Introduction to Soft Computing
Prof. Debasis Samanta
Department of Computer Science & Engineering
Indian Institute of Technology, Kharagpur

Lecture - 13
Fuzzy logic controller (Contd.)

So, there are 2 broad approaches, so far the fuzzy logic controller design is concerned. One approach is Mamdani approach and another is Takagi Sugeno approach. 2 approaches are different, the way they treated the fuzzy logic controller design. We have discussed the Mamdani approach and we see that the Mamdani approach follow a rule base and then fuzzy linguistic state and then Fuzzification of the input and then produce the fuzzy output and then crisp value. The method more or less same, they are in the Takagi Sugeno approach, but the though the way they treat the fuzzy inference engine is different.

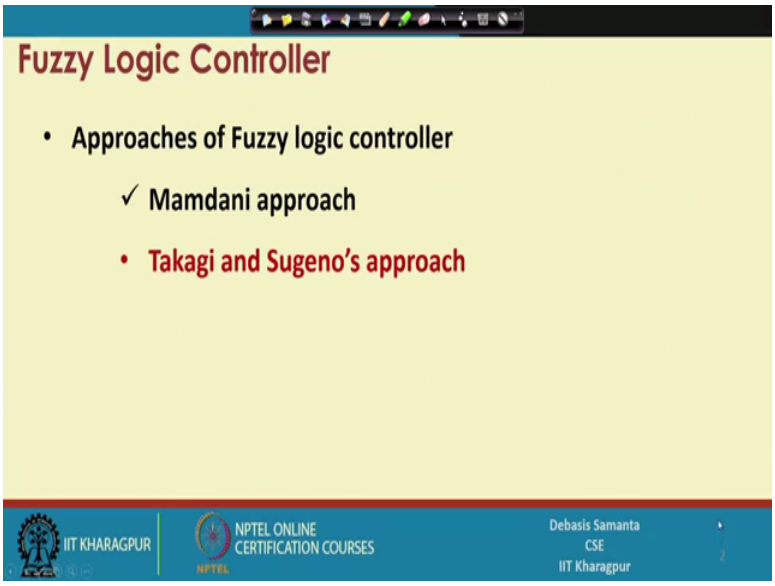
Now, fuzzy inference engine rather more what is called the interpretable so far the Mamdani approach is concerned. However, in case of Takagi Sugeno approach, it is less interpretable. So, interpretation means, anybody can see from the design how it works, but Takagi Sugeno approach is bit difficult because it follows certain mathematical treatment so far the inference engine is concerned.

Now, so far the output quality is concerned, the Mamdani approach is less accurate, whereas Takagi Sugeno is give more accurate output and output calculation if we consider according the to the 2 different approaches, Mamdani approach follows the standard Fuzzification method. So, Fuzzification method if we follow COG, it is computationally expensive, whereas the Takagi Sugeno approach follow the simple numerical calculation and it is faster. So, we can broadly can say that Mamdani approach is easy to interpret, but less accurate and computationally bit expensive. On the other hand, Takagi Sugeno approach is difficult to interpret, accurate; more accurate than the Mamdani approach and then calculation is fast.

So, if you want to design a fast and accurate fuzzy logic controller, then we can follow Takagi Sugeno approach, but there is one issue, Takagi Sugeno approach as it needs some mathematical treatment. So, whatever the rules are there they needs to be stored in the form of a, some mathematical representation and that is a big challenge for the designer. So, if the designer is not so much experienced, then they can follow certain difficulty in this direction, whereas Mamdani approach is very easy to frame the rules and then rule base and then inference engine.

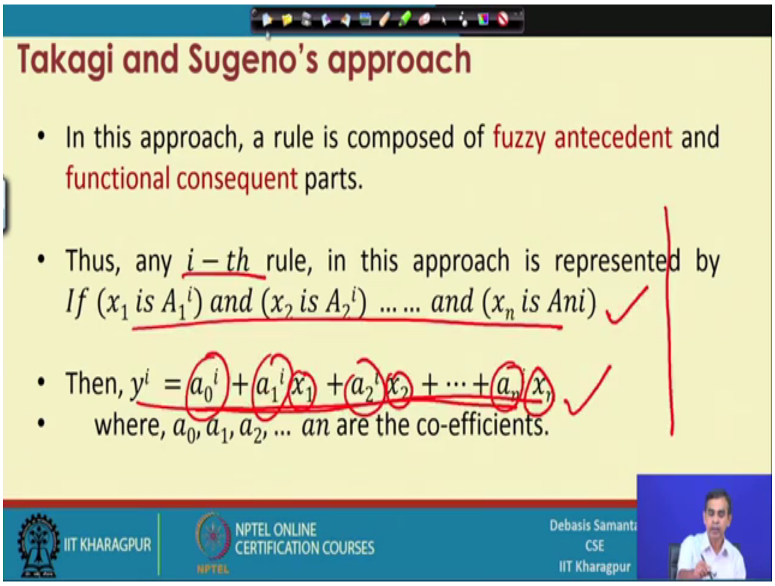
So, these are the 2 difference are there. Obviously, there is a trade off. Now, let us see how the Takagi Sugeno approach so far the fuzzy logic controller is concerned and then how it works.

