Statistical Learning for Reliability Analysisπ∞βα√σμ
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Lecture – 22
Statistical Inference (Part 1)

Hello everyone warm greetings. So, today we will start a new topic a very interesting topic, I am sure you all will enjoy, learning this topic, this is statistical inference. This is what basically what we are talking from the first class our main aim of statistical learning, statistical method is for statistical inference. In fact, sampling distribution, what we have learned? Sampling distribution is the basically the backbone of statistical inference.

Like, what I want to say is that like in computer all of you even if you know from computer science background, you know how a computer works. So, if we do programming, if we write a program, the main part that means your CPU, the CPU does all the job, but for the CPU to do all the jobs, there are some other parts also which have some activities is not it? So, like that is what if I tell take the whole thing that is the statistical inferences like for solving executing the program, the different steps.

And the main part is the CPU as I told you, so main part in statistical inference is a sampling distribution. Sampling Distribution is not a standalone something that basically we have learned sampling distributions so that we can do statistical inference. So that is what we will be learning in this topic. So, statistical inference is a long chapter. So, we will be doing in many parts so this is the part 1.

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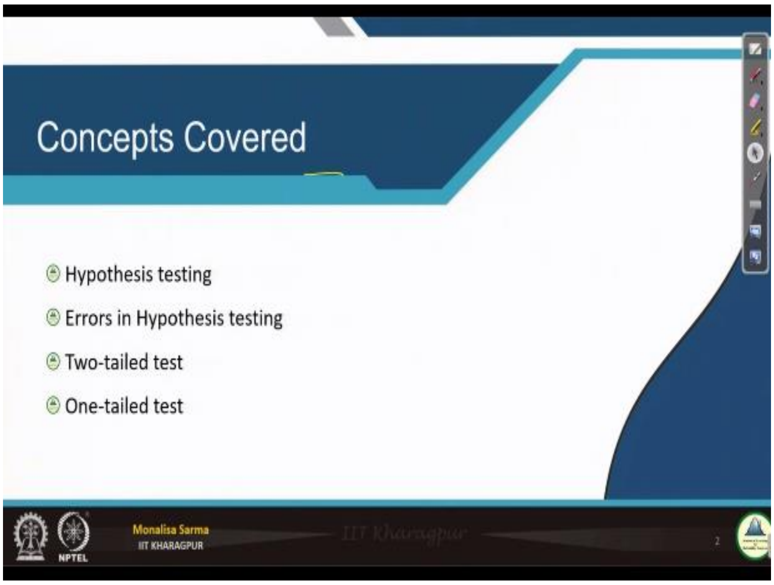

So, here in this today's lecture, we will be learning what is hypothesis testing, what do we mean by errors in hypothesis testing, then what is a 2-tailed test and what is a 1-tailed test?

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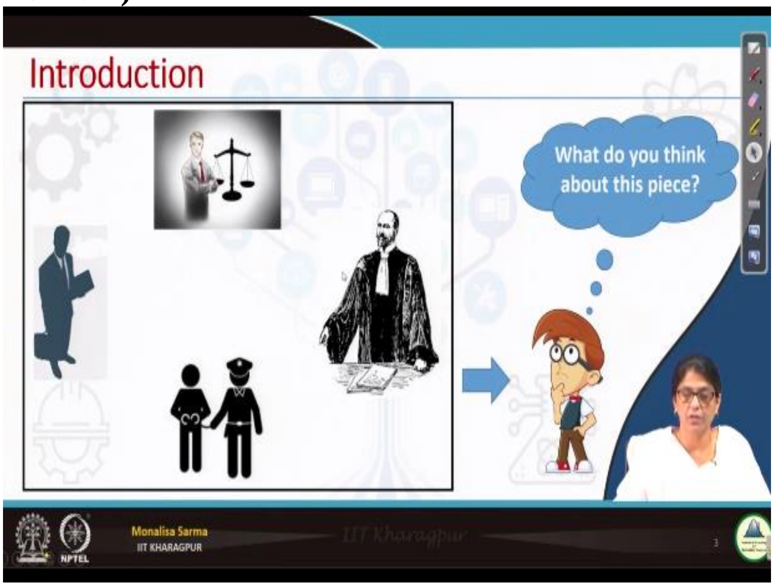

So, now what do we think about this space? If you see here, from this picture, what you can see? So, it is like so here, what we see? Here we can this is a public prosecutor, maybe this may be a public prosecutor, this may be a defence lawyer and this is a suspected criminal and this is maybe the police resolving this criminal and this is the judge is not it? So, from this piece, what you can infer?

I will come to this piece, again, this piece of painting, whatever you say, I will come to this again, after a few slides. Then I am sure all of you will be able to understand why I have in the beginning I have kept this picture here.

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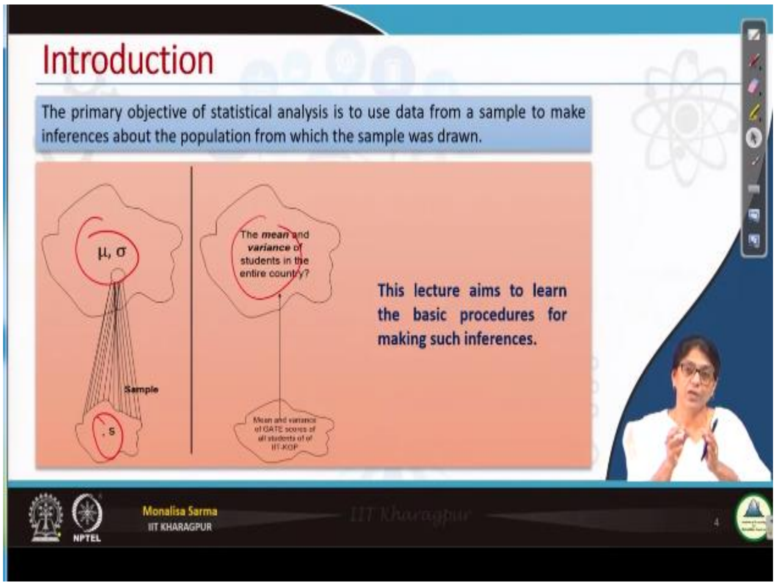

So first, so now introduce it first is the primary objective of statistical analysis is to use data from sample to make inference about population from the sample that was drawn, we have repeatedly I am to telling this again and again, the primary what is my primary objective, we will use data from the sample to infer about this population is not it? So, this is one such example is from sample we will get some item sample mean or sample variance from there we will try to infer about maybe this is a so this is a population.

This is my sample from the sample I am inferring another population. Similarly, like suppose I know the mean and variance of the GATE score of all the students of IIT Kharagpur, say from this sample this I am taking it a sample mean score of the students of IIT Kharagpur. From this, I am trying to predict something about the mean and variances of the students of the entire country get results. I am not have all the things in exams. So, this is what we call statistical inference.

So, this lecture aims to learn the basic procedures for making such inference. So, how do we make some inference? There are some procedures, there is certain steps which you follow to make this inference. Of course, sampling distribution is the main part that is the backbone, but where we use sampling distribution, how do we use? What are the steps that we follow? That we will be discussing here.

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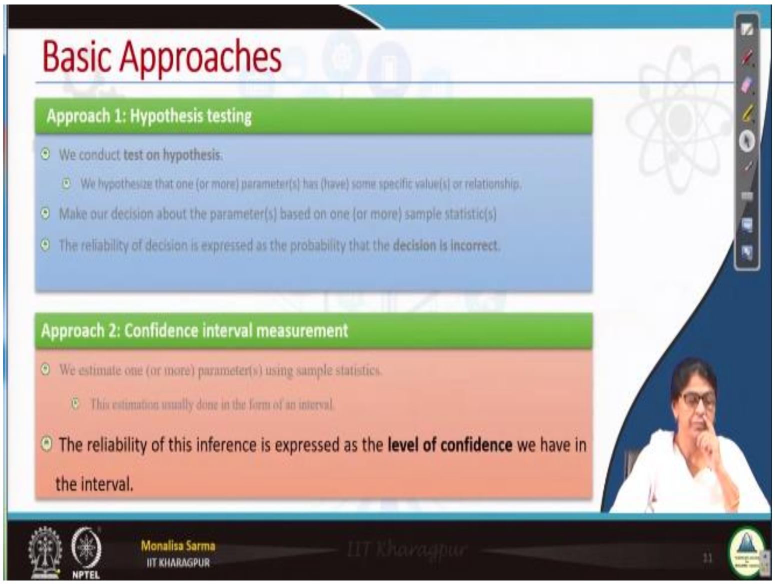

So, to the basic approach, when we talk of doing this statistical inference, so there are 2 approaches, the first approach is called hypothesis testing. So, we conduct tests on hypothesis that means what we conduct based on hypothesis means what we hypothesize that one or more parameters have some specific value or relationships. So, initially, when I was discussing sampling distribution, I was always using the word estimated or from past results or whatever it is in some way, I have also mentioned that.

I do not want to use the term hypothesis here because we will be talking about that time later. see from here on, I will not be using estimated on there from here on I will be using the word hypothesize that so, what we do we hypothesize that one or more parameters have some specific value like when we told when we in previous question, the mean of the population is this mean of the variance of the population is this that means, population mean and population variance.

