Introduction to Wireless and Cellular Communication
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Lecture - 39
Fading Channels - Diversity and Capacity
Channel State Information, Optimum Power Allocation

Good morning. We begin lecture 38. Our goal is to get a good understanding of capacity we have already made some end roads into understanding capacity in terms of the situations when information regarding the channel state is available at the receiver. Today we will complete that discussion and also look at the optimum approaches when the information regarding the channel state is available at the transmitter as well.

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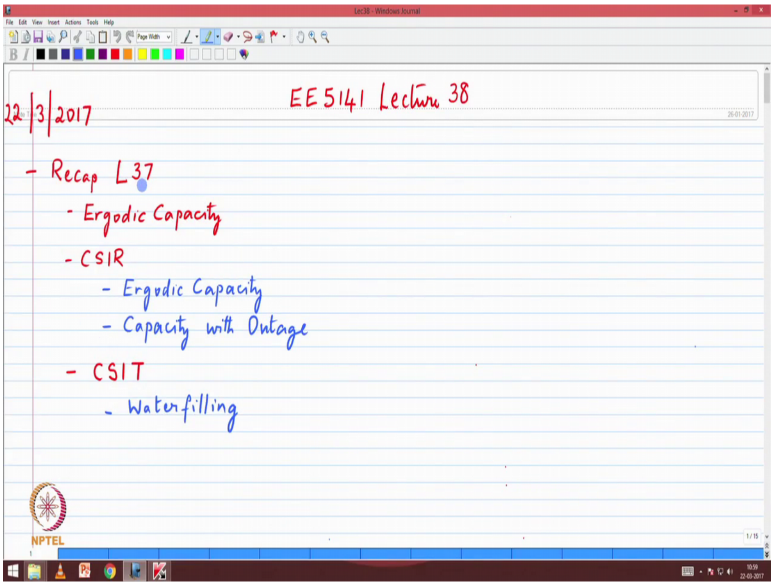

So, the goal of today's lecture will be to cover our you know reinforce our understanding of a ergodic capacity, how do we achieve capacity, what are the ways in which we can achieve capacity if the information is known at the receiver and then what happens if it is known at the transmitter.

So, to set the stage let me just quickly review some slides from yesterday I again because of the there was few questions after the lecture I just want to make sure that the information is clarified.

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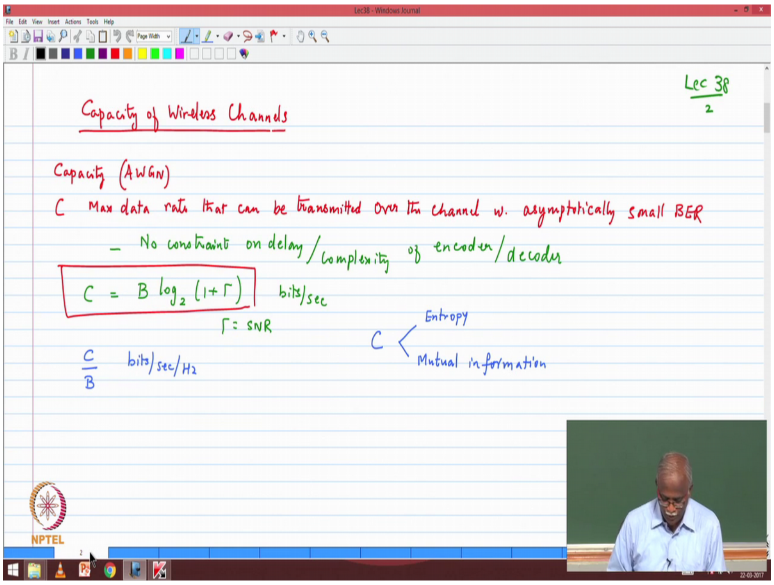

And also we have a good starting point for today's lecture. We start of with our understanding of capacity as defined by Shannon for AWGN channels and that capacity is given by B times logarithm base 2 of 1 plus SNR. And the unit is are bits per second if you want to look at it in terms of normalized capacity.

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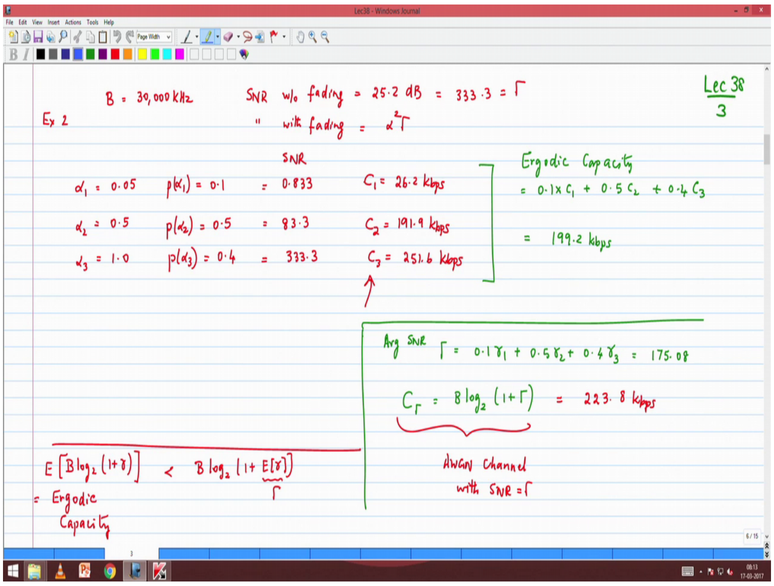

Then it will be in bits per second per hertz and the example that we looked at was a case where we had 3 different channel conditions. Each of which producing a corresponding capacity and we then said that the ergodic capacity or the capacity of this channel would be obtained through this summation of the capacities weighted by their appropriate probabilities. And we got a expression for the ergodic capacity.

