones. So, if you want to calculate some numbers you should use activated ones, okay. They in real life atoms may go more but activation may be smaller. So use solid solubility this is temperature against solid solubility. And you should be this is already written solubility limits and this is electrical active. So use solubility limit  $N_0$  calculations should be performed from these graphs, okay. So, if you are doing pre-deposition 1000 degree for boron so go at from 1000 to somewhere here. And this is  $2 \times 10^{-20}$  is the concentration for  $N_0$  at 1000 degree centigrade. Is that clear? So any ((01:08:15) you calculating first take diffusion constant from the other graph, take solubility limit from this graph.

So these values will not be specified as I just now give you values. They will be actually given temperature this and this sheets will be provided. Is that okay? This is second graph which I will provide you. Of course, this is slightly different we will come back to it this graph because we are not done oxidation so far. During oxidation what happens to diffusion? Okay. So there is an issue there. So these are different graphs will show you, okay. Then there is also a case that impurity concentrations which is essentially is how bad they are how far away from intrinsic carrier concentration. This is very important, if  $N$  is equal to  $N_i$ , so is  $P$  will be equal to  $N_i$ , okay.

So what does that mean that the material is not doped. Is that clear to you? If N or P is equal to Ni, it is intrinsic. So there is no junction possible, okay. If everywhere Ni then nothing is possible. So at a given temperature this, what is the Ni, Ni value is called Ni e as such that you must know. So you cannot have impurities which are lower than that value at that template. Otherwise, they will not actually diffuse. So you had to verify, of course, that you do not right now, but maybe I will show you how it hurts you many times. There is some term called field dependent diffusion and that helps.

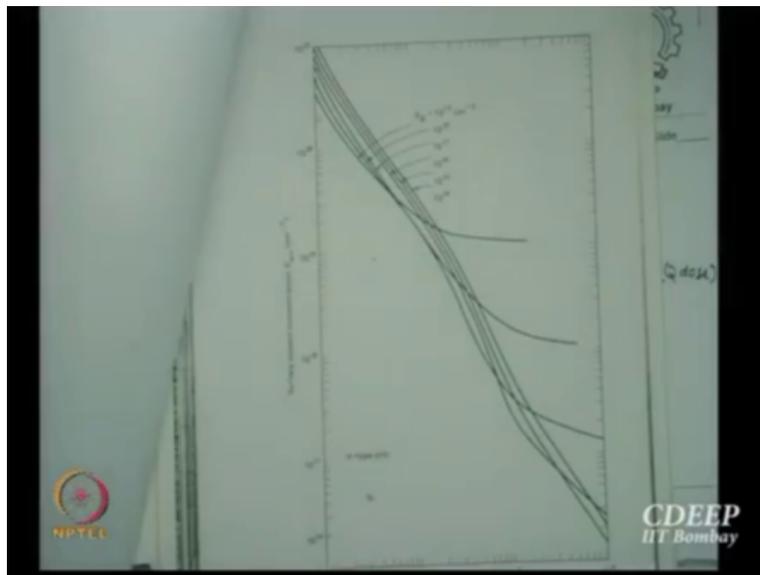
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Now, these are the two profiles Gaussian and error function. The lower one is complementary function and the upper one is assuming right now same surface concentrations. This is Gaussian profile, okay. Is that okay? So this error function profile and complementary error function profiles look similar, but slightly different because complementary error function never changes its shape. It may keep on at different time temperature, it may increase further and further but the profile will remain. In case of Gaussian, it starts flattening.

In complementary this does never, this will never change. In the Gaussian because net impurities are fix, so, it will go down and down and down, okay. Surface concentrations keep on reducing, okay. And depth will be larger and larger, okay. So this is the second this graph ,of course, next time I will show you this more importantly there is graph between surface concentration versus sheet resistance. However, we have not done it but will - what I can monitor is sheet resistance and junction depths monitored and also in problem what I say, I want a junction at 0.6 micron. I gave you value, so  $x_j$  will be given to you.