(Refer Slide Time: 03:15)



Now, we will consider one example, so that you can understand Takagi Sugeno approach and mainly in terms of input example case studies rather we will explain the step, whatever the methods and technologies are there, I will discuss in time.

(Refer Slide Time: 03:29)



Anyway, let us proAceed here the Takagi Sugeno approach. Now, according to this approach, every rule is represented as it is in the same as Mamdani approach using the, if then clause.

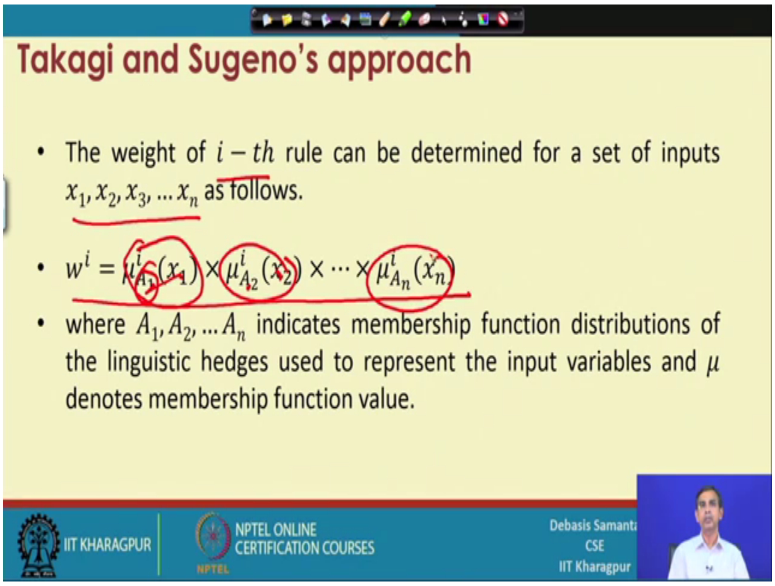
Now, for a system if there are n input, then a rule in the Takagi Sugeno approach take this form. It is the rule that is with the n input x 1, x 2 and x n. So, if n input at any instant having it is own value and if these are the input, which basically related to the fuzzy state A 1, these are input fuzzy state A 2 and these are n th input, the fuzzy state A n, then the rule will take this form.

Now, this is the rule and if this is the rule, then the output of the rule also can be expressed mathematically and then for this input the output is shown here. So, it is basically input x 1, x 2, x n and a 0, a 1, a 2 and dot dot a n are the coefficients. Now, how these coefficients can be obtained, I will discuss it. Anyway, this is the coefficient will be supplied by the fuzzy engineer based on the different rule.

So, for the i-th rule, so these are the coefficient will be totally different and this is a difficult job time to time for engineer to decide right value of this coefficient, because the output of a rule depends on the right choice of the this coefficient value. Anyway, so if fuzzy, we can depends on the wisdom of the fuzzy engineer and let the fuzzy engineer suggest these are the coefficients for the i-th rule and therefore, for any input which satisfies this, the rule we can calculate the output. So, these are the basic thing and this is very difficult job for the engineer to decide this one, once you decide it then rest of the things is very straightforward and simple.

Now, let us see how the rest of the part can be for the 2 rules, for the rules that we have discussed which takes this form like.

