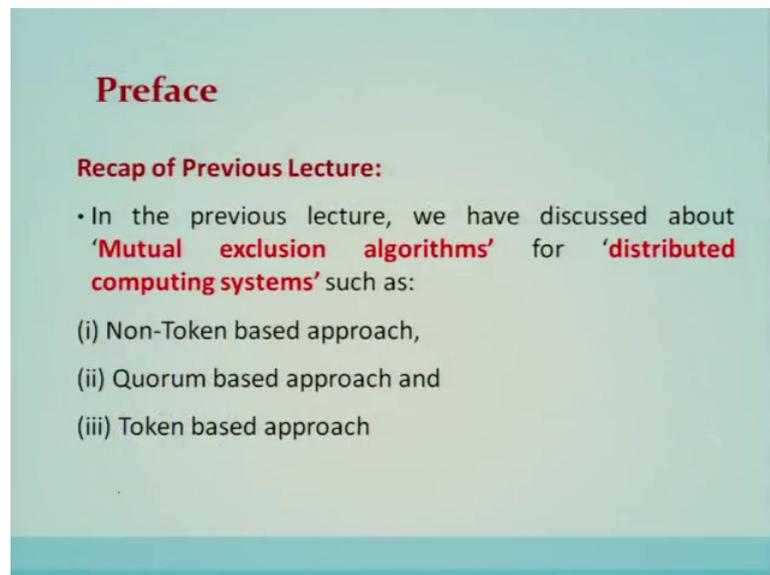


Distributed Systems
Dr. Rajiv Misra
Department of Computer Science and Engineering
Indian Institute of Technology, Patna

Lecture – 10
Consensus and Agreement Algorithms

Lecture 10, Consensus and Agreement Algorithms.

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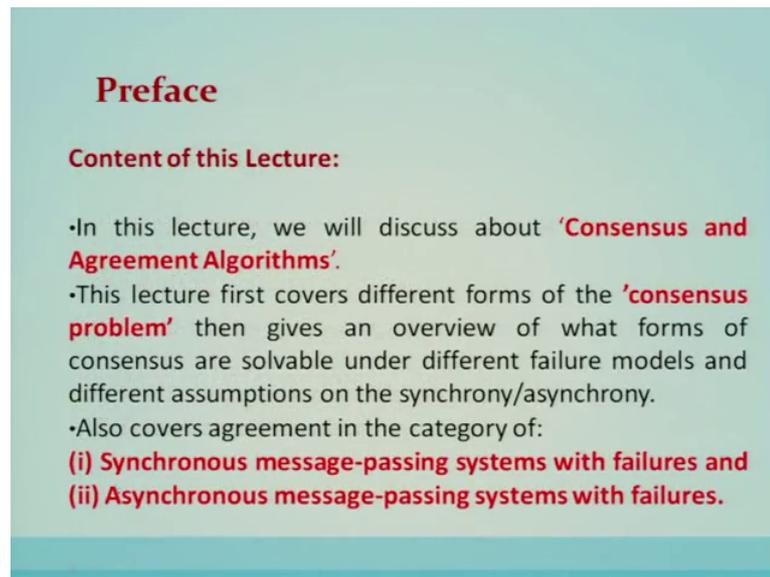
Preface

Recap of Previous Lecture:

- In the previous lecture, we have discussed about **'Mutual exclusion algorithms'** for **'distributed computing systems'** such as:
 - (i) Non-Token based approach,
 - (ii) Quorum based approach and
 - (iii) Token based approach

Preface, recap of previous lecture - in previous lecture we have discussed about mutual exclusion algorithms for a distributed computing system such as non-token based algorithms, quorum based algorithms and token based algorithms.

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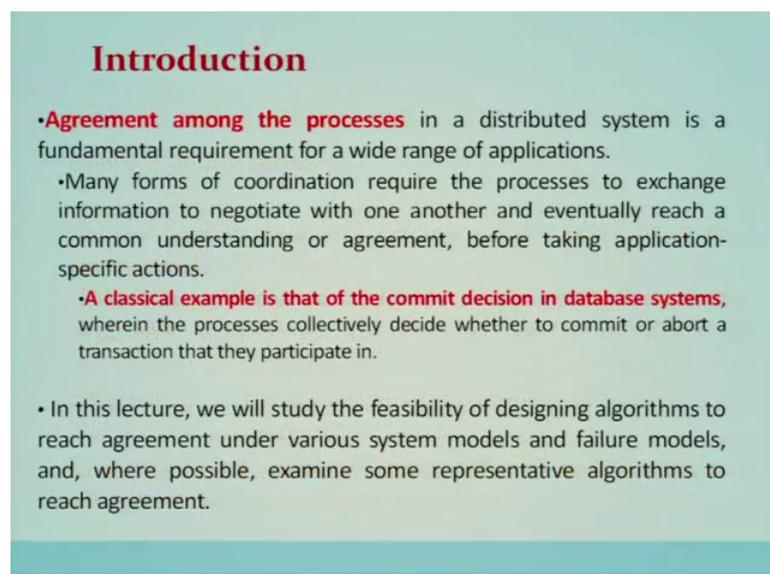
Preface

Content of this Lecture:

- In this lecture, we will discuss about '**Consensus and Agreement Algorithms**'.
- This lecture first covers different forms of the '**consensus problem**' then gives an overview of what forms of consensus are solvable under different failure models and different assumptions on the synchrony/asynchrony.
- Also covers agreement in the category of:
 - (i) **Synchronous message-passing systems with failures and**
 - (ii) **Asynchronous message-passing systems with failures.**

Content of this lecture, in this lecture we will discuss about consensus and agreement algorithms. This lecture first covers different aspects of consensus problem and then gives an overview of what forms the consensus are solvable under different failure models and different assumptions on synchrony and asynchrony. Also it covers the agreement in the category of synchronous message passing system with failures and asynchronous message passing system with failures.

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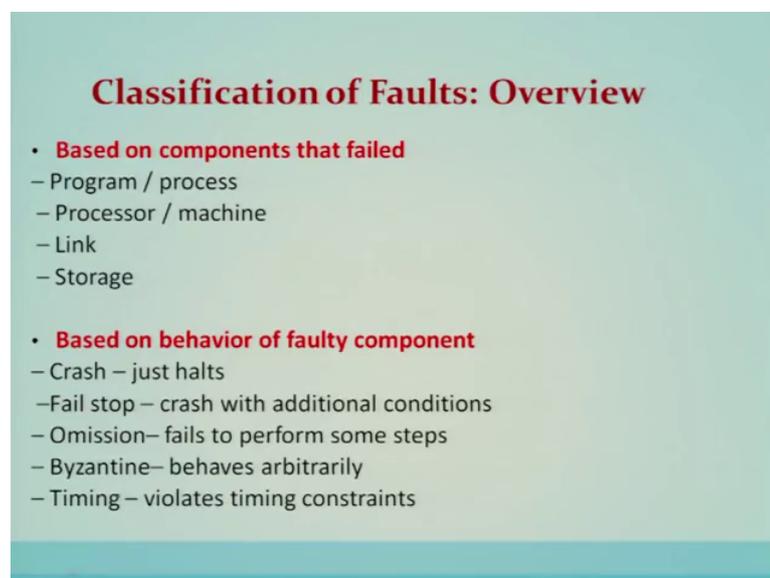
Introduction

- Agreement among the processes** in a distributed system is a fundamental requirement for a wide range of applications.
 - Many forms of coordination require the processes to exchange information to negotiate with one another and eventually reach a common understanding or agreement, before taking application-specific actions.
 - A classical example is that of the commit decision in database systems,** wherein the processes collectively decide whether to commit or abort a transaction that they participate in.
- In this lecture, we will study the feasibility of designing algorithms to reach agreement under various system models and failure models, and, where possible, examine some representative algorithms to reach agreement.

Introduction, agreement among the processes in a distributed system is a fundamental requirement for a wide range of applications. Many forms of coordination require the processes to exchange information to negotiate with one another and eventually reach a common understanding or an agreement before taking and application specific actions a classical example is the commit decision in the database system where in the process collectively decide whether to commit or abort a transaction that they participate in.

In this lecture we will study the feasibility of designing algorithms to reach agreement under various system models and failure models and we are possible examine some of the representative algorithms to reach an agreement.

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Classification of Faults: Overview

- **Based on components that failed**
 - Program / process
 - Processor / machine
 - Link
 - Storage
- **Based on behavior of faulty component**
 - Crash – just halts
 - Fail stop – crash with additional conditions
 - Omission – fails to perform some steps
 - Byzantine – behaves arbitrarily
 - Timing – violates timing constraints

Now, we are going to see some of the fault models. So, let us classify the faults. So, based on the component that failed the components which are basically failing in the distributed systems are classified as either the program or a process or a processor or a machine or a link or a storage. So, they are prone to the failures and we assume that in this discussion that one of these components if it is failing how the algorithms are basically going to lead to a consensus or agreement which is required to run the applications. So, based on the behavior of faulty component, so the fault models different fault models we will be classified as follows.

So, the crash, crash fault there we assume that the system just halts after the problem. Fail stop, fail stop will crash with some additional conditions, crash is means that it will

halt. So, omission means it will fail to perform some by step; that means, some of the step will omit. Byzantine fault behaves arbitrary is the most general fault model. Timing, it violates the timing constraints. These are the different faulty components and they are basically used to model down failure models.

