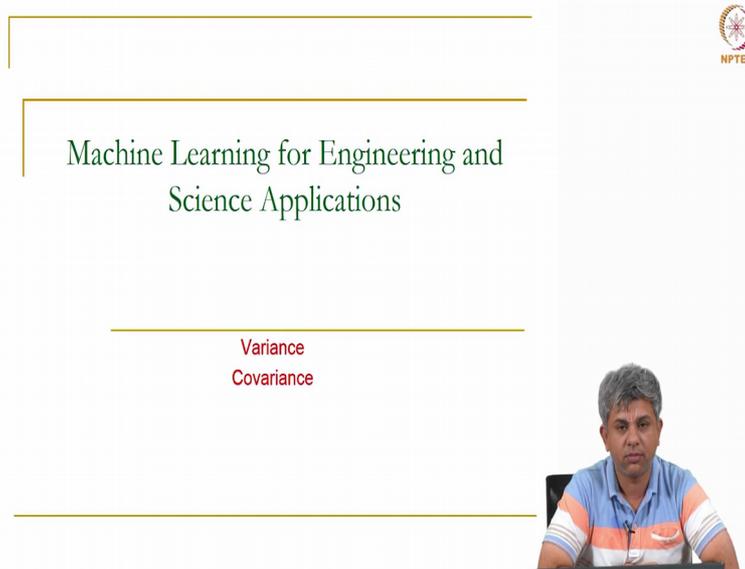


Machine Learning for Engineering and Science Applications
Professor Dr. Balaji Srinivasan
Department of Mechanical Engineering
Indian Institute of Technology Madras
Variance Covariance

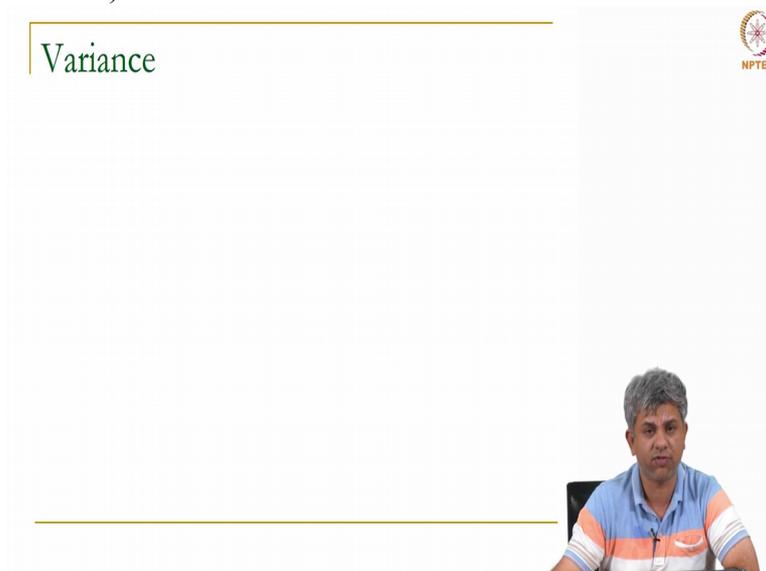
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The slide features a white background with a yellow border. At the top right is the NPTEL logo. The main title 'Machine Learning for Engineering and Science Applications' is centered in green. Below it, the words 'Variance' and 'Covariance' are listed in red. A small inset video of Professor Dr. Balaji Srinivasan is visible in the bottom right corner of the slide.

In the previous video we had looked at the idea of Expectation. In this video we will be looking at some additional ideas which are called variance and covariance. Again you will be familiar with some of these

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The slide features a white background with a yellow border. At the top right is the NPTEL logo. The word 'Variance' is written in green. A small inset video of Professor Dr. Balaji Srinivasan is visible in the bottom right corner of the slide.

from school and some of these might be unfamiliar.

So let us continue

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution



our discussion from where we had our expectation discussions. Once again the same

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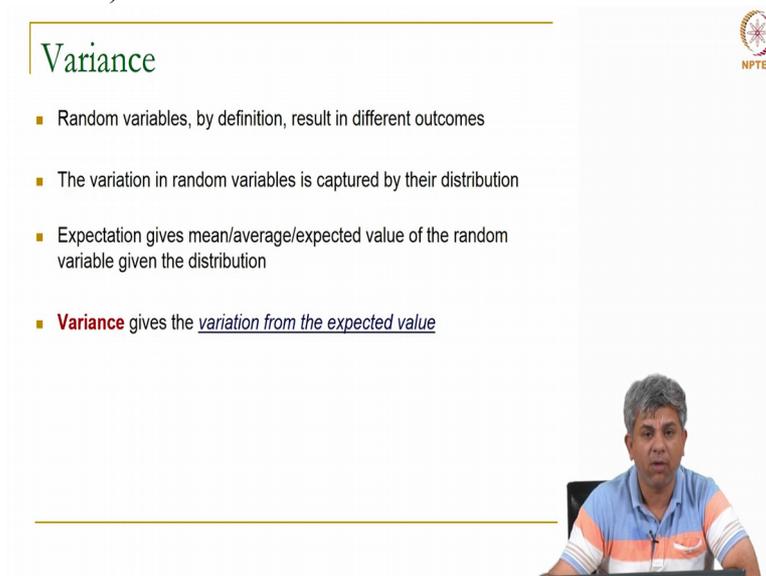
Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution



idea continues from before. If you have a random variable it is going to vary, Ok. And its variation is actually captured by its distribution. What expectation does is, on an average what can you expect? Ok this is what it will tell you.

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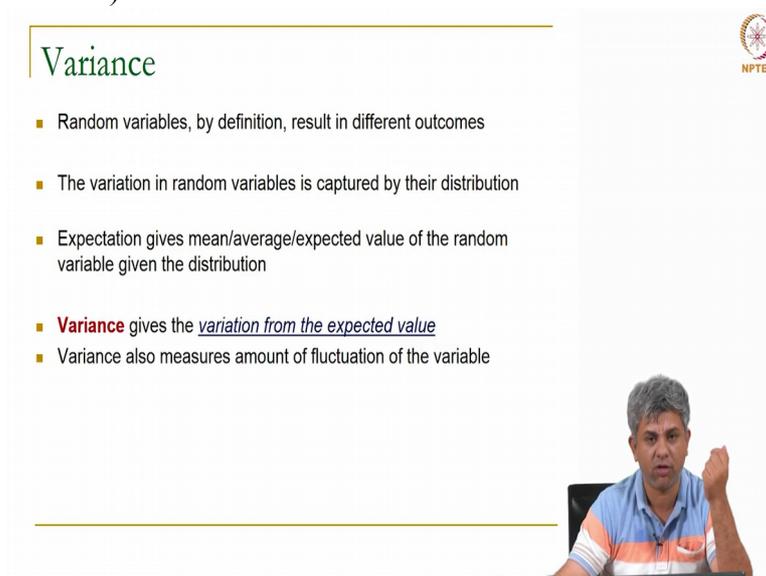


The slide is titled "Variance" in green text. It features a list of four bullet points. The first three are in black, and the fourth is in red. The NPTEL logo is in the top right corner. A video inset in the bottom right shows a man with grey hair wearing a blue and white striped polo shirt.

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value

But what we want to know is variance. Variance is, Ok, it is given that this is the expected value. For example expectation for dice throw is 3 point 5 but how much more can it go, how much less will it go? What is the variation from the expected value? What is the variation from the mean, from the expectation? This is what variance talks about,

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The slide is titled "Variance" in green text. It features a list of five bullet points. The first four are in black, and the fifth is in red. The NPTEL logo is in the top right corner. A video inset in the bottom right shows the same man as in the previous slide, now with his right hand raised.