How do we know it is such from the huge thing, from the huge thing how we do know the mean and the population? Definitely, we have hypothesized that value now what do we mean by hypothesis? First let me come to what do you mean by I will come to this slide again.

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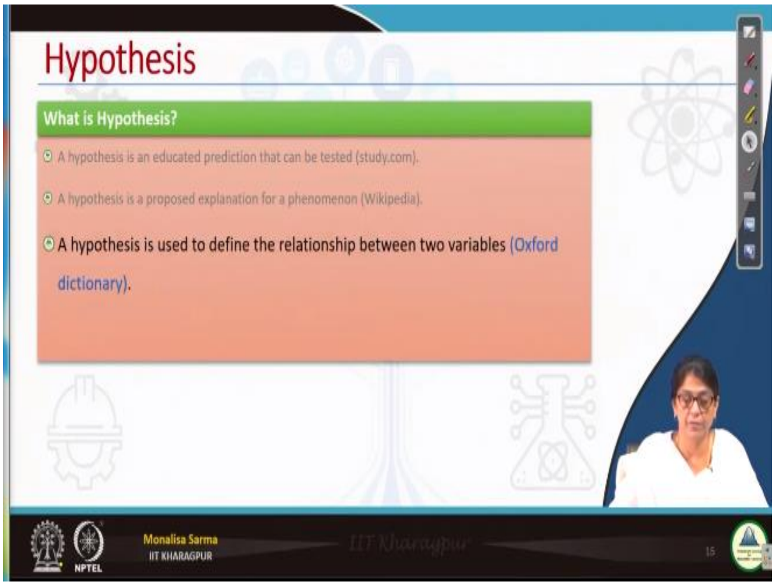

So, let me take a quick see what is hypothesis? If you just Google it and just Google it you will see hypothesis you will see different definition. So, what one definition is says a hypothesis is an educated prediction that can be tested is a prediction. Educated prediction means you are predicting not just like that you are predicting, you are predicting the best in some knowledge. Again hypothesis is a proposed explanation for a phenomenon.

Again dictionary what Oxford dictionary tells a hypothesis is used to define the relationship between 2 variables that of course, is not very much for our use. Then what does Walpole? Walpole is a standard book on statistical methods what does it say a supposition or proposed explanation made on the basis of limited evidence as a starting point for further investigation on the basis of limited investigation, on the basis of whatever evidence past experience whatever we have.

We give some value, we predict will specify certain value and that is basically a starting point for further investigation, this value can be anything it is a value or relation between 2 variables whatever it is, so but why do we so that we can start further investigation for in starting investigation, it is better if we have some value from there, we will go up. That is what Walpole says so, that is hypothesis.

So, now coming back to this so, the first hypothesis testing what is that we hypothesize that one or more parameters have certain values or relationship. So, we then what happens? First, we hypothesize that then we make our decision about the parameters based on the sample statistics. That is what we have seen in sampling distribution? Sampling distribution, we did not use that what hypothesis, but here I am using it.

So, we hypothesis some value then we collect the sample from the sample we get the data from whatever data we collect, based on that we give our decision whether whatever value we have hypothesis is correct or not. Then now the reliability of the decision whatever decision we give that it is not the problem, it may not be this, the mean may not be this whatever is estimated or variance may not be this whatever it is specified the decision.

The reliability of the decision is expressed as the probability that a decision is incorrect. What is the probability that a decision what we have given what is the probability that is incorrect that also we specify, we just do not tell that this is we do not agree with this hypothesis parameter, the hypothesis parameter may be wrong or hypothesis parameter is right. We just cannot we just only do not say that we also explicit in terms of reliability, what is this reliability? This reliability is the probability that the decision is incorrect.

The reliability of the decision our we explicit and the probability that a decision is incorrect. There is one more approach this is the first approach for statistical inference and this approach we call it hypothesis testing second approach confidence interval measurement confidence, what we do we basically estimate the value of the parameters based on the sample statistics means like in hypothesis testing, we hypothesize the value and then based on that we do the further investigation.

But in confidence interval we do not hypothesis any value we just try to estimate the value of the parameter based on the sample values. So, this we will not be discussing here this I will go in details when we will be discussing and I think after 3, 4 lectures I will come to this confidence interval measurement. So, let us not discuss about this now so, this we have already seen.

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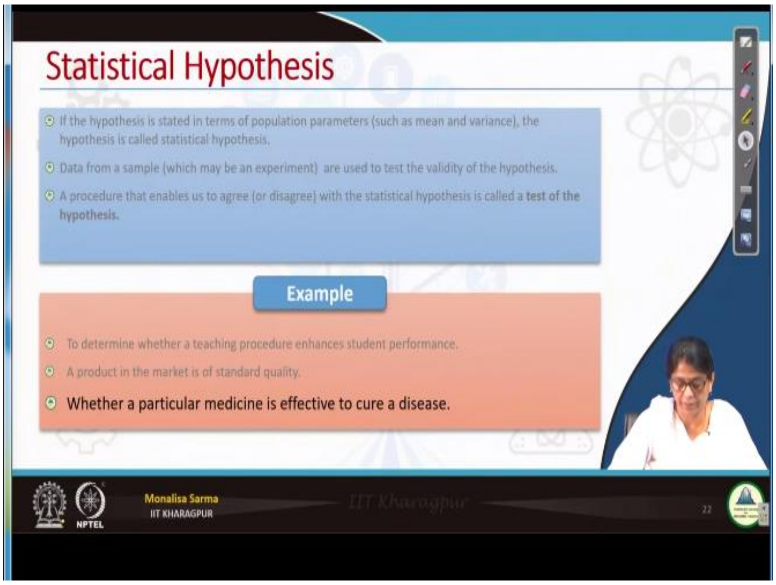

Now, we know what is hypothesis we have known now? Now what is statistical hypothesis because this hypothesis, maybe we may hypothesis about anything. So, what is statistical hypothesis? If the hypothesis is stated in terms of population parameter such as mean and variance, the hypothesis is called statistical hypothesis. So, that means we will be talking about statistical hypothesis only. So, data from the sample are used to test the validity of the hypothesis.

Validity of the hypothesis means we will talk about the reliability of a decision how will give a reliability of a decision? That is the probability this may be incorrect? A procedure that enables us to agree or disagree with a statistical hypothesis is called a test of hypothesis that we will see what is the test of hypothesis? So, some example of statistical hypothesis like someone maybe we want to determine whether teaching procedure enhances student performance.

So, whether new teaching procedure has come to market whether the teaching procedure will enhance the student performance what, how we will do maybe we will take a conductive test before us introducing the testing procedure and after introducing the teaching procedure, maybe after some time, we will again take a test and we will evaluate the score and what is the score before this introduction of this new procedure.

After the introduction of this new procedure does base on the mean score of this we will be able to find out whether the teaching procedure is good or not. So, that is one way of there is one example of statistical hypothesis. Another example maybe product is the market is of standard quality, then when I talk of standard quality the maybe the time it last may any is suppose the LED bulbs, LED bulbs how put is maybe the average working average life of the bulb.

So, the standard quality means it has to it is average has to be a particular value say x. So, what we take a sample from there we find out whether it satisfies that or not. So, first we hypothesize a valid and from we take a sample and whether it is correct or not that is again another example of statistical hypothesis. One more example whether a particular medicine is effective to cure a disease these are some of the examples, there is many such examples.

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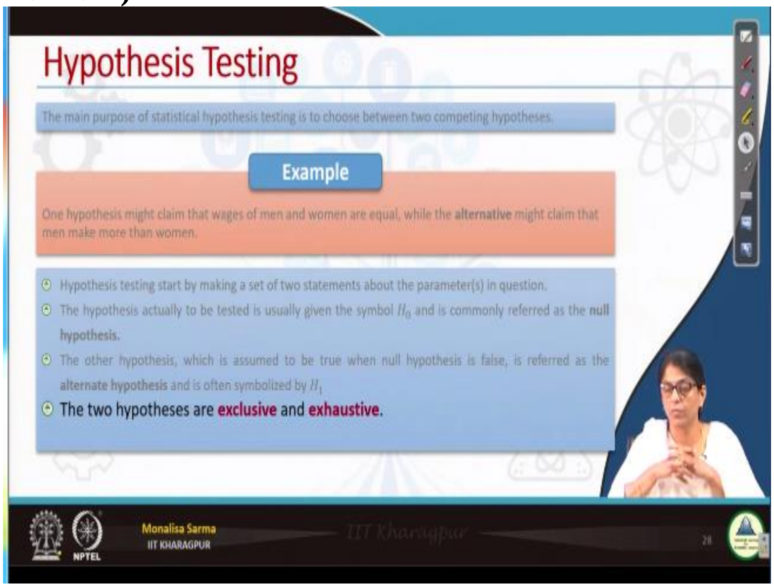

Now, when we talk about hypothesis testing, basically, we have 2 different hypotheses. So that is what the main purpose of statistical hypothesis is to choose between 2 competing hypotheses, 2 hypotheses and both are competing hypotheses, we have to choose between 2 we will see. One example 1 hypothesis might claim that wages of men and women are equal that is 1 hypothesis. Another hypothesis, maybe men make more money than women. So, it has 2 competing hypotheses. So, we will have to choose between 2.

