**Algorithmic Game Theory**
**Prof. Palash Dey**
**Department of Computer Science and Engineering**
**Indian Institute of Technology, Kharagpur**

**Lecture - 03**
**Example of Games**

Welcome. So let us continue our Example of Games. We defined formally what is called normal form games and in last lecture we have seen many examples of normal form games. Let us continue in that line and see some more examples.

So, till last class all our examples are what is called two players game, the games involve two players, but in general games can involve many players.

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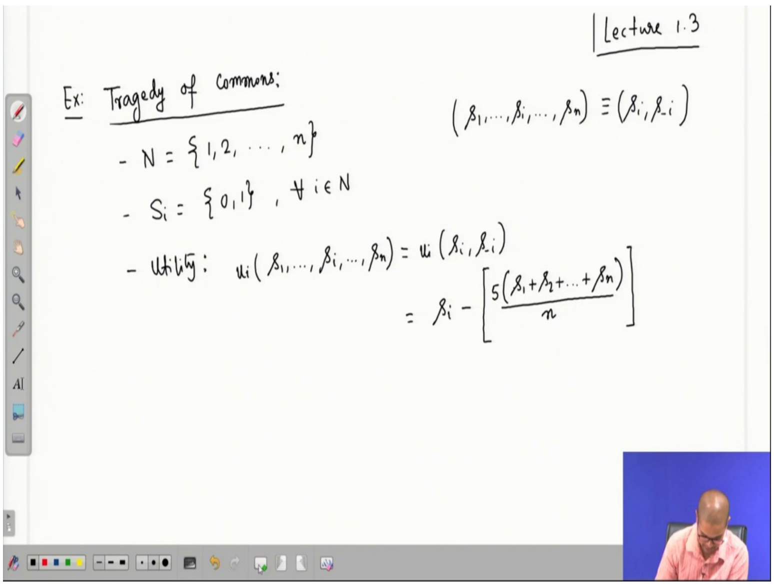


So, let us see. So, our first example is what is called tragedy of commons, this is an example.

So, what is the set of players N is suppose we have n players 1, 2. And strategy set $S_i$ for each player is 0 1. So, this is for all $i \in N$. So, what is the setting think of some action which has some benefit, but which has some harmful effects also and the benefit will be will be gained by only the player who is playing that particular action, but the harm will be shared by all of them that is why the name tragedy of common.

For example, setting up a setting up of a factory. So, if a company if a player sets up a factory then there are benefits and money will be generated and those utilities will be gained by the player who set up who sets up that factor. But there are there may be environmental harms which will be borne by all the players in the society everyone in the society, that is why the name tragedy of commons.

So, what is the utility? So, utility of player i, lets highlight player i strategy suppose this is $S_i$. Now there is a shorthand which we will use in this course and which is very common in game theory any book or any literature. When we focus on player i this we will often denote as $(s_i, s_{-i})$.

So, this is same as same as $u_i(s_i, s_{-i})$ this is just a notation. And suppose this is $s_i$ if player i plays 0 that means, for example, it does not set up a factory it does not get any utility. If it plays 1, suppose if it sets up a factory then it derives a utility of 1. But there is a total damage to the society and that is suppose.

So, what is the number of factory setup $(s_1 + s_2 + ... + s_n)$ the number of players this is the number of players who are setting up the factory and suppose the damage done to the society is by n times 5. Suppose then this much, so say suppose 5 times damage happens and this will be shared by all the players there are n players. So, this is the damage that needs to be that affects the utility of player i.

So, this is what is called tragedy of commons.