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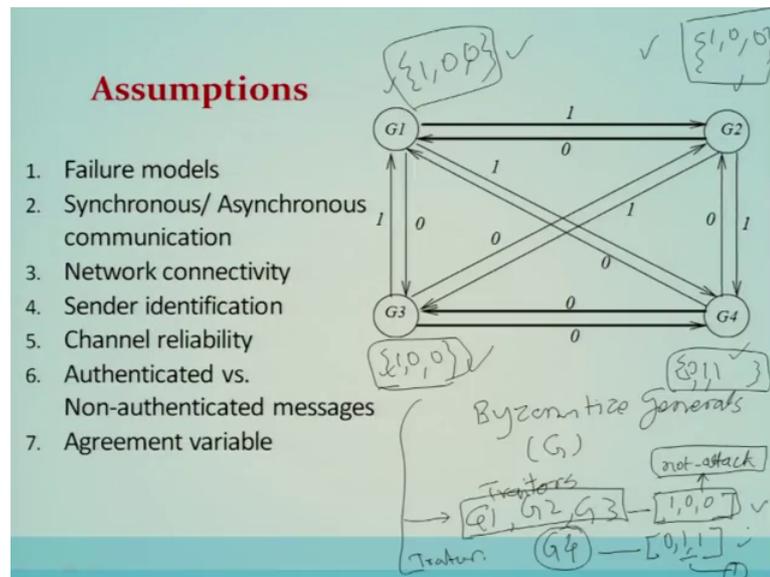
Classification of Tolerance: Overview

- **Types of tolerance:**
 - **Masking** – System always behaves as per specifications even in presence of faults.
 - **Non-masking** – System may violate specifications in presence of faults. Should at least behave in a well-defined manner.
- **Fault tolerant system should specify:**
 - Class of faults tolerated
 - What tolerance is given from each class

Now, classification of tolerance type of tolerance masking the system always behaves as per the specification in the presence of faults; that means, faults are masked. Non masking that system may violate specification in the presence of faults should at least behave in a well defined manner.

Fault tolerant system should specify the class of faults tolerated what tolerance is given from each class. So, these are basically some of the requirements.

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Now, as far as different models are concerned different assumptions which we are going to use in this particular algorithm design the assumptions are we have to assume some failure models, so we have to see what are the different failure models possible. Then we have to make an assumption about synchronous and asynchronous type of communication.

Then another resumption we have to make about its network activity sender identification channel reliability authenticated versus non a authenticated message and agreement values. These are the different assumptions which we have to make in the algorithm we are going to discuss one by one in the next coming slides.

So, before that let us see the problems of an agreement in this particular figure which is shown who are here G 1 is basically this particular problem is about byzantine general problem that is why it is basically G, it is represented as G. So, here there are four general and they are leading the troops and that is why they are the group 1, group 2, group 3 and group 4 different generals they are located in the byzantine city and this particular hill is located in Istanbul nowadays.

So, at four different sites where they will not be able to see each other and located on the hill these four different byzantine generals they have to take the decision either to attack or not to attack. And if they take the decision simultaneously then they are going to win if they are not going to take decision simultaneously then basically they are going to

lose. Now some of these particular byzantine generals are the traitors and they will communicate whether to attack or not to attack they have to communicate with the help of message. So, here the message are nothing, but they are communicated through the messengers and the messengers are the messages are lost means if these messengers are caught by the army camp. So, here this G 1 G 2 G 3 G 4 they are byzantine generals and they are basically representing their troops at four different locations and these communication links are nothing, but they are the messengers they are able to they will be sending or they will be communicating with these different generals and once they are communicated securely then they have to follow the action whether to attack or not to attack.

.So, here you can see that there are some generals which are traitors also. So, depending upon how many generals traitor they have to take the decisions - 0 and 1, 0 means no attack and 1 means attack. So, let us assume that, so after the set of message exchanges G 4 will receive the messages 0 then 1 and 1 and G 3 will receive the messages from the other generals as 1 then 0 and 0 and G 2 will receive the messages as one then 0 and 0 and G 1 will receive the messages as 1 then 0 and then 0. Now can we just see that the messages which they received if it is coded in some form; that means, neither means it is very difficult to take this particular decision why because, so here it is 1 0 0 here it is 1 0 0. So, you can see that G 1 G 2 and G 3 they have reached to the same values, but as far as G 4 is concerned G 4 has a different value.

So, if let us say it is interpreted as G 1 G 2 G 3 will have the same value and the majority of it is 0s that is not to be attacked. As far as G 4 is concerned if we go by this then it will have 1 and let us say that it is having decision to attack. So, basically here we can conclude that this is basically a traitor and other non traitor that is the generals they are basically lead to a common value and that is not to attack. So, this basically values are arrived at after several message rounds. So, in still this particular problem is unsolved why because here G 1 G 2 G 3 they reach to common value and G 4 is not. So, everyone is not agreeing on a common value. So, the most of the applications they have to basically depend upon how the agreement is to be arrived at agreement of same value or agreement or set of values. So, these are the problems and this app has a wide applications and we are going to see how this particular agreement problem is going to be solved using algorithms and in what are the models and what are the system models

failure models where we are going to solve them. So, we are going to see these assumptions one by one.

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1) Failure models

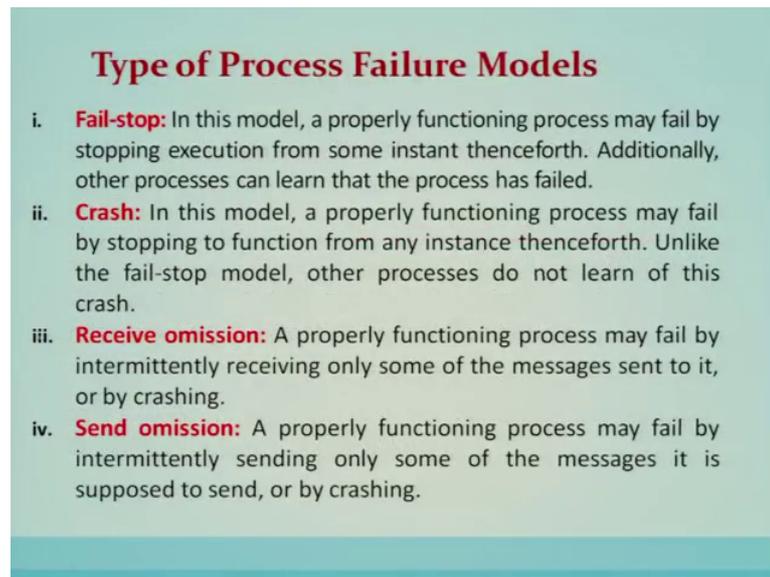
- A failure model specifies the manner in which the component(s) of the system may fail.
- There exists a rich class of **well-studied failure models**. The various process failure models are: (i) Fail-stop, (ii) Crash, (iii) Receive omission, (iv) Send omission, (v) General omission, and (vi) Byzantine or malicious failures
- Among the n processes in the system, at most f processes can be faulty. A faulty process can behave in any manner allowed by the failure model assumed.

n
 f

A failure model, a failure model specifies the manner in which the component of a system may fail. There exist a rich class of well studied failure models the various failure models are fail stop, crash, receive omission, send omission, general omission and byzantine or malicious failures.

Now here in failure models among n processes in the system, we represent it using n small n and we can also assume that there are at most f different processes can be faulty. So, a faulty process can behave in any manner allowed by the failure model which basically is to be assumed in the particular problem setting.

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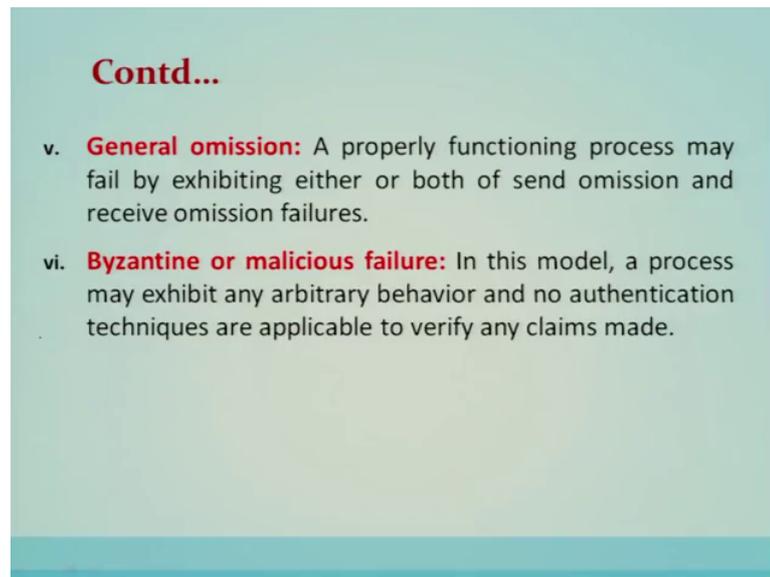
Type of Process Failure Models

- i. **Fail-stop:** In this model, a properly functioning process may fail by stopping execution from some instant thenceforth. Additionally, other processes can learn that the process has failed.
- ii. **Crash:** In this model, a properly functioning process may fail by stopping to function from any instance thenceforth. Unlike the fail-stop model, other processes do not learn of this crash.
- iii. **Receive omission:** A properly functioning process may fail by intermittently receiving only some of the messages sent to it, or by crashing.
- iv. **Send omission:** A properly functioning process may fail by intermittently sending only some of the messages it is supposed to send, or by crashing.

The types of failure models. So, the first type is called fail stop in this model a properly functioning process may fail by stopping the execution from some instance thenceforth, additionally other processes can learn that the process has failed this is called fail stop.

Crash, crash failure model in this model a properly functioning process may fail by stopping to function from any instance thenceforth unlike the failed stop model other processes do not learn of this crash. Third one is receive omission failure model a properly functioning process may fail by intermittently receiving only some of the messages sent to it or by crashing. Send omission a properly functioning process may fail by intermittently sending only some of the messages it is supposed to send or by crashing.