How will you choose? We will choose based on the sample data. So, hypothesis testing how do we start? We start by making a set of 2 statements about the parameters in questions. So, hypothesis testing firstly, that we always start by making 2 statements about the parameters in question by what I mean maybe, if we want to infer about the mean. So, we will make 2 statements about the mean of the population. If we want to infer about the variance, we will make 2 statements about a variance of the populations.

So, the hypothesis actually to be tested easily given by a symbol H 0 and is commonly known referred as null hypothesis. So, there are 2 hypotheses one is called null hypothesis the null hypothesis is usually the status quo, usually that whatever it is already what is existing. So that is specified as null hypothesis. And we want to test it whether what we are claiming or, what we want to be correct whether that is true or not, that we specified null hypothesis and the alternate is what we want to test it.

Whether, I should not have what we want to test it whether what may mean this, which is assumed to be true when null hypothesis is false. So, what actually to be tested is given in the null hypothesis that means what we actually want it to be true, we want something to be true, whatever we are claiming we want that claim to be true. There are reasons I will come to there so that is called null hypothesis.

And another one which actually we want to test. We do not want that to be true, that might be true, we want to test it so that we call it as alternative hypothesis. So, null hypothesis is the status quo that means maintaining the same status. So, null hypothesis symbolizes at H 0 and alternate hypothesis symbolize at H 1. So, these 2 hypotheses are exclusive and exhaustive, exclusive and exhaustive I am sure all of you know the meaning of what this means or exclusive means both the hypothesis cannot be true.

Both the hypothesis cannot be true and there are only these 2 hypotheses is possible that is why it is exhaustive and both cannot be true only one has to be true. One of the hypotheses has to be true.

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Now, the same picture that we have seen in the first slide. Now we can understand so this may be the null hypothesis which we want to calculate the suspect. We do not want to be convicted. So, there is a null hypothesis alternative hypothesis or we can say the other way around, then this is the based on this maybe this is the sample or some witness or whatever it is maybe the suspects of what to say whatever the suspect has to say, and based on that what we infer. So, now you can understand the significance of this picture.

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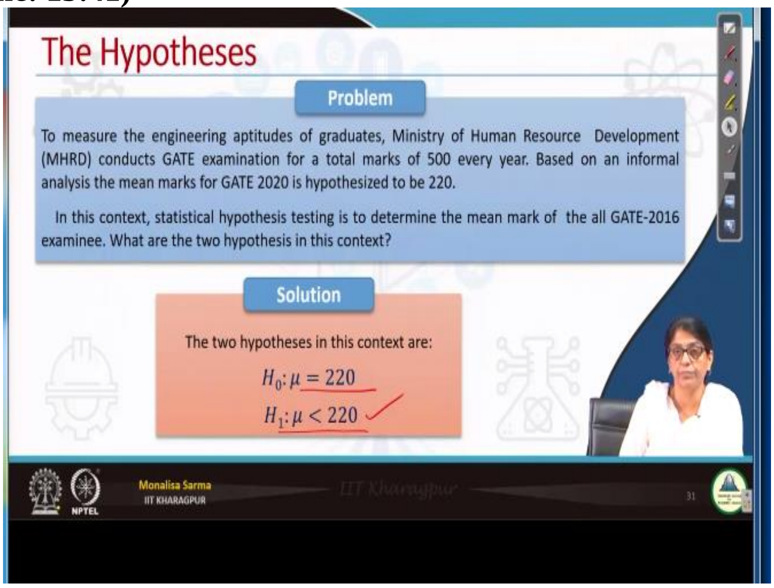

So, one simple example like on what I have already given in the first slide, a second slide maybe. So, to measure the engineering aptitude of graduate Ministry of Human Resource Development MHRD conducts GATE examination for total marks of 500 every year. Based on the informal analysis, the main marks for GATE is hypothesized to be 220 from an analysis were done and it is founded mean marks to be around 220.

So, in this context, statistical hypothesis testing is to determine the mean marks of all GATE examinee. What are the 2 hypotheses? In this context what maybe there are 2 hypothesis? The 2 hypotheses first is the null hypothesis definitely, μ that is the mean means μ we are talking about a population. So, we write as μ. So, $\mu = 220$ this is my null hypothesis. Now, my alternate hypothesis maybe if we want to test it, whether it is less than 220 or we want to test it whether it is greater than 220.

Accordingly, I will specify the alternate hypothesis. Now, suppose I want to test it I have a doubt though I have assumed it 220 but I have a doubt it is it may be less than 220 then my alternate hypothesis 220 less than 220.

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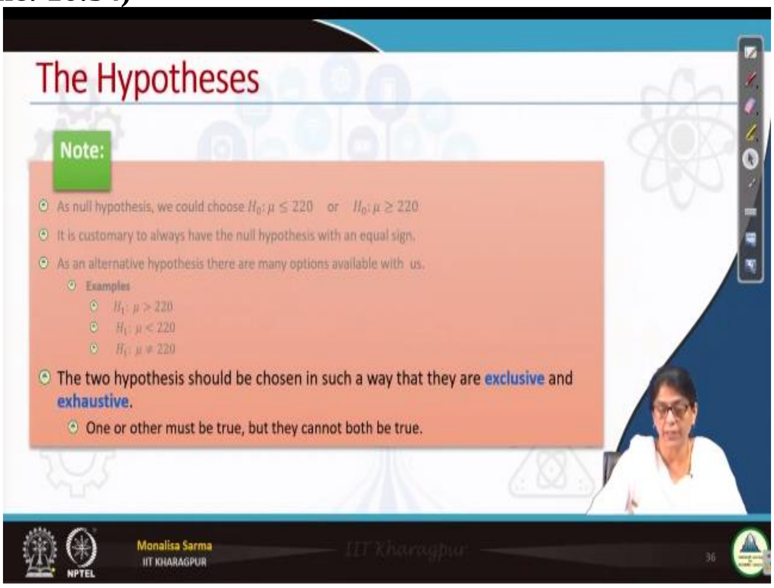

So, now and null hypothesis always it is customary to always add a null hypothesis with an equal sign. So, the null hypothesis we have specified to take it is always necessary the null hypothesis should be with an equal sign, why I will not explain this again this point will be discussing I think the next lecture or maybe next to next lecture I will discuss because dates need some other clarification. So, but you remember till them null hypothesis always it has to be with an equal sign.

So, whenever, I say this question $\mu = 220$, $\mu < 220$ that means, $\mu = 220$ because, I told us 2 hypotheses are exhaustive. So, this does not mean exhaustive there. What about a greater than 220 that means, this $\mu = 220$ it is basically it is telling that μ is greater or equal to 220. So, we could choose less equals to or greater equals to whatever it is or simply equals to if it is simply equals to then alternate hypothesis will be not equal to.

So, it has to be exhausted it is as an alternate hypothesis there are many options available greater or less than if not equal to, but null hypothesis is always equal to should be the other along with equal along with less than or greater than or equal to has to be there we will see where one or other must be true but they cannot both be true very important.

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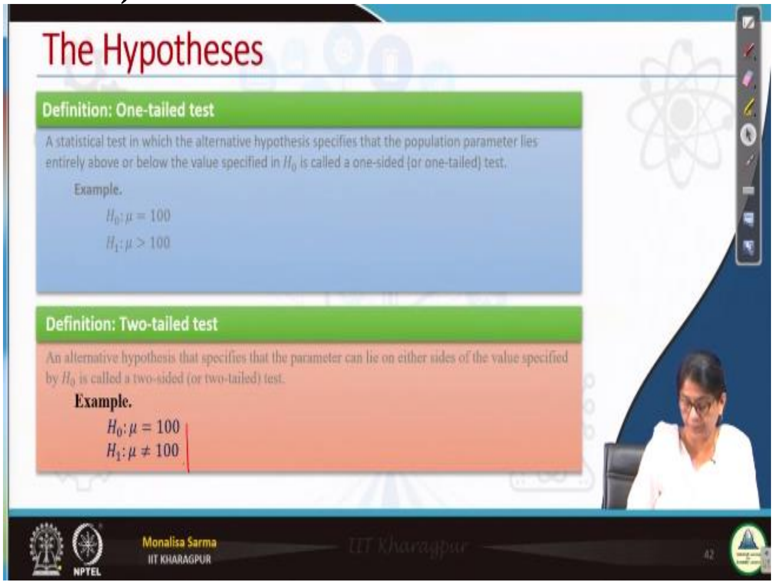

So, now, what is one-tailed test? One-tailed test is a statistical test in which the alternate hypothesis specified that the population parameter lies entirely above or below the values specified in H 0 whatever we have specified in H 0 it specifies the population parameter lies either above that or below that then we call it is one-tailed test. The last example what we have seen that a get value GATE score is less than 220 that is a one-tailed test if I would have written not equals to 220 that is a two-tailed test. So, this is an one-tailed test example.