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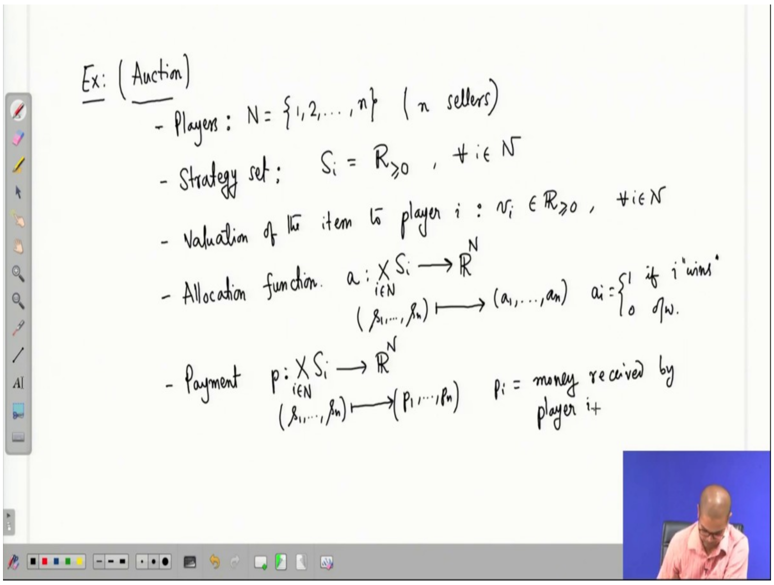


Our next example is another very famous and we will come to this in come to this example many more times in our course is what is called say auction. Any book on game theory devotes substantial time and effort discussing auctions.

So, suppose we are thinking of a buying auctions and the players. So, who are the players? Players is suppose $N=\{1,2,\ldots n\}$. Suppose these are the players and suppose these are the sellers, these are n sellers. Think of an organization who wants to buy something and there are n potential sellers and they are asked to bid for their bid for the item.

So, that is their strategy set. So, strategy set $S_i$, suppose $S_i$ is they are free to bid any non negative real number this is for all $i \in N$ ok. And now we want to now we will explain what is the utility to it to define the utility let us define few auxiliary terms. So, each item there is a intrinsic valuation of the item to each of the players. So, that and let us call that $v_i$.

So, valuation of the item to player i. Let us call it $v_i$ and suppose these are real number again non negative real number ok. And once all the players bid for the value then there is some something called allocation function, which decides who has owned the auction and who is going to sell the item. So, allocation function.

So, what is an allocation function? It is a function denoted by a it is a it maps from strategy profiles the tuple of all the strategies to the allocation vector $R^n$ is a set of all n tuples. So, it maps each strategy profile $(s_1, \ldots, s_n)$ to $(a_1, \ldots, a_n)$. And $a_i$ is 1, so what is the semantic? Semantic is $a_i$ is 1 if i wins the auction.

That means, it is going to sell the item and 0 otherwise. So, ideally if for deterministic auctions this will be only those vectors whose all components are 0 except 1 non-zero component and that will be 1. Then there is something called payment how much each player will get paid, again like that like allocation.

Let us define payment, from strategy profiles to this payment vector $(s_1, \ldots, s_n)$ goes to $(p_1, \ldots, p_n)$ and what is the semantic? Semantic is $p_i$ is the money received by player i ok. So, that typically typical auctions allocate the item to the highest to the lowest bidders whoever is ready to whichever seller is ready to sell the item at the lowest possible amount that seller is declared winner.

So, that is how the allocation function is typically defined and payment function is there are two popular payment functions there of course, can be many other.

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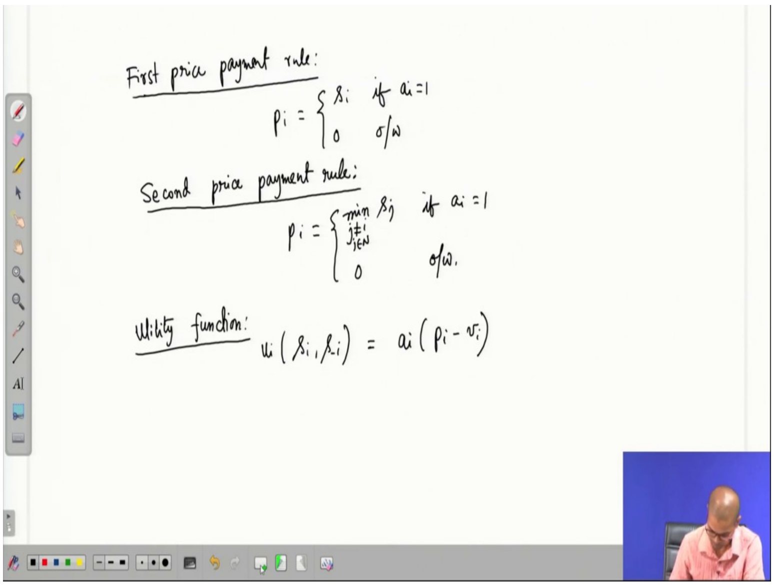


The first one is called first price auction, payment for first price payment rule first price payment rule. What is the first price payment rule? That winner gets paid its bid. So, $p_i = s_i$ if i wins; that means, if $a_i = 1$ and 0 otherwise losers does not receive anything.