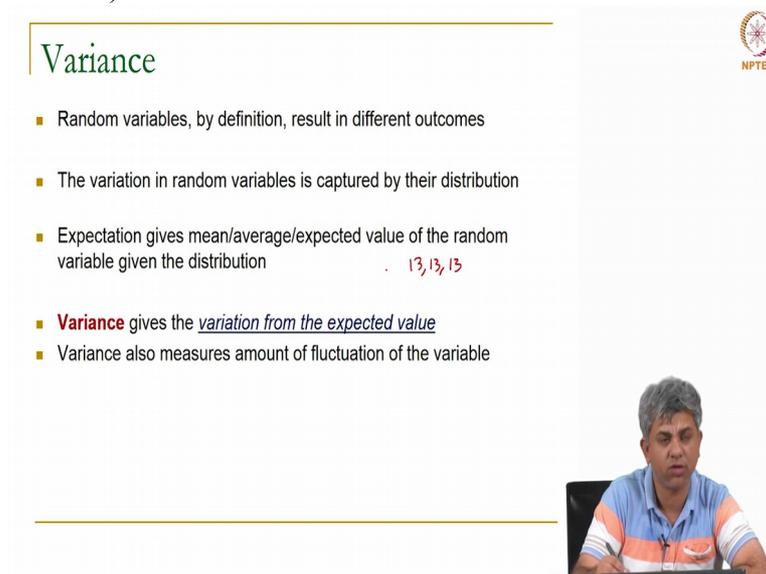
- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

Ok.

Variance also measures; in many cases you can also measure how much does the quantity fluctuate? Ok. So it is possible, entirely possible for two distributions to have the entirely same expectation but different variances.

For example if I take the value 13, 13, 13 or, Ok

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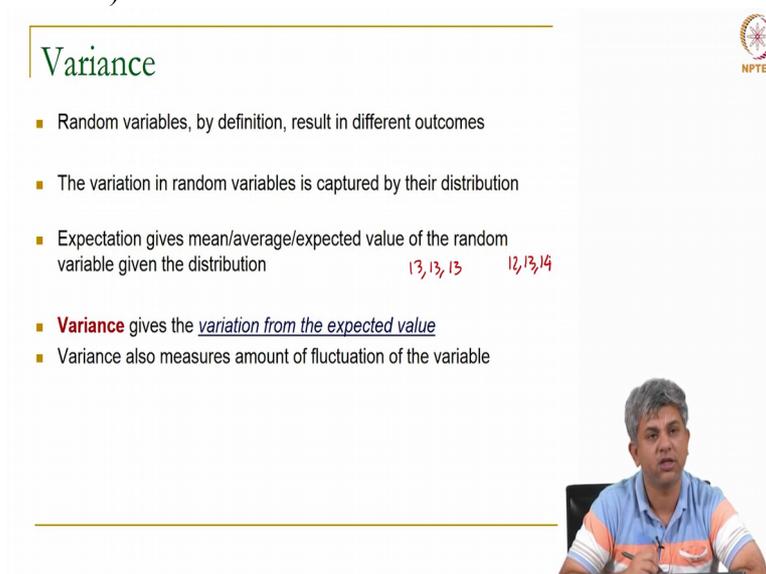
Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution $13, 13, 13$
- **Variance** gives the *variation from the expected value*
- Variance also measures amount of fluctuation of the variable

The slide features a video inset of a man in a blue and white striped shirt sitting at a desk with a microphone. The NPTEL logo is in the top right corner.

so let us say there is some random value which takes the value 13, 13, 13. In this case the expectation is 13. Another one

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution $13, 13, 13$ $12, 13, 14$
- **Variance** gives the *variation from the expected value*
- Variance also measures amount of fluctuation of the variable

The slide features a video inset of the same man from the previous slide. The NPTEL logo is in the top right corner.

is 12, 13, 14. In these cases is the same expectation but

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

13, 13, 13 12, 13, 14
Same Different



different variance, Ok.

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

13, 13, 13 12, 13, 14
Same Different



We see this one varies

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

Handwritten notes: $13, 13, 13$ and $12, 13, 14$ are circled. Below the first set, it says "Same distribution". Below the second set, it says "Different variance".



more from the mean and this one in fact does not vary

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

Handwritten notes: $13, 13, 13$ and $12, 13, 14$ are circled. Below the first set, it says "Same distribution". Below the second set, it says "Different variance".



from the mean at all and we actually get variance is equal to 0. So

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

Examples:

- Variance in returns on a certain investment in the market (Risk measure)

Handwritten notes: $13, 13, 13$ and $12, 13, 14$ are circled. Below them, it says "Same expectation" and "Different variance".



once again some practical example, if we go back to the investment example, expected returns is one thing but you would also like to know how much can change, Ok

So your expected returns might be x y z amount of Rupees, but your actual variation, you could actually have a profit or you could actually have a loss, even though your expected value might be a profit, Ok.

So some times and in fact lot of times in modern, what is called, modern portfolio theory usually the variance is what is used to find out what is the risk in investment.

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Variance

- Random variables, by definition, result in different outcomes
- The variation in random variables is captured by their distribution
- Expectation gives mean/average/expected value of the random variable given the distribution
- **Variance** gives the variation from the expected value
- Variance also measures amount of fluctuation of the variable

Examples:

- Variance in returns on a certain investment in the market (Risk measure)
- Variance in rainfall during the coming monsoon

Handwritten notes: $13, 13, 13$ and $12, 13, 14$ are circled. Below them, it says "Same expectation" and "Different variance".



Though this is a little bit controversial but anyway, people, many people do use variance as a measure of risk.

So similarly I gave the example of the expected value of rainfall during the coming monsoon. But you could also look at the variance in rainfall during coming monsoon.

Not only do you want to know what the rainfall is but you also want to know what is the maximum it can go, you know in case you have to plan for, in a flood, Ok. Or what is the least that it can go in case you have to plan for drought, Ok. So variance measures these ideas.

So let us look

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Univariate variance

- The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P



at mathematically what variance is. Once again we will start with a univariate case. Remember univariate simply means scalar. So if I take a single value x or a single random variable x which is a scalar, you look at two sorts of quantities, variance and standard deviation.

Standard deviation is usually denoted by sigma

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Univariate variance

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

σ = √variance

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if you want to be very precise you will say V drawn for x drawn from P of f of x,

(Refer Slide Time: 04:14)

Univariate variance

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$

σ = √variance

NPTEL



Ok.

Once again if P is clear

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Univariate variance

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$

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from the context we drop P and simply say $V x$. If x is also clear we say V of f

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Univariate variance

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

NPTEL



and usually, and this is unlike the expectation, this is the usual notation which is used, which is either V of f or Var of f , Var stands for variance.

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Univariate variance $\sigma = \sqrt{\text{variance}}$

- The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

- Mathematically,



Mathematically

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Univariate variance $\sigma = \sqrt{\text{variance}}$

- The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

- Mathematically,
$$V[f(x)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$
$$= \mathbb{E}[(f(x) - \bar{f}(x))^2]$$



variance of f of x is the variation or the difference between f of x and the expected value of f of x ,

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Univariate variance $\sigma = \sqrt{\text{variance}}$

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

■ Mathematically,

$$V[f(x)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$
$$= \mathbb{E}[(f(x) - \bar{f})^2]$$


Ok. So this, I will, we use the notation

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Univariate variance $\sigma = \sqrt{\text{variance}}$

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

■ Mathematically,

$$V[f(x)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$
$$= \mathbb{E}[(f(x) - \bar{f})^2]$$

$E[x] \equiv \bar{x}$



just for simplification \bar{x} to denote expectation of x . So similarly expectation of f can be written as \bar{f} .