So, two-tailed test an alternative hypothesis that specifies the parameter can lie on either sides of the value specified by H 0 whatever we have specified H 0 that it can lie on both sides then it is called 2 sided that is two-tailed test. So, this is an example of two-tailed test $\mu = 100$ $\mu \neq 100$ this is an example of two-tailed test. We will see one-tailed, two-tailed what is it we will see then get (())(19:30) using the what 2 numbers tailed mean here? Tailed definitely I think you could understand I am talking about a distribution tailed.

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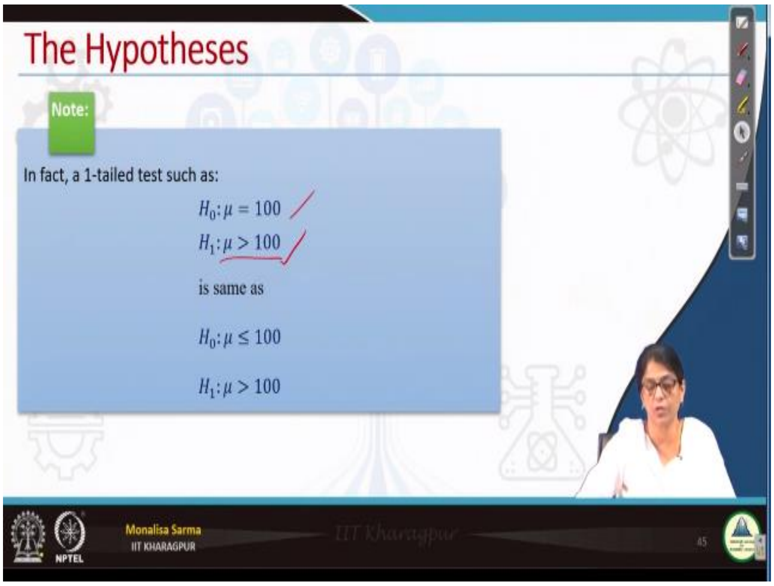

So, in fact a 1-tailed test such as this as same as I told you it has to be exhaustive. So, if I am writing a greater that means equal that means I am actually meaning less than equals to, but I am not very bothered whether it is less than or what, I just want to test whether it is greater than 100 or not. I am assuming this 100 and I want to test whether it is greater than 100 or not means less than 100 I am not giving any importance to that. So, when I am talking equals to, that means I am specify actually, I am meaning less than equals 200 only.

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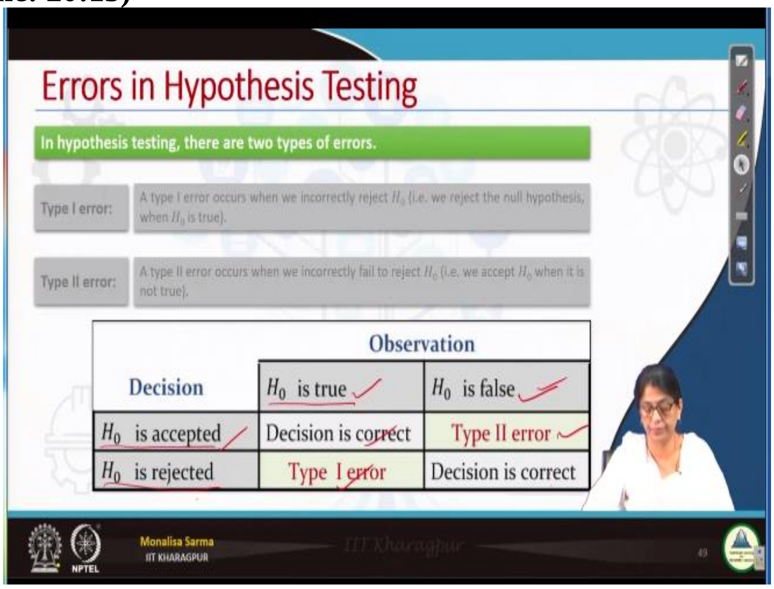

Again, in hypothesis testing, there is one unescapable part there is an error, error in our decision whatever decision we take in hypothesis testing as a tool, we will be there 2 steps, 1 is the null hypothesis another is the alternative hypothesis. So, we take a sample, from the sample we get the data from the gate data, we tell whether null hypothesis is true or the alternate hypothesis is true means we basically give a decision on based on these 2 hypotheses.

Now, this decision, always there is a probability that there may be some error in this decision. So, doing an error in then on escapable part in this statistical inferences, this is something we cannot escape there may not be any error, but still the probability of error will always be there. So, basically, there are 2 different types of error what are the 2 errors first is called a type 1 error, see what is type 1 error? So, we will be using this type 1 error type 2 error for coming few classes. So, please remember this again.

So, type 1 error occurs when we incorrectly reject H 0 that means, our null hypothesis whatever we have specified in the null hypothesis, that like in an example, for GATE score, we are specified null hypothesis is mu = 220 the actually if you see population μ is 220. But whatever sample we got from the sample, we found that mean ≠ 220 probability of mean = μ very very less from the sample what we got, that is why we have rejected the hypothesis.

But actually if we say if there was some mechanism to find out I mean, actually it is mean is 220 only. So, that is what so then that means, we have made it type 1 error. So, type 1 error occurs when we incorrectly reject H 0 that is we reject the null hypothesis when H 0 is true, that is type 1 error. Now, what is type 2 error? Type 2 error occurs when we incorrectly fail to reject H 0 actually H 0 is false, but we fail to reject H 0 like to get example, actually mean ≠ 220 what is an alternate hypothesis?

Alternate hypothesis was greater or less than 220. So, actually if you see the actual result actually mean is actually less than 220. But on a sample what we got sample indicates that the mean = 220 that means, it type 2 error occurs that means, we incorrectly because we take the decision based on a sample and sample statistics or variable for different samples we may get different, different sample statistics is value, as well as what matters most of the sample is there biased sample or unbiased sample.

Sometimes we may incorrectly take a biased sample as well and sometimes what because the selection what we have selection? We have selected even random for us to make a select the sample unbiased, our selection has to be random, but however while picking taking randomly picking different subjects also it is it may so, happen that our sample is not very diverse sample so, we may come up with a wrong result is not it?

So, that is what so, a type 2 error occurs when we incorrectly fail to reject H 0 actually H 0 is false, but the simple result indicators no H 0 is correct so, that is type 2 error. Some examples will make it very clear. So, what is this is an observation you see. So, these are a decision H 0 is accepted H 0 is rejected. There are 2 cases H 0 is accepted 1 if H 0 is true and we are accepting the decision that means this is a correct, when H 0 is false and we are accepting H 0 then it is a type 2 error. We are rejecting H 0 whereas H 0 is true and this is a type 1 error H 0 is rejected is false and decision is correct.

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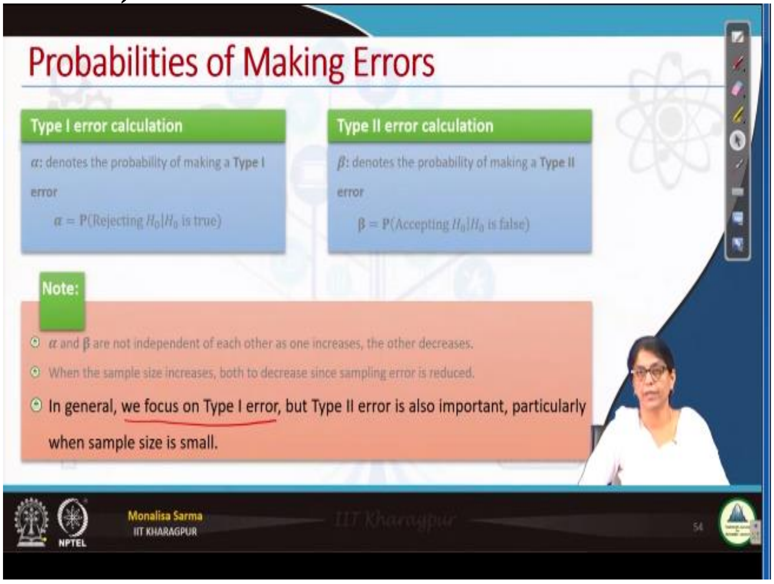

So, now this type 1 error and type 2 error. So, we have some representation for type 1 and type 2 error we usually indicate type 1 error by α type 2 error by β. So, here type 1 how to calculate this type 1 error. Now question is how to calculate the type 1 error? Type 1 error α denotes the probability of making a type 1 error. So, what is α? Alpha is nothing but probability of rejecting H 0 given H 0 is true see we have talked learn conditional problem is not it? What is conditional probability, what is the probability of A given B?

So, what is the probability of rejecting H 0 given H 0 is true that is α. Similarly, what is β? Beta is the type 2 error probability of accepting H 0 given H 0 is false is H 0 is false and we accepting H 0 what is that probability of β. So, we will see with some example the formation of different types of hypotheses formation of α β and region. So, α and β are not independent of each other as one increases the other decreases if α increases β decreases, β increase α decreases.

When the sample size increases both decreases, because sampling error is reduced that we have seen. So, in general we focus on type 1 error but type 2 error is also important, particularly when sample size is small anyway, in general we focus on type 1 error why we focus on type 1 error, this thing also I will come later. So, there are 2 points in this today's class there are 2 points which I mentioned that I will be discussing in some other class because now you do not know all the technical details to understand this.