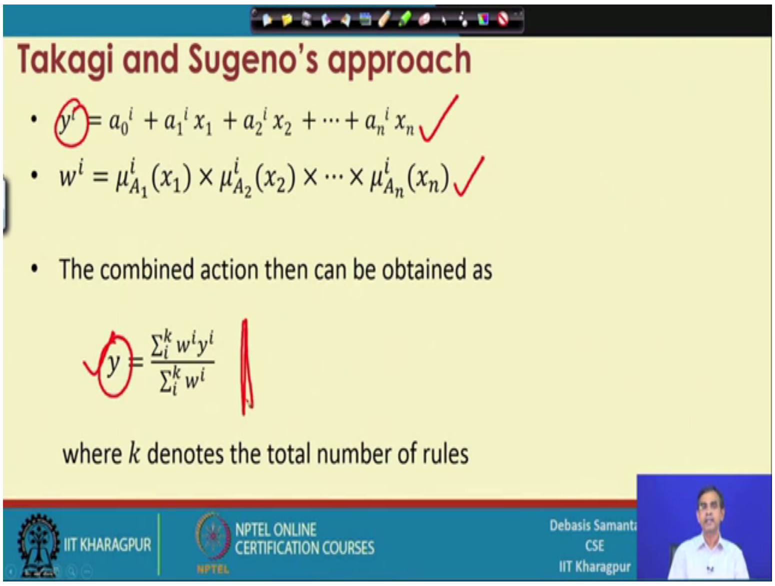
(Refer Slide Time: 05:56)



Now, according this approach, it first calculate weight of a rule. For example, for the i-th rule that we have discussed with the nth input, the weight; the i-th weight can be calculated using this formula. This formula it is like this, x 1, which is belongs to the fuzzy state A 1 taking it is membership value, so mu A 1 and similarly for the, another x 2 which is in the fuzzy state A 2, taking this membership value as mu A 2 and so on.

For the n-th input taking the fuzzy state A n and taking it is membership value. So, it basically takes the product of the different membership values for all inputs, belongs to the different fuzzy state. So, taking these are the values, we can calculate the weight of the, i-th rule. So, for whatever the rules are relevant for each rule we will be able to calculate weight.

(Refer Slide Time: 07:03)

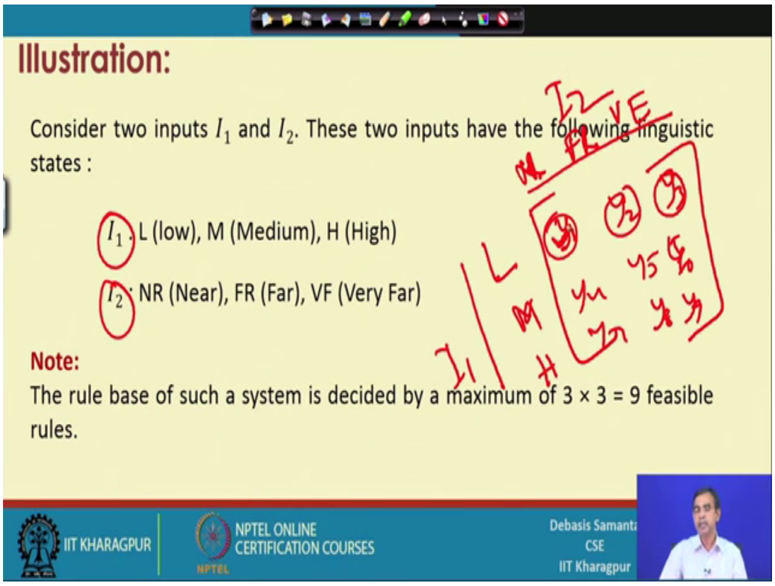


Now, after the weight is calculated, the next step is to calculate the output value these call the y i. As I told you, so y i is associated with each rule and this is y i value for the i-th rule and this is a weight for the i-th rule, that we have calculated in the last step and taking these values for all the rules.

Suppose there are k rules and then taking the sum of product of all the w i into y i and divided by the sum of all weights, it will give you the final output and that is the output and you can say that this output y, is basically the crisp output and this is the one difference from the Mamdani approach to the Takagi Sugeno approach, where we need to calculate the fuzzy output first and from the fuzzy output we have to calculate the crisp output.

But here, directly from the fuzzy input as the rule we can calculate the weights for each rule and then the output value of this one and then using this formula the final output, which is in the crisp value can be calculated. So, this way this is first, because straight away we can avoid the Fuzzification module there and then we can directly come into the rule.