(Refer Slide Time: 05:06)

Univariate variance

$\sigma = \sqrt{\text{Variance}}$

- The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

- Mathematically,

$$\begin{aligned}
 V[f(x)] &= \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2] \\
 &= \mathbb{E}[(f(x) - \bar{f})^2]
 \end{aligned}$$

$E[x] = \bar{x}$
 $E[f] = \bar{f}$



So what is variance of f , it is the expectation of f minus \bar{f} square,

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Univariate variance

$\sigma = \sqrt{\text{Variance}}$

- The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

- Mathematically,

$$\begin{aligned}
 V[f(x)] &= \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2] \\
 &= \mathbb{E}[(f(x) - \bar{f})^2]
 \end{aligned}$$

$E[x] = \bar{x}$
 $E[f] = \bar{f}$
 $V[f] = E[(f - \bar{f})^2]$



that is the variance from the mean square. So this is

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Univariate variance $\sigma = \sqrt{\text{variance}}$

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

■ Mathematically,

$$V[f(x)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$
$$= \mathbb{E}[(f(x) - \bar{f})^2]$$

$E[x] = \bar{x}$
 $E[f] = \bar{f}$ mean square
 $V[f] = E[(f - \bar{f})^2]$



mean square value. So variance is essentially mean square.

We can also define the

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Univariate variance $\sigma = \sqrt{\text{variance}}$

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

■ Mathematically,

$$V[f(x)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$
$$= \mathbb{E}[(f(x) - \bar{f})^2]$$

$E[x] = \bar{x}$
 $E[f] = \bar{f}$ mean square value
 $V[f] = E[(f - \bar{f})^2]$

The standard deviation is given as $\sigma[x] = \sqrt{V[x]}$



standard deviation. Standard deviation is square root of variance, Ok. So standard deviation is root mean square, Ok. Variance is mean square;

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Univariate variance $\sigma = \sqrt{\text{variance}}$

■ The **variance** and its square root, **standard deviation** of some function $f(x)$ with respect to a probability distribution $P(x)$ measure how much the value of $f(x)$ varies for various samples when x is drawn from P

Denoted by $V_{x \sim P}[f(x)]$

- If P is clear from the context $V_x[f(x)]$
- If x is also clear from the context $V[f(x)]$
- **Usually**, simply denoted as $V[f]$ or $\text{Var}[f]$

■ Mathematically,

$$V[f(x)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])^2]$$
$$= \mathbb{E}[(f(x) - \bar{f})^2]$$

$E[x] = \bar{x}$
 $E[f] = \bar{f}$ mean square value
 $V[f] = E[(f - \bar{f})^2]$

The standard deviation is given as $\sigma[x] = \sqrt{V[x]}$ \rightarrow Std Deviation: Root mean square



standard deviation is root mean square.

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Covariance (univariate)



Now there is another idea, some of you might actually be unfamiliar with this idea. This is the idea of covariance. Now covariance is something we are interested in, if instead of having one variable x , you now have two variables, Ok. When I say

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$



univariate what it means is each of x and y are individually univariate we look at a multivariate case a little bit later, Ok.

So notice that when we defined variance of x this was expectation of, I have not taken f so variance of f , x is simply x minus x bar square, mean square deviation. Now you can write it as expectation of x minus x bar multiplied by x minus x bar.

So this idea can now be

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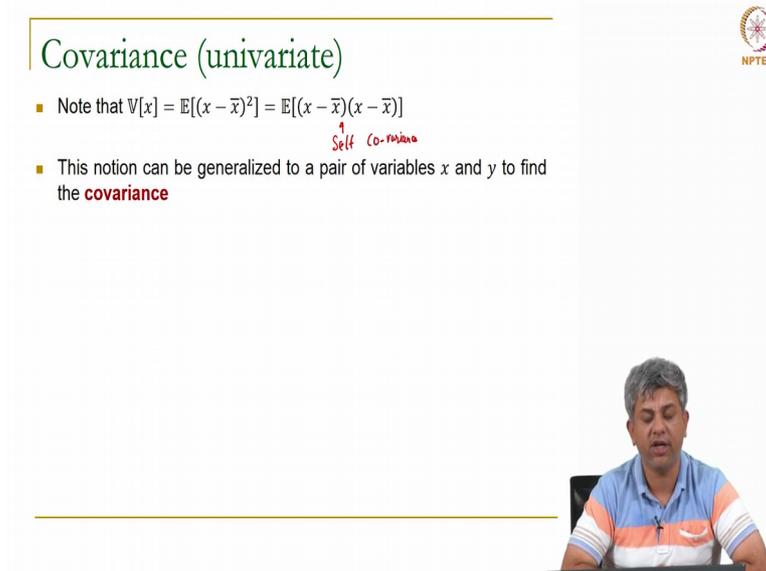
Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
- This notion can be generalized to a pair of variables x and y to find the **covariance**



generalized to a pair of variables, Ok. This I can call self covariance. The meaning will become

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Covariance (univariate)

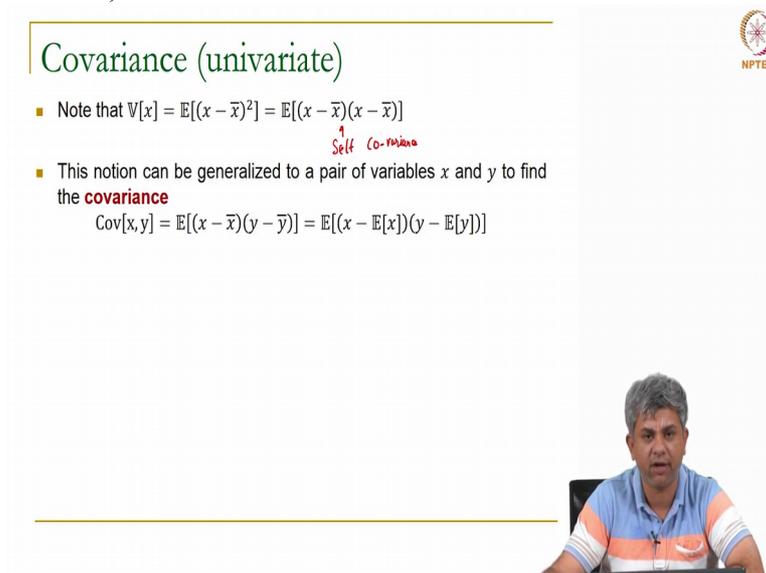
- Note that $V[x] = \mathbb{E}[(x - \bar{x})^2] = \mathbb{E}[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**



shortly clear as we go through the next couple of slides, Ok.

So suppose instead of one variable, you have two variables, we simply generalize this idea

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Covariance (univariate)

- Note that $V[x] = \mathbb{E}[(x - \bar{x})^2] = \mathbb{E}[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$



and we define covariance of two variables x and y as expectation of x minus x bar

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-Variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$



multiplied by y minus y bar, Ok.

What does that mean? This says that how much does x vary from its mean when y varies from its own mean, Ok. So this is what is called covariance. That is how much do x and y vary together, Ok.

Now just like we defined it, again I use x bar just for compact notation, just like we can define covariance of x and y , you can also define covariance of functions of x and functions of y , all you need to do

(Refer Slide Time: 07:51)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-Variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$



is replace

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
 \uparrow
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$



in these expressions x by f of x

(Refer Slide Time: 07:56)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
 \uparrow
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$



and y by g of y

(Refer Slide Time: 07:58)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$



and you will get the corresponding covariance functions for f of x and g of y .

(Refer Slide Time: 08:05)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$



(Refer Slide Time: 08:06)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as



Now closely related quantity to covariance is something called the correlation.