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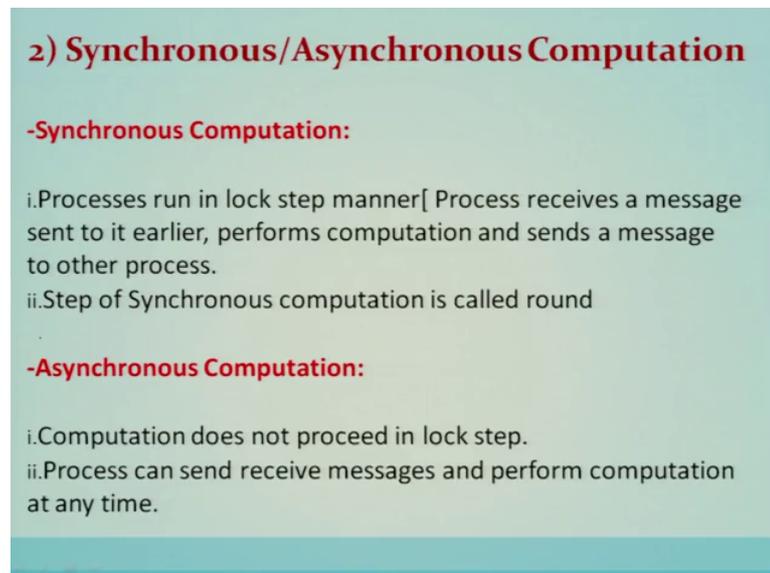


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- v. **General omission:** A properly functioning process may fail by exhibiting either or both of send omission and receive omission failures.
- vi. **Byzantine or malicious failure:** In this model, a process may exhibit any arbitrary behavior and no authentication techniques are applicable to verify any claims made.

General omission a properly functioning process may fail by exhibiting either or both of send omission and receive omission failures. Byzantine or malicious in this model a process may exhibit any arbitrary behavior and no authentication techniques are applicable to verify any claims made.

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2) Synchronous/Asynchronous Computation

-Synchronous Computation:

- i.Processes run in lock step manner[Process receives a message sent to it earlier, performs computation and sends a message to other process.
- ii.Step of Synchronous computation is called round

-Asynchronous Computation:

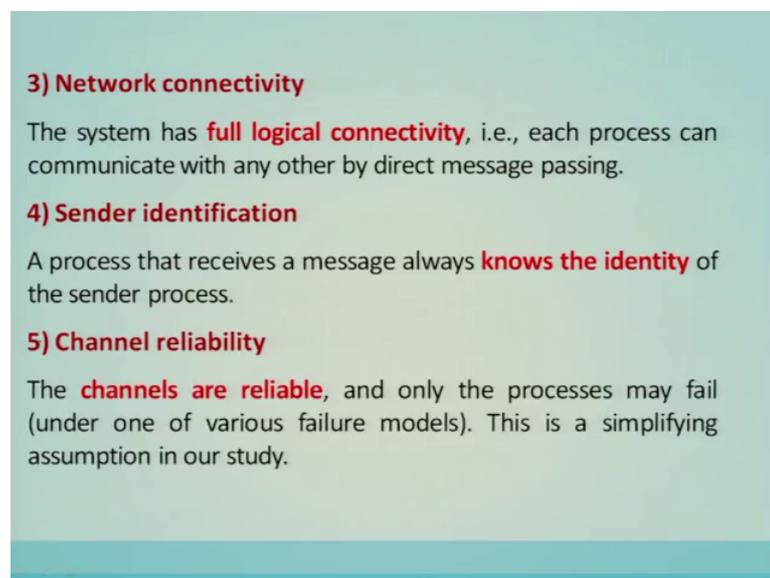
- i.Computation does not proceed in lock step.
- ii.Process can send receive messages and perform computation at any time.

Now, the next assumptions in these algorithms are called about the synchronous oblique asynchronous computations Synchronous computation a process runs in a lock step

manner that is a process receives a message sent to it earlier performs the computation using those message and send the message to other process.

So, this is basically lockstep which the process runs in this particular discrete sequence. So, steps of synchronous computation is called around. A synchronous computation, computation does not proceed in this strict lockstep manner the process can send receive messages and perform the configuration at any point of time there is no bound in these delays of the messages which is assumed in a synchronous communication.

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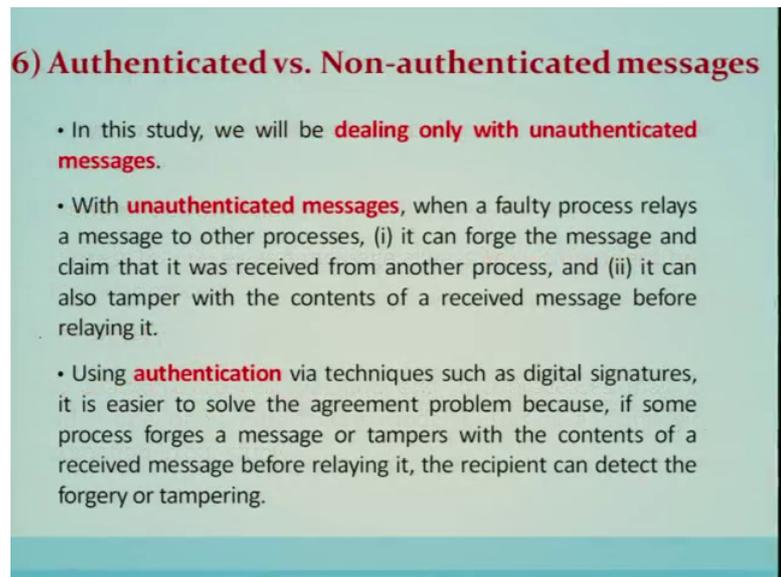
3) Network connectivity
The system has **full logical connectivity**, i.e., each process can communicate with any other by direct message passing.

4) Sender identification
A process that receives a message always **knows the identity** of the sender process.

5) Channel reliability
The **channels are reliable**, and only the processes may fail (under one of various failure models). This is a simplifying assumption in our study.

Third type of assumption is called about network connectivity; system has full logical connectivity that is the processes can communicate with a any other process by direct message passing. Fourth assumption is about sender identification a process that receives a message always moves the identity of the sender. Fifth channel reliability the channels are reliable and only processes may fail this is very important assumptions. So, the failure about processes we are going to or processes we have we are assuming under different failure models. So, this will simplify our study about different algorithms in this setting.

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6) Authenticated vs. Non-authenticated messages

- In this study, we will be **dealing only with unauthenticated messages**.
- With **unauthenticated messages**, when a faulty process relays a message to other processes, (i) it can forge the message and claim that it was received from another process, and (ii) it can also tamper with the contents of a received message before relaying it.
- Using **authentication** via techniques such as digital signatures, it is easier to solve the agreement problem because, if some process forges a message or tampers with the contents of a received message before relaying it, the recipient can detect the forgery or tampering.

Authenticated versus non authenticated messages: In this part of the discussion we will be dealing only with the unauthenticated messages, with unauthenticated message when a faulty process relays the message to other processes it can forge the message and claim that it was received from another process and it can also tamper with the content of the received message before relaying it. Using authentication via the technique such as digital signature it is easy to solve the agreement problem because if some of the process forges the message or tampers with the content of the received message before relaying it the recipient can detect the forgery or tampering. So, we are going to use only the unauthenticated messages as per the algorithms are concerned.

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7) Agreement variable

- The agreement variable **may be boolean or multivalued**, and need not be an integer.
- When studying some of the more complex algorithms, we will use a boolean variable.
- This simplifying assumption does not affect the results for other data types, but helps in the abstraction while presenting the algorithms.

Agreement variable, the agreement variable may be a boolean or a multivalued and need not be an integer when studying some more complex algorithms we will assume it as boolean variable. The simplifying assumptions does not affect the result of other data type, but helps in abstraction while presenting the more details of the insight of the algorithm.

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Performance Aspects of Agreement Protocols

Few Performance Metrics are as follows:

- (i) **Time:** No of rounds needed to reach an agreement
- (ii) **Message Traffic:** Number of messages exchanged to reach an agreement.
- (iii) **Storage Overhead:** Amount of information that needs to stored at processors during execution of the protocol.

Performance aspects of the agreement protocol, few performance matrix matrices for the agreement protocols are as follows first is the time that is number of rounds needed to

reach an agreement message traffic that is the number of messages exchanged to reach an agreement, then storage overhead amount of information that needs to store at the processor during the execution of the protocol.

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Problem Specifications

1. Byzantine Agreement Problem (single source has an initial value)

- Agreement:** All non-faulty processes must agree on the same value.
- Validity:** If the source process is non-faulty, then the agreed upon value by all the non-faulty processes must be the same as the initial value of the source.
- Termination:** Each non-faulty process must eventually decide on a value.

2. Consensus Problem (all processes have an initial value)

- Agreement:** All non-faulty processes must agree on the same (single) value.
- Validity:** If all the non-faulty processes have the same initial value, then the agreed upon value by all the non-faulty processes must be that same value.
- Termination:** Each non-faulty process must eventually decide on a value.

Problem specifications. So, the problem is specification under this agreement and consensus algorithms are as follows - the first one is byzantine agreement problem here the single source has the initial value. So, it has three conditions the agreement that is all non faulty processes must agree on the same value. Validity, if the source is non faulty than agreed upon value by all non faulty processes must be the same as the initial value of the source. Termination each non faulty process must eventually decide on a value.

Consensus problem - all processes have an initial value agreement all non faulty processes must agree on the same that is a single value. Validity, if all the on faulty processes have the same initial value then agreed upon value by all non faulty process must be that same value. Termination, each non faulty process must eventually decide on a value. So, may be that you have seen the difference if you have not noticed then again I am underlining it.