There is another one is called second price auction, second price payment rule. $p_i$ is if it loses, so if it wins then only it will get paid and if it is it loses then it would not get paid anything, but if it wins how much it will get paid? It would not get paid its bid it will get paid the second minimum, so minimum over the rest of the bid.

Minimum over $s_j, j \neq i, j \in N$. That is why the name second price auction or second price payment rule it is the second one second lowest. Of course if there is a tie between lowest then the lowest and second lowest is same. And its so you may wonder, why you know why this crazy second price auction what is the rational behind it and that the answer you will get in coming weeks.

And now we can define utility function. Utility function of player i in this strategy profile $(s_i, s_{-i})$ what is $(s_i, s_{-i})$? It is nothing but the strategy profile $s_1, s_2, \ldots s_n$ we are just focusing on $s_i$ this is a nice convenient shorthand for this for that big strategy profile.

This is nothing but $a_i \times p_i - v_i$. So, the utility is amount of money received minus its valuation because the seller is giving the good and in return is getting paid $p_i$ amount of money. So, it is $p_i - v_i$ and this is the utility only if it wins; that means, that is why we are we have multiplied with $a_i$ and if it loses its utility is 0.

So, with this sort of variety of examples let us move on to our first question, that can we predict the outcome of the game? What will be the outcome of the game and what? Sorry the first question is each player what each player will play? What is the action from each player's point of view? What action he or she would play?

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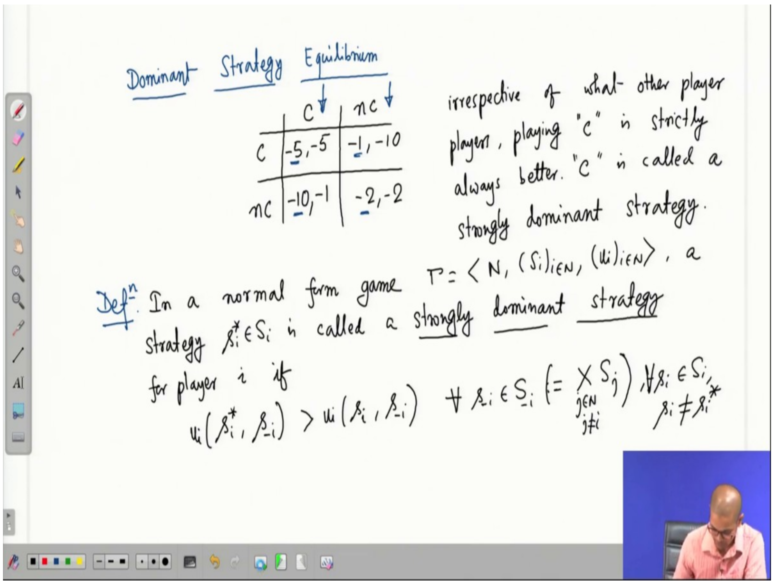


So, towards that we define what is called dominant strategy equilibrium let me introduce to you the concept of dominant strategy equilibrium, what is it? So, let us before defining it formally let us explain this with the example and the best example is, what is prisoners dilemma.

Let me explain what each player will play let us recall prisoners dilemma involved two players and both of them has the option of confessing or not confessing, confess not confess. If both of them confess both of them gets a jail term of 5 years if both of them does not confess both of them gets a jail term of 2 years.

If row player does not confess and column player confesses then row player gets a utility of minus 10 gets a jail term of 10 years and column player gets a utility of minus 1 and symmetric situation happens in the other case ok. Now you see let us analyze this game and say from row players perspective, what the row player will play? Let us argue.

So, suppose the column player plays C, suppose the column player plays confesses. Then the row player has two options either it can confess and in that case it will gets a utility of 5 and or it cannot it may not confess in that case it is its utilities minus 10.