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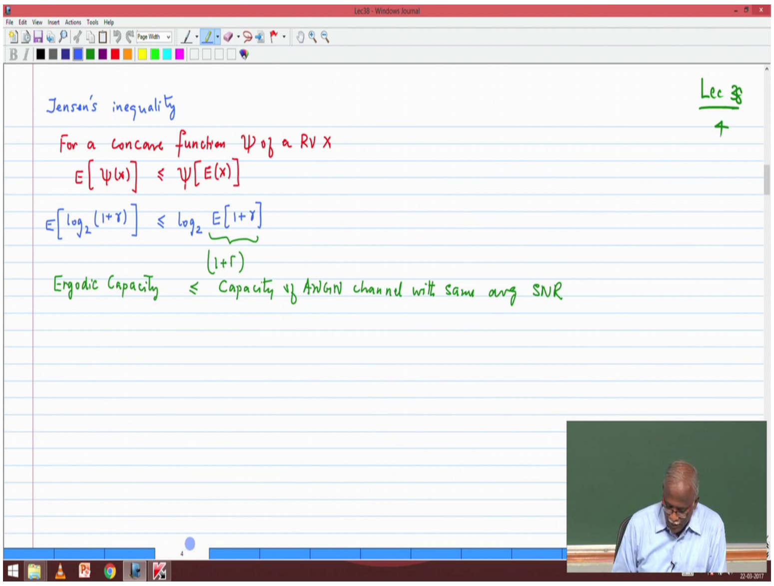

Subsequently, we did brief discussion on Jensen's inequality which basically said that function expected value of a function of a concave function of a random variable x is less than or equal to the function evaluated at the expected value of the random variable and this basically translated into a result that is useful for us. It is the one that enables us to compare ergodic capacity with the capacity of an AWGN channel which has the same average SNR both the ergodic the fading channel and the AWGN channel have the same average SNR. One is a one situation where the capacity is constantly fluctuating. The other one is an a channel where for the average SNR you get a fixed number as the capacity.

So, given these 2 we find that the ergodic capacity is less than the capacity of the corresponding AWGN channel.

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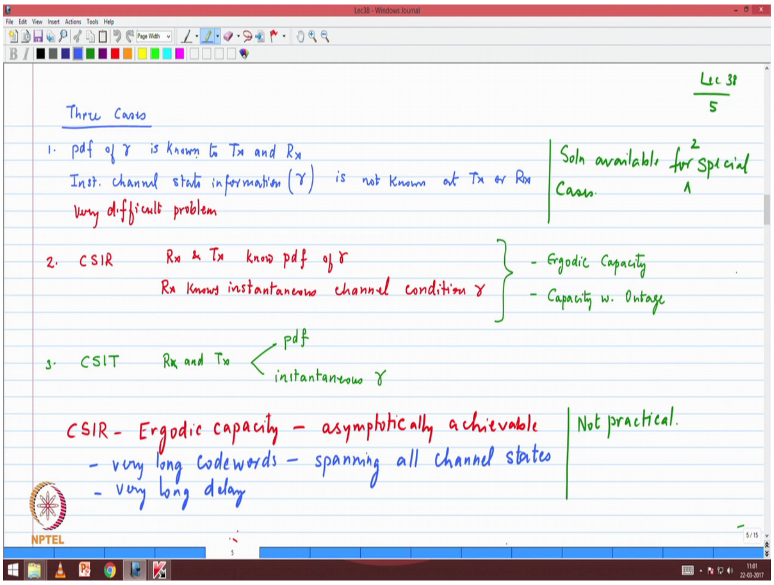

Then we said there are 3 cases that we can study in the context of a computing capacity. First one where a we know only the PDF of the distribution or the distribution of the of gamma and this is known both to the transmitter and to the receiver I just wanted to clarify one point that basically this turns out to be very difficult problem. It is known for 2 special cases and Rayleigh fading channels is one of them, but in general this is a very difficult problem that is not something that we focus on. What we have instead focused on are these 2 scenarios, which are closer to practice. And CSIR and CSIT. And CSIR is the first case that we looked at this is a situation where the receiver and the transmitter known the PDF of gamma and the receiver knows the instantaneous channel condition gamma.

We said that under this assumption we can then look at 2 possibilities one computing or how do you achieve ergodic capacity. So, under CSIR it is possible to achieve ergodic capacity under a set of assumptions we will just high light that. We will also look at a situation where we say not looking at ergodic capacity, but I want to look at very practical realizable schemes. So, I would look at capacity with outage. So, that is the and of course, the CSIT.

Now, CSIR the case 2 that we studied the clarification or the a point to be taken away is, if I say that I have I have CSIR and I want to achieve ergodic capacity, what is the method? The transmitter does not know; what are the channel conditions. So, it has to transmit at a fixed rate. CSIR situations schemes or it has to transmit at a fixed rate; however, the channel may be good it may be bad; we do not know that ahead of time.

So, the way that ergodic capacity is achieved in the context of CSIR only the receiver knows and the transmitter does not know is that transmitter must use very long code words. Basically asymptotically it could be infinitely long code words which have to span all the channel states the good states and the bad states. And the code words must then be designed such that even in the presence of the bad states you can still recover the information with the with the very low probability of error.

So, this is this is where the catch comes where you have very long code words, very long code words means that there is delay there is complexity associated with that. So therefore, yes it is possible to talk about ergodic capacity, but it is one of those situations where we know that it can be constructed, but how do you actually do it in practice it is it is a very difficult problem. So, again we just sort of make a foot note saying it is good from a concept point of view you know what is what you should be achieving, but it is something that would be very difficult in practice because of the delays that would be involved.

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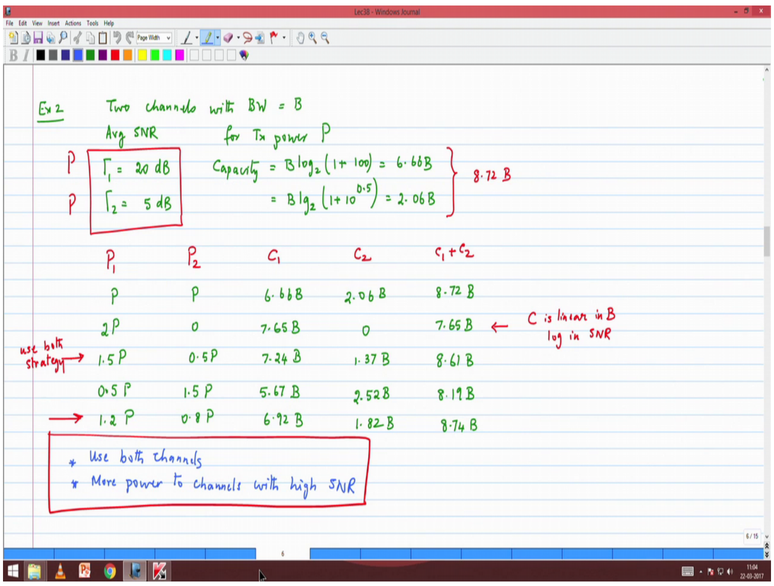

We looked at another example where they were 2 channels with different average SNRs and different capacities. And we distributed the powers we kind of increased the power of one reduce the power of the other, we came up with the following conclusion that we should use both channels it is advantageous to use both channels. There was a question after the class which said you know should I always even if the other channel is very bad. Now, what you will find today in today's lecture is that once we start talking about outage then if the channel becomes very bad, you may not choose you may choose not to use that particular situation.