So, 2 things why we focus on type 1 error and secondly, why it is what to say null hypothesis has equality sign needed for null hypothesis in this 2 we will be discussing later.

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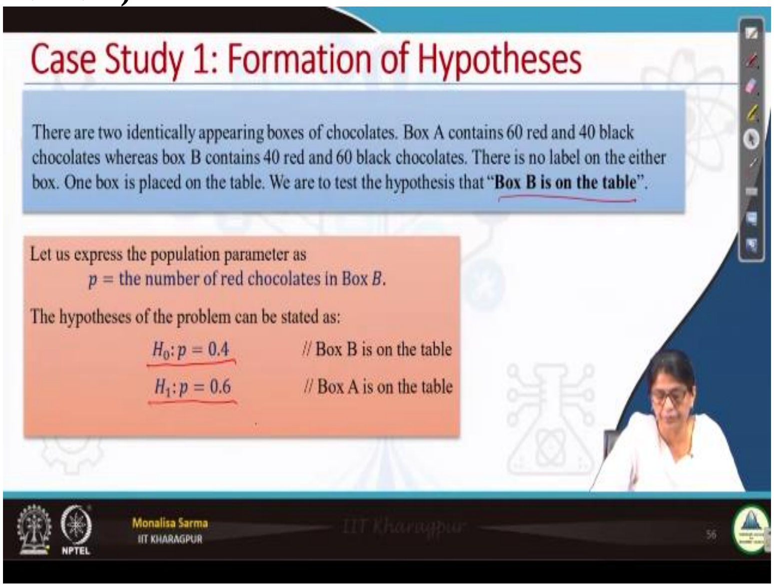

So, now, with some very simple example not very technical type of example, we will see how we form the hypothesis. So, we will take 2 examples basically. So, first example, as the simply a box of chocolates there are 2 boxes, box A and box B. Box A contains 60 red and 40 black chocolates, the 60 red chocolate and 40 black chocolate, box B contains 40 red and 60 black chocolate, if our interest in identifying the boxes based on the number of red chocolate it has.

So, probability the box number of red chocolate boxes has what is that probability is 0.6. But for Box B it is 0.4. So, now there is no level in both the boxes one box is placed on the table, we are to test the hypothesis that box be on the table box B on the table that we are to test that hypothesis whether this is correct or not. So, it is a very disturbing non technical example. We will take a technical example also. So, let us start with this.

So, now, for what will be my null hypothesis? Null hypothesis is probability that box B is on the table we will consider the main deciding factor is the red chocolate. So, p is the number of red chocolate in the box. So, my null hypothesis is p = 0.4 and if I write it informally, informally, I can write our null hypothesis is the box B is on the table instead of writing that I am writing as pie = 0.4 taking that direct chocolate as the deciding factor for the boxes. So, my alternate hypothesis is p = 0.6.

Now, I have to find out whether my null hypothesis is true or not I will test null hypothesis the one which we will test which you want to be correct here. This is a whether boxes in A is on the table B is on the table. It does not make any difference, but anyway, but in actually in real example, we want the null hypothesis to be true and we are testing on null hypothesis. So, we will take the sample from the sample we will take this we will find out which hypothesis is true.

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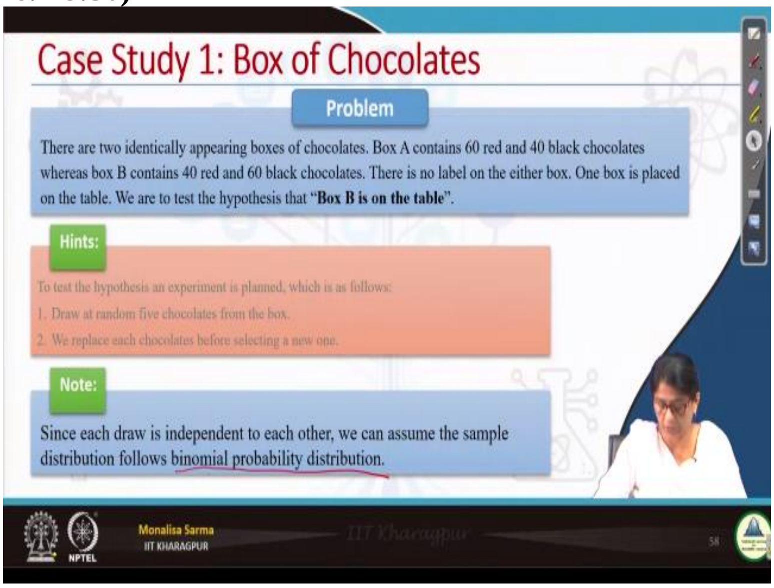

So, first thing is that to find out which hypothesis is true from the sample what we have to find out we always first will have to decide a rejection region what do we mean by rejection region? Rejection region means the probability for what probability we will tell that this is not true. Remember we while doing sampling distribution we are specified when we get very less probability we tell that whatever we have estimated whatever we have guessed or whatever it is conjectured, it is not true when we get a very less probability.

Now, what is this less probability? This less probability meaning we will specify this as a rejection region. So, if I take this sampling distribution of mean and this is a normal distribution, my rejection region will be this very definitely this is a very less probability is not it? Probability is nothing but the area of this under this, is not it? So, this is very less, but this is my rejection region. So, now for this simple example, similarly, I will have to find my rejection region.

Rejection region means from the sample I will have to calculate a test statistics from the sample I have to calculate the test statistics. And I will have to find out the probability of that test statistics. So, here so what I will do, what I am interested in proving? I am interested in proving that box B is on the table, is not it? This is box B is on the table I want to put that box B is on the table that means box with just less than number of red chocolates.

So, what I will do? I will do my experiment how I have designed an experiment, draw at random 5 chocolates from the box we replace each chocolates before selecting a new one. Selected one put it back; selected one put it back that means this is nothing but a binomial distribution simple. This choice independent since each draw independent to each other we can assume simple distance using we can assume that a sample distribution follows binomial probability distribution.

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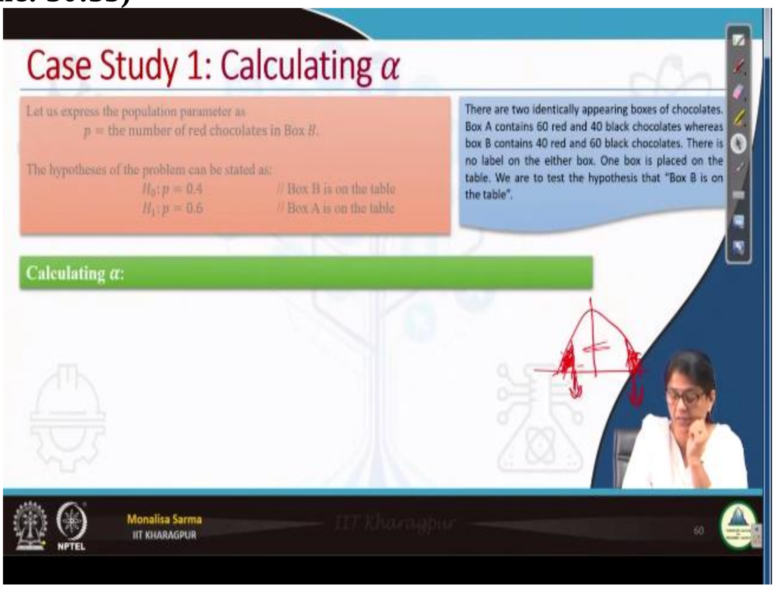

Now, we need to calculate α. So, how do we calculate α? For calculating α that means probability of type 1 error. Type 1 error means what is type 1 error remember that probability that given that H 0 is true and we are rejecting H 0. So, when that will happen, that will happen when we get some when we will reject H 0 we will get reject H 0 when we will get some very less probability of happening is not it?

So, this portion it may value have falls in this area then I will reject it. So, this is nothing my α if the sample results fall in this region, maybe it actually it is correct, but from the sample I got this result, because I have just taken from one sample is not it? Maybe you would have to add many more sample I would have seen this it does not fall here it falls in this region. So, this is my α this region basically. So, I have to calculate α.

So, for calculating α first I will have to find out which is my rejection region at one point α means there has to be some point corresponding to this then only this portion I call it as α what is the point specific today that means, my rejection region starts from here. So, this is the starting of my rejection region, this point what is this point? This is my rejection region. So, for this simple example, what rejection region we may consider.

