Algorithmic Game Theory
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Lecture - 40
Mechanism Design Intro

Welcome. So, in the last lecture we have finished the first part of the course which is like game theory. We studied various ways various types of games to model various real life situations and how to study various real life situations using games and using the notions of game theory how we can predict what will be the outcome as a player, how to analyse games and how to play games.

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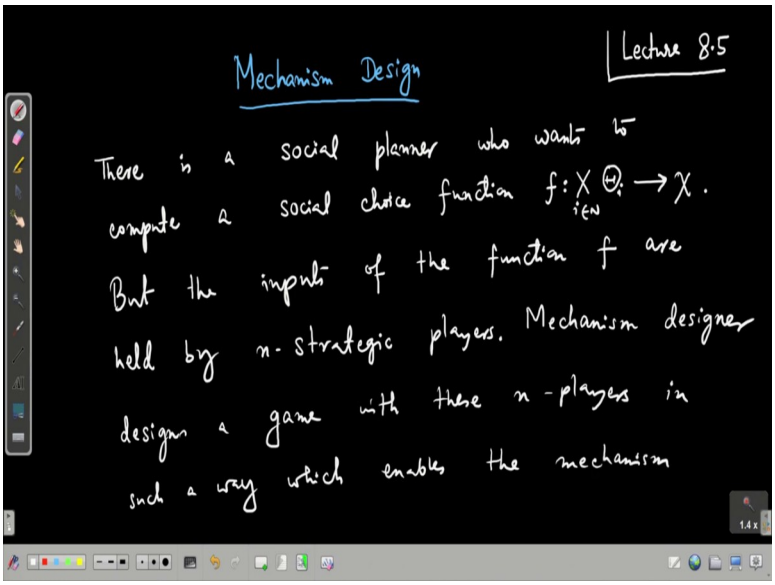

Now, in the we are starting the second part of the course which is Mechanism Design which is also called reverse game theory. So, the high level idea is this that you know there is a social planner. Think of some higher authority or government who wants to compute social choice function if from type profiles to X.

So, think of an auction scenario again auction is a very generic scenario which is say you which motivated various notions various theories of game theory and suppose the auctioneer wants to allocate the good who values it the highest, but the valuations are private quantities we do not know the valuations.

So, in the auction the a set of all type profiles could be the set of all valuation profiles each type set is the set of all possible values of the item for the player. So, a particular type is a particular value for the item for that of that player and from the type profile we want to we want to find an allocation, which gives the item to the player who values it most, but you know the because these are types these are private information we do not know we do not know them know them explicitly.

And so, we want to the social planner wants to compute the social choice function, but the inputs of the function f are held by n strategic player. So, $f(\theta,\ldots,\theta_n)$, but θ_i is held by player i and player is strategic and rational and intelligent.

So, merely asking player i to reveal its value its type may not work it may not be the best interest in the best interest of the player i to reveal its true type θ_i. And if it does not reveal its true type then how will we going to come going to able to compute this function f and that is where the entire field of mechanism design comes into picture.

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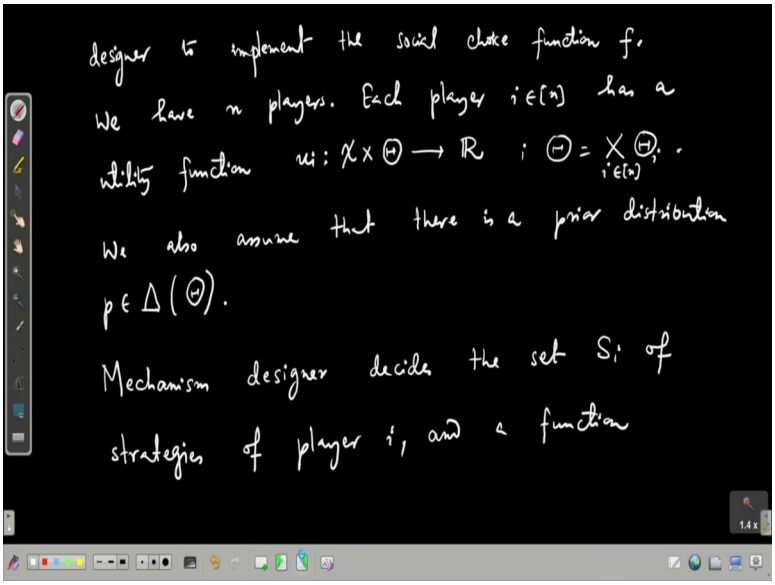

So, mechanism design mechanism designer designs a game with these n-players in such a way which enables the mechanism designer who is same as the social planner mechanism designer to compute or in the terminology of mechanism design it is called implement; mechanism designer to implement the social choice function f ok. So, we have players and each player. So, we have n players each player i has a utility function ui from set of outcomes and type profiles to real numbers.