So, in this particular example it was advantageous for us to use both channels again keep that in mind. You know as much as possible we try to use all the channels that are available; however, we will try to optimize the power allocation, that is a very important element you do not you know distribute equal power or do not go in the intuition saying that the channel with poor SNR requires more power and the that that is actually not the, right; strategy. Strategy will be gives more power to the stronger the channels with better SNR. So therefore, we get the advantage of that, ok.

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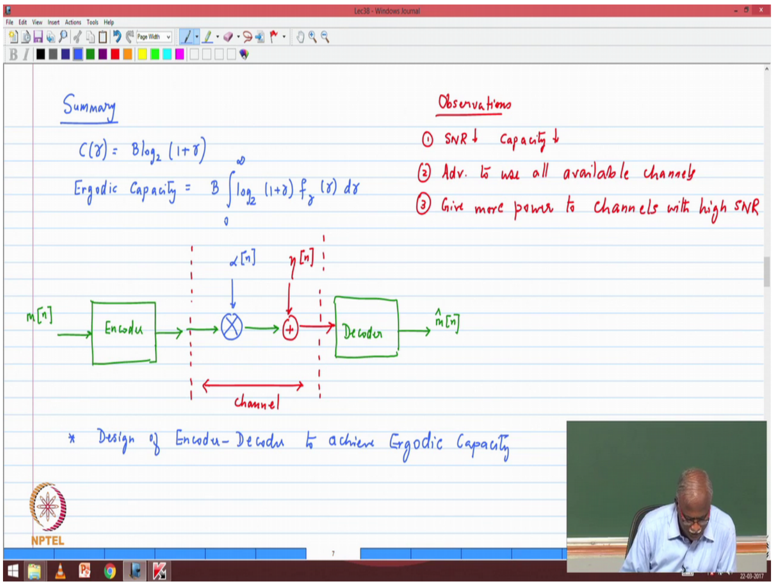

So, we with that background let just quickly summaries all the information that that we have. So, basically this would be a summary. Summary of the key points that we want to take forward from here; so we say that our capacity is not a fixed number it is a function of the SNR, B times logarithm base 2 1 plus gamma. And ergodic capacity is a number which is like the expected value of the capacity ergodic capacity is given by B times integral 0 to infinity logarithm base 2 1 plus gamma f gamma of gamma d gamma. Now this is what we want to achieve. This is the max this is the best that we can do under the fading channel we know that this is going to be upper bounded by the AWGN capacity of a equivalent channel with the same average SNR.

So, the observations that we have made so far observations, the first one is that SNR the capacity is very sensitive to SNR. If SNR goes down the capacity is going to go down that is something that we have to be aware of. Second it is advantageous to use all available channels, all available channels. That is what we saw in the in the example. All available channels and the third point is that it is 2 our advantage to give more power, give or allocate more power more power to channels with higher SNR.

So, you kind of creating a more imbalance, but the idea is to achieve capacity and to achieve capacity this seems to be the direction in which the examples are pointing us channels with high SNR. So, this is at a high level the key points that we have observed now may be a picture is very helpful for us. What is it that we are trying to do? We are trying to decide a encoder decoder combination, encoder at the transmitter which will take my information, let me call that as M of n. than at the other end I am trying to decode the counterpart decoder, such that I can recover M hat of n. This is the and the channel that the encoder decoder has been studied very extensively in terms of Shannon capacity is that is the AWGN channel. So, where it is additive noise and what we have is a situation where there is another impairment that comes in the in the process it is a multiplicative impairment and that is the feeding coefficient.

So, basically what we have is a situation where there is fading and then AWGN and So for us the definition of the channel is from the output of encode to the input of the decoder. So, this my channel, this is the one in which the fading and all the impairments are happening I need to and this has a certain capacity and I am trying to design the encoder decoder to maximize that capacity and to reach that capacity that is the channel that is that is present for us.

So, the problem statement for us is the design of the encoder decoder, design of encoder decoder pair, decoder to achieve ergodic capacity. So, that is the problem statement and the so far what we have said is, if only the PDF is known it is a very difficult problem if the receiver knows this information, it will end up being that the encoder decoder becomes very complex. Because you will have to use very long code words which will which can operate even under bad channel conditions and still give you good error performance.

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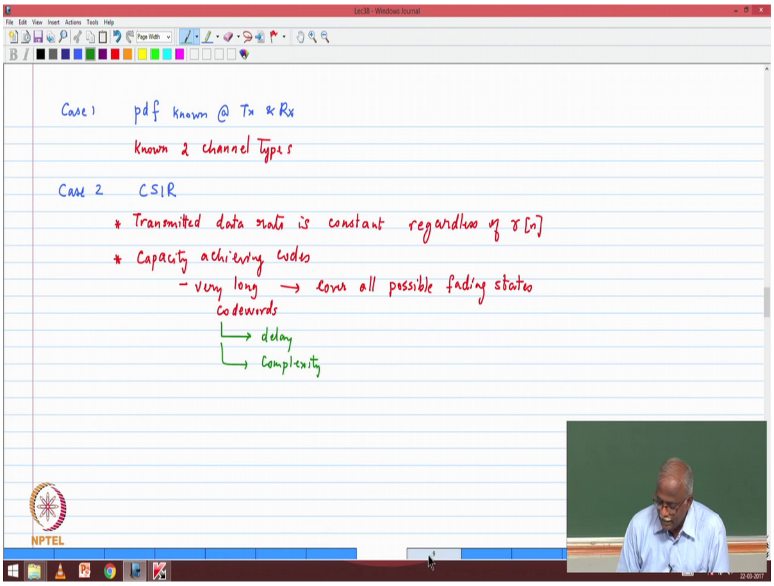

So, few more statements about; so case one is PDF only that is both transmitter and receiver PDF known at T x and R x this is the difficult problem we are not going to spend too much time on that; however, to mention that it is known for 2 channel types. So, we know the solution for 2 channel types, but in general very difficult problem. So, our focus is on case 2 which is CSIR the following points need to be kept in mind again it is may seem a little repetitive, but important that we do not forget this. The transmitted data rate is constant; we are not varying the transmitted rate.