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as
$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}}$$



You will see that it is simply the covariance divided by the square root of variance of x and variance of y . Another way to say it is remember square root of variance of x is standard deviation of x .

(Refer Slide Time: 08:35)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as
$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}} = \frac{\text{Cov}[x, y]}{\sigma_x \sigma_y}$$



So essentially correlation is normalized, Ok.

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as
$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}} = \frac{\text{Cov}[x, y]}{\sigma_x \sigma_y}$$

Normalized covariance



What does normalized mean? Normalized means, you know just like we normalize the vector and make it into unit normal, similarly you are normalizing covariance so that its size stays between certain limits. So as it turns out, covariance will always, sorry correlation will always lie between minus 1 and

(Refer Slide Time: 09:10)

Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as *Normalized Covariance*
$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}} = \frac{\text{Cov}[x, y]}{\sigma_x \sigma_y}$$

-1 ≤ Corr ≤ 1



1, Ok.

Now we had looked at the

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as *Normalized Covariance*
$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}} = \frac{\text{Cov}[x, y]}{\sigma_x \sigma_y}$$

-1 ≤ Corr ≤ 1
- Measures how linearly correlated the two random variables are



ideas of variance and covariance.

(Refer Slide Time: 09:20)

Covariance (univariate)

- Note that $V[x] = \mathbb{E}[(x - \bar{x})^2] = \mathbb{E}[(x - \bar{x})(x - \bar{x})]$
Self Co-variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

- Similarly, $\text{Cov}[f(x), g(y)] = \mathbb{E}[(f(x) - \mathbb{E}[f(x)])(g(y) - \mathbb{E}[g(y)])]$

- A related quantity is the **correlation**, defined as *Normalized Covariance* $-1 \leq \text{Corr} \leq 1$

$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}} = \frac{\text{Cov}[x, y]}{\sigma_x \sigma_y}$$

- Measures how linearly correlated the two random variables are
- Note $\text{Cov}[x, x] = \text{Var}[x]$ and $\text{corr}[x, x] = 1$



What does correlation indicate? Correlation indicates how linearly correlated, the word linearly will become clear in the next slide, how linearly correlated the two variables, the two random variables are.

For example let us say x is height and y is weight, Ok. What you expect is, as height increases weight also increases. Of course there is randomness here. There are tall people who can be very, very, very slim and there can be short people who could actually have higher weight than the taller person but nonetheless you can see that overall the trend will be that as x increases y also increases, Ok.

Notice that by definition, covariance of x comma x , Ok which is simply expectation of x minus x bar, x minus x bar it will simply give you variance of x , that was the way it was designed. And if I find the correlation of x and x , it makes complete sense that this is 1, Ok.

So you can see that since the numerator itself is variance of x , square root of variance of x multiplied by variance of x you will get correlation of

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Covariance (univariate)

- Note that $V[x] = E[(x - \bar{x})^2] = E[(x - \bar{x})(x - \bar{x})]$
Self Co-Variance
- This notion can be generalized to a pair of variables x and y to find the **covariance**
$$\text{Cov}[x, y] = E[(x - \bar{x})(y - \bar{y})] = E[(x - E[x])(y - E[y])]$$
- Similarly, $\text{Cov}[f(x), g(y)] = E[(f(x) - E[f(x)])(g(y) - E[g(y)])]$
- A related quantity is the **correlation**, defined as *Normalized Covariance*
$$\text{corr}[x, y] = \frac{\text{Cov}[x, y]}{\sqrt{V[x]V[y]}} = \frac{\text{Cov}[x, y]}{\sigma_x \sigma_y}$$

-1 ≤ Corr ≤ 1
- Measures how linearly correlated the two random variables are
- Note $\text{Cov}[x, x] = \text{Var}[x]$ and $\text{corr}[x, x] = 1$ ✓



x comma x as 1,

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Interpreting Covariance and correlation

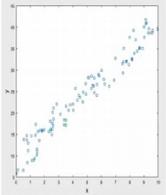


Ok

So let us now in this slide look at an interpretation of what covariance and correlation mean. I have already given you a slight

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Interpreting Covariance and correlation

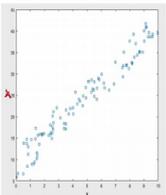


The slide features a title 'Interpreting Covariance and correlation' at the top left. Below the title is a scatter plot with a grid. The x-axis is labeled from 1 to 10, and the y-axis is labeled from 10 to 40. The data points are blue dots showing a clear upward trend. In the top right corner, there is a circular logo with a gear and the text 'NPTEL'. At the bottom right, there is a video feed of a man with grey hair wearing a blue and orange striped polo shirt, sitting at a desk.

indication. So let us say this direction here is x and here it is y.

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Interpreting Covariance and correlation



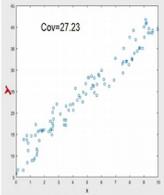
This slide is identical to the previous one, but with a red 'x' marking a specific data point on the scatter plot at approximately x=1 and y=10. The rest of the slide content, including the title, NPTEL logo, and speaker view, remains the same.

And let us say x and y are random variables. And you see this kind of variation amongst them, Ok.

You can see that even though both are random, as x increases, y also increases. That is at least the overall trend, Ok. Now if I calculate the covariance in this case it comes to some 27 point something.

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Interpreting Covariance and correlation



Cov=27.23

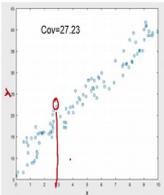
- Positive covariance means when x increases, y is expected to increase too.



Now the general rule of, not rule of thumb, the general rule is that if you have positive covariance then it means that as x increases y is also expected to increase, even though in a certain few cases it can happen that, you know at this given x, you have a y

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Interpreting Covariance and correlation



Cov=27.23

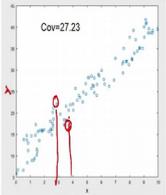
- Positive covariance means when x increases, y is expected to increase too.



and at a slightly higher x, you have lower

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Interpreting Covariance and correlation



- Positive covariance means when x increases, y is expected to increase too.



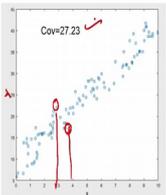
NPTEL

y.

Even though that expectation could be violated, overall the trend is that as x increases, y increases. And that is what is indicated by

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Interpreting Covariance and correlation



- Positive covariance means when x increases, y is expected to increase too.

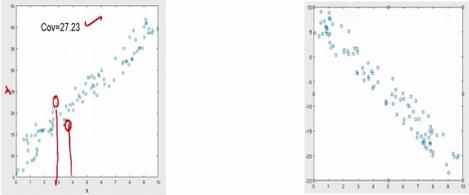


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covariance, the value of covariance, Ok or the sign of covariance.

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Interpreting Covariance and correlation



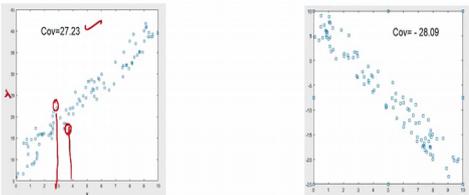
■ Positive covariance means when x increases, y is expected to increase too.



Now you can see a counter case. In this case what you notice is as x increases, y decreases.

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Interpreting Covariance and correlation



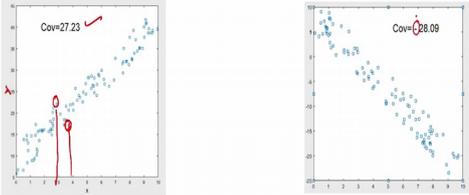
■ Positive covariance means when x increases, y is expected to increase too.