So, capital theta is the set of all type profiles ok. Because there must be utility functions without utility functions players being strategic this does not make any sense. So, players must have implicit or explicit utility function and because they like or dislike various allocation various outcomes differently they like various outcomes differently and that is encoded by this utility function u_i.

Without this utility function players being strategy rational intelligent these things does not make any sense. We also assume that assume that there is a prior distribution prior distribution p over the type profiles as we have assumed in the Bayesian game and this common type p is a common knowledge ok.

So, in general when we talk about mechanism design, we do not explicitly write that each player has a utility function unless needed. So, it is assumed because without this there is no strategic behavior is possible and without this then the mechanism design does not make any sense it is just the then computing a function ok.

Now loosely speaking what does the mechanism designer does? Mechanism designer defines two things mechanism designer designs a game now, but what do you mean by game there are already players and mechanism designer designs a strategic form game.

Now, what is a strategic form game other than players i also need to have set of strategies and utility function it already have. So, we now somehow have to use this utility function. So, mechanism designer decides the set S_i of strategies of player i ok that it decides, but how will this strategies this set S_i is currently to the outcome and? So, this is this S_i mechanism designer decides and a function, which maps strategy profiles.

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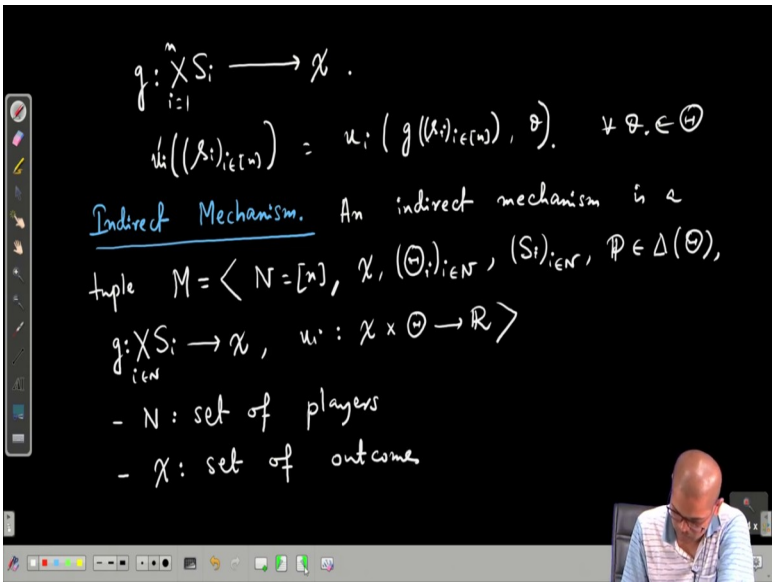

Function say $g: \times_{i=1}^{n} S_i \rightarrow X$ this also decides. Now, then what is the utility function? Utility function in this normal form game $u_i((s_i)_{i\in N})$ is basically. So, let us call this $u_i^{'}((s_i)_{i\in N}) = u_i(g((s_i)_{i\in N}), \theta)$ is there ok. This is for all $\theta \in \Theta$. So, this is the intuition. So, now, let us define it formally what is a mechanism and the mechanism that we have defined is called indirect mechanism because mechanism designer is not directly asking each player to reveal its true type.

It is sort of indirectly designing a game and asking the players to participate in the game and hope that indirectly by the behavior of the players in the game, it will be able to infer enough information about their type, which enables the mechanism designer to implement the social choice function f that is it its going via an indirect route that is why it is called an indirect mechanism it is not directly asking each player to simply tell their type.

Indirect mechanism and indirect mechanism is a tuple M which has a set of players capital M equal to N, then a set of outcomes, then type set for each player $(\Theta_i)_{i\in N}$ then set of all strategies for each player $(S_i)_{i\in N}$ then a prior distribution P over the type profiles theta then this function g which the mechanism designer designs cleverly which maps strategy sets to outcomes and then utility function u_i from outcome and types type profile to real numbers ok.