So, basically transited data rate is constant, is constant and the reason because of that is because the transmitter does not know what the current channel conditions are, is constant regardless of the instantaneous value of gamma n, regardless of gamma n. So, this is very important element. And of course, the capacity achieving codes capacity achieving codes are very complex, achieving codes are very long because they have to be long enough to cover all the possible fading states, very long too to cover all possible, all possible fading states.

All the way from very bad channel conditions to very good channel conditions possible fading states and the minute you have long code words; that means, the they are they are going to long very long code words, it is not just, long very long code words and very long code words means the impact will be in terms of the delay and in terms of the complexity, the decoding complexity.

So, both of these are factors that we have to have to keep in mind. So, the code that has been designed must work at a constant rate under all channel conditions and give you a very low probability of error. So, that is the CSIR, CSIR situation. What we would know like to look at is the second one where we are not trying to achieve the ergodic capacity, but we are trying to look at a outage, channels with outage.

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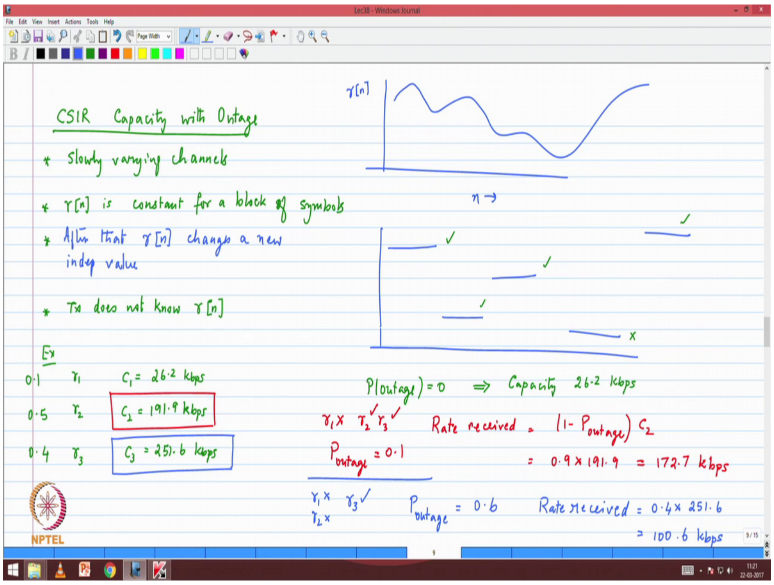

So, CSIR is the scenario we are going to look at capacity with outage, with outage. The channel model is going to change slightly for this again please pay close attention to this particular the assumptions being made. So, this is primarily looked at or studied in the context of slowly varying channels, slowly varying channels. As opposed to the previous case where you were looking primarily at a reasonably fast fading channels because you must cover all possible states.

Now, slowly fading channels with a specific characteristic; the characteristic is that the gamma of n that is the instantaneous SNR is not changing symbol to symbol it is constant for a block of symbols, is constant for a block of symbols, block of symbols and then gamma changes it is constant for a block and then after that it changes, after that gamma n changes to a new value, independent value. Gamma n changes to a new independent value and remains constant for a block of symbols, new independent value.

It is from a particular distribution it could be a Rayleigh distribution or rician distribution basically, but the gamma of n. So, the way to visualize it is in practice what we see is an SNR that that will change right, that this is how this is how gamma of n changes as a function of n in practice, but the channels that we are we are assuming are of this type it is fixed for a period of time than randomly changes to some other value again fixed for a period.

So, basically it looks like it is fixed for a period of time and then randomly changes and again it goes up and down based on that. Now the transmitter does not know the gamma n is just that this is a type of gamma n that we have transmitter does not know. Because we have assumed only CSIR, does not know gamma of n.

So, the question is how do I design my system such that I get through the data most of the time, but under some channel conditions I am going to say I am going to accept outage I am not I am not going to accept the that there is going to be a error at errors the signal will not get through at on the some channel conditions. So, basically this is the outage. So, let us say that you know may be for this particular channel condition I am going to say I am going to tolerate outage for really bad SNR, but all of these must go through without any without any errors. So, basically that would be my design approach.

So, here is a way that you could start to think about the achieving channel capacity with outage. Again it is very important to tie it to the channel model that we are assuming. Because this is a channel which remains constant for a period of time and then randomly changes according to that is distribution. So, will go back to our example, example I do not remember this example to or 3 it is probably example 2. There are 3 SNRs the 3 SNRs gamma 1 gamma 2 gamma 3 I am not sure if I have those values, but you can fill in those the corresponding capacities C one was 26.2 kbps C 2 was 199.9 kbps. This one had a probability of 0.1 this one had a probability of 0.5 this one had a probability of 0.4. C 3 was 251.6 kbps ok.

So, now if this is a situation where gamma 1 gamma 2 gamma 3 follow this particular pattern. Randomly switching around and I tell you that I have to design a system for 0 outage. So, basically if I say that the probability of outage. Probability of outage equal to zero; that means, under all channel conditions you must get my data through then I must design my system for the worst case which means that this in this case the capacity that I can achieve, capacity that I can achieve will be 26.2 kbps. Because that is the system that will be designed to transmit with gamma 1 it may be gamma 2 gamma 3, but again the gamma over the system design for gamma 1 will work and therefore, it satisfies ok.

Now, if I were to say that I am willing to forgo, forgo gamma 1. So, gamma 1 I am going to consider as outage gamma 2 gamma 3 must be transmitted without error, now what is the CIST, what is the through put that I can achieve? In that case I can achieve a capacity of 199.9 kilobit is I can transmit at that rate. So, but the rate received some parts some portion of the time it will the data will not get through. So, because of the gamma 1 scenario this will be equal to 1 minus probability of outage into C 2.

So, probability of outage, probability of outage will be 0.1 because I have designed my system to work with gamma 2. If it if the situation is gamma 1 data will not get through that is outage that is 0.1, 0.1 probabilities the rest of the time it will go through. I am transmitting information at C 2 rate C 2 that is 191.2, but 0.1, 0.1 probability the data will not get through So, the effective rate at the receiver will be one minus probability of outage in to C 2 that basically is 0.9 times 199.9 which is 172.9 kbps, kbps.