And sure enough the covariance in this case is negative, Ok. It is negative

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Interpreting Covariance and correlation



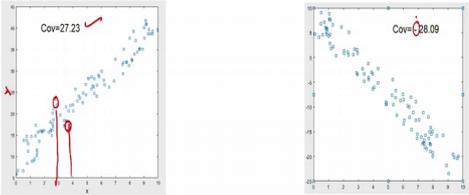
■ Positive covariance means when x increases, y is expected to increase too.



28 point 0 9. And what negative covariance

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Interpreting Covariance and correlation



■ Positive covariance means when x increases, y is expected to increase too.

■ Negative covariance means when x increases, y is expected to decrease



indicates is that as x increases, y actually decreases.

Let us look at the third case. You will see that you really cannot say anything about any trend. It seems like x and y have no relation whatsoever. If you find out the covariance in this case, it still gives the positive value. It is 8 point 5, so 8 point 5 3. But we would like to qualitatively distinguish between these two cases,

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Interpreting Covariance and correlation

■ Positive covariance means when x increases, y is expected to increase too.
■ Negative covariance means when x increases, y is expected to decrease

Ok.

Here covariance is positive but you see a clear trend. Here covariance is positive even though it is a smaller value, we can notice that it is a smaller value but it is all over the place, Ok. So can we relate these two? It turns out we can by looking at

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Interpreting Covariance and correlation

■ Positive covariance means when x increases, y is expected to increase too.
■ Negative covariance means when x increases, y is expected to decrease

the correlation.

So notice that the correlation in this case

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Interpreting Covariance and correlation

■ Positive covariance means when x increases, y is expected to increase too.
■ Negative covariance means when x increases, y is expected to decrease

is point 9 7. This is what normalization means. Correlation is simply the numerator. It tells you what the trend is. But how positive is positive? We will not know unless we compare it with something and the comparison metric here is the variance of these two quantities.

So if you find out the correlation, that is normalize the covariance divided by sigma x sigma y, then you get the correlation of point 9 7 which is very, very close to 1. Remember that if I had simply taken a linear relationship x versus x I would have got 1.

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Interpreting Covariance and correlation

■ Positive covariance means when x increases, y is expected to increase too.
■ Negative covariance means when x increases, y is expected to decrease
■ Correlation close to 1 means strongly, positively correlated

So correlation which is close to 1 means that there is a strong positively

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Interpreting Covariance and correlation

- Positive covariance means when x increases, y is expected to increase too.
- Negative covariance means when x increases, y is expected to decrease
- Correlation close to 1 means strongly, positively correlated
- Correlation close to -1 means strongly, negatively correlated

correlated, linearly positively correlated, Ok strong positively linear correlation between the two variables.

Similarly if I find out the correlation in this case, I find out that it is minus point 9 8.

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Interpreting Covariance and correlation

- Positive covariance means when x increases, y is expected to increase too.
- Negative covariance means when x increases, y is expected to decrease
- Correlation close to 1 means strongly, positively correlated
- Correlation close to -1 means strongly, negatively correlated

And it is strongly negatively correlated linearly. Now what we would expect is, in a case of this sort our correlation should be very low, Ok even though covariance is positive.

And indeed this is the case. So correlation in this case is point 1 4 which is

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Interpreting Covariance and correlation

- Positive covariance means when x increases, y is expected to increase too.
- Negative covariance means when x increases, y is expected to decrease
- Correlation close to 1 means strongly, positively correlated
- Correlation close to -1 means strongly, negatively correlated

lim

much smaller in comparison to point 9 7. In fact typical rule of thumb, if you take correlation below point 3 or even point 5, you might as well assume that the variables are not correlated. Of course the lower the correlation gets, the lesser actually the relationship between the two variables,

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Interpreting Covariance and correlation

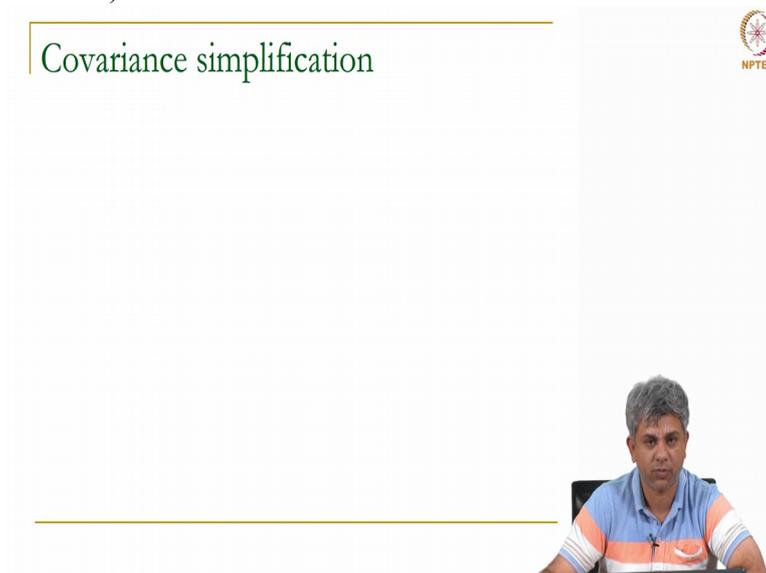
- Positive covariance means when x increases, y is expected to increase too.
- Negative covariance means when x increases, y is expected to decrease
- Correlation close to 1 means strongly, positively correlated
- Correlation close to -1 means strongly, negatively correlated
- Correlation close to 0 means no (linear) correlation

lim

Ok.

So correlation which is close to 0 means there is no real correlation between the two variables.

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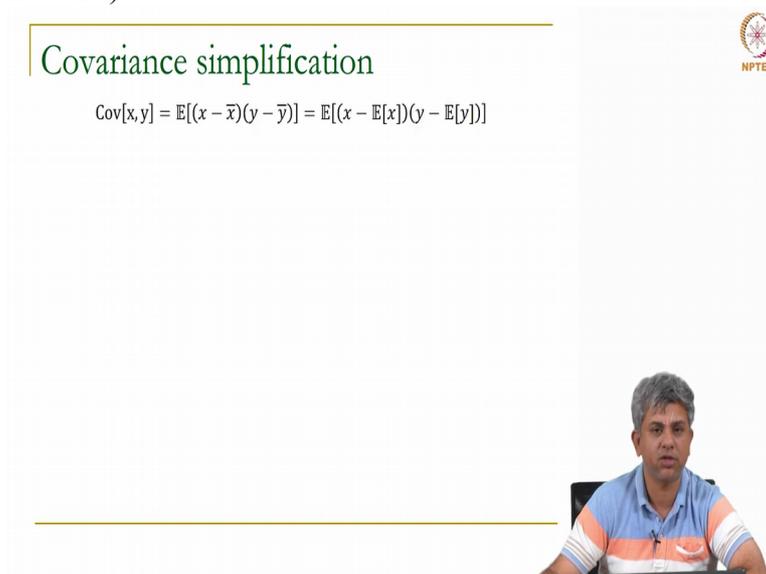
Covariance simplification

NPTEL

The slide features a title 'Covariance simplification' in green text at the top left. In the top right corner, there is a circular NPTEL logo. At the bottom right, there is a video feed of a male lecturer with grey hair, wearing a blue and white striped polo shirt, sitting at a desk.

So let us look at some mathematical simplification of covariance which we will be using

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

NPTEL

The slide features a title 'Covariance simplification' in green text at the top left. Below the title, the covariance formula is displayed in black text. In the top right corner, there is a circular NPTEL logo. At the bottom right, there is a video feed of the same male lecturer as in the previous slide.

multiple times through this course.

So recall that covariance of x comma y is expectation of x minus x bar multiplied by y minus y bar. You can

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$



simplify this. This is usually useful for calculations, that is covariance of x y is x y bar minus x bar y bar. Notice that the two are not the same.