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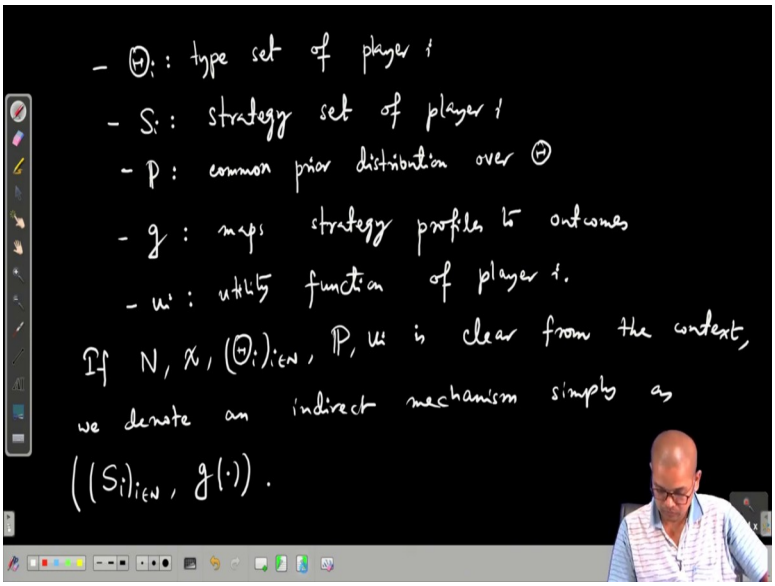

So, let us write what are these things N is the set of players, X is the set of outcomes Θ_i is the type set of player i then we have S_i is the strategy set of player i then P is the common prior distribution, which is a common knowledge entire gamma is a common knowledge including P common prior distribution over type profiles theta then g maps strategy profiles to outcomes and u_i is the utility function of player i.

Now, typically many of these are clear from the context and we will omit for brevity. So, if N set of players set of outcomes set of types $(\Theta_i)_{i\in N}$ then common prior distribution P and utility functions and if everything other things is clear from the context.

We denote an indirect mechanism simply as what mechanism designer has to do? Mechanism designer has to tell what is the strategy set of each player i in N and what is the function mapping strategy profiles to outcomes. So, this is indirect mechanism now what is direct mechanism?

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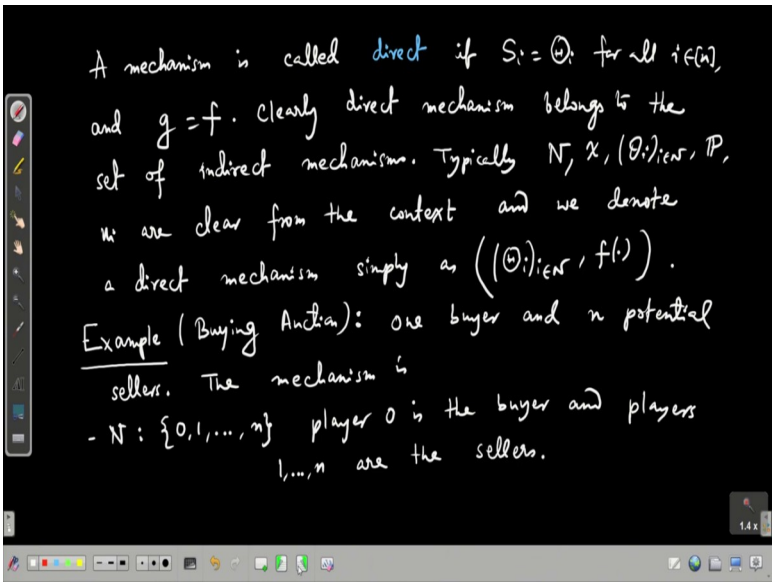

So, a mechanism is called direct is called direct if mechanism designer is not designing something superfluous it just ask each player to reveal its true type. That means, is to say that S_i the strategy set is simply typeset theta i ok. This is for all if S_i equal to say for all and it simply tells that we will we are going to calculate this function f there is no intermediary function g and this function $g=f$. If this happens then we call that mechanism direct mechanism.

So, clearly the set of direct mechanisms is a direct mechanism belongs to the set of indirect mechanism it is a special case of indirect mechanism ok and typically as usual typically the other thing. So, in the mechanism design tuple in the mechanism tuple in then N, X outcome then types $(\Theta_i)_{i \in N}$ and prior distribution P and the utility function u_i typically these are clear from the context.

And we denote a direct mechanism simply as the same notation as indirect mechanism, but here strategy sets are simply type sets and $f=g$ ok. So, you see that you know how simple is this direct mechanism and you know there is nothing to do for the mechanism designer to design ok. Now let us see an example of again a canonical example of an auction, which can be viewed as a mechanism say a buying auction. So, there is a buyer and n one buyer and n potential sellers and so, what is the mechanism we have?

And so the mechanism is what are its components its slightly different from our models when we when model using games. So, we have you know in the set of players we

include both the buyer and the sellers. Suppose player 0 is the buyer and 1 to n are the seller. So, player 0 is the buyer and players 1 to n are the sellers.