Now I can, let us elaborate the Takagi Sugeno approach with an example, so that you can understand. So, this is the method actually by which the Takagi Sugeno approach calculates the output and this is basically the mechanism, where the fuzzy inference engine work. Now, whatever the rule base and then Fuzzification and everything that is the same as Mamdani approach, now let us consider, as an illustration, one example. (Refer Slide Time: 08:48)

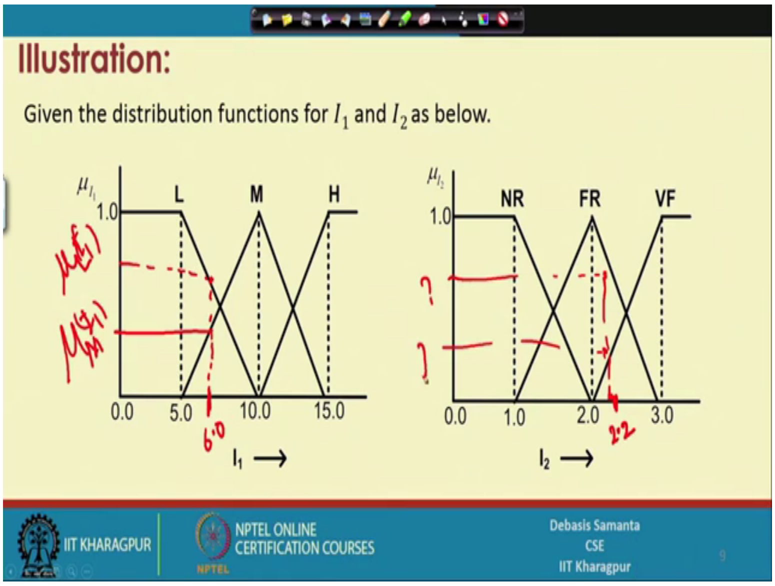


So, let us consider is a 2 input system. 2 inputs are denoted as I 1 and I 2, they let us consider there are abstract input and also consider that the 2 inputs I 1 and I 2 has the fuzzy linguistic, namely low, medium and high. That means, I 1 can be considered as a fuzzy state low, fuzzy state M or fuzzy state high with the different values of the membership or the different values of I 1.

Similarly, the other input I 2 has the 3 fuzzy linguistic again. They are called near denoted NR and far denoted FR and very far denoted as VF. So, there are 3, linguistics for the first input and then 3 linguistics state or fuzzy state for the second input. Now again if we follow the rule base matrix for this and you can understand the rule base matrix has, how many rules? Total 9 rules. So, if this is the L M and R, sorry it is a, so rule base system it is like this, so L, M and H. So, the 3 input for I 1, these are the I 1 and another 3 input for I 2. So, this is the I 2 and has NR, FR and then VF.

Now, here in this rule it is basically, so there are 9 rules. So, rule 1, the y 1 output, y 2 and y 3, similarly y 4, y 5, y 6, y 7, y 8 and y 9. Now, in the Mamdani approach, we have observed that all the rules values here in terms of fuzzy linguistic, where as in the Takagi Sugeno approach all the rule values in terms of mathematical representation by which the y 1, y 2,y 3 all this things can be calculated. So, if there is a rule if I 1 is L and I 2 is NR, then the output y 1, will be calculated using some mathematical notation. Now, for this again example, see what is the mathematical notation that we can consider out of these 9 rules, it is expressed here.

(Refer Slide Time: 11:29)

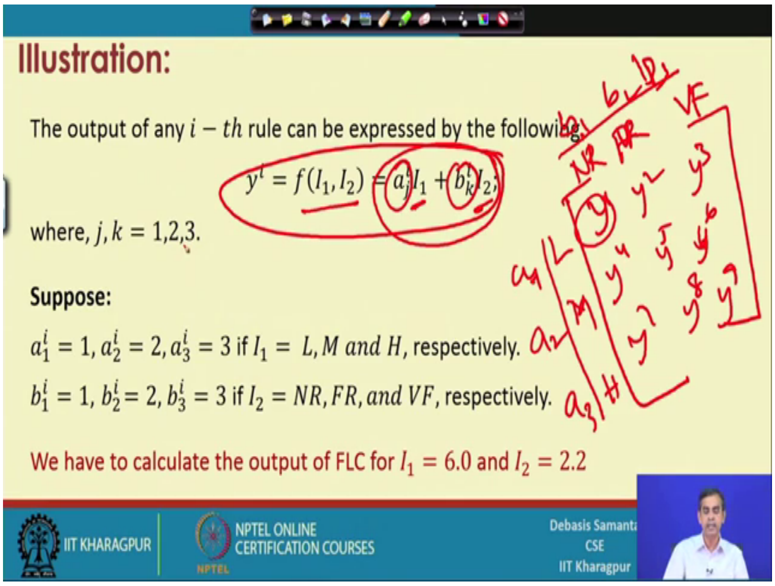


Now, before going to this, again let us consider the fuzzy linguistic state for 3 fuzzy states. So, far the, I 1 input is concerned, which are denoted as L M and H is shown here. So, it is the fuzzy state for the L M and H, the fuzzy membership function which is like this, for L it is like this and for H it is like this. So, these are the fuzzy state defined for the, I 1 and the input of I 1, is in the range 0 to 15 as it is mentioned here.