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$



In terms of expectation notation, expectation of x y minus expectation of x expectation of y, Ok.

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to show

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{\bar{x}} = \bar{x}$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

So let me quickly prove this. We will use the fact that the expectation of expectation or the average of an average is simply the same as the average because the average is one single value, Ok. At least it is useful to think of it that way.

So if I have already taken average of 10 values I have got one simple number. If I take average above that, it does not make any difference whatsoever. Ok so let us use this.

So remember that covariance of x y is expectation of x minus x bar y minus y bar;

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to show

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{\bar{x}} = \bar{x}$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$
 $\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})]$

this is expectation of, let us open this bracket up, x y minus x bar y minus x y bar plus x bar y bar,

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

- This can be simplified as $\text{Cov}[x, y] = \bar{x}\bar{y} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$

Two are not to sum
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - \bar{x}y - x\bar{y} + \bar{x}\bar{y}]$$



we can read each of these quantities separately.

Expectation of x y minus expectation of x bar y minus expectation of x y bar plus expectation of x bar y bar,

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

- This can be simplified as $\text{Cov}[x, y] = \bar{x}\bar{y} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$

Two are not to sum
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - \bar{x}y - x\bar{y} + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[\bar{x}y] - \mathbb{E}[x\bar{y}] + \mathbb{E}[\bar{x}\bar{y}] \end{aligned}$$



Ok. So the first quantity is simply expectation of x y. We cannot do anything with it. Now in this quantity x bar is a constant, it can be brought out, so you can write this as x bar times expectation of y.

Here you can bring out y bar, y bar times expectation of x Ok and this is, these are already average quantities so you can write it as expectation of x bar,

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to sum

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - \bar{x}y - x\bar{y} + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[\bar{x}y] - \mathbb{E}[x\bar{y}] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \mathbb{E}[\bar{x}]\mathbb{E}[\bar{y}] \end{aligned}$$



simply $\bar{x}\bar{y}$, Ok. Expectation above and beyond this does not make sense.

So I will write this as expectation of xy minus $\bar{x}\bar{y}$, expectation of y is \bar{y} , minus $\bar{y}\bar{x}$, expectation of x is \bar{x} plus $\bar{x}\bar{y}$. These cancel

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to sum

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - \bar{x}y - x\bar{y} + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[\bar{x}y] - \mathbb{E}[x\bar{y}] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \mathbb{E}[\bar{x}]\mathbb{E}[\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} - \bar{y}\bar{x} + \bar{x}\bar{y} \end{aligned}$$



so you can write this as expectation of xy ,

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to sum

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$
$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - \bar{x}y - x\bar{y} + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[\bar{x}y] - \mathbb{E}[x\bar{y}] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} - \bar{y}\bar{x} + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \end{aligned}$$



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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to sum

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$
$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - \bar{x}y - x\bar{y} + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[\bar{x}y] - \mathbb{E}[x\bar{y}] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} - \bar{y}\bar{x} + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \\ &= \bar{xy} - \bar{x}\bar{y} \end{aligned}$$



expectation of x y minus expectation of x times expectation of y.

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

\leftarrow Two are not to sum

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[x\bar{y}] - \mathbb{E}[\bar{x}y] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} - \bar{y}\bar{x} + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \\ &= \bar{xy} - \bar{x}\bar{y} \quad \text{OR} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] \end{aligned}$$



So we have proved the result, Ok.

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

\leftarrow Two are not to sum

- This can be simplified as $\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[x\bar{y}] - \mathbb{E}[\bar{x}y] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} - \bar{y}\bar{x} + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \end{aligned}$$

\nearrow Proved

$\text{Cov}[x, y] = \bar{xy} - \bar{x}\bar{y} \quad \text{OR} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$



There is a special case of this which we tend to use, which is if you said, if x equal to y then we have covariance of x comma x, remember covariance of x comma x is simply, by

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to show

- This can be simplified as $\text{Cov}[x, y] = \bar{x}\bar{y} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[x\bar{y}] - \mathbb{E}[\bar{x}y] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \end{aligned}$$

Proof

$\text{Cov}[x, y] = \bar{x}\bar{y} - \bar{x}\bar{y} \quad \text{OR} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$

If $x=y$, $\text{Cov}[x, x] = \text{Var}[x]$



For $x = y$, this relation gives $\text{Var}[x] = \mathbb{E}[x^2] - (\mathbb{E}[x])^2$

definition variance of x.

This now becomes expectation of x square minus expectation of x multiplied by expectation of x

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

Two are not to show

- This can be simplified as $\text{Cov}[x, y] = \bar{x}\bar{y} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$
- Proof : We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[x\bar{y}] - \mathbb{E}[\bar{x}y] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \end{aligned}$$

Proof

$\text{Cov}[x, y] = \bar{x}\bar{y} - \bar{x}\bar{y} \quad \text{OR} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$

If $x=y$, $\text{Cov}[x, x] = \text{Var}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]\mathbb{E}[x]$



For $x = y$, this relation gives $\text{Var}[x] = \mathbb{E}[x^2] - (\mathbb{E}[x])^2$

which is expectation of x square minus expectation of x

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Covariance simplification

$$\text{Cov}[x, y] = \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[(x - \mathbb{E}[x])(y - \mathbb{E}[y])]$$

- This can be simplified as $\text{Cov}[x, y] = \overline{xy} - \bar{x}\bar{y} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y]$

- Proof: We use the fact that $\bar{x} = \mathbb{E}[x]$ i.e. $\mathbb{E}[\mathbb{E}[x]] = \mathbb{E}[x]$

$$\begin{aligned} \text{Cov}[x, y] &= \mathbb{E}[(x - \bar{x})(y - \bar{y})] = \mathbb{E}[xy - x\bar{y} - \bar{x}y + \bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \mathbb{E}[x\bar{y}] - \mathbb{E}[\bar{x}y] + \mathbb{E}[\bar{x}\bar{y}] \\ &= \mathbb{E}[xy] - \bar{x}\mathbb{E}[y] - \bar{y}\mathbb{E}[x] + \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \\ &= \mathbb{E}[xy] - \bar{x}\bar{y} \quad \text{OR} = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] \\ \text{If } x=y, \text{ Cov}[x, x] &= \text{Var}[x] = \mathbb{E}[x^2] - \mathbb{E}[x]^2 = \mathbb{E}[x^2] - (\mathbb{E}[x])^2 \end{aligned}$$

For $x = y$, this relation gives $\text{Var}[x] = \mathbb{E}[x^2] - (\mathbb{E}[x])^2$

the whole square, Ok. So this is also a definition that we are, or equivalence that we will often use.

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Independence and covariance

Let us come now; there is a certain point of covariance and independence that I wish to discuss.

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



So we already saw that when x and y are two independent random variables then covariance of x and y is actually 0. So if x varies in one way and y varies in a completely independent way you will actually get covariance of x comma y is equal to 0.

You can actually prove it from the previous slides results also

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



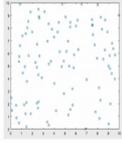
but I will not do that. I will just appeal to your intuition, Ok. So if you have,

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Independence and covariance



- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true



you know sort of random spread of points of this sort, then covariance of x comma y is 0.