In byzantine agreement only one source has the initial value, in consensus problem all the processes not only single, but all the process have the initial values and in both of the cases they have to decide on a particular value, in both the cases they have to decide on a particular value. So, the only the problem setting is different here in consensus all the

process have their initial values we are in magenta in agreement only the single source has the initial value.

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3. Interactive Consistency Problem (all processes have an initial value)

Agreement: All non-faulty processes must agree on the same array of values $A[v_1 \dots v_n]$.

Validity: If process i is non-faulty and its initial value is v_i , then all non-faulty processes agree on v_i as the i th element of the array A . If process j is faulty, then the non-faulty processes can agree on any value for $A[j]$.

Termination: Each non-faulty process must eventually decide on the array A .

Third type of problem is called interactive consistency problem here all the processes have the initial values. So, agreement all non faulty process must agree on the same array of values. This you have to notice that this is the difference they are the earlier two methods they have to agree on a single value here they are agreeing on a set of values that is an array of values validity. If a process i is non faulty and its initial value is v_i then all non faulty processes agree on v_i as i th element of the array. If p_j process j is faulty then the non faulty processes can agree on any value for A_j .

Termination each non faulty process must eventually decide on an array. So, the array of a particular element shows that this value if they agree if it is a non faulty then they have to agree on that initial value and if it is a faulty then they have to agree on any value. So, this particular array will be built for n different processes 1 and so on up to n and so at the termination all non faulty process must decide on this particular array this array will be exchanged at all the non faulty. So, this is called interactive consistency problem.

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Equivalence of the Problems

- The three problems defined above are equivalent in the sense that a solution to any one of them can be used as a solution to the other two problems. This equivalence can be shown using a reduction of each problem to the other two problems.
- If problem A is reduced to problem B, then a solution to problem B can be used as a solution to problem A in conjunction with the reduction.
- Formally, the **difference between the agreement problem and the consensus problem** is that, in the agreement problem, a single process has the initial value, whereas in the consensus problem, all processes have an initial value.
- However, the two terms are used interchangeably in much of the literature and hence we shall also use the terms interchangeably.

Among three problems there is an equivalence. So, three problems defined above are equivalent in the sense that a solution to one of them can be used as a solution to the other two problems this equivalence can be shown using a reduction of each problem to the other two problem. For example, if a problem A is reduced to a problem B, then the solution to a problem we can be used to solve the problem A in conjunction with the reduction.

Formally, the difference between the agreement and the consensus is that an agreement problem a single process has the initial value like in a byzantine agreement whereas, in consensus problem all the processes have the initial values. However, and they are equivalent; that means, if you know how to solve a byzantine agreement problem then basically the solution of that byzantine agreement problem can be used to solve the consensus problem and interactive consistency problem.

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Overview of Results

- **Table 10.1** gives an overview of the results and lower bounds on solving the consensus problem under different assumptions.
- It is worth understanding the relation between the consensus problem and the problem of attaining common knowledge of the agreement value. For the “no failure” case, consensus is attainable.
- Further, in a synchronous system, common knowledge of the consensus value is also attainable, whereas in the asynchronous case, concurrent common knowledge of the consensus value is attainable.

So, overview of the results, the following table gives an overview of the results and the lower bound on solving the consensus problem under different assumptions. It is worth understanding the relation between the consensus problem and the problem of attaining common knowledge of the agreement value. For the no failure case consensus is attainable that we will see in the next table.

Further in the synchronous system common knowledge of the consensus value is also attainable whereas, in the asynchronous case concurrent common knowledge of the consensus is attainable in no fault situation that we can see in this particular in this figure.

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Overview of Results		
Failure mode	Synchronous system (message-passing and shared memory)	Asynchronous system (message-passing and shared memory)
No failure	agreement attainable; common knowledge also attainable	agreement attainable; concurrent common knowledge attainable
Crash failure	agreement attainable; $f < n$ processes $\Omega(f+1)$ rounds	agreement not attainable
Byzantine failure	agreement attainable; $f \leq \lfloor (n-1)/3 \rfloor$ Byzantine processes $\Omega(f+1)$ rounds	agreement not attainable

Table 10.1: Overview of results on agreement. f denotes number of failure-prone processes. n is the total number of processes.

In a failure-free system, consensus can be attained in a straightforward manner

So, if the failure model is no failure and in the synchronous model the agreement is attainable and common knowledge is also attainable. If it is a synchronous message passing and a shared memory system then agreement is attainable and also the concurrent common knowledge is attainable, but if the failure model is a crash fault or a crash failure then agreement is attainable in the synchronous system and f the number of faulty processor is assumed to be less than the total number of processors. And it will be resolved that is agreement can be attainable in the f plus 1 of a lower bound number of rounds; that means, minimum f plus 1 round is required to basically achieve the agreement.

However, in a synchronous model even for the crash fault or a crash failure model agreement is not attainable at all. If the fault model is byzantine then the agreement is attainable in the synchronous system where the faulty number of processor is less than or equal to the floor of n minus 1 divided by 3. And this particular agreement is achieved in the lower bound of f plus 1 number of round where f is the faulty number of nodes.

However in a synchronous model the agreement under byzantine failure is also not attainable because if it is not attainable in the crash fault then obviously, it will not be attainable in the byzantine fault.

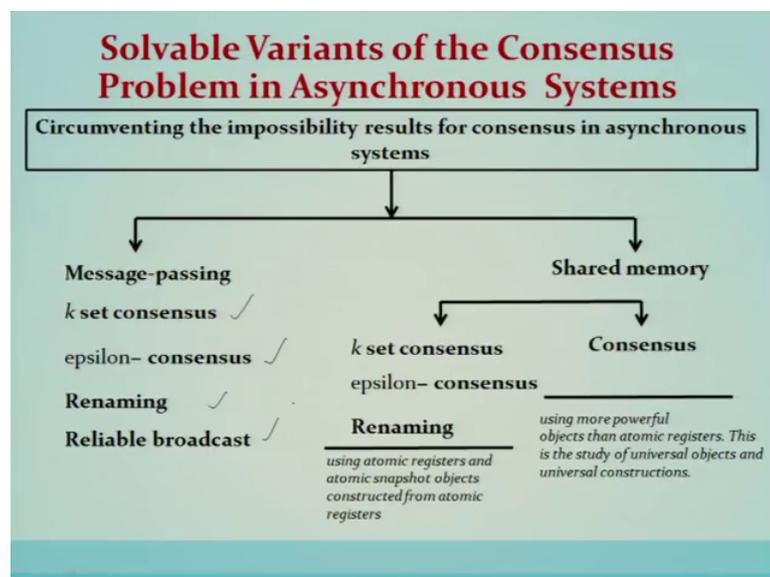
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- **Consensus is not solvable in asynchronous systems** even if one process can fail by crashing. ✓
- **Figure 10.1** shows further how asynchronous message-passing systems and shared memory systems deal with trying to solve consensus.

So, consensus is not solvable in a synchronous system even if one processor can fail by the crashing. So, the next figure for that shows how a synchronous message passing system and a shared memory system deal with trying to solve the consensus problem.

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Now, since you know that if possibility of the consensus problem under asynchronous system, so what we can do is basically we can solve a variance of this consensus problem in this model that is in a synchronous system and that is shown here. So, under message passing system a variant of the consensus problem is called k set consensus, epsilon

consensus renaming and reliable broadcast that we will see in the next few slides in the more detail about them.

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Weaker Consensus Problems in Asynchronous System	
Consensus Problem	Description
Terminating reliable broadcast	It states that a correct process always gets a message even if the sender crashes while sending. If the sender crashes while sending the message, the message may be a null message but it must be delivered to each correct process.
k-set consensus	It is solvable as long as the number of crash failures f is less than the parameter k . The parameter k indicates that the non-faulty processes agree on different values, as long as the size of the set of values agreed upon is bounded by k .
Approximate agreement	Like k -set consensus, approximate agreement also assumes the consensus value is from a multi-valued domain. However, rather than restricting the set of consensus values to a set of size k , ϵ -approximate agreement requires that the agreed upon values by the non-faulty processes be within ϵ of each other.
Renaming problem	It requires the processes to agree on necessarily distinct values.
Reliable broadcast	A weaker version of reliable terminating broadcast (RTB), namely reliable broadcast, in which the termination condition is dropped, is solvable under crash failures.

Now, the weaker consensus problem in synchronous system we are just trying to understand what these problems are a terminating reliable broadcast it is states that a correct process always gets a message even if the sender crashes while sending it. k -set consensus it is solvable as long as the number of crash failures f is less than the parameter k the parameter k indicates that non faulty processes agree on a different values as long as the size of the set where our values agreed upon is bounded by k that is called k set consensus. Approximate agreement like k set consensus approximate agreement also assume the consensus value is from multi value domain; however, rather than restricting the set of consensus values to a set, set of size k it says that epsilon approximate agreement requires that the agreed upon value by the non faulty processor is within epsilon of each other.

Renaming problem it requires the processes to agree on necessarily a distinct value. A reliable broadcast a weaker version of a reliable terminating broadcast, namely the reliable broadcast in which the termination condition is dropped is solvable under the crash fault.

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- To circumvent the impossibility result, weaker variants of the consensus problem are defined in **Table 10.2**.
- The overheads given in this table are for the algorithms described.

To circumvent the impossibility results the weaker variants are defined in the next table.

