Introduction to Soft Computing
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Lecture – 05
Fuzzy relations

So, in the last lecture we have learnt about different operations on fuzzy set. So, different operation on fuzzy set that we have learnt is basically given A 2 fuzzy set or A fuzzy set how another fuzzy set can be obtained. Now we are going to learn another concept in fuzzy logic it is called the fuzzy relation. So, by means of fuzzy relation we say that if 1 element belongs to A fuzzy set then how this element is related to another fuzzy set. So, this basically the relation; between the 2 elements, which belongs to the 2 different fuzzy set.

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So, fuzzy relations in many way related to the crisp relation crisp relations means the relation those are there on the crisp set. So, will first learn about the crisp relation and then whatever the operations those are possible on crisp relation is basically semi applicable to the fuzzy relations also , but there are certain difference. So, it will be better if will learn first operations on crisp relation and then the operation that can be applied to the fuzzy relation some examples also will be considered in word to understand the crisp relation and then we will be in A position to know fuzzy relation, some examples operations those are possible on fuzzy relation and finally, we clear our idea with some examples.

So, these are the topics that we are going to cover in this lecture.

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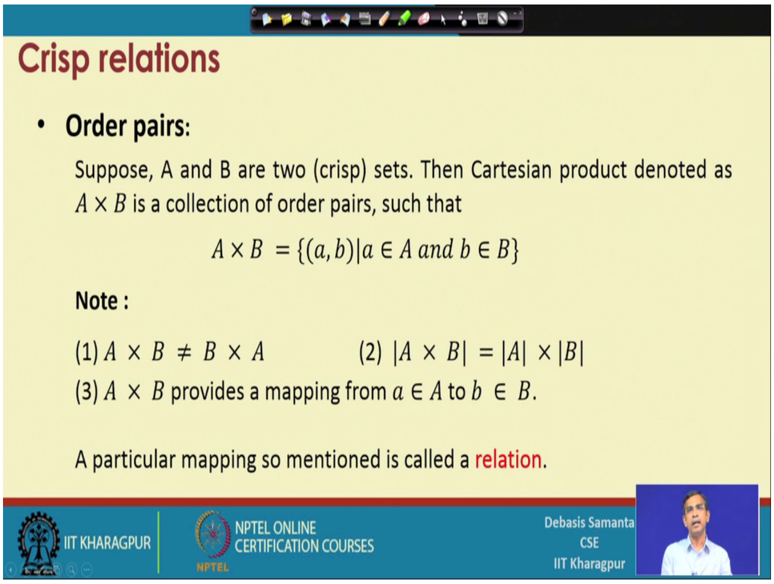

So, let us first discuss about the crisp relation now crisp relation basically is an collection of order pair. So, if A and B are the 2 sets then it is order pair is denoted by the Cartesian product A cross B and it basically gives the collection of order pair a b such that A belongs to the set A and B belongs to the set B .

So, this order pair relation is important to understand the crisp relation. Now A particular mapping is basically belongs to A particular relation and you know. So, for the crisp relation is concerned these are the different property holds good the first property is that A cross B is not equals to B cross A; that means, they are not commutative and here is basically the number of elements which is belong to the product is same as the number of elements belongs to the constituent set A and the product of the number of elements belongs to the set B.

So, this also the equation hold goods and as I told you A cross B essentially provides A mapping the from A set from an element belongs to set A to another element B belongs to set B. So, it is basically A mapping and this mapping is expressed by means of an order pair and this particular mapping is called A relation.

Now, we can understand this relation better if we consider an example.

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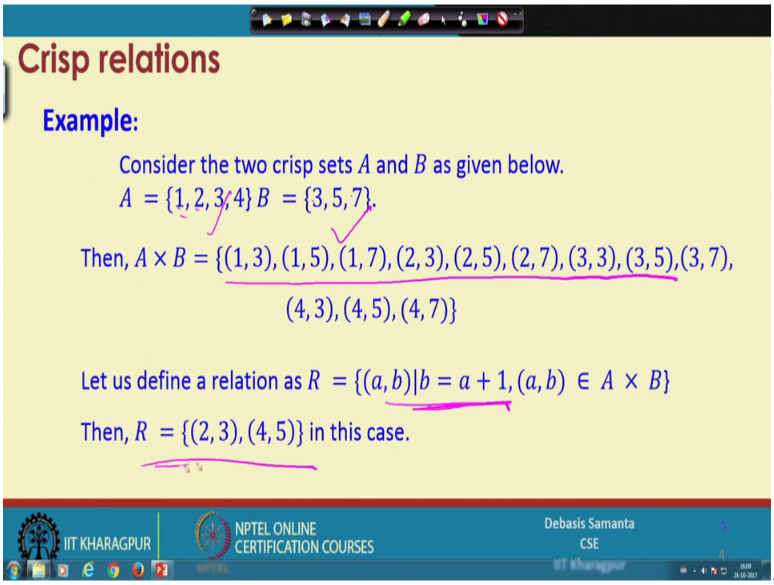

So, example suppose 2 crisp set A and B and A is represented by this form and B is 3, 5, 7. So, these are the 2 sets A and B and we can obtain they are Cartesian product Cartesian product we can obtain for this 2 phrase is all possible order pairs. So, it is shown here this is the Cartesian product of that 2 fuzzy set A and B. So, here for given 1, 2, 3, then 1 5 1 7 then 2 3 2 4 5 to 7 3 5 3 3 3 5 3 7 and so on.

So, these are the different what is called the elements which belongs to the Cartesian product of the 2 fuzzy sets A and A and B and then the relation, I told you the relation is basically A particular mapping now here we express the relation and this is A this is this is suppose the relation that we have discussed here.

So, relation between the 2 order elements in A order pair should satisfy the these equation, if it is satisfy then it gives A particular set or it is basically A relation which is shown here for example, if this is A relation hold good for every element then the relation that can be obtained is basically this one.

So, A relation is basically A collection of order pairs which satisfy A particular mapping or A particular definition.