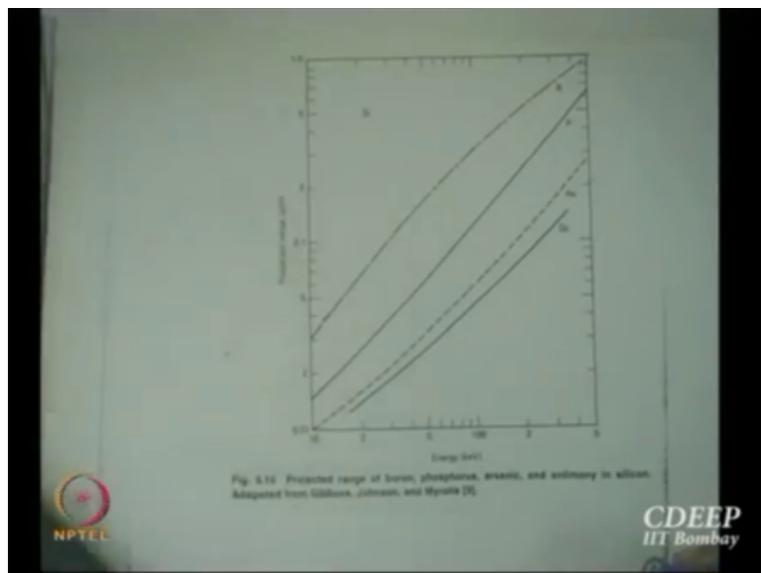
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$R_s$  you can evaluate or I will provide. I will show you how to do that. So given  $R_s$  for given substrate concentration that is surface concentration you can actually directly monitor how much is this. Is that clear? Because, these are all temperature time cycle  $R_s \times j$  products, okay. So given the  $R_s \times j$  product, I will be able to know for a given base concentration where is, because examine junction is fixed. So what is the surface concentration can be directly rate from here? Okay. You can calculate every time but this graphs allow you to read this. So what I can say, this is the surface concentration and this is the sheet resistance monitored.

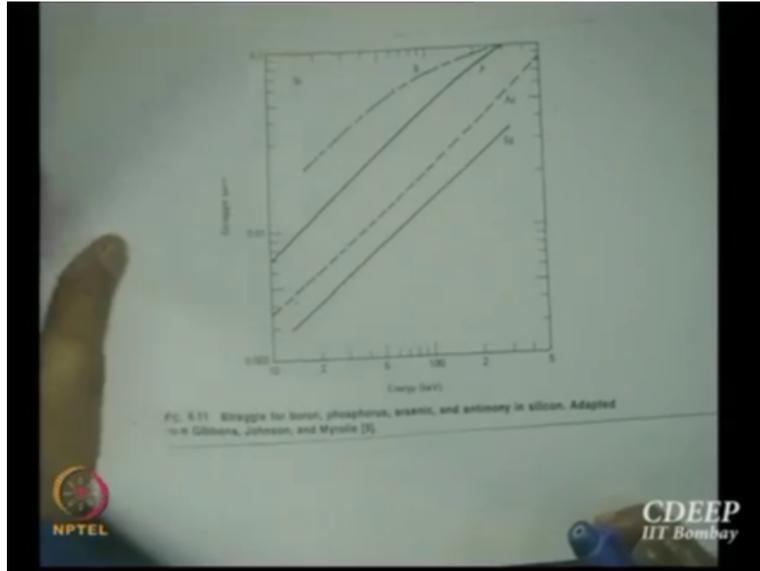
So essentially I am saying junction depth is given to you, okay, either way. The problems which I ask are something like this is called convoluted situations. Something is given in some other format but it is available, okay. So you have to look into this I can get from this graph, okay. So I know how this value. Is that okay? Similarly, this has been done for both what I should say, p-type impurity, n-type. 4 graph are there Gaussian n-type, p-type and both error function graphs are available. Of course, then there is a oxidation graphs, we will see later. Then there will be oxidation for 1 0 0 wafer 1 1 1 then there are graphs for implants.

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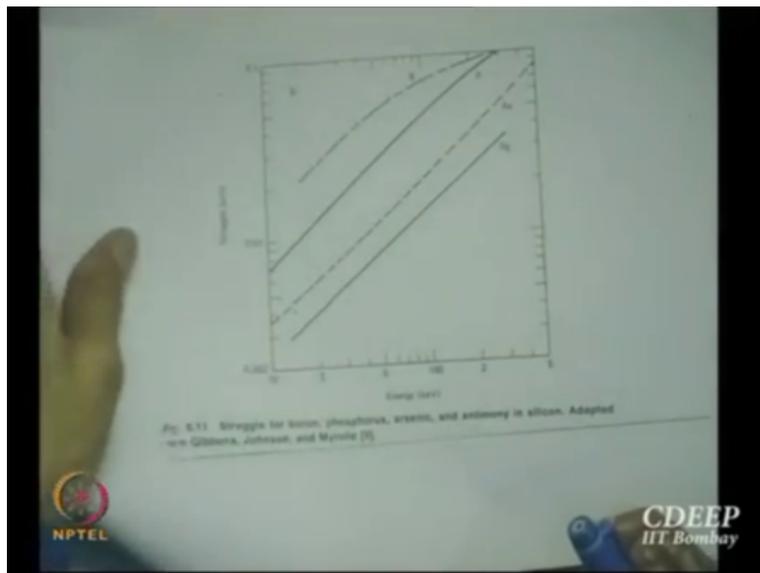
This is (01:12:52) versus energy.