Now, if I want to get a little bit more aggressive and say you do not know I want to see if I can get even higher rate. So, then I say gamma 1 I consider as outage gamma 2 also as outage gamma 3 I will assume that is the one that will get through so; that means, I can transmit at 251.6 kbps that is the rate at which I am going to transmit, but keep in mind that the probability of outage probability of outage is when gamma 1 or gamma 2. So, that basically becomes 0.6. So, the rate received actual rate at which you your sending information is that 0.6 with the probability 0.6 it is an outage. So, only 0.4 is going to get through. So, this will be 0.4 into the capacity that we have 251.6 which actually gives you a lower number 100.6 kbps.

So, some observations with outage you can have some advantages, because you are going to say that those bad channels I am going to sort of remove from going from impacting my performance. So, that is a important element that we are able to work with and to and to get out of our system. However, we need to be carefully that that we do not become too aggressive in terms of outage because then the total through put will get will get effected.

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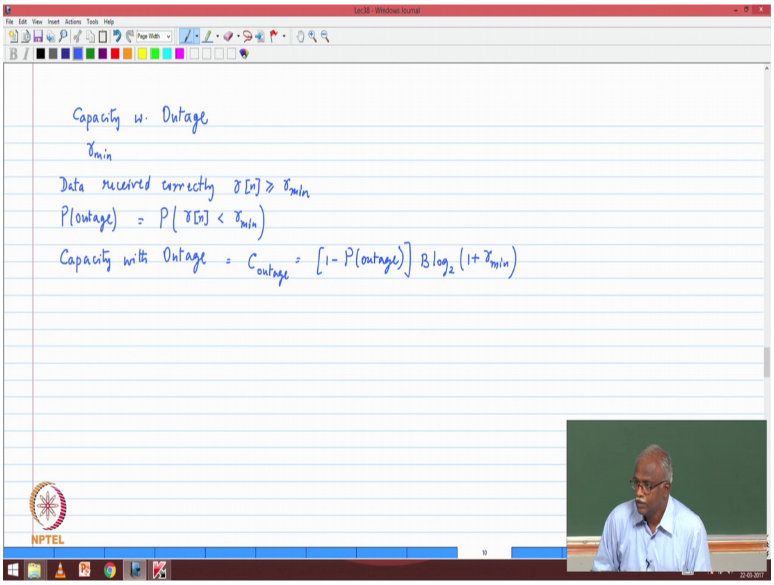

So, a summary of the points of capacity with outage; just a few highlights basically, we decide; what is gamma min above which you want the information to get through.

So, data will be received correctly. Data received correctly if gamma, if gamma n is greater than or equal to gamma min. And outage which is something that we know exist becomes we have designed the system accordingly this happens if the probability with the probability that gamma of n is less than gamma min. And the capacity with outage capacity, with outage is given by, because this is a the assumption are that it is a slowly varying channel and it sort of a what is we call as a block fading channel.

So, under that we have designing the capacity with outage. So, let us call this as C subscript outage; that means, with outage this is equal to 1 minus probability of outage into B logarithm base 2 1 plus gamma min, right? Because gamma min is my threshold above for which I have designed anything above that the data will get through ok.

So, in a nutshell, what we have achieved with the discussions with regard to capacity with outage? Is that you have removed the effect of these very bad channels. And what you are left with is are those channels under which you can pump a lot of the information through. And you can also visualize thinking of it as saying what happens to the data that got lost I will retransmit. So therefore, the there is a reduction in my capacity it is not the capacity number that I got for gamma 2, but it is something slightly less than that because some parts of the, some portions of the time the channel are the system is in outage. So, this is a good point for us to sort of take to pause and say do we have a reasonable handle on capacity.

Student: Do we have the information of the probability of each state all the time?

So, the question do we know the probability of each other state? The assumption is that the PDF is known. So, yes you do have the probability of the different states, because that is very important for us to decide where you want to draw the threshold for outage. Because you do not want to like for example, in the in the in the example that we looked at if I set the threshold at gamma 3 for example, actually it hurt me because gamma 2 has high probability of occurring and therefore, it will it will actually hurt me to choose that.

So, yes you must know the PDF so that you can draw the line appropriately good question of. So, this is this is a way for us to understand what is a one way of achieving capacity in a practical system where we allow outage to occur. Now the key information is now what happens if we move from CSIR to CSIT.

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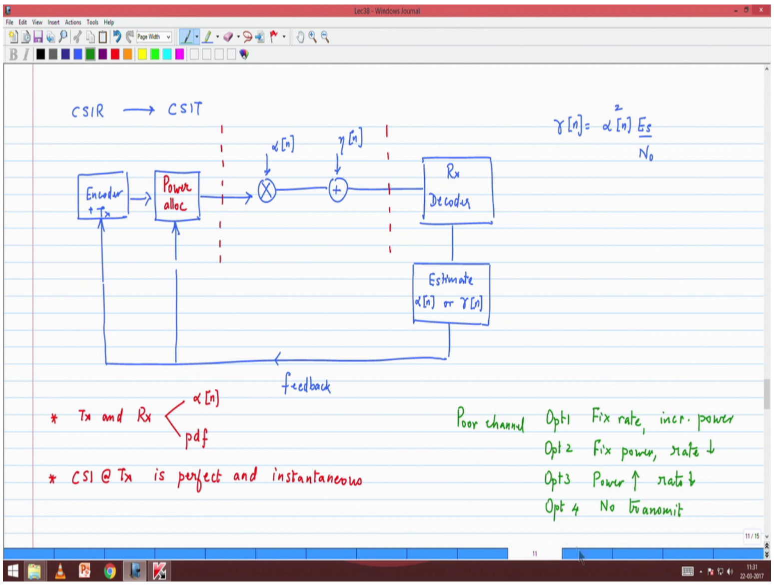

We move from CSIR; that means, the information is also known to the to the transmitter. So, here is the scenario that we would we have. So, encoder and then as we have drawn before the alpha of n is present, that is the fading the noise AWGN. And then at this point there is a decoder. So, I will call it as the receiver slash decoder this is also encoder plus transmitter.