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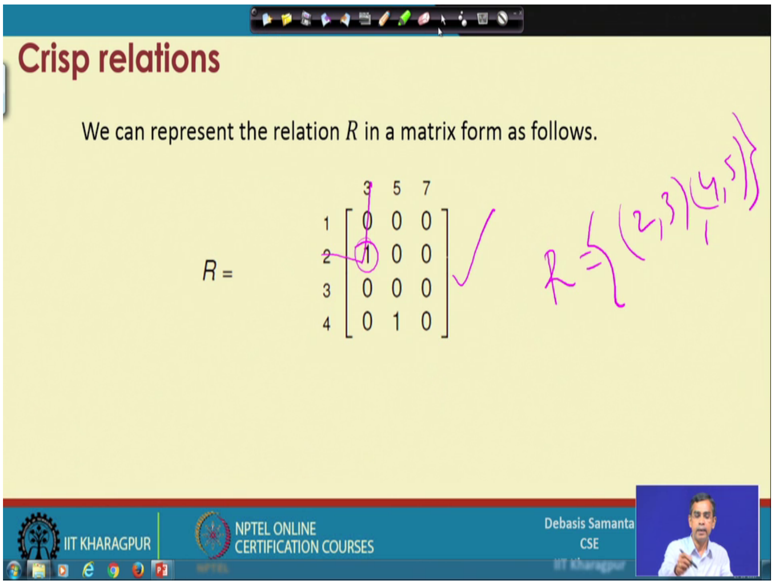

Now, so this is the crisp relation and such a relation can be expressed in A more compact way and this compact way it is called the matrix representation of A relation, now the matrix representation of the relation which we have learned earlier. So, R is 2 3 and 4 5 is denoted by this matrix you can see 1 and 3 which is not belong to the set. So, it is 0 and the elements, 2 and 3 belongs to the set. So, it is 1. So, here 0 and 1 are the entries in the relation matrix 0 indicates that that order pair is not belongs to this relation and 1 indicates that that order pair belongs to the relation.

So, this way A relation can be represented by means of A relation matrix.

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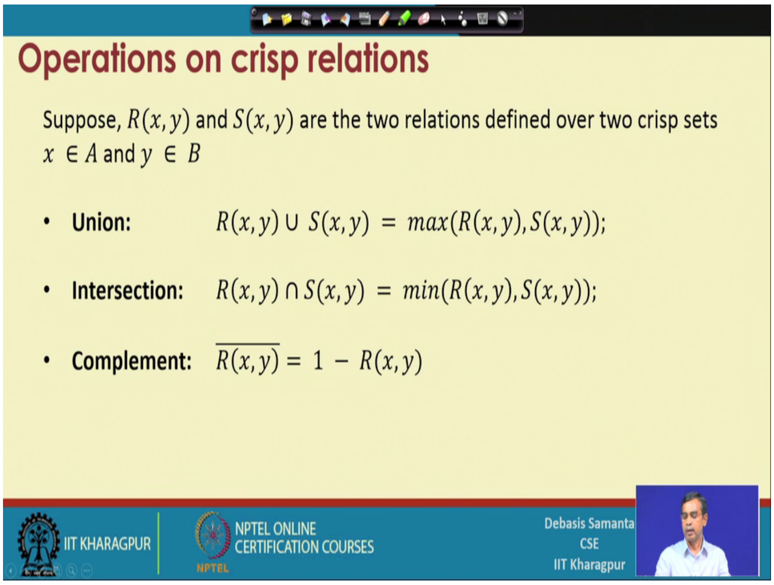

Now, so operation on crisp relation so these are the relations if it is available to us we can apply different operations on it, say suppose R and S are the 2 operations defined over x and y where x is the somes some elements belongs to the universe of discourse x and y is the element belong to the universe of discourse y. So, R x y can be obtained as A relation matrix similarly S x y can be obtained by means of an another relation matrix.

So, if the 2 relation matrix, are available to us then we can apply many operations on them. So, the operation on them that can be a applied can be applicable is union intersection complement like this. Now you can find the difference between the union of 2 fuzzy sets and union of 2 fuzzy relations. So, union operation of 2 fuzzy set is expressed by this form. So, R is the relation and B is the S is the another relation right and this relation obtained over the 2 crisp set say A and B then the union of the 2 relations can be defined as A max of the 2 entries in x and y in both relation.

So, 1 example can be given here. So, this is the union operation likewise the intersection operation is basically minimum values of the entries and the complement is 1 minus the entries in each elements.

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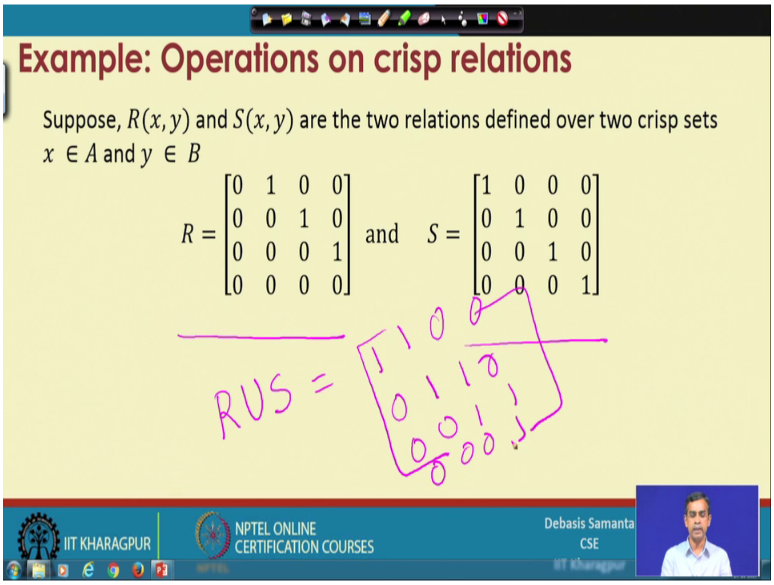

Now, 1 example that can be consider here suppose this is the 1 relation that can be obtained over A and B and this is the another relation obtained over the sets A and B. So, it is basically A cross B with some relation it is also A cross B with different relation and then we want to obtain union of the 2. So, union of the 2 we can write R union S. So, it is basically 1 matrix now union operation as I told you it is A max so 0 and 1. So, you have to take the 1 1 0 it is A 1 then 0 0 for the first rows similarly 0 1 1 0 0 0 1 1 and 0 0 0 1.