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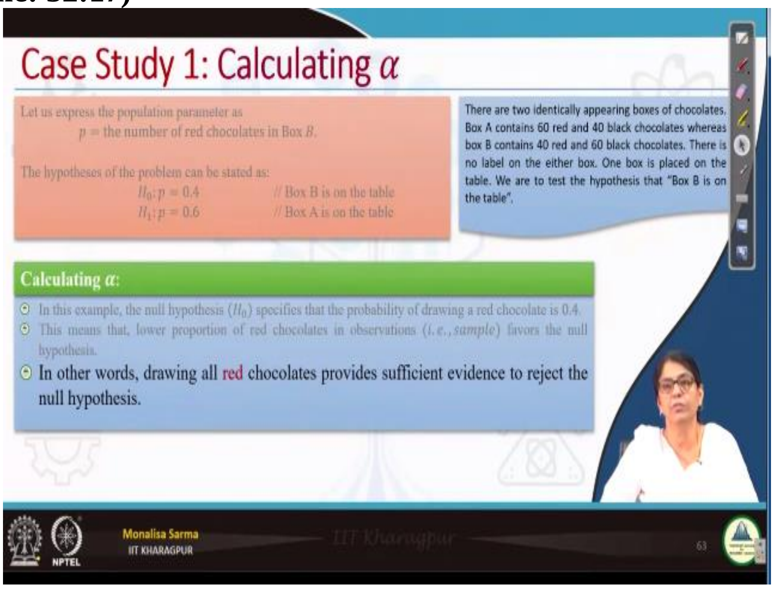

So, in this example, null hypothesis specifies that a probability of drawing a red chocolate is 0.4. This means that the lower proportion of red chocolates in observation favours the null hypothesis. So, let us take the rejection region maybe if we since it blocked box B in the table that means very less red chocolates are there. So, if you let us take our rejection region we may assume here if we draw all the written that if we are drawing how many chocolates? 5 chocolates.

So, you are doing 5 when you draw 5 chocolates, all the 5 chocolates if we get red, so, that is a rejection region if all the 5 chocolates we get red, then we can the conclusion that box B is not on the table box A is on the table because we get so many red chocolates we are picking but we got all the chocolates red that may be a rejection region see here. Now we can understood the concept of type 1 error say I have decided my rejection region as if I get all my chocolates red then I will decide that is not blocked box B but box A is on the table.

Because boxes more number of red chocolate. But see there may be the case actually blocked box B is on the table. But when I am picking it randomly I pick all the red chocolates, is not it? That thing is there so that is the probability of error that is the type 1 error I have committed. So, now, what is the probability of type 1 error that means what is the α that we need to calculate?

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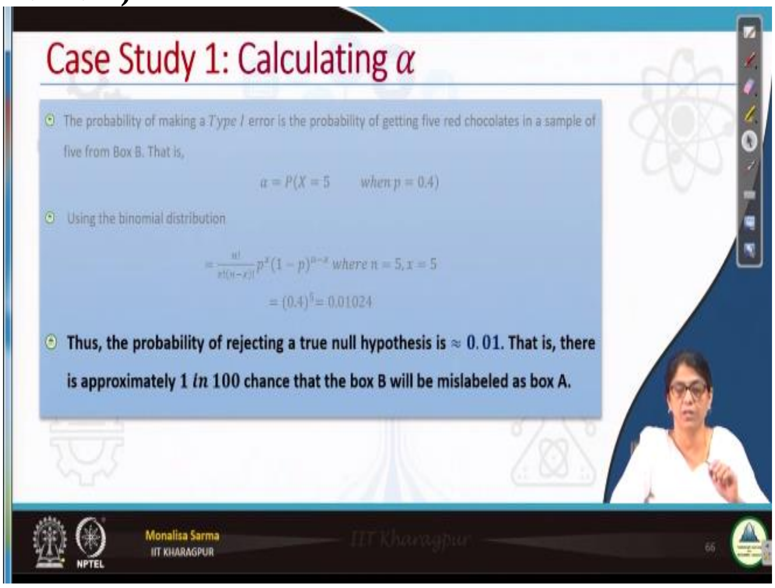

So, what is α? A is probability of that X = 5 when p = 0.4, I have specified my rejection region, rejection region is all the 5 chocolates are there. So, now I can calculate what is α that is the type 1 error actually hypothesis is null hypothesis is true, but we sample is also that it is not true that means we have rejected a null hypothesis. So, α is probability that x is = 5 when p = 0.4 this is my α type 1 error.

So, using simple we can use binomial distribution to find out what is this probability. So, if X = 5 so from 5 draw 5 chocolates, but it will be using this formula I will be getting this. So, this is my value simple binomial distribution, I will not go into the details of this. I am sure you know that we have done this sort of thing binomial distribution, so this is my value of α. So, there is the probability of rejecting a true null hypothesis.

If that is my rejection region, true null hypothesis is 1% that is there is approximately 1 in 100 chance that a box B will be mislabelled as box A there is very less sense if this is the thing if I consider if this is my rejection reason I consider that there is very lessons that it is actually though it is box B is on the table, I say that no it is not box B box A is on the table.

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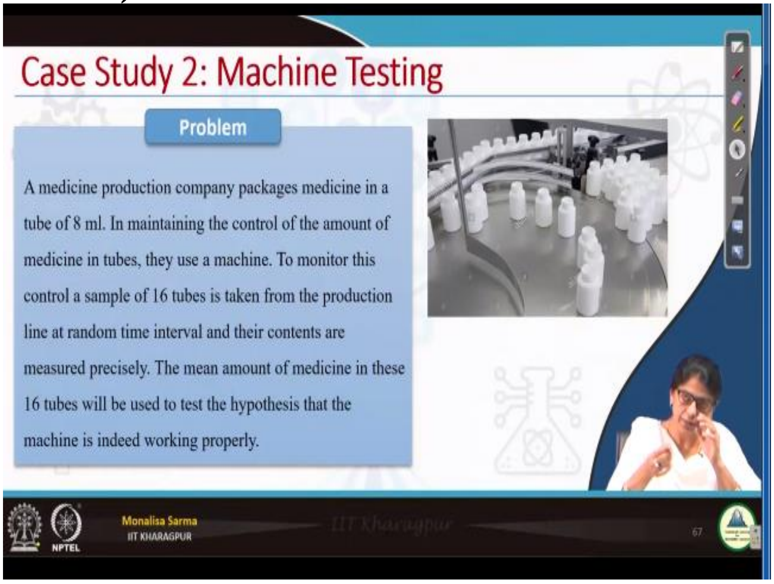

So now, that is how we find out type 1 error that is 1 example where we have seen how we can form the hypothesis, how the type 1 error is calculated. Now, you may ask why type 2 error we did not see we will come later, for this question type 2 error calculation will be very easy that I think I have in this lecture only, but actually calculation of type 2 error is very difficult we will come since next I think next to next lecture.

Now, let us take a bit technical problem earlier problem was a very simple toy game so, think toy example. So, a bit technical problem example. So, what is a medicine production company? We are not doing statistical inference till now we are just going step by step just reaffirming the hypothesis and try to find out the probability of type 1 error? That is α a medicine production company packages medicine in a tube of 8 ml.

So, in maintaining the control of the amount of medicine in tubes they use a machine to monitor this control a sample of 16 tube is taken from the production line at random time interval and the contents are measured precisely what we are done in medicine production company packages medicine in a capsule of say 8 ml. So, that means, it is expecting that the liquid there has to be 8 ml it is expecting that that is the expected value. So, whether this is correct or not.

So, what is the some to monitor this it has taken a sample of 16 and from each sample it is it tries to measure the content. And the contents or measure precisely the mean amount of medicine in this 16 tubes.

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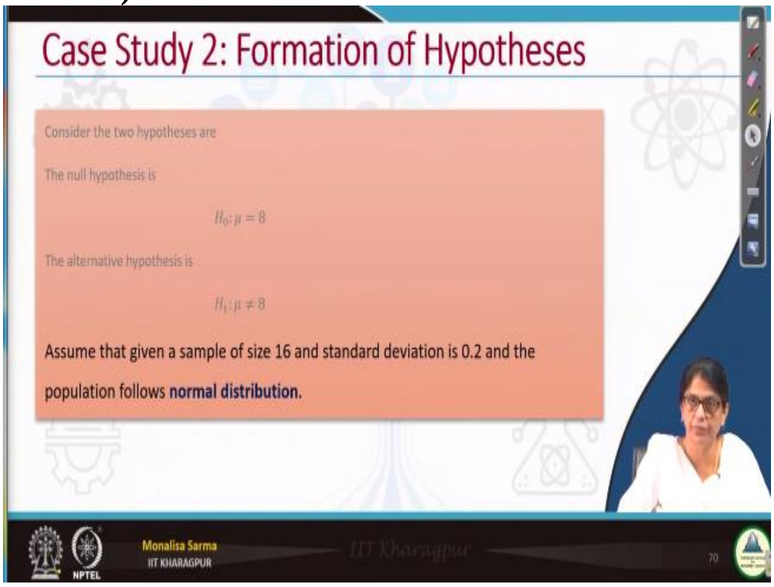

It is not given here the data are not given. So, the mean amount of medicine in the 16 tube will be used to test the hypothesis that the medicine machine is indeed working properly. So, we have to test the hypothesis that whether it is working properly or not. So, that we will know from the sample if we test the values. So, here how what will be our null hypothesis because we want to be true, we wanted 8 ml should be true to our $\mu = 8$ that is why null hypothesis.

What is my alternate hypothesis? It is not equals to 8 because if it is less than 8 then also it is because it is a medicine that means it will affect the general population and general people and if it is greater than 8 and also it will affect the population. If we talk of Institute of medicine if we talk away fruit drinks, so, if it is some say 100 ml and if it is less than 100 ml then it is cheating to the customers if it is more than 100 ml customers will be happy, but the company manufacturing company will be at a loss. So, both ways not needed.