Recall the assumption of selfishness or rationality tells us that player will play which maximizes its utility. So, if by some reason row player knows that the column player is going to play C then playing C is the best option best strategy for row player. Now let us see if what will happen if column player plays NC. If column player plays NC row player again have has two options either playing C or playing NC.

And row player again sees that playing C is better it is strictly better for row player because it will give a utility of minus 1 against a utility of minus 2. So, what row player observes is a crucial observation is irrespective of what other players same situation you know holds true for column players also. So, irrespective of what other player plays playing C is strictly better strictly always better.

Such in such a case, that particular strategy is called a strongly dominant strategy. So, C is called a strongly dominant strategy ok. So, what is a strongly dominant strategy a strategy is a particular strategy is called a strongly dominant strategy, if irrespective of what other player plays play that strategy always give strictly more utility. Then that particular strategy is called a strongly dominant strategy.

So, let us formally define it definition. In a normal form game $\Gamma = \langle N, (S_i)_{i \in N}, (u_i)_{i \in N} \rangle$, a strategy. Let us call it $s_i^*$ in the strategy profile of player i a strategy $s_i^*$ is called a strongly is called a strongly dominant strategy for player i, if the following holds. If what holds the utility of player i when it plays $s_i^*$ when other players is playing any other strategy $s_{-i}$ this is strictly more than $u_i$ when player when the other players continues to play their other strategies and player i deviates from $s_i^*$ to $s_i$.

So, then it is called a strongly dominant strategy it should hold for all $s_{-i} \in S_{-i}$. What is $S_{-i}$? This is by definition $\times_{j \in N, j \neq i} S_j$ this is $S_{-i}$. And this small $s_i$ varies this is for all small s i in S i of course, because it is strict equal inequality $s_i \neq s_i^*$ .

So, this is called a strongly dominant strategy.

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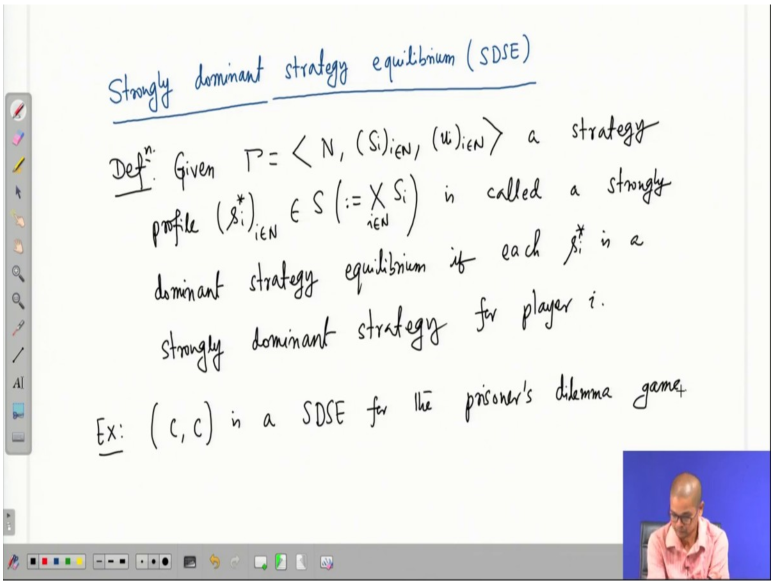


Now, from this we define what is what we mean by a strongly dominant strategy equilibrium. We write strongly dominant strategy equilibrium SDSE in short.

So, what is the definition? So, again given a normal form game $\Gamma = \langle N, (S_i)_{i\in N}, (u_i)_{i\in N}\rangle$, a strategy profile $(s_i^*)_{i\in N} \in S$ . So, what is S? This is another notation which we will use. So, S is the set of all strategy profiles $\times_{j\in N} S_j$. So, strategy profile is called a strongly dominant strategy equilibrium. If each $s_i^*$ is a strongly dominant strategy for player i, if each $s_i^*$ is a strongly dominant strategy for player i ok.

So, and what is an example that both player confessing $(C, C)$ is a strongly dominant strategy equilibrium for the prisoners dilemma game ok. So, let us stop here we will continue from here next time.