Now, one of things that we have to do is we have to estimate alpha of n, estimate either alpha of n or gamma of n. Both are essentially the same information because gamma of n is equal to alpha of n square times the average SNR or E s by n not. So, average SNR multiplied by alpha square is the instantaneous SNR and that is what we want to estimate and this is what we want to feed back to the transmitter. So, basically the feedback channel, feedback channel is what we have here.

Now why did not I draw the line all the way through because encoder? So, what are sum of things that the encoder will change if it knows what the channel conditions are. What are things that it will change?

Student: (Refer Time: 29:15).

It will change the rate that is n captured inside the n coder the other thing that it can change is the amount of power that is allocated. So, that is why I have an additional block here. So, let me call this as power allocation, how much power you allocate, right? Because the transmitter now knows how much the channel is (Refer Time: 29:36 so that you can take advantage of with the in the channel.

So, this is what we would have. Now this feedback information is also needs to be given to the power allocation because the channel SNR is going to determine how much power you are going to allocate. Where is my channel? Channel like before is encaptured between this. So, there is feedback I can change the coding I can change the power allocation that is for that particular transmission and then this is the scenario in which we are going to achieve this in.

So, here are the assumptions that we make and. So, the assumptions are that both T x and R x they know both alpha of n and they know the PDF of alpha of n. Both of which are known and this is known through the feedback. And we also assume that CSI at the transmitter that is CSIT that information is perfect; that means, there is no error in estimation there is no error in the feedback channel. It is perfect and it is instantaneous; that means, the information that you have is valid for that time that you are doing the transmission.

Basically if this is the frame work that under which we are assuming then here are the possible options that the transmitter can do if it a notices that there is a bad channel. So, if you now look at the situation of a poor channel or a bad channel here are the options that the transmitter can do. Option one it can fix the rate; that means, do not change the encoder or the decoder, but it can increase the power increase the power of transmission that that is the power allocation.

So, that you can see whether the rate will go through or it can go for option 2 where you fix the power do not adjust the power, but you reduce the rate; that means, reduce the information rate. So, put more coding on that. So, basically put more coding therefore, the likely hood of the channel option 3 I can increase the power and reduce the rate. I can do these 3 options of course, there is an option 4 where I can choose not to transmit, 4 possible options are present.

Now, the key thing to note is that the since the transmitter knows what the channel conditions are and it knows what are it is options in terms of coding and decoding, it can choose not to transmit at certain points of time and therefore, save power and therefore, achieve the achieve better performance. The basically we transmit only

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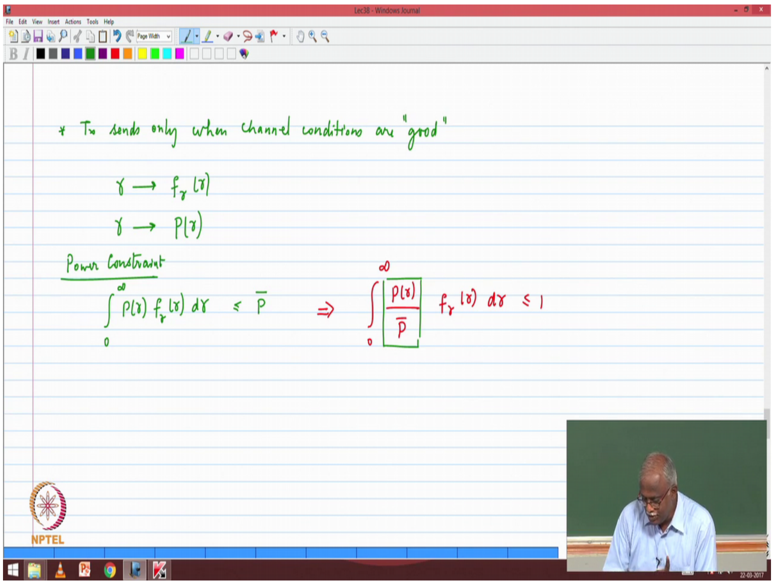

When channel conditions are good T x sends only when channel conditions are good or when it can ensure that the transmission will get through, only when channel conditions are good. Again there is a qualitative statement, but it is very important that you know we quantify it in a in a in a rigorous manner when the conditions are good. Let us quickly move into the aspects of CSIT and then we will. So, one of the things that that we will like to do is we know that the SNR which occurs in the channel has got a certain distribution f gamma of gamma.

Now, for each of these SNR scenarios we can then come up with a corresponding power allocation. So, just like we have whenever for a particular gamma we can say that the power allocation is going to be based on is a going to be some function of gamma. Good SNR more power bad SNR less power some rule you are going to follow, but if this cannot be something that is arbitrary.

So, you must have a power constraint. The power constraint is very important for us to keep in mind because only then we can have a fair comparison between different schemes. This basically says 0 to infinity P of gamma that is the power that you allocate based on the probability f gamma of gamma d gamma; that means, for all the possible SNR states you will allocate some kind of power and this when you when you take it with the appropriate probabilities, must be less than or equal to some average power level. You cannot exceed average power level. So, basically you cannot keep adding more and more power on for some channel. Some where your P bar will get exceeded if you if you if you do not allocate correctly.

So, this is a very very important element we do have an average power constraint only then we can compare with AWGN type channels we can compare with the all of the other scenarios that, because see this power allocation is now coming in the context of CSIT. So, we should not lose height of that. Now this can actually be written in a more effective form which can be expressed as follows 0 to infinity P of gamma divided by P bar f gamma of gamma d gamma less than or equal to 1; so very nice constraint. So, basically what it says is do not look at P of gamma as an independent quantity look at it as a scaled number.

Now, P gamma of by P bar can be greater than one or less than 1, but at the end of the day when you look at all the PDF of gammas this should be upper bounded by 1. You cannot basically have a gain in power. So, you have to adjust your normalized power to satisfy this constraint.