So, this is the another relation matrix that can be obtained using the operation relation operation of the 2 relations R and S. So, this way we can obtain the relation.

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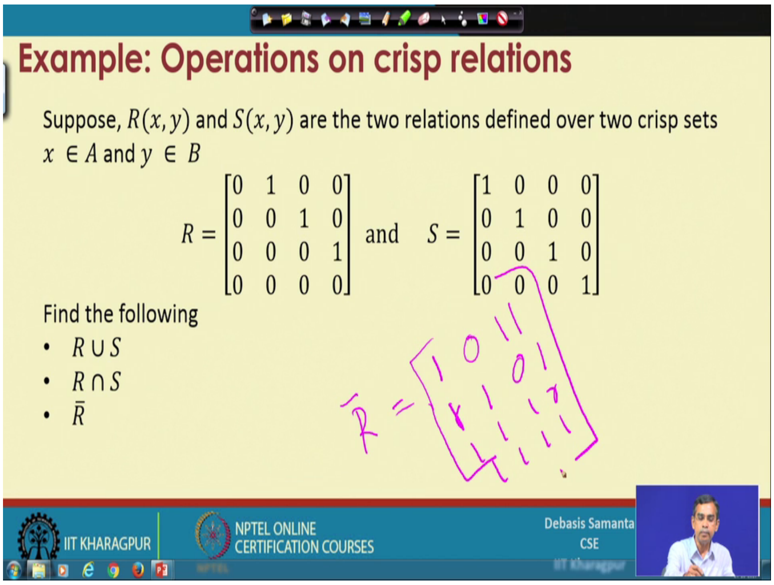

Now, using the same concept you can find easily the union of 2 intersections of 2 and then complement. Now, so far the complement operation is concerned R complement this is equals to it is basically complement value. So, if the 0 then it will be 1 and it is 1 then 0 so you can complement of these is this this this and then 1 1 0 1 1 1 1 0 and 1 1 1 1. So, this is the complement of the relation R.

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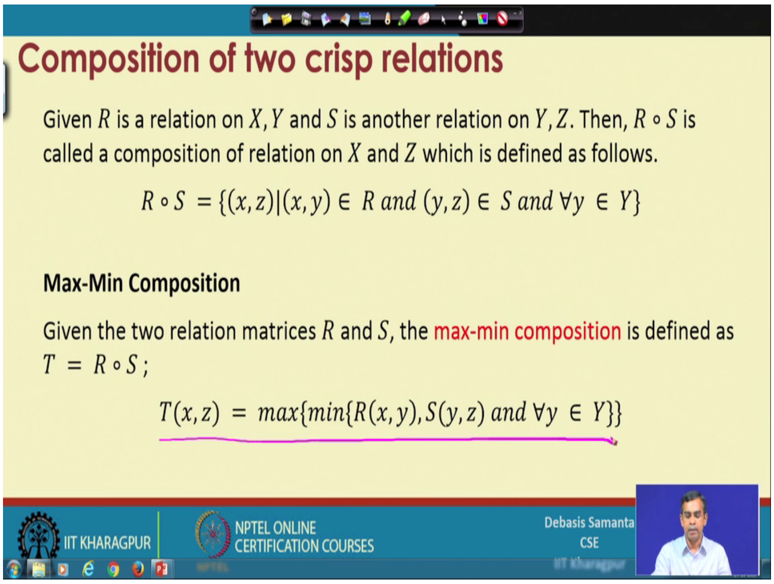

Now, there is another important relation it is called the composition and composition relation is why they applicable in the context of fuzzy relation. So, composition operation it is denoted by this\\is symbol R composition S; that means, from 2 relation we can find another relation, but. So, these R relation suppose over 2 set A and B and S relation is over another say A and C. So, here C is the 1 common set then we can obtain R composition S basically relation from A to B via C. So, this is the concept that is called the composition.

Now, composition operation can defined mathematically using max min calculation the max min it is called the max min composition this is y and is denoted by these expression, max min composition is basically this 1 now it is little bit difficult to understand at the movement. So, I can give an example. So, that you can understand it basically follows the similar concept of product of 2 matrix actually. So, it basically take the first minimum of corresponding entries and then take for A particular entries the maximum value.

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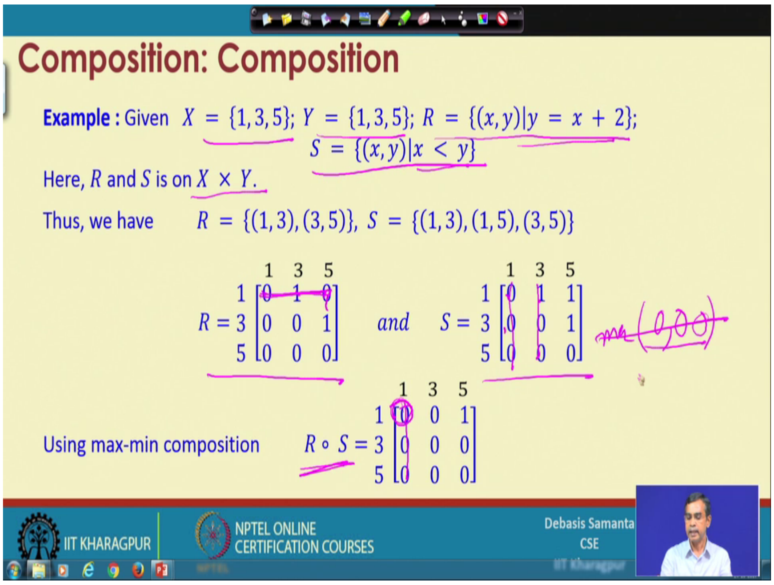

So, let us have some example. So, that we can understand the max min composition or simply A composition operation to relation now this is A 1 example we have to consider carefully. So, suppose x is the 1 universe of discourse y is another and the relation R xy defined over x and y and S is also another relation defined over the same discourse x and y, the relation that is there for A R it is basically this is the relation and this is the relation that is there in S.

Now, based on this thing we can easily obtained the Cartesian product and then applying this relation we can obtain these matrix and these matrix. So, these are the 2 relations obtained from the 2 fuzzy set 2 crisp set x and y now having this relation we can find the composition of the 2. Now composition basically we take first row wise and then column wise just like A product to obtain the first element here sorry. So, this is the row and this is the 1 to obtain the first element here.