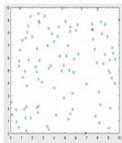
However if you want to go the other way, Ok which is you want to say that if covariance is 0, x and y are independent

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Independence and covariance



- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



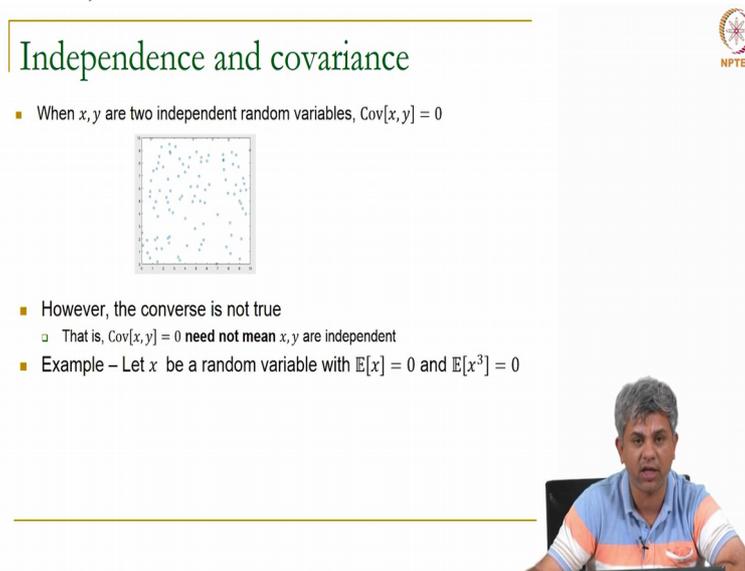
- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent



this is not true, Ok. So covariance of x being, x comma y being 0 need not necessarily mean that x and y are independent of each other.

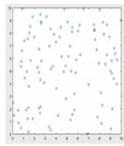
Let us see

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The slide is titled "Independence and covariance" and features the NPTEL logo in the top right corner. It contains the following text:

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



The scatter plot shows a square grid with a random distribution of blue dots, representing independent random variables.

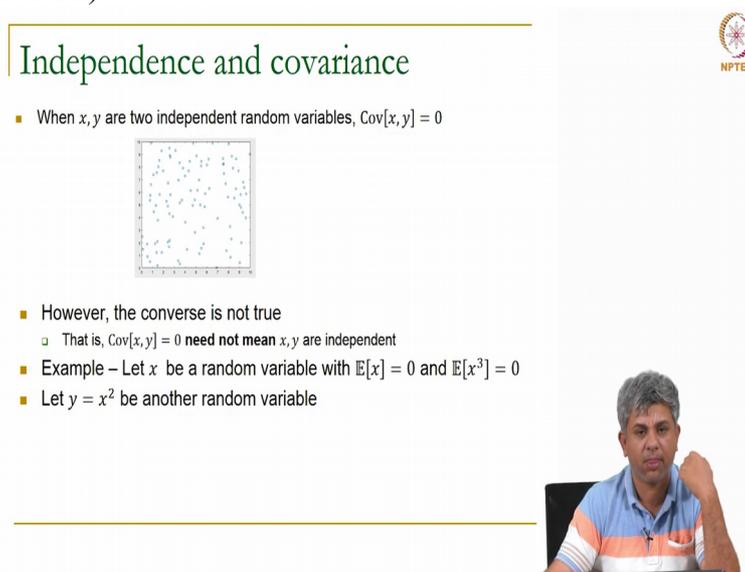
- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$

A speaker is visible in the bottom right corner of the slide.

an example, ok. So let us say x is a random variable with expectation of x is 0 and also expectation of x cube being 0, Ok. This need not always necessarily be the case, but you can easily generate a set of random numbers for which this is true, Ok.

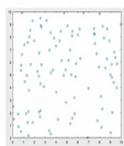
For example if I take x is a random variable, a uniform random variable with values going from minus 10 to 10 these are uniformly spaced and all values are equally possible then expectation of x will be 0, expectation of x cube will also be 0 as you can quite easily see,

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The slide is titled "Independence and covariance" and features the NPTEL logo in the top right corner. It contains the following text:

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



The scatter plot shows a square grid with a random distribution of blue dots, representing independent random variables.

- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable

A speaker is visible in the bottom right corner of the slide.

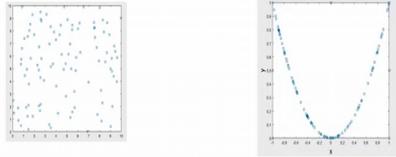
Ok.

Now let us take y as x square, Ok. Now y is another random variable.

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable

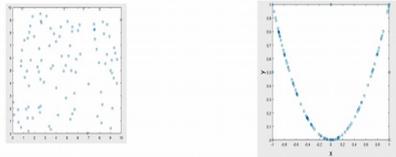


So if x is a random variable and y is simply x square you know you will find the distribution of x and y like this, Ok. Now clearly x and y are not independent, Ok. Obviously the value of y

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable *→ x, y are not independent*



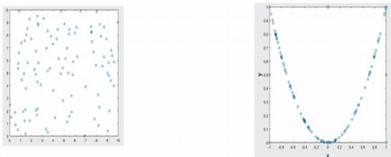
depends on the value of x .

However if we try and find out

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable $\rightarrow x, y$ are not independent
- Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$



covariance of x comma y , the expression that we had in the previous slide was the expectation of $x y$ minus expectation of x multiplied by expectation of y .

Now this is expectation of x cube minus expectation of x , expectation of x square. By construction

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable $\rightarrow x, y$ are not independent
- Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$

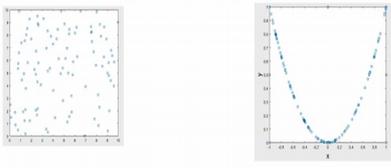


expectation of x cube was 0,

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ need not mean x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable $\rightarrow x, y$ are not independent
- Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$



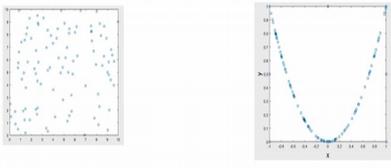
again by construction expectation of x was 0. So the covariance actually is 0.

So for this case even though you can see a nice relationship between the two variables, covariance is actually 0,

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$



- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ need not mean x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable $\rightarrow x, y$ are not independent
- Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$
 - That is, covariance is zero even though the variables are not independent

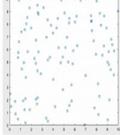
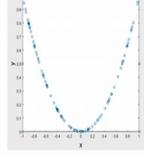


Ok. So that is, the covariance is 0 even though the variables are not independent of each other,

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$
 -  
- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable $\rightarrow x, y$ are **not** independent
- Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$
 - That is, covariance is zero even though the variables are not independent
 - It turns out zero covariance only means that there is no *linear* relationship



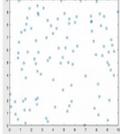
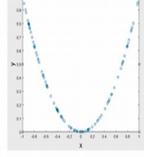
Ok.

So what turns out is covariance is 0 only means that there is no linear relationship, Ok. So

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$
 -  
- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
- Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
- Let $y = x^2$ be another random variable $\rightarrow x, y$ are **not** independent
- Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$
 - That is, covariance is zero even though the variables are not independent
 - It turns out zero covariance only means that there is no *linear* relationship
- In summary, Independence \Rightarrow Zero Covariance but Zero Covariance \neq Independence



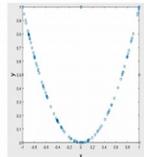
this can get a little bit confusing. So let me summarize this.

In case two variables are independent, Ok in this direction, so if independence is there, then there is zero covariance. But it does not necessarily mean that if there is zero covariance

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Independence and covariance

- When x, y are two independent random variables, $\text{Cov}[x, y] = 0$
 - 
 - 
- However, the converse is not true
 - That is, $\text{Cov}[x, y] = 0$ **need not mean** x, y are independent
 - Example – Let x be a random variable with $\mathbb{E}[x] = 0$ and $\mathbb{E}[x^3] = 0$
 - Let $y = x^2$ be another random variable $\rightarrow x, y$ are **not** independent
 - Then, $\text{Cov}[x, y] = \mathbb{E}[xy] - \mathbb{E}[x]\mathbb{E}[y] = \mathbb{E}[x^3] - \mathbb{E}[x]\mathbb{E}[x^2] = 0$
 - That is, covariance is zero even though the variables are not independent
 - It turns out zero covariance only means that there is no *linear* relationship
- In summary, Independence \Rightarrow Zero Covariance but Zero Covariance \nRightarrow Independence



that there is independence.