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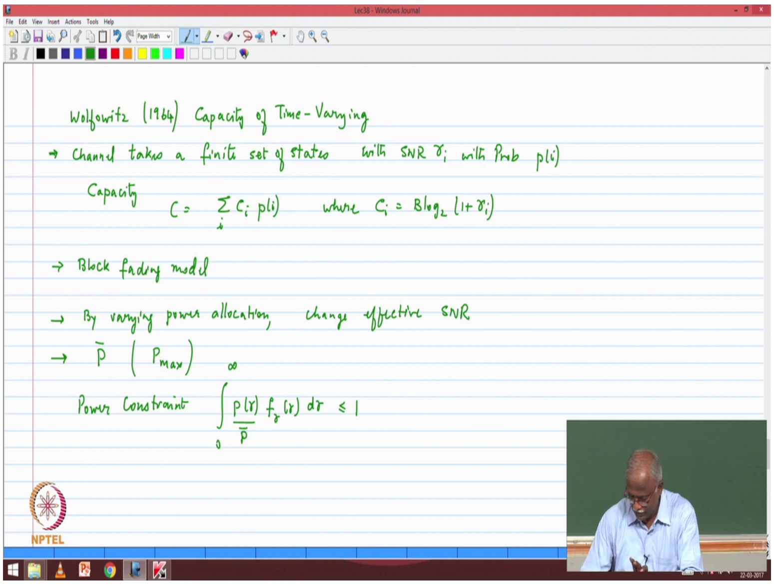

So, now comes the problem statement of CSIT. The problem statement was posed by Wolfowitz. Wolfowitz 1964, where he was looking at the capacity of a time varying channel. Which is exactly what we are looking at where the transmitter and receiver knows the PDF and the channel assumes that you can get the, the both transmitter and know the instantaneous SNR ok.

So, the assumptions or the problem statement of Wolfowitz was as follows. He said that the channel takes on, channel takes on a finite set of a SNRs, takes a finite set of states. Again it is a simplified model before we attack the key problem takes on a finite set of states that that is the SNRs with SNR alpha gamma i. So, gamma i can take a finite set of values with probability with known probabilities.

And basically we would know like to compute the we have been using P for powers. So, I just need to be a little bit care full with this. So, with known probability let me put lower case P of i, basically the capacity that we are interested in is the summation over i C i P i, basically this looks like the ergodic capacity where C i is equal to B time logarithm base 2 1 plus gamma i.

Basically this is the a ergodic capacity, we are going to assume the block fading model, the block fading model is what we are going to assume. Basically what we are saying is that you know what the SNR that is occurring in the channel. You can you we have no control over the alpha what happens in the channel, but we can by allocating power we can adjust the SNR of the effective channel. So, basically by varying power allocation, by varying the power allocation, power allocation what we would like to do is, you can impact the SNR, you can change the effective SNR, because that is going to come in to play.

So, but there is a fixed P average, P bar. We cannot exceed P bar. And in some cases there may be a P max also, P max also. So, which means that you know you cannot suddenly apply very large amount of power. So, basically there may be constraints definitely there is a constraints on P bar there may be a constraint on P max also again for now we will assume that our allocations are reasonable. So therefore, P bar is sufficient for us ok.

So, the power constraint as we have already mentioned, power constraint is integral 0 to infinity the power that is assigned f gamma of gamma d gamma divided by P bar is less than or equal to 1. Now comes the problem statement that is a very very important. So, so the problem that Wolfowitz was trying to to address was,

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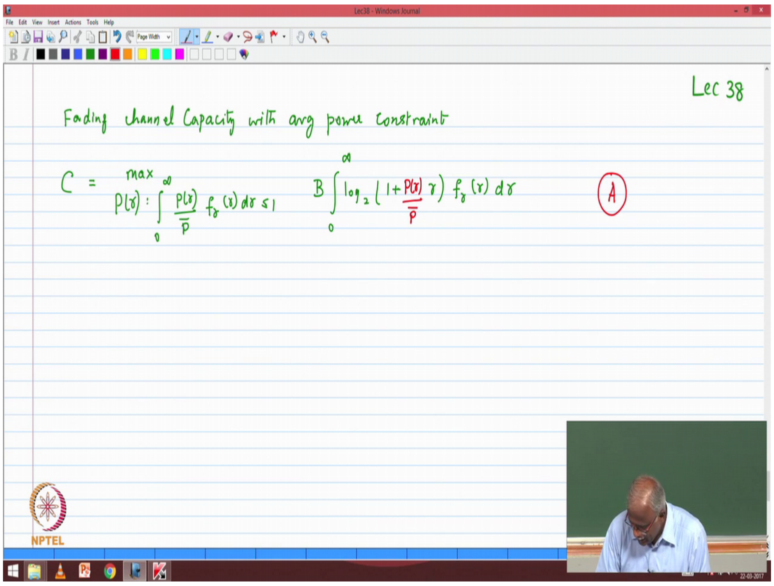

The fading channel capacity, fading channel capacity C s I t scenario with average power constraint, with average power constraint is the following problem statement; the capacity where you are allowed to choose the power allocation such that the capacity is maximized.

So, the capacity is maximized over all possible power allocations P of gamma you are allowed to choose any allocation you want your allowed to do, provided you satisfy the power constraint integral 0 to infinity P of gamma divided by P bar f gamma of gamma d gamma is less than or equal to 1, that is the power constraint. Your you can choose any power allocation scheme is allowed and the and the reason you choose that is So, that we can maximize the capacity B times integral 0 to infinity logarithm base 2 1 plus gamma f gamma of gamma d gamma i. Have left a little space there because gamma was the SNR the t channel gave you, but you modified it using power allocation. So, the power allocation that you are doing is P of gamma divided by P bar, very important ok.

So, basically that is a number that can be less than 1 greater than 1. So, which means that it will affect gamma and because you have done this power allocation it will result in a certain capacity and your allowed to maximize you can try all combinations of power allocations such that you can satisfy and you can get the capacity.

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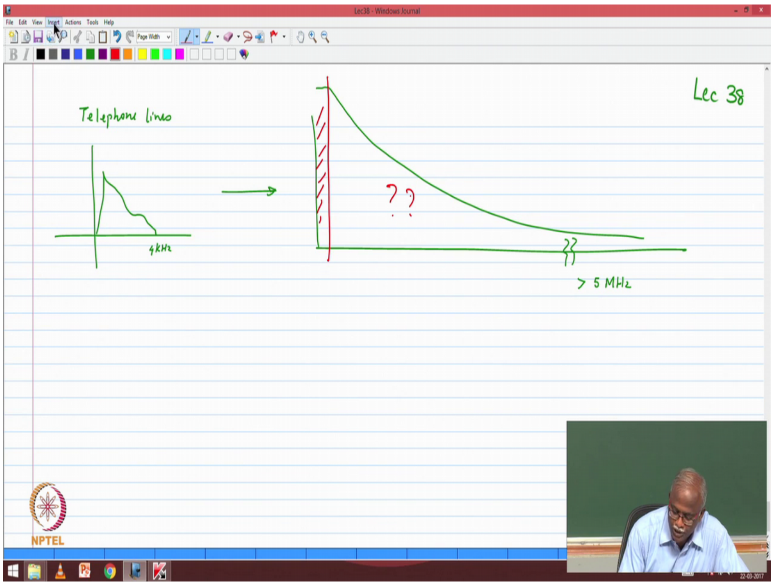

Let us call this as A. Now the question is do we know an answer for this particular type of a A problem statement. And it turns out that yes we do have a answer for this in a very very intuitive and very satisfying manner. So, here is the solution to this particular problem. And I will leave you with a to think about that ok.