So, is basically take like. So, 0 and 0 take the minimum so minimum is 0. So, 0 is A minimum then 1 and 1 as 1 and 0 then it is A minimum is 0 then 0 and 0. So, minimum is 0 and then take the max max of this so this is 0. So, this is A 0 for the next element we can obtain so these and then these one. So, 0 and 1 so further next further next these 1 and then these one. So, you can get this element. So, this and this we can go so; that means, 0 and 1 take the minimum it is 0.

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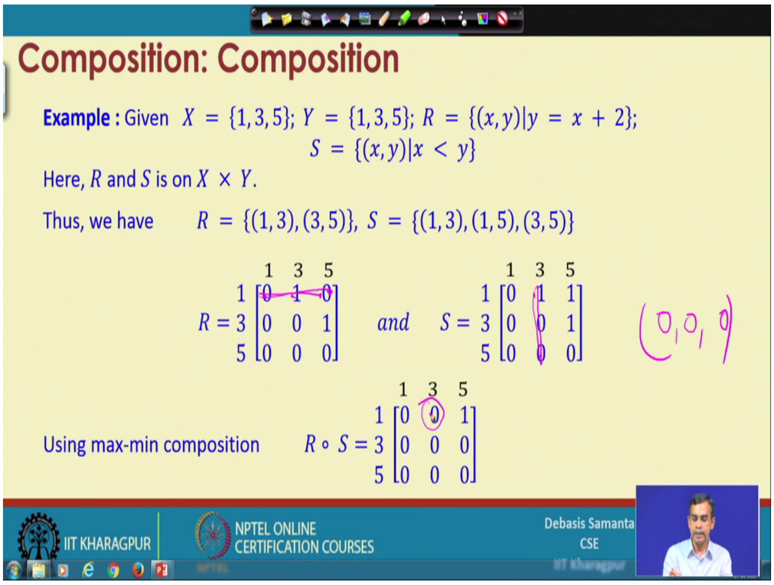

And then 1 and 0 take the minimum it is 0 and then 0 and 0 take the minimum this 1 and then maximum. So, it is 0 1.

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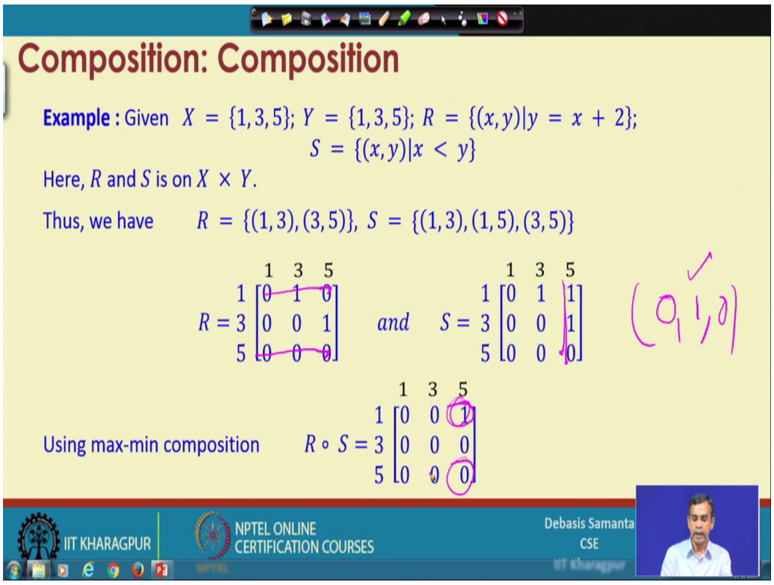

Now, let us see how this element can be obtained. So, again we have to take this 1 and then this one. So, first 0 and 1 0 then 1 and 1 1 and 0 and 0 it is 0 and taking the maximum this one. So, it get this one. So, these element, again we can be obtained if we take these 1 and the this last element can be obtained. So, this way we can obtain the relation called the max min composition of the 2 relations R and S.

So, it needs A little bit practice to understand it. So, it will basically take the mean corresponding to this traversing and then taking the max of all these will give A particular element. So, this is the idea and now let us see, how these operations though they are applicable to the crisp is also applicable to the fuzzy, but in A different one.

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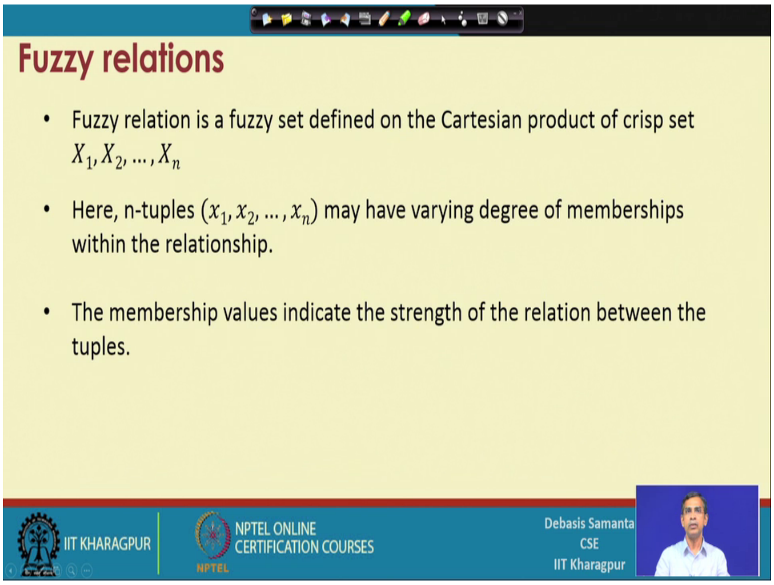

So, difference between the fuzzy relation and crisp relation lies in the terms of increase in the relation matrix in case of crisp relation the entries in the matrix is either 0 or 1 whereas, in case of fuzzy relation the entries in the matrix is any value in between 0 and 1 both inclusive.