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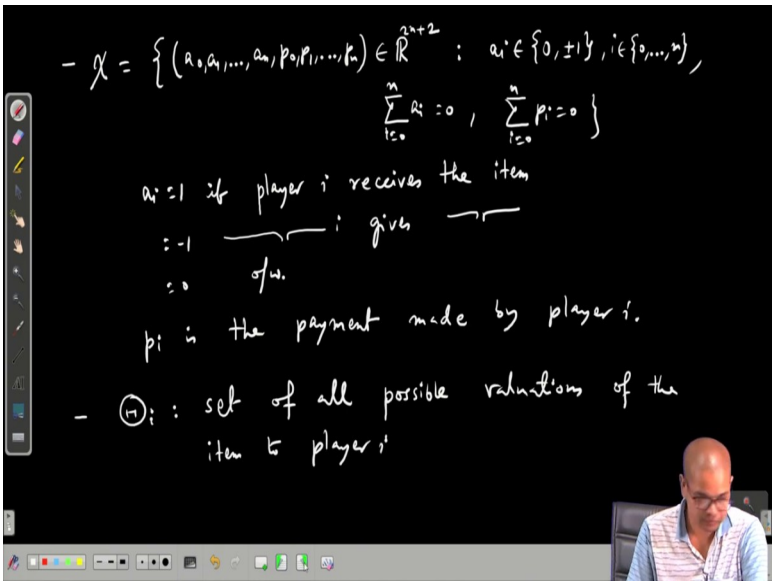

Then the set of outcomes $X=\{(a_0,a_1,\ldots,a_n,p_0,p_1,\ldots,p_n)\in\mathbb{R}^{2n+2}: a_i\in\{0,\pm1\}, i\in\{0,\ldots,n\}, \sum_{i=0}^{n} a_i=0, \sum_{i=0}^{n} p_i=0\}$. So, plus 1 means, receiving the item minus 1 means giving the item and for a_i and p_i positive means paying some amount and negative means receiving that that amount.

So, a_i equal to 1 if player i receives the item minus 1 if player i gives the item. So, minus 1 is possible only for a 0 and 0 otherwise. Similarly, p_i is the payment made by is the payment made by player i. So, if player i pays then it is positive if it receives money then it is negative then we have θ_i is the set of all valuations all possible valuations of player i of the item 2 player i ok.

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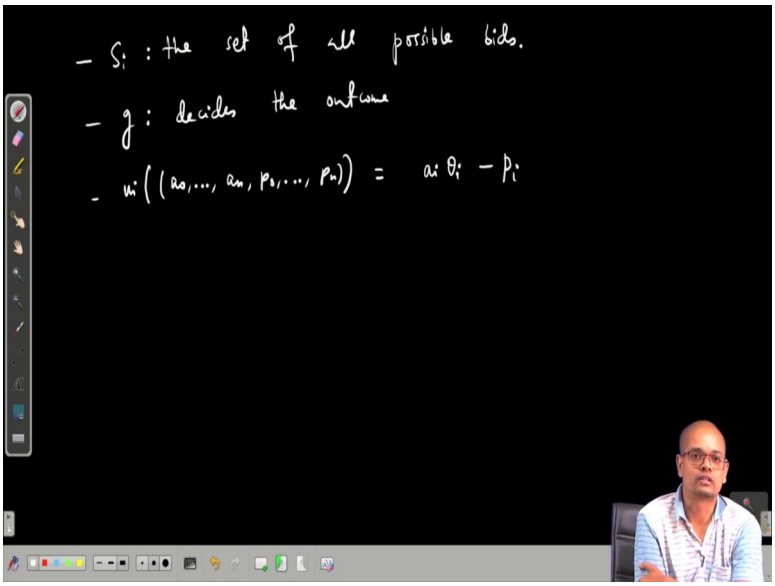

And the strategy set S i is the set of all possible bids function g decides the outcome. That means, whether the whether some seller is going to supply the item and the buyer is going to buy the item and what is the payment made by the buyer and which seller receives that these are all the outcomes and the utility $u_i(a_0,..,a_n,p_0,\ldots,p_n)=a_i\theta_i-p_i$

So, $\theta_i a_i$. If it loses then it is sort of giving the item if $a_i=-1$ then; that means, it is giving the items. So, it has given something which the player values θ_i and p_i is the amount of money that player i pays in the play pays to the auctioneer and the auctioneer may be some maybe give that item to the to the seller who in turn supply the item ok.

And so, you see that this mechanism this indirect mechanism and direct mechanism they are very closely resembles the Bayesian games and indeed there is a very deep connection and it is very similar. So, in the next class we will see what is the connection of mechanism design with Bayesian games and formalize the notions of implement ability what do we mean that this social choice function f is implementable by a direct or indirect mechanism because you know the recall each theta i is held by strategic players.

So, we need to incentivize the players to ensure that it is in the best interest of the players to play in such a way that the their strategy profile from their strategy profile we get to compute the function value f at any $(\theta_1,..,\theta_n)$. So, these things we will see in the next class ok.

Thank you.