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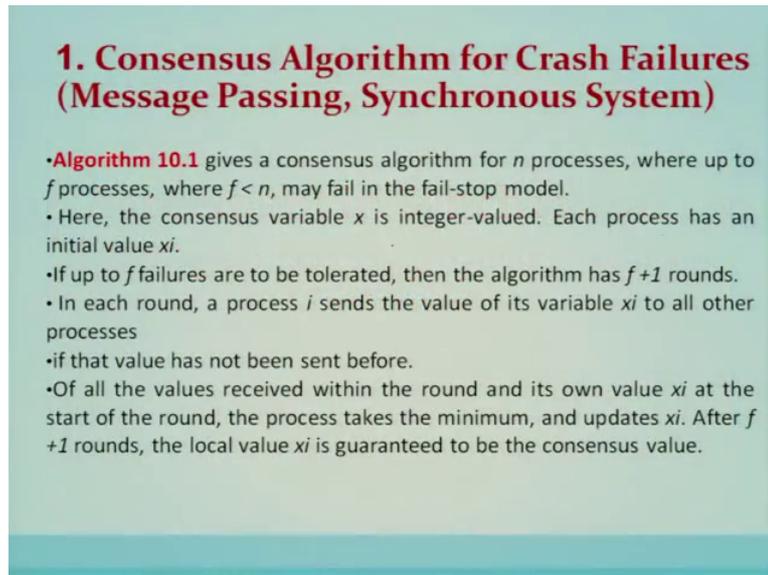
Some Solvable Variants of the Consensus Problem in Asynchronous Systems

Solvable Variants	Failure model and overhead	Definition
Reliable broadcast	crash failures, $n > f$ (MP)	Validity, Agreement, Integrity conditions
k-set consensus	crash failures, $f < k < n$. (MP and SM)	size of the set of values agreed upon must be less than k
ϵ -agreement	crash failures $n \geq 5f + 1$ (MP)	values agreed upon are within ϵ of each other
Renaming	up to f fail-stop processes, $n \geq 2f + 1$ (MP) Crash failures $f \leq n - 1$ (SM)	select a unique name from a set of names

Table 10.2: Some solvable variants of the agreement problem in asynchronous system. The overhead bounds are for the given algorithms, and not necessarily tight bounds for the problem. Here MP- Message Passing, SM- Shared Memory

So, reliable broadcast failure model if it is crash fault it requires the over the condition n is greater than f similarly k set consensus. Then basically the value of k should be more than f and should be bound less than less than n . Epsilon agreement crash failures this particular model is working n is should be greater than equal to $5f + 1$. Renaming up to f fail stop processes it can sustain and n should be greater than $2f + 1$ and for the crash fault f is less than $n - 1$.

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1. Consensus Algorithm for Crash Failures (Message Passing, Synchronous System)

- **Algorithm 10.1** gives a consensus algorithm for n processes, where up to f processes, where $f < n$, may fail in the fail-stop model.
- Here, the consensus variable x is integer-valued. Each process has an initial value x_i .
- If up to f failures are to be tolerated, then the algorithm has $f + 1$ rounds.
- In each round, a process i sends the value of its variable x_i to all other processes if that value has not been sent before.
- Of all the values received within the round and its own value x_i at the start of the round, the process takes the minimum, and updates x_i . After $f + 1$ rounds, the local value x_i is guaranteed to be the consensus value.

Agreement in synchronous message passing systems with failures: Consensus algorithm for crash failures message passing synchronous system. So, algorithm given in the next slide it gives the consensus algorithm for n processes where up to f processes where f is less than n may fail in a fail stop failure model. Here the consensus variable x is integer value; each process has initial value x_i . If up to f failures are to be tolerated than algorithm has f plus 1 rounds, in each round a process I sense the value of its variable x I to all other processes if that value has not been sent before.

So, of all the values received within that round and its own value x_i at that start of the round the process takes minimum and updates x_i occur f plus 1 rounds the local value x I guaranteed to be the consensus value.

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Consensus Algorithm for Crash Failures

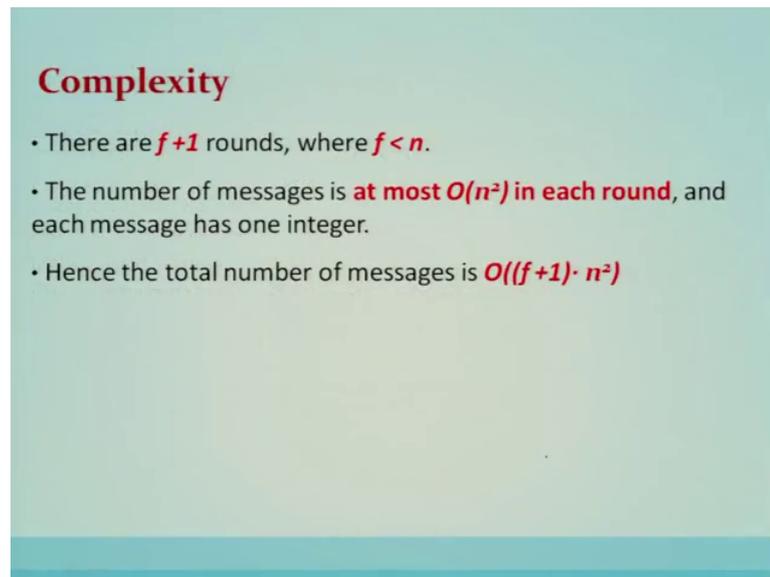
```
(global constants)
integer: f; // maximum number of crash failures tolerated
(local variables)
integer: x ← local value;
(1) Process  $P_i (1 \leq i \leq n)$  executes the Consensus algorithm for up to  $f$  crash failures:
(1a) for round from 1 to  $f + 1$  do
(1b) if the current value of  $x$  has not been broadcast then
(1c) broadcast( $x$ );
(1d)  $y_j \leftarrow$  value (if any) received from process  $j$  in this round;
(1e)  $x \leftarrow \min(x, y_j)$ ;
(1f) output  $x$  as the consensus value.
```

Algorithm 10.1 Consensus with up to f fail-stop processes in a system of n processes, $n > f$. Code shown is for process P_i $1 \leq i \leq n$

Let us understand it using this particular algorithm that is explained earlier. So, we can understand that if let us say that we have three different processors and let us say one is faulty. So, f is equal to 1 here in this case. So, the agreement requires $f + 1$ that is equal to two rounds. If it is faulty let us say it will send 0 to 1 process and 1 to another process i, j and k . Now, on receiving one on receiving 0 it will broadcast 0 over here and this particular process on receiving 1 it will broadcast 1 over here. So, this will complete one round in this one round and this particular process on receiving 1 it will send 1 over here and this on the receiving 0 it will send 0 over here. So, you can see that here it will receive it will receive 0 and 1 and this will receive 0 and 1, this will also receive 0 and 1 and the minimum if you take this will be having 0 0 and 0.

So, if let us say that it sends 1 back to him if it receives one then let us say it receives one in that case. So, it is 0 0 and 1. So, this is round number 1. So, similarly if you take another round of these particular messages and if this is having value 1 this is having value 0 this is having 0 if they exchange with each other and take the minimum then all of them will have the value 0 and they will reach to a consensus after two rounds.

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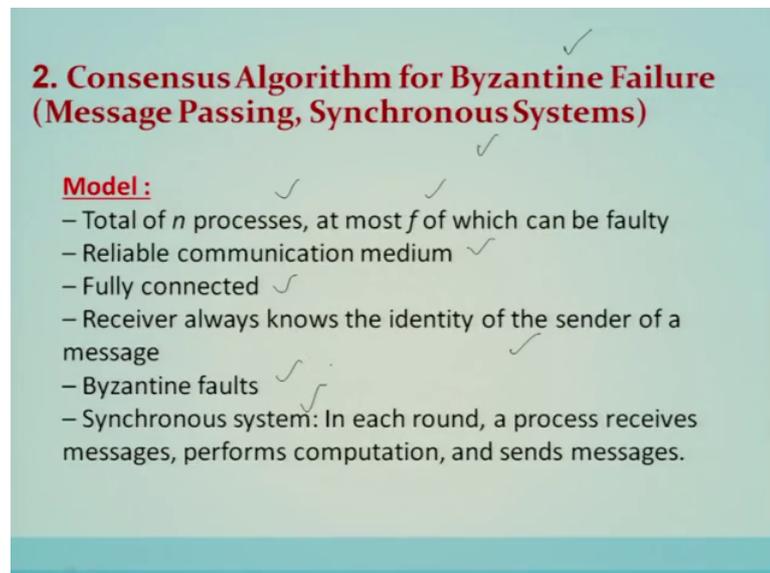


Complexity

- There are $f+1$ rounds, where $f < n$.
- The number of messages is **at most $O(n^2)$ in each round**, and each message has one integer.
- Hence the total number of messages is **$O((f+1) \cdot n^2)$**

So, the complexity of this particular algorithm is it requires f plus 1 rounds where f is less than n and the number of messages is order n square in each round and each message has one integers hence the total number of messages is f plus 1 into n square f plus 1 is the total number of rounds and in each round n square messages are required.

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2. Consensus Algorithm for Byzantine Failure (Message Passing, Synchronous Systems)

Model :

- Total of n processes, at most f of which can be faulty
- Reliable communication medium
- Fully connected
- Receiver always knows the identity of the sender of a message
- Byzantine faults
- Synchronous system: In each round, a process receives messages, performs computation, and sends messages.

Now, the next most important algorithm in today's lecture is given here that is consensus algorithm for byzantine failures this algorithm is given by Leslie Lamport this is called Shostak Pease Lamport algorithm this is for byzantine failures. So, it is assuming a

synchronous system and the failure model is byzantine and we are going to see this particular algorithm.