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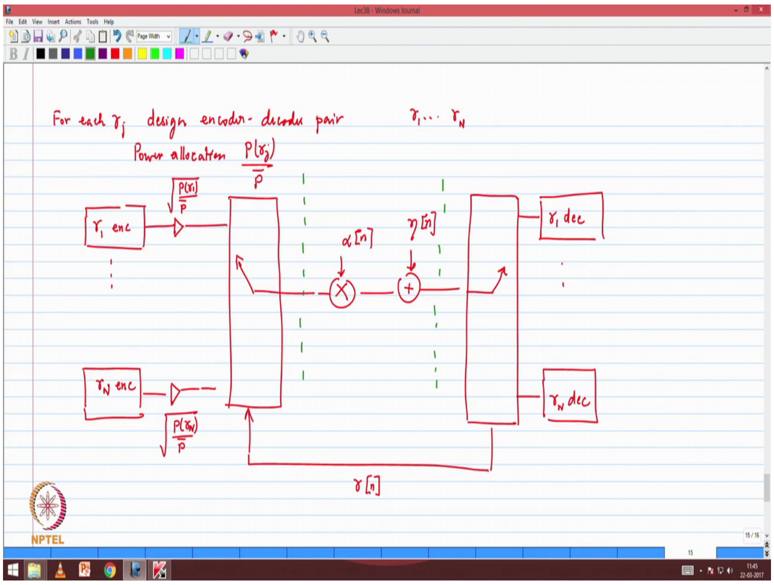

What we are going to do is since there are a finite number of gammas. So, basically for each gamma, for each gamma j design an encoder decoder pair, design the encoder decoder pair. And you also have the option of adjusting the power location. So, basically the power allocation for gamma j power allocation will be P of gamma j divided by P bar that will be the power allocation.

So, here is what the model is going to do. It is going to design let us say there is gamma 1 through gamma n, there are n states. Here is the gamma 1 encoder, gamma 2 encoder this is gamma n encoder. Each of these will get power allocation and that power allocation when it looks when it look at it in terms of amplitude this will be square root of gamma 1 divided by P bar this will be square root of P of gamma n divided by P bar, that is an amplitude scaling when you take the power you will get the appropriate SNR scaling.

And here we have just a multiplexing switch, this goes into the channel will give you the multiplicative factor alpha of n the additive noise eta of n and at the receiver I have a simple de multiplexing switch which says, this is gamma 1 decoder gamma 1 decoder all the way to gamma n decoder, gamma n decoder. And what am I feeding back? I am feeding back what is gamma n.

So, this is and where is my channel; channel is still the same. Notice it is a very elegant design which says if I have a finite set of gammas, I must find out what is the optimum power allocation, that is I have not I have not yet solved that problem. Once I have solved that problem then that the realization of the capacity becomes fairly elegant and simple where we say that design the optimum encoders, encoder decoder pairs apply the appropriate power allocation. And then all you need to do is estimate what is the current channel conditions you transmit using that particular encoder. Encoder and at the other end using the same information you would you make sure that you pass it through the appropriate decoder and you achieve the type of. So, this is a very good way to understand now the key point that we now have to address is, how do the power allocation.

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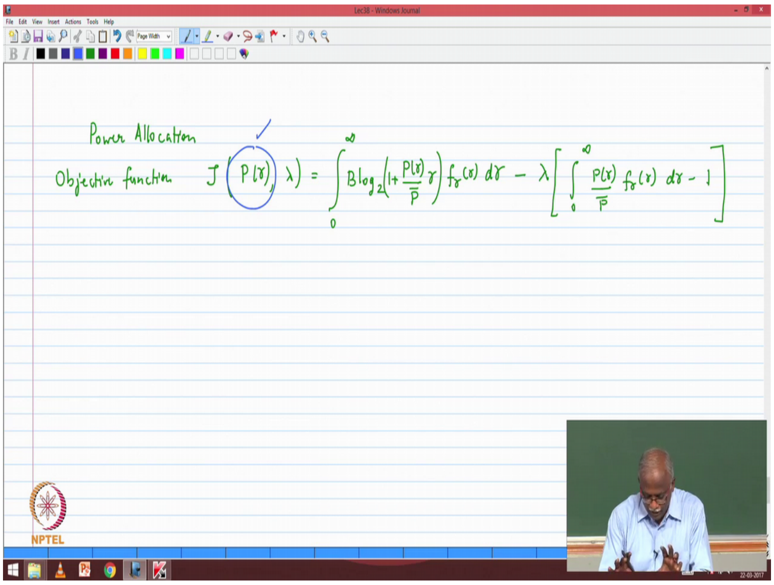

So, basically the question now then goes down to how to do the power allocation that is a very very important problem. And we now have to pose the question in the following manner. So, here is the problem statement and we will then solve it in the next class. So, now, what are we saying that we now had define a objective function, the objective function will call it as j it is based on the power allocation P of gamma and the Lagrangian variable lambda, such that you want to you want to look at the maximization of this particular quantity, 0 to infinity B times log base 2 P of gamma by P bar into gamma 1 plus f gamma of gamma d gamma. That is a objective function that is what I want to maximize subject to a power constraint. So, minus lambda times integral 0 to infinity P gamma by P bar f gamma of gamma d gamma minus 1. Or if you if you write it as P gamma f gamma of gamma d gamma it will minus P bar.

So, basically this is the objective function. This is what we need to work with and basically, we want to find out that optimum power allocation that will give me the desired result. So, think about this, we once you know the power allocation you know how to achieve the optimum capacity.

Now the key the last step is how do you find out the optimum power allocation. And this is very important for us because, this may turn out to be a somewhat counter intuitive final result, but it will also tell us how to achieve the capacity of a telephone line channel.

Thank you.