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Then there is a graph for struggle versus energy struggle is the variation. We will see that, Delta struggle versus energy for all impurities of choice. Then there will be transfer struggle, lateral struggles is this.

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This is transfer struggle.

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Table A.1 Error Function of  $z$

$z$	$\text{erf } z$	$\text{erfc } z$	$\text{erf } z$	$\text{erfc } z$
0.00	0.000000	1.000000	0.000000	1.000000
0.01	0.003989	0.996011	0.003989	0.996011
0.02	0.007958	0.992042	0.007958	0.992042
0.03	0.011917	0.988073	0.011917	0.988073
0.04	0.015876	0.984104	0.015876	0.984104
0.05	0.019835	0.980135	0.019835	0.980135
0.06	0.023794	0.976166	0.023794	0.976166
0.07	0.027753	0.972197	0.027753	0.972197
0.08	0.031712	0.968228	0.031712	0.968228
0.09	0.035671	0.964259	0.035671	0.964259
0.10	0.039630	0.960290	0.039630	0.960290
0.11	0.043589	0.956321	0.043589	0.956321
0.12	0.047548	0.952352	0.047548	0.952352
0.13	0.051507	0.948383	0.051507	0.948383
0.14	0.055466	0.944414	0.055466	0.944414
0.15	0.059425	0.940445	0.059425	0.940445
0.16	0.063384	0.936476	0.063384	0.936476
0.17	0.067343	0.932507	0.067343	0.932507
0.18	0.071302	0.928538	0.071302	0.928538
0.19	0.075261	0.924569	0.075261	0.924569
0.20	0.079220	0.920600	0.079220	0.920600
0.21	0.083179	0.916631	0.083179	0.916631
0.22	0.087138	0.912662	0.087138	0.912662
0.23	0.091097	0.908693	0.091097	0.908693
0.24	0.095056	0.904724	0.095056	0.904724
0.25	0.099015	0.900755	0.099015	0.900755
0.26	0.102974	0.896786	0.102974	0.896786
0.27	0.106933	0.892817	0.106933	0.892817
0.28	0.110892	0.888848	0.110892	0.888848
0.29	0.114851	0.884879	0.114851	0.884879
0.30	0.118810	0.880910	0.118810	0.880910
0.31	0.122769	0.876941	0.122769	0.876941
0.32	0.126728	0.872972	0.126728	0.872972
0.33	0.130687	0.869003	0.130687	0.869003
0.34	0.134646	0.865034	0.134646	0.865034
0.35	0.138605	0.861065	0.138605	0.861065
0.36	0.142564	0.857096	0.142564	0.857096
0.37	0.146523	0.853127	0.146523	0.853127
0.38	0.150482	0.849158	0.150482	0.849158
0.39	0.154441	0.845189	0.154441	0.845189
0.40	0.158400	0.841220	0.158400	0.841220
0.41	0.162359	0.837251	0.162359	0.837251
0.42	0.166318	0.833282	0.166318	0.833282
0.43	0.170277	0.829313	0.170277	0.829313