So, in this algorithm the model which is assumed is saying that there are total n processes and f can be the faulty the communication is reliable and it is fully connected topology receive always knows the identity of the sender and here the fault model is assumed as very general fault model that is byzantine and the synchronous system in each round a processor receives a message performs computation and sends. So, synchronous concentration model is assumed here.

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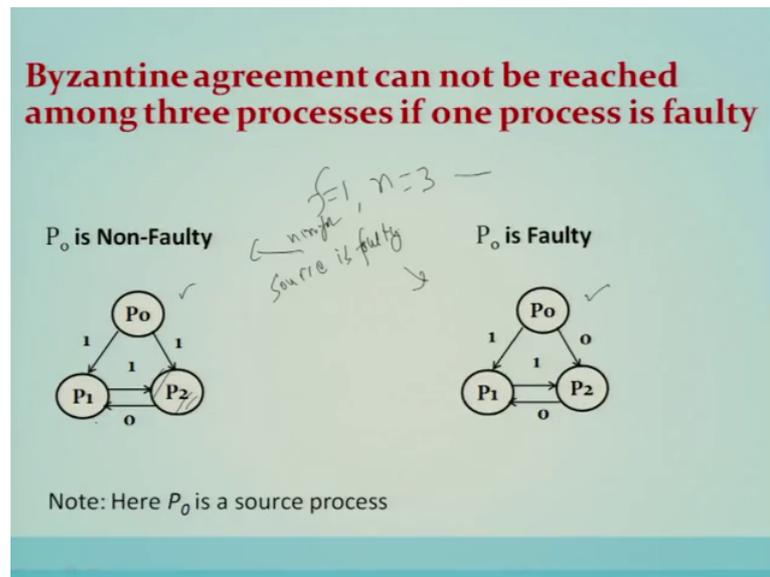
Solution for Byzantine Agreement Problem

- The solution of Byzantine Agreement Problem is first defined and solved by **lamport**.
- Pease showed** that in a fully connected network, **it is impossible to reach an agreement if number of faulty processes 'f' exceeds $(n-1)/3$ where n is number of processes**

$f \leq \lfloor \frac{n-1}{3} \rfloor$

So, solution of the byzantine agreement is first defined by first defined and solved by Leslie Lamport. So, Lamport Shostak Pease algorithm it is. So, Pease shows that show that fully connected network it is impossible to reach an agreement if the number of faulty processor f exceeds n minus 1 divided by 3 that is f should always be less than or equal to n minus 1 divided by 3.

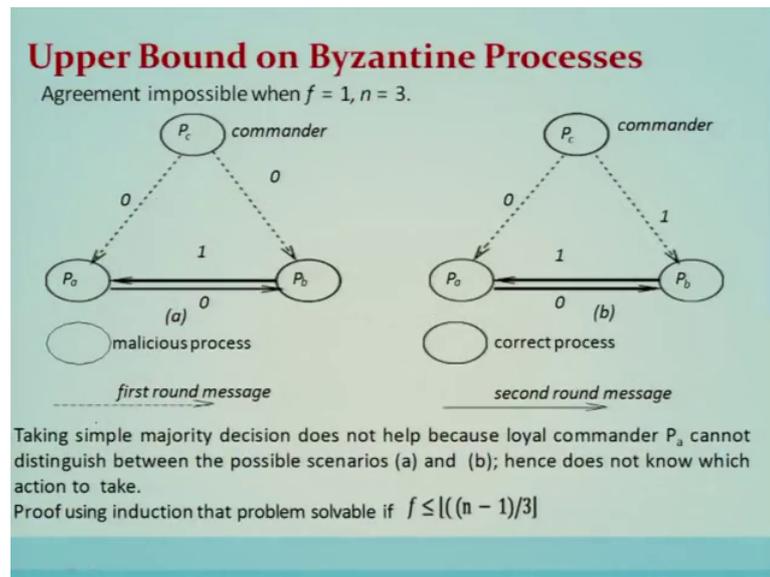
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So, let us take this particular example whether this example shows that the condition where f is less than n minus 1 by 2 is violated over here; that means, if f is equal to 1 and n is equal to 2 this particular assumption is violated; that means, n minus 1 by 2 is not 1 in that case, but we are assuming 1 so obviously, as per the previous condition agreement byzantine agreement is not possible and we can see in this particular example.

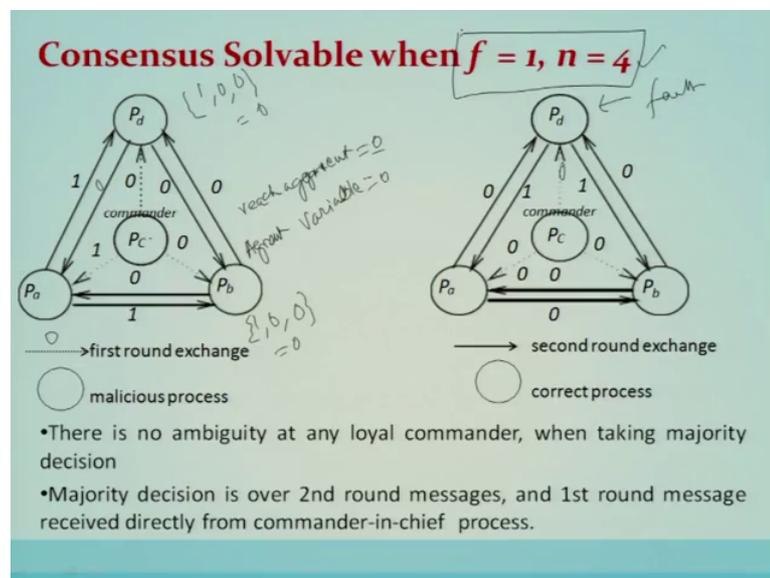
Here P_0 is faulty is non faulty and here P_0 is faulty so that means P_0 is the source, the source is faulty here in this case and source is non faulty in the other case. So, source is non faulty, but some other process is faulty let us say that P_2 is faulty. So, P_1 will send because it is non faulty same values to P_1 and P_2 and as far as the P_2 s concerned it will send a different value because it is a faulty.

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So, this agreement will not be reached here in this particular example. So, same example we can see that agreement is not possible.

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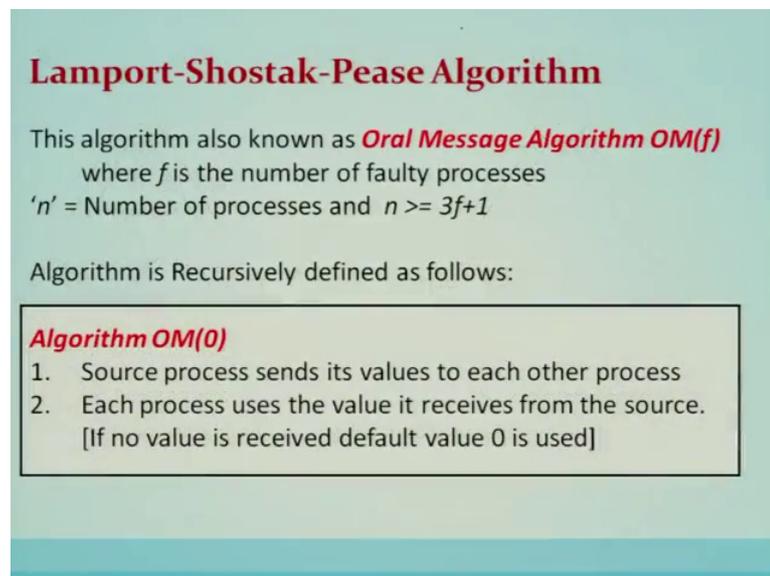
Now, agreement is possible when f is equal to 1 and the total number of processor is 4. So, agreement we can see how it is possible we can see about the commander P_c . So, this is the source it will send the message 0 since it is faulty. So, it will send 0 to P_d 0 to P_b , but 1 to P_a in the first column. So, P_a after receiving this one it will send one to both

the neighbors, similarly P b after receiving 0 it will send 0 why because it is not faulty, similarity P d will send after receiving 0 at both the ends.

So, if we take these values which will be received here it is 1 and basically it is 0 and this is also 0. So, the majority is basically 0 here in this case here also if you see the values 1 0 and 0. So, the majority is 0 and here also majority is 0. So, in this particular case even if the source is faulty, it will reach to an agreement, reach an agreement and that value will be agreed upon value or agreement variable will be equal to 0.

Similarly, here in this example also let us assume that this is there are some other node is faulty in that case. So, for example, you will receive 0 and this receives yeah this is faulty in this case why because after receiving 0 after receiving 0 it will send 1 and 1 in both the cases. So, we will see that in the same manner this also we will reach to an agreement and. So, as far as Lamport substrate Pease condition is concerned if the condition is satisfied with the number of faulty processor then the agreement is possible.

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Lamport-Shostak-Pease Algorithm

This algorithm also known as **Oral Message Algorithm OM(f)** where f is the number of faulty processes
'n' = Number of processes and $n \geq 3f+1$

Algorithm is Recursively defined as follows:

Algorithm OM(0)

1. Source process sends its values to each other process
2. Each process uses the value it receives from the source.
[If no value is received default value 0 is used]

So, the algorithm is called as the Lamport's for as take Pease algorithm this algorithm also known as oral message algorithm OM f, f is the number of faulty processors. So, the number of processors n should be greater than or equal to 3 f plus 1. The algorithm is recursive and the base of the recursion that is OM 0 says that the source process sends its values to each other process. Now each process uses its value, value it receives from the source if no value is received the default 0 is assumed.

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Algorithm OM(f), $f > 0$

1. The source process sends its value to each other process.
2. For each i , let v_i be the value process i receives from source. [Default value 0 if no value received]
3. Process i acts as the new source and initiates **Algorithm OM($f-1$)** where it sends the value v_i to each of the $n-2$ other processes.
4. For each i and j ($j \neq i$), let v_j be the value process i received from process j in STEP 3. Process i uses the value **majority** (v_1, v_2, \dots, v_{n-1}).