Now, similarly for the other input I 2, the range of values that is in 0.02, 3 .0 and all of and the fuzzy state that we have considered here, it is shown here. For NR it is look like this, for FR it is like this the membership function and for VF the membership function is like this. So, this is the fuzzy state for the 2 inputs I 1 and I 2, mu I 1 is the membership values for the different fuzzy state belongs to the input I 1 and mu I 2 is a membership values for the different fuzzy state belongs to the different input as I 2.

Now,. So, this is the Fuzzification module that is here or it is basically the fuzzy design is there. That means, all the input should be represented in the form of a fuzzy linguistic and we have discussed the 3 different fuzzy linguistic related to this illustration. Now having these the fuzzy linguistics there, now we have to calculate for a particular instant, that means, for a particular values of I 1 and for a particular values of I 2 at any moment, how the fuzzy input can crisp input can be converted to the fuzzy input.

(Refer Slide Time: 13:23)

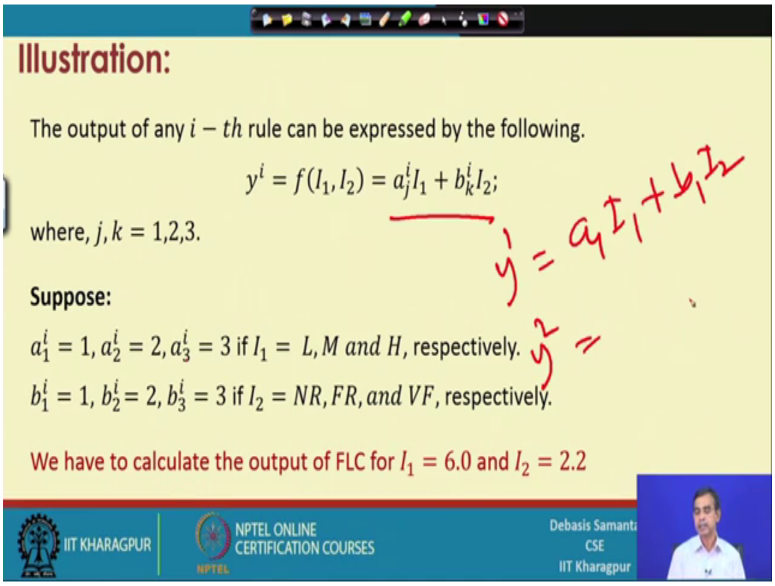


It is basically the same method that we have discussed there and we can follow it. Now here, in this slide say suppose each fuzzy rule that is denoted here in this expression. As I told you, again the same thing I can write it. So, it is basically y 1, y 2, y 3 right, y 4, y 5, y 6, y 7, y 8 and y 9, 9 rules are there. For each rule, that can be discussed in terms of the input value I 1 and I 2 and suppose we assume these are the, what is called the mathematical representation of output values.

Now, if we see, so this is the input I 1 and I 2 and then these are the coefficient. Now, all this coefficient needs to be decided by the designer. So, it is basically they are, basically how many coefficients are there L M and H. If we consider 1 coefficient a 1 and a 2, a 3 for this, similarly here NR then FR and VF, then we can consider b 1, b 2, b 3 so there.

Now, so we have to decide the different values for this rule, if a 1 is 1, b 1 is this and this one. So, there are 9 different values to be considered here, for j equals to 1, 2, 3 and some k equals to say, 9 different values to be considered there. Now for simplicity, suppose we consider the values a i like this one, a 1 for any rule I 1 is 1, a 2 i for any rule is 2, a 3 for any rule I 1 is 3.