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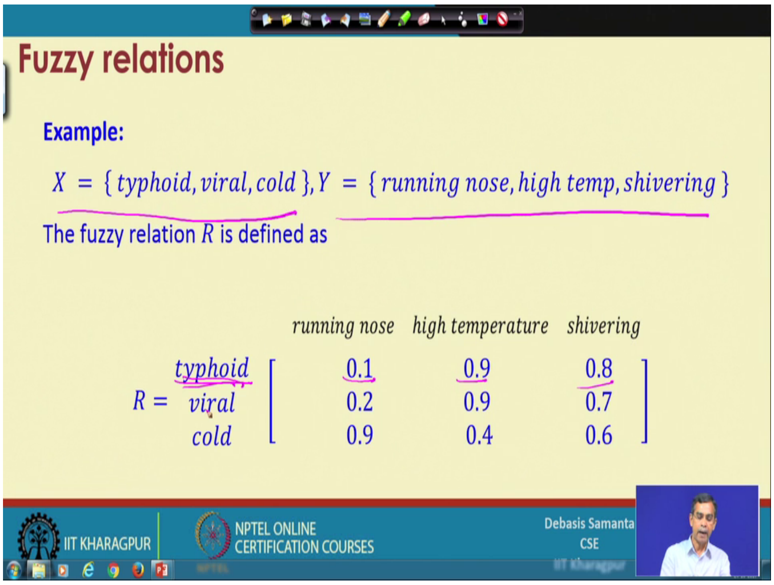

Now, let us start with an example about the fuzzy relation and suppose 2 fuzzy sets which is described over the 3 elements which is here 1 fuzzy set is X and another fuzzy set is Y the different elements in the fuzzy sets are here in X is typhoid viral cold in fuzzy set Y, the elements are they are running nose high temperature and shivering now you can understand what is the meaning of these to fuzzy sets and the relation then.

So, basically here if the disease is typhoid then what are the difference symptoms are there. So, if the disease is typhoid the symptom that running nose is, but with the strain 0.1 high temperature 0.9 and shivering 0.8.

So, every disease and the different symptoms with the different membership values is represented and by means of A relation matrix. So, if it is A viral then what about the shivering the shivering is 0.7, if it is A viral then running nose, but running nose with little bit less uncertainty than the shivering that is 0.2 and 0.7 . So, the relation matrix basically shows the different element which are belongs to the different sets how they are related to each other. So, this basically the physical significant of the fuzzy relations and now one thing it is clear that the element in the fuzzy relations; that means, the entries in the relation matrix is basically any value in between 0 and 1 both inclusive.

So, this is the only difference between the fuzzy relation and the crisp relation, otherwise every operation though we have defined in case of crisp also equally applicable to the fuzzy sets now let us see what are the different operations they are possible for the fuzzy relations.

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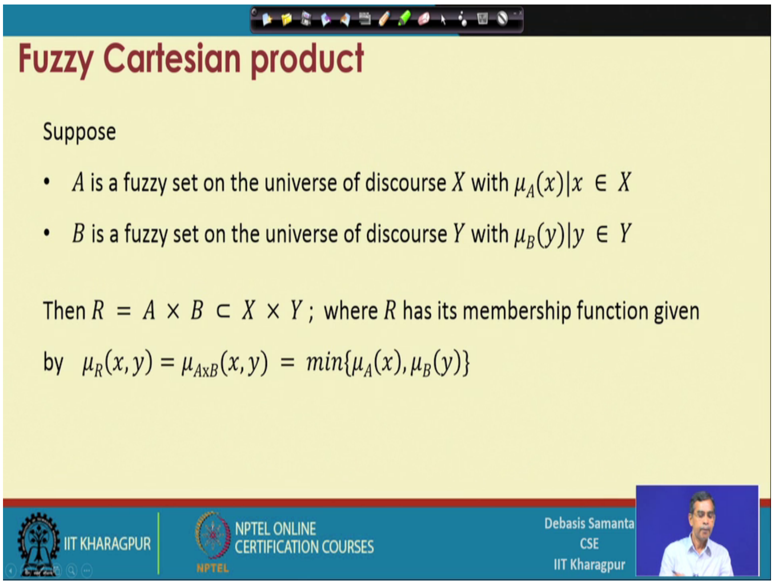

So, here so the fuzzy relation again defined as in the crisp relation like the min operation. So, it is A min actually. So, this is the min operation.

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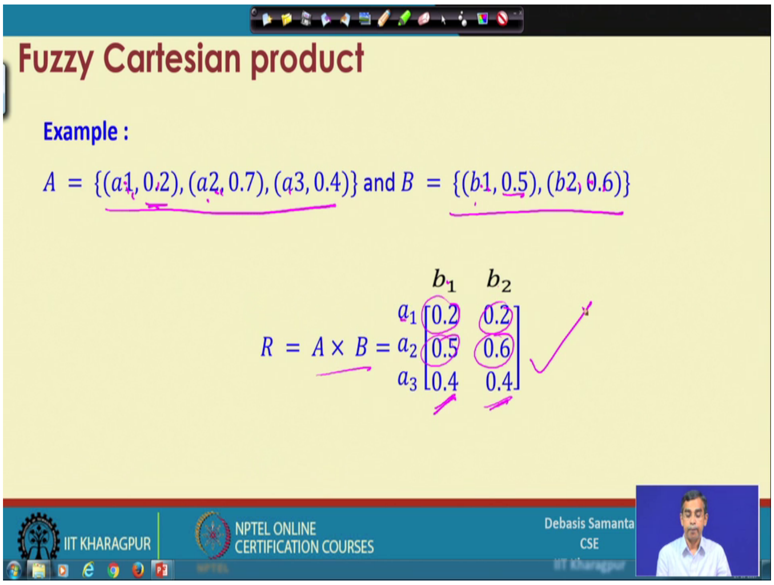

Let us see 1 example. So, here say A and B are the 2 fuzzy set A and B are the 2 fuzzy sets now I can find A relation the relation can be obtained as we have discussed here, now relation operation that can be defined over 2 fuzzy sets it is basically represented by this expression that nu R x y where is A membership values belong to relation and it is denoted a A cross B as I told you Cartesian product of x y and it basically takes the minimum of 2 corresponding values in both the set A and B for x and y respectively. So, it is basically taking the minimum.