"The function majority(v_1, v_2, \dots, v_{n-1}) computes the majority value if exists otherwise it uses default value 0."

Now, recursion, recurse procedures of this algorithm OM, OM f , f is greater than 0 then the first steps is that source process sends its values to each other process now for all for each I let v_i be the value the process I receives from the source now default 0 if the value is not received. Process i as the new source and initiate algorithm OM f minus 1 where it sends the value v_i to each of n minus 2 process.

Now finally, after this particular for loop is over in step number 2 then fourth step says that for each i and j , $j \neq i$ which is not equal to i j is not equal to let v_i be the be the be the process I received from the process j in step number 3, process i uses, the values using the majority function we want to v_{n-1} the function majority computes the majority value if exist otherwise it uses the default value as 0. So, this majority function is application dependent.

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(i) Solving the Byzantine agreement, for $f = 1$ and $n = 4$

Round number	A message has already visited	Aims to tolerate these many failures	And each message gets sent to	Total number of messages in round
1	1	1	$4 - 1 = 3$	$4 - 1 = 3$ ✓
2	2	$1 - 1 = 0$	$4 - 2 = 2$	$(4 - 1) \cdot (4 - 2) = 3 \cdot 2$ ✓

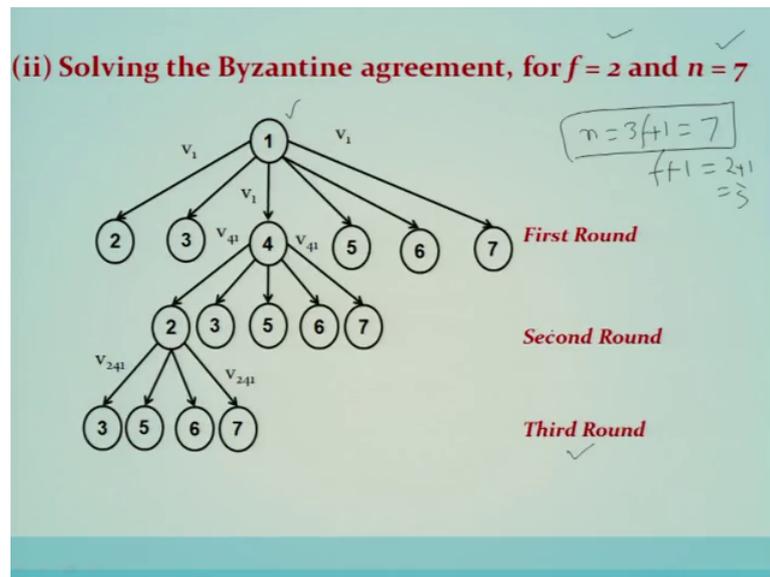
• Number of messages for agreement on one value is: $3 + 2 \cdot 3 = 9$ messages ✓

So, this basically will be the algorithm which is applied under the byzantine fault model and under the synchronous communication model, system model.

So, as we have seen that f is equal to 1 n is equal to 4 the same algorithm we have applied in the previous slide. Now, let us see how many messages, how many rounds will be there. Now as you know that it will require how many two rounds will be required. So, in each round the total number of messages, total number of messages in each round will be three for example, a source will send to its neighbors. Similarly, in the second round now each of these will send to its corresponding neighbors. So, basically 6 messages in second round and 3 messages in the first round, so total nine messages will be required here and the total number of number of rounds will be the 2 that is f plus 1 that is equal to 2 and the messages will be will be s equal to n minus 1 plus n minus 1 multiplied by n minus 2, that is n minus 1 means 4 minus 1 minus 1 that is 3 n 3 plus 6 that is 9 total 9 messages will be there.

So, if we generalize this particular total number of messages. Now if f is equal to, if f is equal to 2 then n is equal to $3f + 1$ that is equal to 7 .

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So, 7 is required and for the number of faulty processes are 2. So, this is particular model or the in this particular case byzantine agreement is possible. And how many number of rounds it is going to take? f plus 1 that is 2 plus 1 that is 3 rounds. So, here just see that in this particular picture 3 rounds are shown. And let us understand that in the first round the source will send the message to all other neighbors except itself. So, it will send to 2 3 4 5 6.

Now, every node will do this way; that means, we have to see how the node 4 is now doing it. Node 4 after receiving this v_1 it will send to its neighbors. So, its neighbors other than the previously defined neighbors or previously received messages or not to be included in that. So, it will send the message for 2 3 5 6 7, now among them to will in turn will send to its neighbors other than from where it has earlier previously received the message that is 3 5 6. So, third round when every node is complete the algorithm will finish and let us analyze how many number of messages will be communicated over here.

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$f=1$
 $f=2$

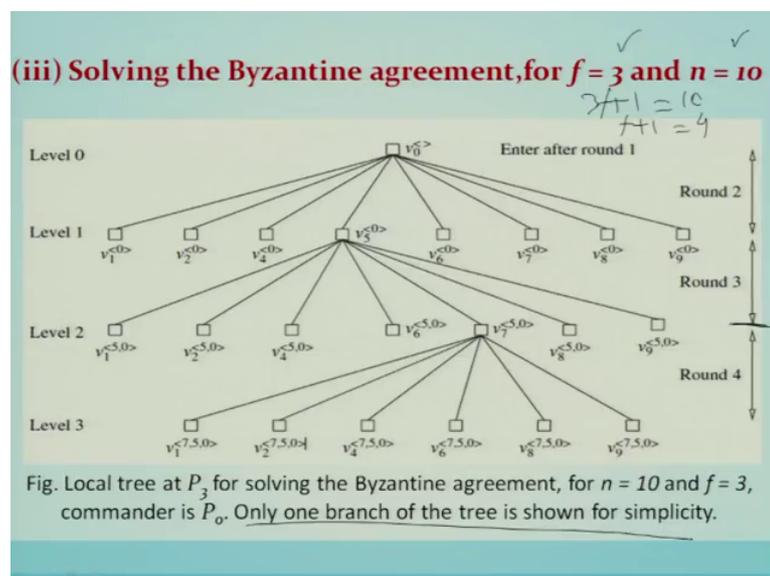
Round number	A message has already visited	Aims to tolerate these many failures	And each message gets sent to	Total number of messages in round
1	1	2	$7-1=6$	$7-1=6$ ✓
2	2	$2-1=1$	$7-2=5$	$(7-1) \cdot (7-2) = 6.5$
3	3	$2-2=0$	$7-3=4$	$(7-1) \cdot (7-2) \cdot (7-3) = 6.5.4$

• Number of messages for agreement on one value is: $6+6.5+6.5.4 = 156$ messages

$\text{rounds} = 3$
 $\text{Total messages} = \binom{n-1}{1} + \binom{n-1}{2} + \binom{n-1}{3}$
 $= 156 \text{ messages}$

So, the number of, so total number of rounds as you have seen is equal to 3 and the messages required will be equal to n minus 1 plus n minus 1 times n minus 2 plus n minus 1 times n minus 2 times n minus 3 and if you see because this is round number 1, this is round number 2 and this is round number 3 different messages. So, total number of messages we have to sum total number of messages which are basically communicated in each round. So, if you sum them it comes out to be 156 different messages. So, we have seen two examples where f is equal to 1 and f is equal to 2, now we are going to see a more complicated example where f is equal to 3.

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When f is equal to 3 the total number of nodes will be $10 - 3f + 1$ that is equal to the n and the number of rounds will be $f + 1$ that is $3 + 1$ that is 4.

So, just see that in this particular figure number of rounds are 4 and as we have seen in the previous example that up to round 3 we have seen similarly the round 4 will also continue that every node will send the message to its basically neighbors and will not send to the other neighbors who have previously sent the message to that particular node. So, this example as written over here only one branch of the tree is shown for the simplicity what you can do is you can explore the complete examples at your end.

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Contd...

Round number	A message has already visited	Aims to tolerate these many failures	And each message gets sent to	Total number of messages in round
1	1	3	$10 - 1 = 9$	$10 - 1 = 9$
2	2	$3 - 1 = 2$	$10 - 2 = 8$	$(10 - 1) \cdot (10 - 2) = 9 \cdot 8$
3	3	$3 - 2 = 1$	$10 - 3 = 7$	$(10 - 1) \cdot (10 - 2) \cdot (10 - 3) = 9 \cdot 8 \cdot 7$
4	4	$3 - 3 = 0$	$10 - 4 = 6$	$(10 - 1) \cdot (10 - 2) \cdot (10 - 3) \cdot (10 - 4) = 9 \cdot 8 \cdot 7 \cdot 6$

• Number of messages for agreement on one value is: $9 + 9 \cdot 8 + 9 \cdot 8 \cdot 7 + 9 \cdot 8 \cdot 7 \cdot 6 = 3609$ messages

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Relationship between # Messages and Rounds

Round number	A message has already visited	Aims to tolerate these many failures	And each message gets sent to	Total number of messages in round
1	1	f	$n - 1$	$n - 1$
2	2	$f - 1$	$n - 2$	$(n - 1) \cdot (n - 2)$
...
x	x	$(f + 1) - x$	$n - x$	$(n - 1)(n - 2) \dots (n - x)$
$x + 1$	$x + 1$	$(f + 1) - x - 1$	$n - x - 1$	$(n - 1)(n - 2) \dots (n - x - 1)$
$f + 1$	$f + 1$	0	$n - f - 1$	$(n - 1)(n - 2) \dots (n - f - 1)$

Table: Relationships between messages and rounds in the Oral Messages algorithm for Byzantine agreement.