0.44	0.174236	0.825344	0.174236	0.825344
0.45	0.178195	0.821375	0.178195	0.821375
0.46	0.182154	0.817406	0.182154	0.817406
0.47	0.186113	0.813437	0.186113	0.813437
0.48	0.190072	0.809468	0.190072	0.809468
0.49	0.194031	0.805499	0.194031	0.805499
0.50	0.197990	0.801530	0.197990	0.801530
0.51	0.201949	0.797561	0.201949	0.797561
0.52	0.205908	0.793592	0.205908	0.793592
0.53	0.209867	0.789623	0.209867	0.789623
0.54	0.213826	0.785654	0.213826	0.785654
0.55	0.217785	0.781685	0.217785	0.781685
0.56	0.221744	0.777716	0.221744	0.777716
0.57	0.225703	0.773747	0.225703	0.773747
0.58	0.229662	0.769778	0.229662	0.769778
0.59	0.233621	0.765809	0.233621	0.765809
0.60	0.237580	0.761840	0.237580	0.761840
0.61	0.241539	0.757871	0.241539	0.757871
0.62	0.245498	0.753902	0.245498	0.753902
0.63	0.249457	0.749933	0.249457	0.749933
0.64	0.253416	0.745964	0.253416	0.745964
0.65	0.257375	0.741995	0.257375	0.741995
0.66	0.261334	0.738026	0.261334	0.738026
0.67	0.265293	0.734057	0.265293	0.734057
0.68	0.269252	0.730088	0.269252	0.730088
0.69	0.273211	0.726119	0.273211	0.726119
0.70	0.277170	0.722150	0.277170	0.722150
0.71	0.281129	0.718181	0.281129	0.718181
0.72	0.285088	0.714212	0.285088	0.714212
0.73	0.289047	0.710243	0.289047	0.710243
0.74	0.293006	0.706274	0.293006	0.706274
0.75	0.296965	0.702305	0.296965	0.702305
0.76	0.300924	0.698336	0.300924	0.698336
0.77	0.304883	0.694367	0.304883	0.694367
0.78	0.308842	0.690398	0.308842	0.690398
0.79	0.312801	0.686429	0.312801	0.686429
0.80	0.316760	0.682460	0.316760	0.682460
0.81	0.320719	0.678491	0.320719	0.678491
0.82	0.324678	0.674522	0.324678	0.674522
0.83	0.328637	0.670553	0.328637	0.670553
0.84	0.332596	0.666584	0.332596	0.666584
0.85	0.336555	0.662615	0.336555	0.662615
0.86	0.340514	0.658646	0.340514	0.658646
0.87	0.344473	0.654677	0.344473	0.654677
0.88	0.348432	0.650708	0.348432	0.650708
0.89	0.352391	0.646739	0.352391	0.646739
0.90	0.356350	0.642770	0.356350	0.642770
0.91	0.360309	0.638801	0.360309	0.638801
0.92	0.364268	0.634832	0.364268	0.634832
0.93	0.368227	0.630863	0.368227	0.630863
0.94	0.372186	0.626894	0.372186	0.626894
0.95	0.376145	0.622925	0.376145	0.622925
0.96	0.380104	0.618956	0.380104	0.618956
0.97	0.384063	0.614987	0.384063	0.614987
0.98	0.388022	0.611018	0.388022	0.611018
0.99	0.391981	0.607049	0.391981	0.607049
1.00	0.395940	0.603080	0.395940	0.603080