So, it is we will try for not equality. So, $\mu \neq 8$ is the alternate hypothesis. This is how we form the hypothesis. Assume that given a sample size of 16 and a standard deviation of 0.2 and the population follow normal distribution.

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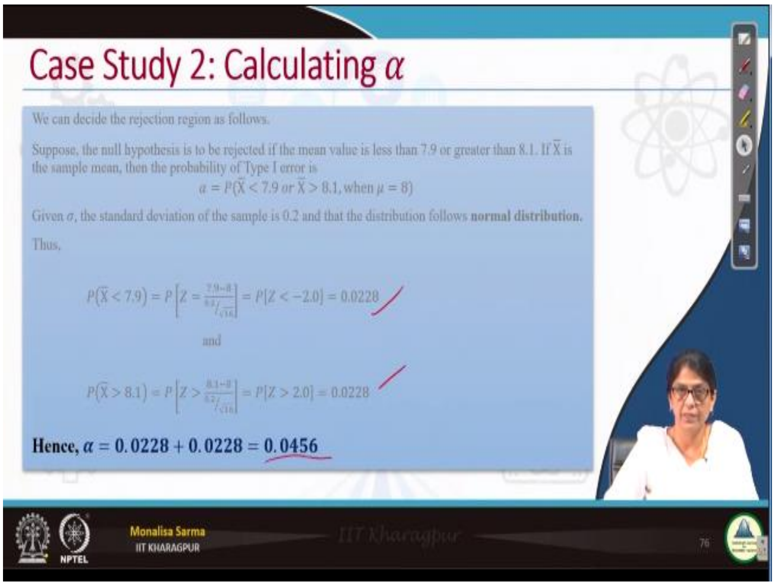

So, now we can decide the rejection region. Now, what is precisely rejection region like earlier case we have taken the rejection region if we get the 5 chocolates now here a rejection region. So, mean we want this 8, but we will accept if it is within 8.1 then within 7.9. So, within 7.9 to 8.1 we will accept it, if it is 8.1 also still it is acceptable if it is 7.9 that also it is acceptable it will not affect a patient 0.1 difference 0.1 ml it will not affect. So, it is 7.9 and 8.1 is acceptable.

So, what is the rejection region? Rejection region is greater than 8.1 and the rejection region is less than 7.9 this is my rejection region. So, what will be my thing type 1 error when my type 1 error will happen my type 1 error will happen that means, actually the medicine is producing correctly, machine is producing correctly only on an average it is around 8 lies within this range only, I will not tell it exactly A.

But it is on an average within this range on the but somehow we have picked some sample there may be some outliers, somehow we have picked some sample and from the sample we found that it is not in this interval but it is either in this interval or in this interval. So, what happens when we found that it is in this or this interval in a lesser or greater interval, then whatever maybe we will scrap the machine totally we will buy in total new machine investing crores and crores of rupees.

Whereas, actually it was the machine was working properly only. So, that is the type 1 error. So, what is type 1 error? Type 1 error is if we have decided that 7.9 and 8.1 is the rejection region. So, my type 1 error is the probability that my X bar is less than 7.9 and greater than

8.1 when μ is actually 8 when the sample mean is actually 8, but what is the probability that I got X bar is 7.9 and less than 7.9 or greater than 8.1 that is my α.

So, how do I calculate X bar getting 7.9 or X bar greater than 8.1 it is nothing but again come find out z distribution from that region then I can get the value from the z table. So, given the standard deviation of the population is 0.2. So, X bar less than 7.9 we know all this we have done lots of this problem, is not it? So, I found probability that is that less than -2 this is the probability.

Similarly, for 8.1 I found this that means if this is 8.1, this is 7.9 that means, this value and this value is this and this, so, this 2 together. So, my α is addition of this and this, this is my α

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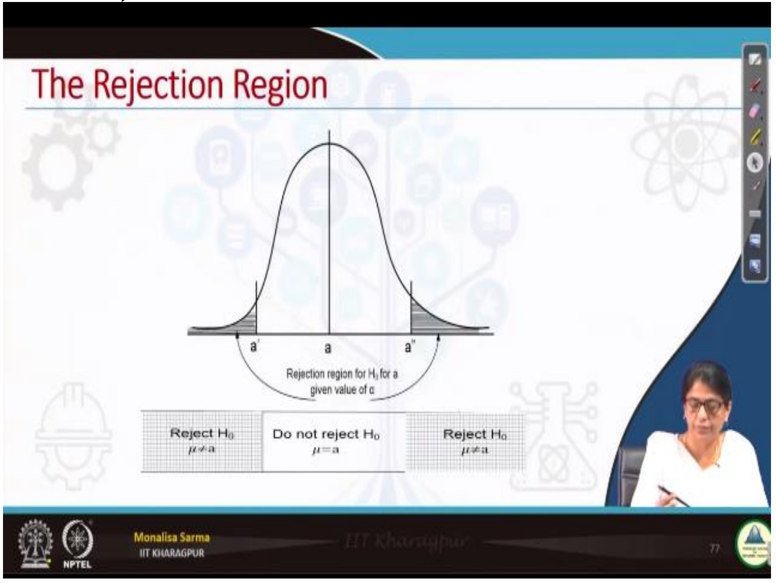

So, this is the rejection region. When I am talking about 2-tailed tests that means I have a 2 rejection region. This is 2-tailed test when I am specifying alternate hypothesis I am specifying is not equal, then I have 2 rejection region this is one reason this is another one region.

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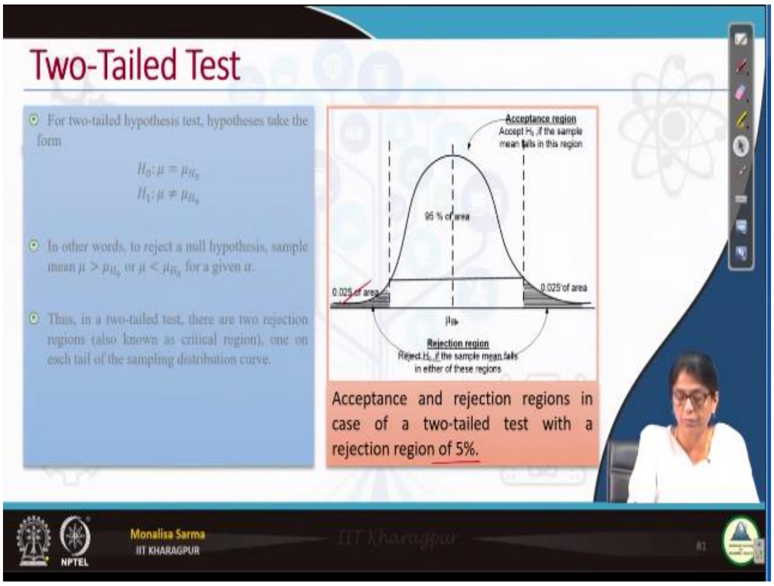

This is same thing I do not have to repeat it here again. So, here you see if my α is 5% α that is type 1 error is 5% and it is a 2-tailed test 5% means 0.025 will be this side 0.025 will be this side, that is in a 2-tailed test there are 2 rejection region and this rejection region is also known as critical region. So, here it is shown if it is a α is 5%. So, it is 0.025 and 0.025.

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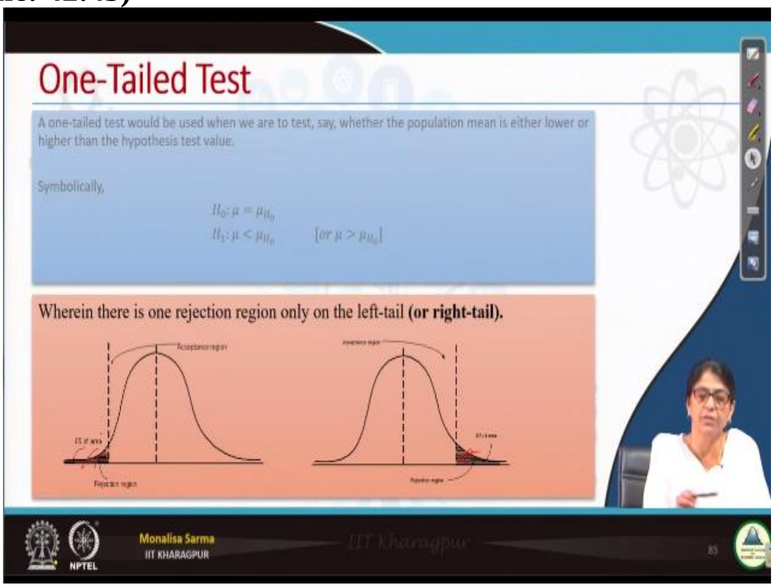

And 1-tailed test to be used when we have to take as I have already specified where we use 1-tailed test when whether the population mean is either low or higher than the hypothesis test, well, then we use 1-tailed test, then 1-tailed test means we will have only 1 rejection region. If I am talking of less than this, then my rejection region is this. And if I am talking I am greater than my rejection region is this alternate hypothesis.