Let us have an example. So, we can clear our idea this is an example can be followed to explain the relation operation for the fuzzy set in terms of Cartesian product A, now here A is the set which is defined like this B is another set which is defined like this and then the relation are is basically Cartesian product as I told you. So, it basically for a 1 and b 1 a 1 and b 2 so a 1 b 1 a 1 b 2 now for a 1 b 1 we have to take the minimum. So, 0.2 and 0.5 take the minimum. So, it is A minimum entries similar a 1 and b 2 0.2 and 0.6 take the minimum so 0.2.

Likewise a 2 b 1 so minimum is 0.5 a 2 b 2 the minimum is 0.6 a 2. So, this is a 2 now a 3 and b 1 so 0.4 and a 3 b 2 0.4. So, these a we can obtain the relation metrics taking the min operation that is there. So, this way we can obtain the relation if the 2 fuzzy sets are given to us.

Now, let us define different operations on fuzzy relation like the different operation in crisp relation. So, like union intersection and then complement these are the operation.

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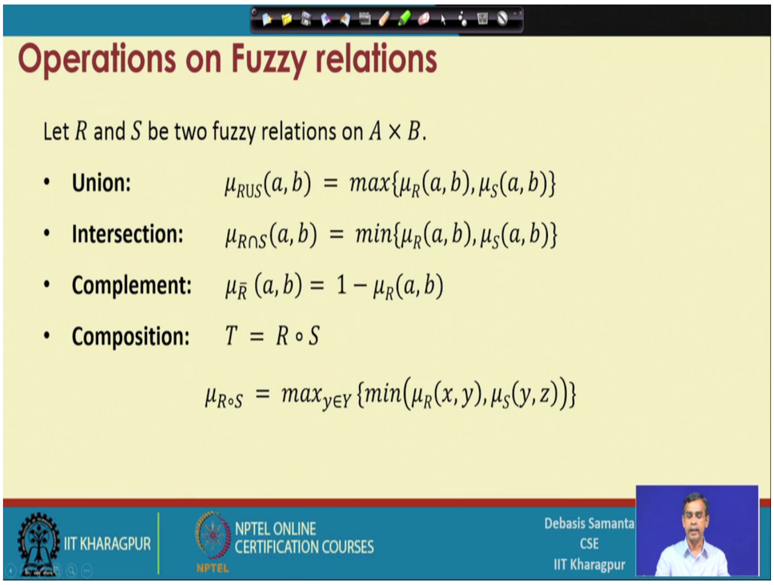

So, union operation on 2 fuzzy sets can be defined using this expression it is basically taking the maximum value of the 2 entries there. So, it will give the new matrix taking the maximum of the corresponding entries intersection basically taking the minimum of the 2 entries and complement it is a unary for 1 operation. So, you take the if it is nu R b then the value or entries in the relation matrix will be the complement; that means, 1 minus nu R a b .

Now, some example can be followed to understand this concept another composition I will discuss the composition operation in details in regards the fuzzy sets it is basically same as max min composition.

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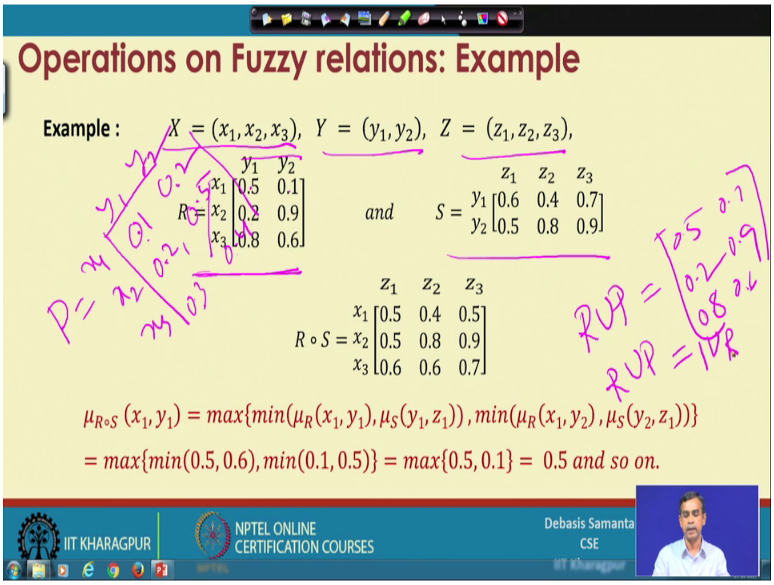

Taking the similar concept now better we if we have 1 example. So, x is A crisp set y is another and Z is another. So, we can consider they are the universe of this course of for the fuzzy sets may be and so R this is the 1 relation defined over 2 sets which is discussed over the universe of discourse x and y. So, this is the relation over the 2 fuzzy sets and A S is the another relation which is defined over y and z.

So, these entries are given to you now if it is given to you then we can calculate R union S easily now again here we can say R union S here basically elements those are not same that is why we cannot apply R union S, but the 2 I mean union operation is applicable if the 2 relations are defined over the same elements. So, if this relations is defined over this one this one then another relation should be defined this one then only we can for example, suppose I defined another relation P and x 1 x 2 and x 3 and y 1 and y 2 and say 0.1 0.2 0.2 and 0.5 0.3 and 0.4. So, this is A relation.

Now, if we want to find the union of the 2 relation. So, R P then relation basically taking the maxima of the corresponding entry so 0.1 and 0.5 so in the first entry 0.5 and then 0.2 and 0.1 so you should take 0.2 0.2 and 0.2. So, 0.2 and 0.5 and 0.9 0.9 then 0.3 and 0.8 0.8 then 0.4 and 0.6 it is 0.6.

So, this is basically the relation obtained over the union operation of the 2 relation R and P and you can note that R P this is equals to same as P R regardiments they holds the commutative property. Now likewise the intersection intersection we have to take in. So, it is basically take the minimum of this on union where is the maximum and union intersection is the minimum.