And this is most important. This you do not know before we quit, okay. This is a table for error function just look at it. Please, because this is something which we just now did, so, I will like to say it the table which is given to you, I do not know whether it is so very visible. It says  $Z$  versus error functions  $Z$ . So,  $Z$  is 0 and the table, which I have used is around 43.99 which is the maximum value you will get any day, no more than this, okay. Now, you can see from here at 0, of course, error function 0 is 0, so that it start 0.

So at 0.1 it is 0.112 but if you start looking for 1 point something 1.17 onwards it is 0.9 and as you increase this  $Z$  it becomes 0.99 larger 99 means higher  $Z$  value. So, for example, error function of 3.99 is 69 rather 7983. Is that were clear? 7983 99999999983, okay, 7983. So what does that mean? So, if I am just now I calculated, so the profile is normally complementary error function. So, what do I do? I take 1 minus this. That is  $x$  you anyway want. So, okay, I put 1 minus this value I subtract, 1 minus 0.0 whatever value I subtract. That will give me 0.999 some value, okay. 1 minus that you got.

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Table A.1 Error Function of  $z$

$z$	$\text{erf } z$	$z$	$\text{erf } z$	$z$	$\text{erf } z$
0.00	0.000000	0.70	0.743058	1.40	0.949243
0.01	0.010000	0.71	0.745244	1.41	0.950438
0.02	0.020000	0.72	0.747430	1.42	0.951633
0.03	0.030000	0.73	0.749616	1.43	0.952828
0.04	0.040000	0.74	0.751802	1.44	0.954023
0.05	0.050000	0.75	0.753988	1.45	0.955218
0.06	0.060000	0.76	0.756174	1.46	0.956413
0.07	0.070000	0.77	0.758360	1.47	0.957608
0.08	0.080000	0.78	0.760546	1.48	0.958803
0.09	0.090000	0.79	0.762732	1.49	0.959998
0.10	0.100000	0.80	0.764918	1.50	0.961193
0.11	0.110000	0.81	0.767104	1.51	0.962388
0.12	0.120000	0.82	0.769290	1.52	0.963583
0.13	0.130000	0.83	0.771476	1.53	0.964778
0.14	0.140000	0.84	0.773662	1.54	0.965973
0.15	0.150000	0.85	0.775848	1.55	0.967168
0.16	0.160000	0.86	0.778034	1.56	0.968363
0.17	0.170000	0.87	0.780220	1.57	0.969558
0.18	0.180000	0.88	0.782406	1.58	0.970753
0.19	0.190000	0.89	0.784592	1.59	0.971948
0.20	0.200000	0.90	0.786778	1.60	0.973143
0.21	0.210000	0.91	0.788964	1.61	0.974338
0.22	0.220000	0.92	0.791150	1.62	0.975533
0.23	0.230000	0.93	0.793336	1.63	0.976728
0.24	0.240000	0.94	0.795522	1.64	0.977923
0.25	0.250000	0.95	0.797708	1.65	0.979118
0.26	0.260000	0.96	0.799894	1.66	0.980313
0.27	0.270000	0.97	0.802080	1.67	0.981508
0.28	0.280000	0.98	0.804266	1.68	0.982703
0.29	0.290000	0.99	0.806452	1.69	0.983898
0.30	0.300000	1.00	0.808638	1.70	0.985093
0.31	0.310000	1.01	0.810824	1.71	0.986288
0.32	0.320000	1.02	0.813010	1.72	0.987483
0.33	0.330000	1.03	0.815196	1.73	0.988678
0.34	0.340000	1.04	0.817382	1.74	0.989873
0.35	0.350000	1.05	0.819568	1.75	0.991068
0.36	0.360000	1.06	0.821754	1.76	0.992263
0.37	0.370000	1.07	0.823940	1.77	0.993458
0.38	0.380000	1.08	0.826126	1.78	0.994653
0.39	0.390000	1.09	0.828312	1.79	0.995848
0.40	0.400000	1.10	0.830498	1.80	0.997043
0.41	0.410000	1.11	0.832684	1.81	0.998238
0.42	0.420000	1.12	0.834870	1.82	0.999433
0.43	0.430000	1.13	0.837056	1.83	1.000628
0.44	0.440000	1.14	0.839242	1.84	1.001823
0.45	0.450000	1.15	0.841428	1.85	1.003018
0.46	0.460000	1.16	0.843614	1.86	1.004213
0.47	0.470000	1.17	0.845800	1.87	1.005408
0.48	0.480000	1.18	0.847986	1.88	1.006603
0.49	0.490000	1.19	0.850172	1.89	1.007798
0.50	0.500000	1.20	0.852358	1.90	1.008993
0.51	0.510000	1.21	0.854544	1.91	1.010188
0.52	0.520000	1.22	0.856730	1.92	1.011383
0.53	0.530000	1.23	0.858916	1.93	1.012578
0.54	0.540000	1.24	0.861102	1.94	1.013773
0.55	0.550000	1.25	0.863288	1.95	1.014968
0.56	0.560000	1.26	0.865474	1.96	1.016163
0.57	0.570000	1.27	0.867660	1.97	1.017358
0.58	0.580000	1.28	0.869846	1.98	1.018553
0.59	0.590000	1.29	0.872032	1.99	1.019748
0.60	0.600000	1.30	0.874218	2.00	1.020943

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Look for this 0.99 in this graph wherever it is and in this figure, okay. Let us say, I get 0.49, so start looking for 49 first. Okay, here is 49. So, okay, from here so 2.27 is the smallest value for 49 values, okay. So, once you start getting that value from there come on this table figure out what is the value of error function  $Z$  0.999, correspondingly, you get the  $Z$  value. So you will have to go back and forth many times just to get exact  $Z$  value not exactly. Then interpolate there exponential function is not linear. I mean ever functions are not linear. Therefore, do not extrapolate. I will take the lower one or upper one and say I took this. It is your choice, okay. In between 2.3 to 5 do not try any values, okay.

You have to really do a calculations or you can do on computer but do not try anything on this. The value is nearest to what value you have choose from there. The reason is all these calculations are only calculations in their life silicon will behave differently. So, why are so worried about? Okay. So these data sheets as I called will be provided on system, by the way, this website is on there are two handouts have been put there, two PPTs have been put there. So please look at it. And this will be also available to you download yourself one copy and maybe I will give it to your this maybe you can use that for Xeroxing but as your choice.