Complexity: $f + 1$ rounds, exponential amount of space, and $(n - 1) + (n - 1)(n - 2) + \dots + (n - 1)(n - 2) \dots (n - f - 1)$ messages

So, we can generalize these particular formula and so number of rounds is very simple calculation that is $f + 1$ rounds, but as far as the messages are concerned it goes to an exponential that is why it is called exponential algorithm because the exponential of the order messages are required here in this particular example, in this particular.

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Agreement in Asynchronous Message-Passing Systems with Failures

Exponential amount of space and messages are required.

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Impossibility Result for the Consensus Problem

Fischer-Lynch-Paterson (FLP) Impossibility Result (By M. Fischer, N. Lynch, and M. Paterson, April 1985)

- **Fischer et al.** showed a fundamental result on the impossibility of reaching agreement in an asynchronous (message-passing) system.
- It states that it is *“Impossible to reach consensus in an asynchronous message passing system even if a single process has a crash failure”*
- This result, popularly known as the **FLP impossibility result**, has a significant impact on the field of designing distributed algorithms in a failure-susceptible system.

Now, agreement in asynchronous message passing system with failures impossible to result for consensus problem Fischer, Lynch and Paterson, FLP impossibility result it is called. So, in 1985 this particular impossibility result is called FLP impossibility result Fischer showed that the fundamental result on the impossibility of reaching an agreement in a synchronous system it states that it is impossible to reach consensus in an asynchronous message passing system even if a single message single process is has a crash failure, this result popularly known FLP impossibility to result has a significant impact in the field of designing distributed algorithm in a failure susceptible systems.

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Weaker Versions of Consensus Problem	
Consensus Problem	Description
Terminating reliable broadcast	It states that a correct process always gets a message even if the sender crashes while sending. If the sender crashes while sending the message, the message may be a null message but it must be delivered to each correct process.
k-set consensus	It is solvable as long as the number of crash failures f is less than the parameter k . The parameter k indicates that the non-faulty processes agree on different values, as long as the size of the set of values agreed upon is bounded by k .
Approximate agreement	Like k-set consensus, approximate agreement also assumes the consensus value is from a multi-valued domain. However, rather than restricting the set of consensus values to a set of size k , ϵ -approximate agreement requires that the agreed upon values by the non-faulty processes be within ϵ of each other.
Renaming problem	It requires the processes to agree on necessarily distinct values.
Reliable broadcast	A weaker version of reliable terminating broadcast (RTB), namely reliable broadcast, in which the termination condition is dropped, is solvable under crash failures.

So, that is why weaker versions of consensus problem in a synchronous system a synchronous model of communication is basically devised and these are called terminating reliable broadcast k-set consensus, epsilon approximate agreement, renaming problem and reliable broadcast. Terminating reliable broadcast problem, a correct process always gets a message even if the sender crashes while sending it.

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(i) Terminating Reliable Broadcast (TRB) Problem

- A correct process always gets a message, even if sender crashes while sending (in which case the process gets a null message).
- **Validity:** If the sender of a broadcast message m is non-faulty, then all correct processes eventually deliver m .
- **Agreement:** If a correct process delivers a message m , then all correct processes deliver m .
- **Integrity:** Each correct process delivers at most one message. Further, if it delivers a message different from the null message, then the sender must have broadcast m .
- **Termination:** Every correct process eventually delivers some message.

So, validity, if the sender of the broadcast message m is non faulty than all the correct process eventually deliver m .

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(ii) Reliable Broadcast Problem

- **Reliable Broadcast** is RTB without terminating condition.
- RTB requires eventual delivery of messages, even if sender fails before sending. In this case, a null message needs to get sent. In RB, this condition is not there.
- RTB requires recognition of a failure, even if no msg is sent
- **Crux:** RTB is required to distinguish between a failed process and a slow process.
- RB is solvable under crash failures; $O(n^2)$ messages

Algorithm: Protocol for reliable broadcast

(1) Process P_0 initiates Reliable Broadcast:

(1a) **broadcast** message M to all processes.

(2) A process $P_i, 1 \leq i \leq n$, receives message M :

(2a) if M was not received earlier **then**

(2b) **broadcast** M to all processes;

(2c) deliver M to the application.

Next problem is called reliable broadcast problem reliable broadcast problem is without terminating condition RTB requires that eventual delivery of the message even if the sender fails before sending it in this case the null message needs to be sent. In reliable broadcast this condition is not there, in reliable in RTB requires the recognition of the failure even if no message is sent. So, is solvable under the crash failure of the order n^2 .

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Applications of Agreement Algorithms

1) Fault-Tolerant Clock Synchronization

- Distributed Systems require physical clocks to synchronized
- Physical clocks have drift problem
- Agreement Protocols may help to reach a common clock value.

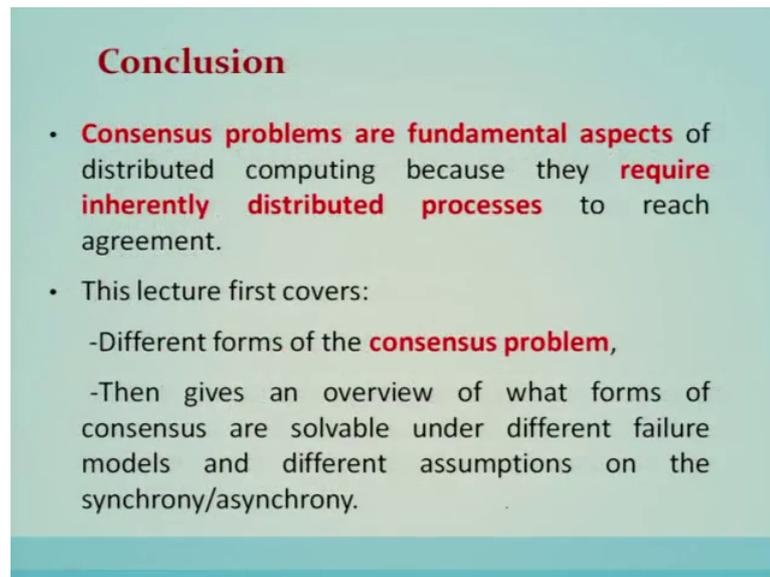
2) Atomic Commit in Distributed Database System (DDBS)

- DDBS sites must agree whether to commit or abort the transaction
- Agreement protocols may help to reach a consensus.

Now, applications of this agreement algorithm are many, so some of them are listed over here as first application is called as a fault tolerant clock synchronization distributed system required physical clock to be synchronized. So, agreement protocol may help them to reach a common clock value.

Atomic commit in distributed database system. So, distributed databases sites must agree on whether to commit or to about the transaction, agreement protocol may help to reach a consensus.

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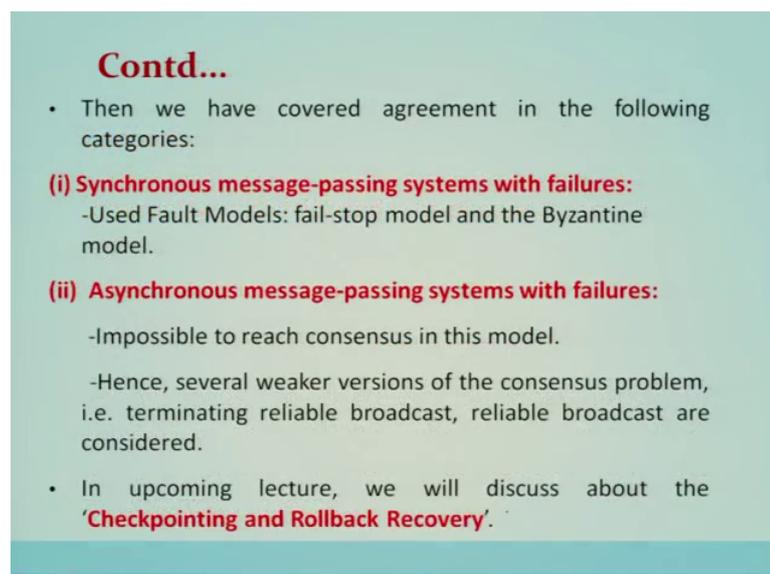


Conclusion

- **Consensus problems are fundamental aspects** of distributed computing because they **require inherently distributed processes** to reach agreement.
- This lecture first covers:
 - Different forms of the **consensus problem**,
 - Then gives an overview of what forms of consensus are solvable under different failure models and different assumptions on the synchrony/asynchrony.

Conclusion, consensus problems are fundamental aspects of distributed computing because they require inherently distributed processes to reach an agreement or a consensus and which is essential in many of the application. So, this lecture covers different forms of consensus problem, then gives an overview of what forms the consensus are solvable under different models, failure models and different computation models.

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Contd...

- Then we have covered agreement in the following categories:
 - (i) Synchronous message-passing systems with failures:**
 - Used Fault Models: fail-stop model and the Byzantine model.
 - (ii) Asynchronous message-passing systems with failures:**
 - Impossible to reach consensus in this model.
 - Hence, several weaker versions of the consensus problem, i.e. terminating reliable broadcast, reliable broadcast are considered.
- In upcoming lecture, we will discuss about the **'Checkpointing and Rollback Recovery'**.

Then we have covered the agreement in the following categories synchronous message passing system with failures use the fault model fail stop and byzantine models. We have seen two different algorithms a synchronous message passing system with failures we have we have shown the FLP impossibility condition in this problem setting, it is impossible to reach the consensus in this model. Hence several weaker versions of the consensus problematic terminating reliable broadcast, reliable broadcast are considered. In upcoming lectures we will discuss about check pointing and (Refer Time: 48:00).

Thank you.