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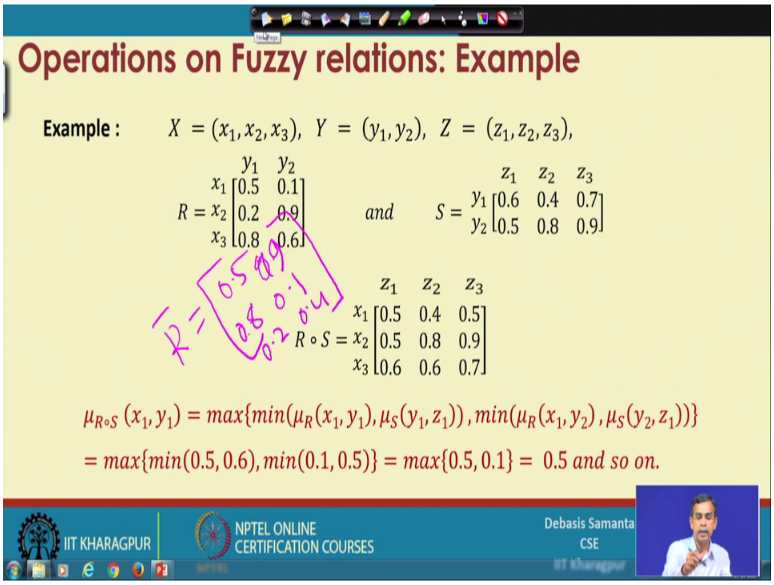

And then complement also can be obtained for example, so, complement operation this R bar that can be obtained like. So, 0.5 then 0.9 then 0.8 then 0.1 0.2 and 0.4 I hope you have understood how it is obtain it is basically taking the complement 1 minus 0.5 1 minus 0.1 and this way.

So, this way the complement operation over A relation R can be obtained now this is another example that I am going to discuss is A composition. So, R and S are the 2 relation.

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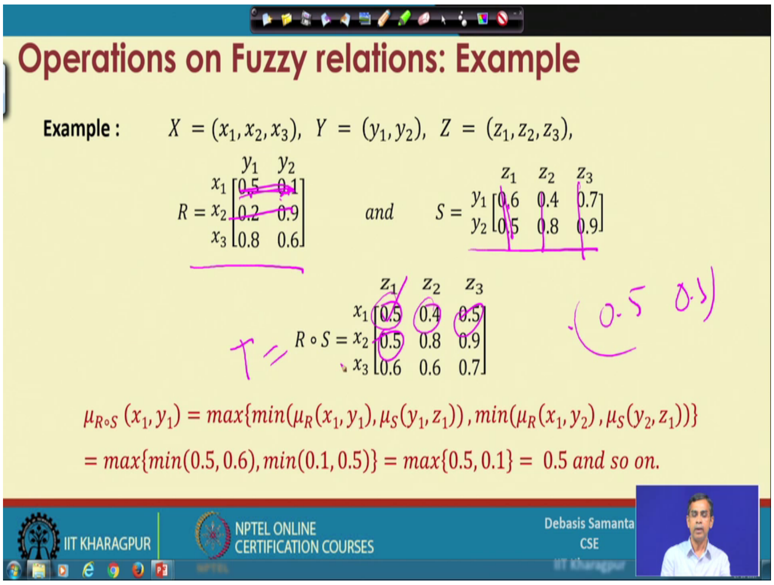

And we want to find another relation T which is A composition of R and S. So, using the max min composition it is the composition that we have discussed about the Cartesian product finding. So, take the minimum first and then minimum of all take the maximum again we can follow it is these 1 and these traverse. So, it will give this 1; that means 0.5 and 0.6 we have to take the minimum. So, it is 0.5 and 0.1 and 0.5 we have to take the minimum 0.1 and taking the maximum or this. So, 0.5 so this so 0.5 and likewise if we traverse this 1 and this 1 the these element can be obtained these 1 and these 1 then this can be obtained these then this element can be obtained and so on.

So, this is the max min composition operation that can be applied on 2 relation R and B.

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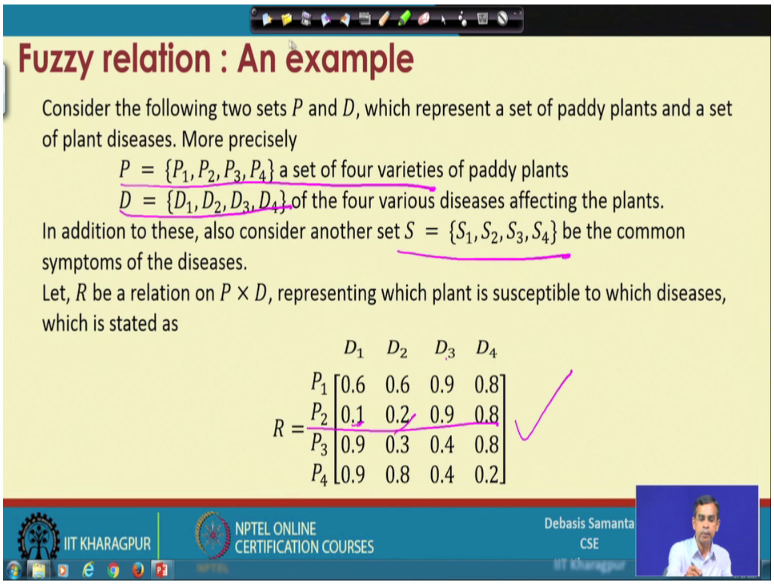

So, this is the relation operation and I can give another last example in this direction here these examples is very interesting to note. So, P is the set with different element P 1 P 2 P 3 P 4 D is the another set with different element say D 1 D 2 D 3 D 4 now in the context of our real application. So, P basically consider A set of varieties of paddy plant; that means, P 1 is A 1 type of paddy plant P 2 is another and so on.

Now, D the set D represent the different diseases where D 1 is A type of disease D 2 is another and so on and say S is the another set with basically set of symptoms the symptoms are S 1 S 2 S 3 S 4. Now how A particular plant is related to the disease that can be given by means of A relation. So, this is basically A fuzzy relation showing that how A particular plant is related to the different disease that may have for example, P 2 is A plant and is susceptible to disease D 1 with the 0.1 certainty D 2 is 0.2 D 3 0.9 and D 4.

So, we can say that P 2 is very much susceptible to the disease d 3 or d 4 and less susceptible disease D 1 D 2 it is the concept. So, these are meaning of the fuzzy relation now having this fuzzy relation are we can obtain another relation by means of composition operation.