If alternate hypothesis if I tell it is less than that then my rejection region will be left if I tell it the alternate hypothesis is greater than my rejection region will be right. Now in this 1-tailed test if I specify my $\alpha = 5\%$, so in 2-tailed it when my α is 5%, both side 0.025, 0.025. If in 1-tailed test α is 5% means it is only 1-tailed 1 side. So, 5% is only this side this is 5%, this is 5% we will do problems on this.

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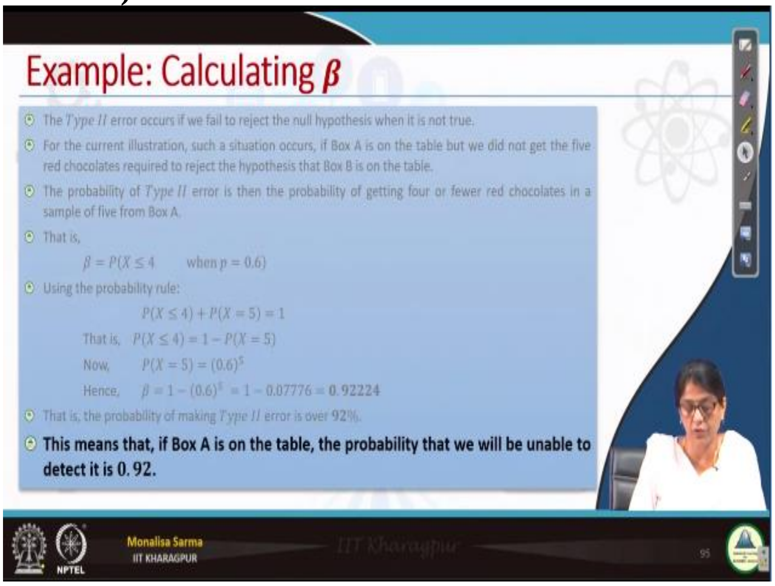

And type 2 error occurs when you fail to reject a null hypothesis when it is not true. So, for the case study 1 box A is on the table, but we did not get the 5 red chocolates required to reject a hypothesis that box B is on the table. And case study 1 what we have seen, actually it was box A was in a table, but we to reject that we needed 5 red chocolates, we did not get the 5 red chocolates. So, we have told that so what we what hypothesis the null hypothesis is correct. That is what we have found out.

But actually it is wrong that is what that is the type 2 error. Type 2 error is when we failed a false null hypothesis. So, for this example, we will be able to calculate β for the other example it is difficult. So, we will solve for this example what is what is β? B is probability of X when X is less than equals to 4 when p is 0.6. Actually, box A is on the table. So, p is 0.6 and when p is 0.6. I am getting X less than or equal to 4.

What is this probability that is my β? So, I found this is my β a very high β that is the problem making type 2 error is over 92%. Now, how can we reduce this β because this is very high β, this problem the type 2 error of 92% is quite high. So, how can we reduce this?

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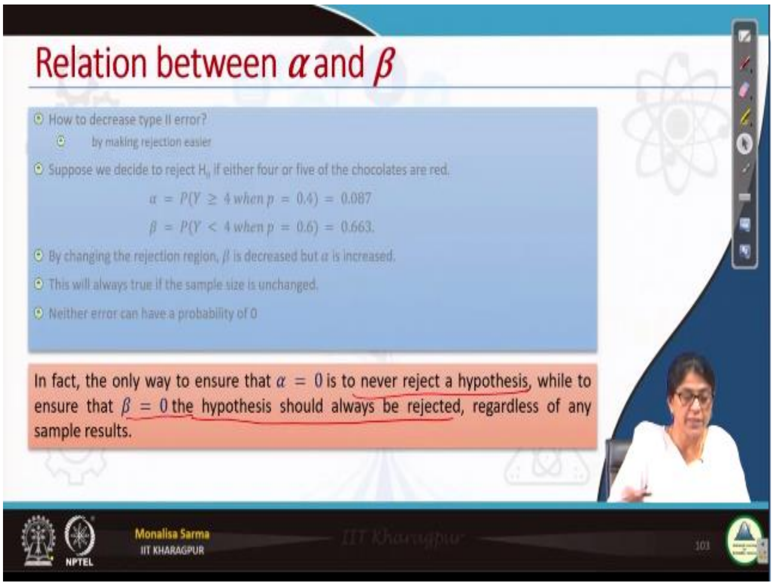

How to decrease type 2 error by making rejection easier, how we can reduce the type 2 error if I made a rejection is easier. That means if we decide to reject H 0 if 4 or 5 of the chocolates are red, earlier what is the rejection region we have considered if 5 added then we will reject. Now, let us make rejection easier that means what we if 4 or 5 of the chocolates are red, then we will reject then what happens?

Then α is probability of Y it is actually X, X is greater than equal to 4 when p = 0.4 this is now my α has become 0.08 earlier my α is 0.01 remember, now my α has increased my β will decrease now for this my β has become 0.663. So, when β increase α decreased, so, by changing the rejection region β is decreased α is increased this is always true and the sample sizes unchanged, however increase the sample size then we will get a better result.

Neither error can have a probability of 0. In fact, the only way to ensure α is 0 is to never reject a hypothesis. If we never reject the hypothesis, then α will be 0 probability of committing a type 1 error will never be there, we are not rejecting it only while to ensure that β = 0 the hypothesis should always be rejected regardless on the sample result this is a hardly deserved a possible this is not at all possibilities, we cannot do that.

And what is the point of conducting the test of hypothesis? So, the neither can have a probability of 0 it may be that there may be no error, but there is always a chance of error.

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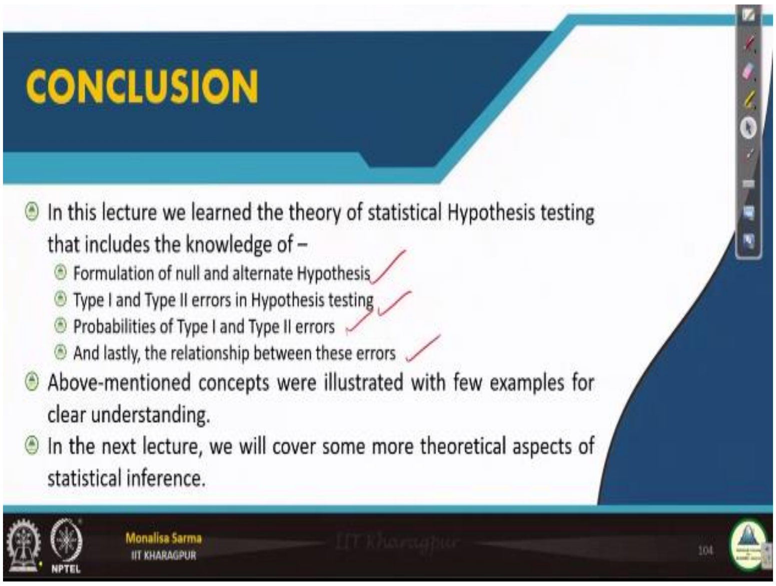

So, here we have calculated β, but actually calculation of β is not easy this is for this toy example we could do that, but calculation of β is difficult. So, we will be learning in I think the next to next lecture what is this. So, now to conclude in this lecture, we have learned about formulation of null and alternative hypothesis, how we formulate the hypothesis, what is type 1 error and type 2 error, what is the probability of type 1 and type 2 error what is α, what is β and lastly the relationship between these error.

We have also illustrated this with some examples. And in the next lecture we will cover some more theoretical aspects of statistical inferences.

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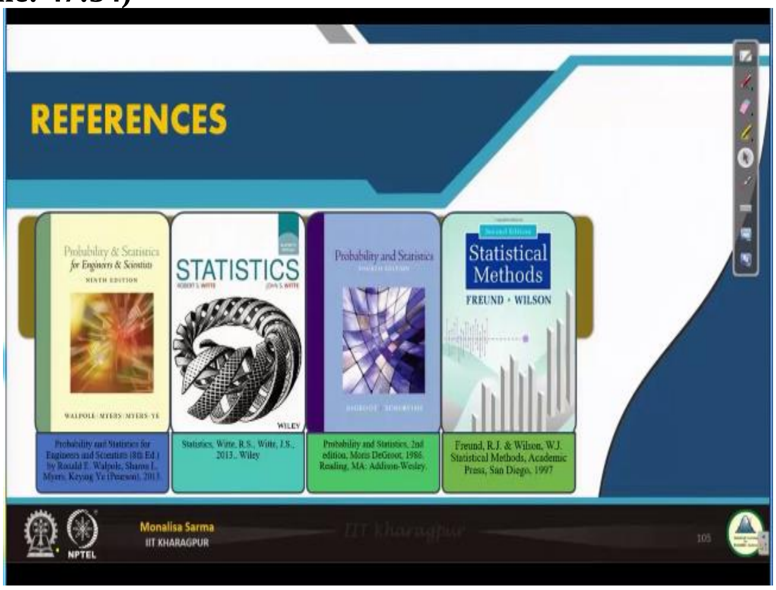

With that these are the references and thank you guys.