If there is zero covariance, all you can say is that they are not linearly dependent on each other, Ok but you cannot talk about general independence, Ok. So covariance basically measures linear dependence of one

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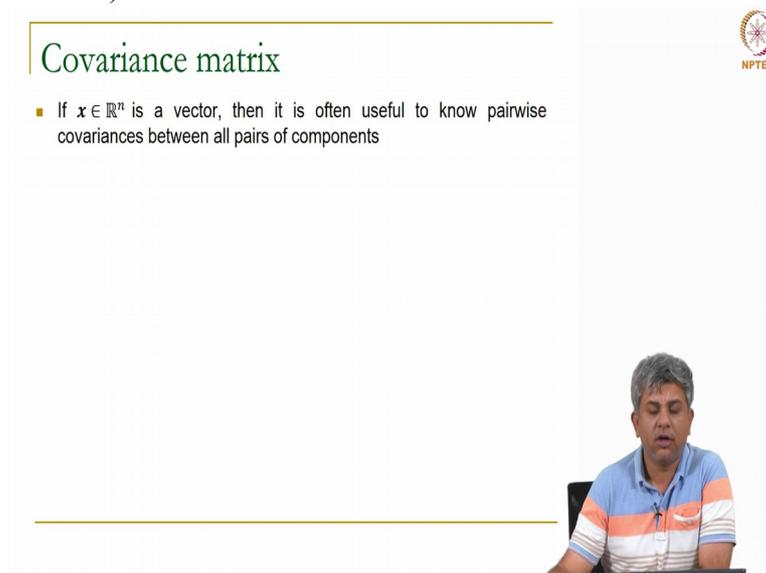
Covariance matrix



quantity on the other.

The final idea that I would like to discuss in this video is that of a covariance matrix. So, so far we have been looking at univariate x .

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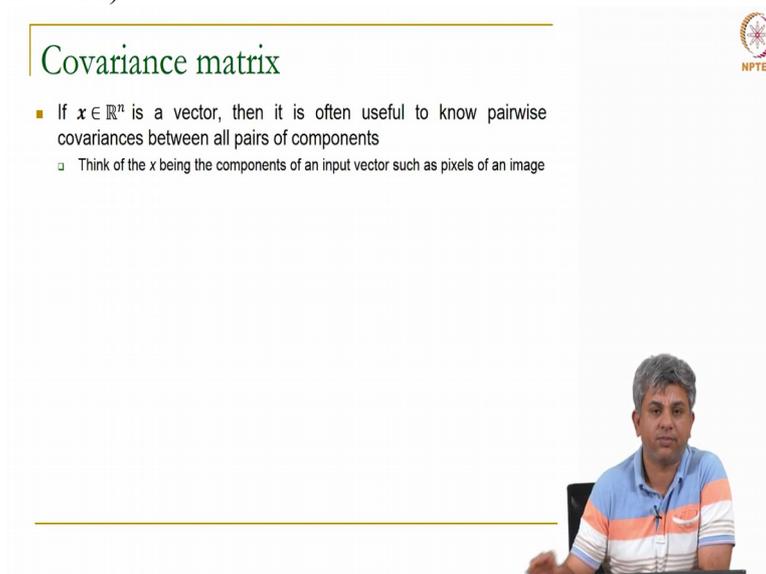
Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components

The slide features a yellow horizontal line above the title and another below the text. In the bottom right corner, there is a video inset showing a man with grey hair wearing a blue and white striped polo shirt, sitting at a desk. The NPTEL logo is visible in the top right corner of the slide.

Now let us consider case such as an image where x is a vector,

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image

The slide features a yellow horizontal line above the title and another below the text. In the bottom right corner, there is a video inset showing the same man from the previous slide. The NPTEL logo is visible in the top right corner of the slide.

Ok. It could be not just an image, of course I will keep on using that example because that is the most intuitive example and we will be using it very often in this course. But it could be anything, Ok.

So not only would you like to know, you know how each, what is the probability of each pixel is, but you would also like to know what is the joint probability of, let us say one pixel being white and the other pixel being black which is usually how you can characterize images.

Or if you look at an input vector such as temperature, pressure, humidity, you know what is the probability that the temperature is so much and the pressure is something else, the joint probability, stuff like that. And how much does temperature vary from the mean given that humidity has varied from its mean. So that is usually how the covariance matrix is used,

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$



Ok.

So if x is a vector, Ok let us say belonging to \mathbb{R}^n . Let us say it has n components. So we define a pair wise correlation or pair wise covariance of each of the variables, temperature with pressure, temperature with humidity, pressure with humidity etc and find out and also find temperature with temperature which is simply a variance etc.

And that is how you define a covariance matrix.

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image

- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \cdots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$



The covariance matrix is, even though it looks big, it is actually very simple to define.

So if you have x which is belonging to \mathbb{R}^n , so let us say, x is x_1 through x_n .

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image

- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \cdots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$

$$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$$



You find pair wise covariances. So the first entry of this matrix is covariance of x_1 with x_1 which is obviously, this is same as variance of x_1 .

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \cdots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$



The second entry here is covariance of x_1 with x_2 , so on and so forth. So covariance in general of x_i with x_j , Ok. This matrix will have size n cross n

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \cdots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$



and

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

\uparrow
variance

\uparrow
Cov

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \cdots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$

- Note that the diagonal elements are simply the variances of individual components, that is, $\text{Cov}[x_i, x_i] = \text{Var}[x_i]$

the diagonal entries of the matrix are simply the variances of the individual components.

So,

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

\uparrow
variance

\uparrow
Cov

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \cdots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$

- Note that the diagonal elements are simply the variances of individual components, that is, $\text{Cov}[x_i, x_i] = \text{Var}[x_i]$

Ok so this one will be variance of x_2 , this is variance of

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Covariance matrix

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \dots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \dots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \dots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$

- Note that the diagonal elements are simply the variances of individual components, that is, $\text{Cov}[x_i, x_i] = \text{Var}[x_i]$





x n. So the covariance matrix is something that we will be using quite often. Sometimes

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Covariance matrix - Σ

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \dots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \dots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \dots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$

- Note that the diagonal elements are simply the variances of individual components, that is, $\text{Cov}[x_i, x_i] = \text{Var}[x_i]$





it is denoted by capital sigma.

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Covariance matrix derived by \sum

- If $x \in \mathbb{R}^n$ is a vector, then it is often useful to know pairwise covariances between all pairs of components
 - Think of the x being the components of an input vector such as pixels of an image
- That is, define $\text{Cov}[x, x]_{i,j} = \text{Cov}[x_i, x_j]$

$x = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_n \end{bmatrix}$
Cov

$$\text{Cov}[x, x] = \begin{bmatrix} \text{Cov}[x_1, x_1] & \text{Cov}[x_1, x_2] & \cdots & \text{Cov}[x_1, x_n] \\ \text{Cov}[x_2, x_1] & \text{Cov}[x_2, x_2] & \ddots & \text{Cov}[x_2, x_n] \\ \vdots & \vdots & \ddots & \vdots \\ \text{Cov}[x_n, x_1] & \text{Cov}[x_n, x_2] & \cdots & \text{Cov}[x_n, x_n] \end{bmatrix} \in \mathbb{R}^{n \times n}$$

Var $[x_i]$



So we will be seeing this in greater detail in future videos. Thank you.