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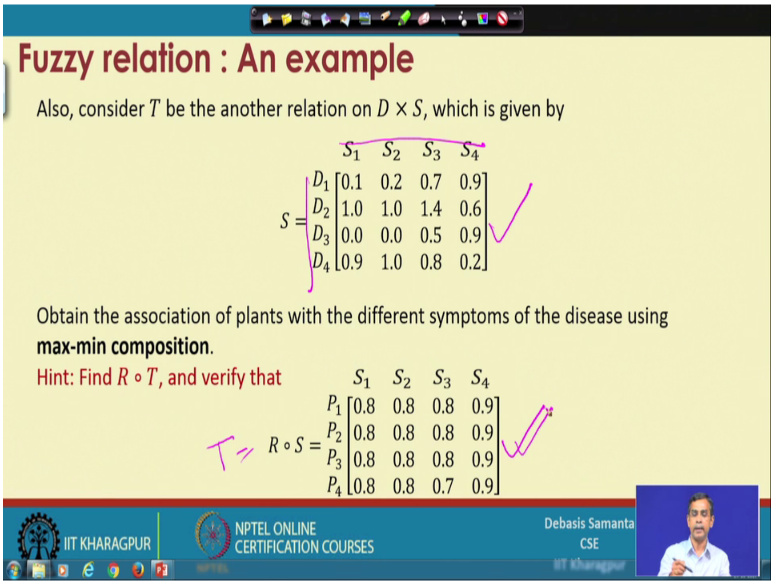

So, S also another relation it basically showing the relation about disease and the symptoms so here different disease are there and different symptoms and this is the metrics showing how particular disease and related to the different symptoms for that, now having this 1 then we can have the composition operation. So, T is A set resultant which is basically R composition S. So, R composition S can be obtain previously we have discuss about R and this is the S and taking the max min composition then you can try and you can check that this is the relation that can be obtained.

So, this relation has the meaning this meaning is that if the R relation shows that which paddy plant is susceptible to which disease and if S denotes the particular disease and what are the symptoms, then R S basically shows A particular plant then what are the symptoms that it basically corresponding to some disease. So, this is the relation showing A plant and then symptoms that they may be affected. So, this is the 1 example and we hope I hope you have understood the concept of relation.

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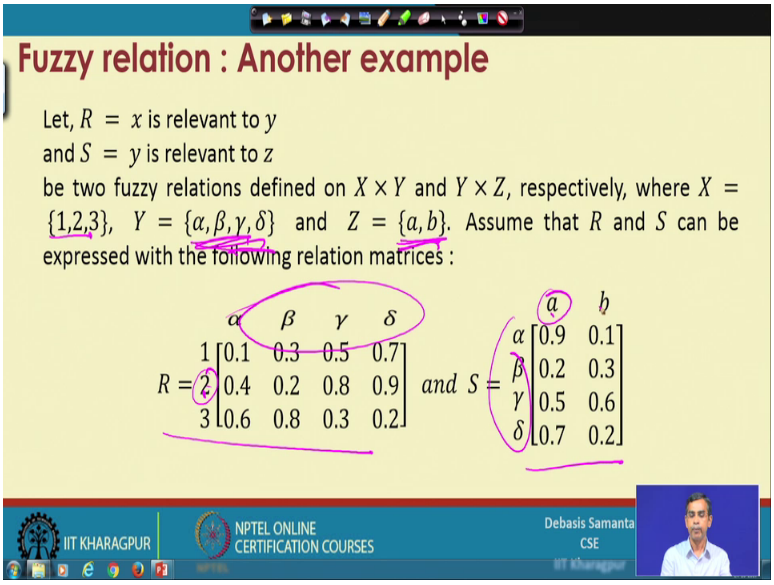

And this is the another example that is the I can give it very quickly. So, there you can understand if R is another relation showing the relation from these sets to these sets sorry this is the R relation is this sets to this set and another relation is showing the relation from these element to this element, if given this 1 then I can find A relation from any 2 anyone. So, bias this one. So, alpha beta gamma basically in between the 2 and then we can find the relation to 2 a or relation 2 to b. So, that can be obtained by means of Cartesian product and then relation composition operation rather not Cartesian product.

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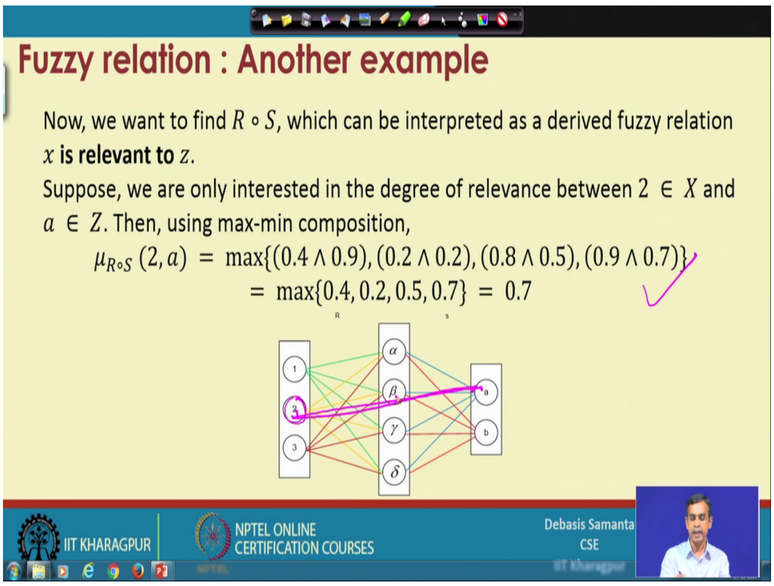

So, it can be like this 1 for example, 2 yeah 2 is the element which is belongs to set and these are the different and a. So, you can find the relation from 2 2 a by means of max min composition or the max min composition is can be calculated which is shown here.

So, it basically here for example, 2 and a there is a relation via other elements alpha beta gamma. So, the relation that 2 is related to A which strength 7 0.7 similarly we can calculate likewise the 2 and a relation between 2 and a we can calculate relation between 1 and b or 1 and a with some what is called the strength. So, this basically shows the relation and it is the meaning it is there. Now, so I think time is over. So, we can stop it here the these portion can be discussed in the next lectures.

Thank you.