(Refer Slide Time: 15:26)

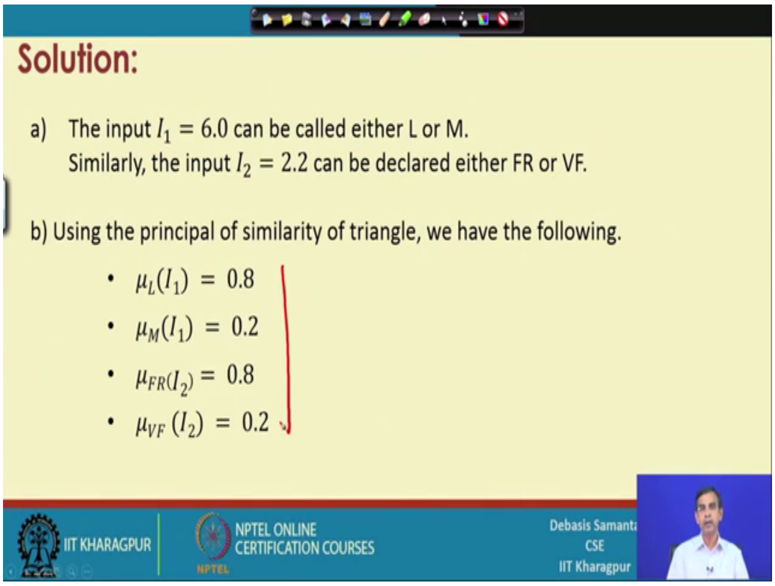


So, you have a simplified assumption and suppose it is decided like this one. So, alternatively I can say like, y 1 this is equals to a 1 into I 1 plus say suppose b 1 into I 2 this is the 1 rule like. similarly y 2 there is another rule also can be considered likewise. So, all this expression, that means, for the output can be represent like this. Now, having this is the representation of a particular rule and corresponding to that particular rule basically the output value, compress the rule base for the fuzzy system according to the Takagi Sugeno approach.

Now, let us consider a particular instant for a certain values of I 1 and I 2. We consider, this is the input at any moment, that I one is 6.0 and I 2 is 2.2. Now given this is the input, we have to calculate the fuzzy output according to the Takagi Sugeno approach. Let us see, how it can be calculated? Now, again for this I 1 equals to 6.0 and I 2, 2.2, we have to calculate the mu values of the mem elements. That belongs to the 2 different fuzzy state. Here, so I 1 is 6.0, so this basically this is a 6.0 and for the I 2 we have considered 2.2, so it is the 2.2 right. So, if this is even, then so this basically 6.0 belongs to that 2 fuzzy state M and as well as another fuzzy state L. So, I 1 is 6.0 belong to the fuzzy state M, has the mu M I 1 value this one.

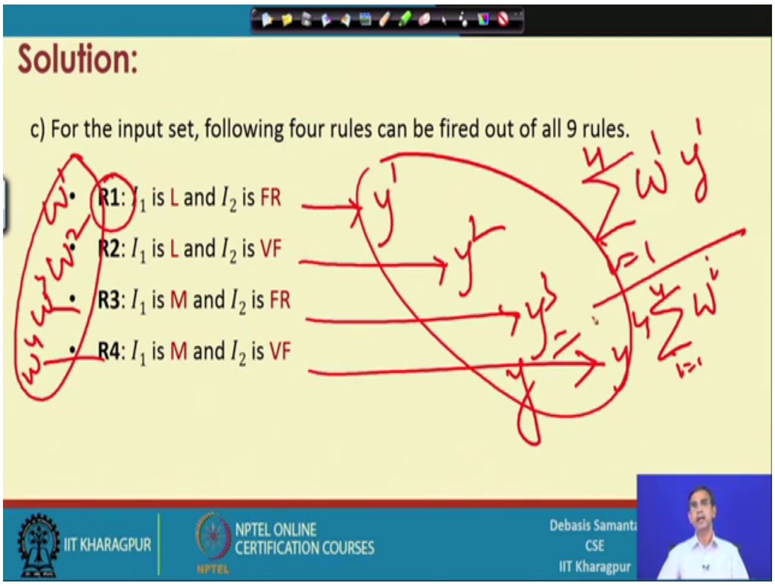
So, it is basically mu M I 1 value this one. Similarly, the 6.0 the input I 1 belongs to the fuzzy state L, having this value. So, it is basically mu L I 1 is the value. Likewise, 2.2 has the 2 fuzzy state namely, VF and then another is FR 2 1. So, this value and this value are to be calculated. So, this mu values can be calculated from this fuzzy description of the state are the same way, as it is followed there in the Mamdani approach, using the principle of similarity of triangles, we can calculate it. So, this mu values will be obtained. Once the, this mu values are known to us, then we can calculate the output value easily.

(Refer Slide Time: 18:06)



Now here, for an example we can calculate the mu value this is for the 2 input, this one as this one, mu 1 mu L I 1, mu M this one and this one.

(Refer Slide Time: 18:08)



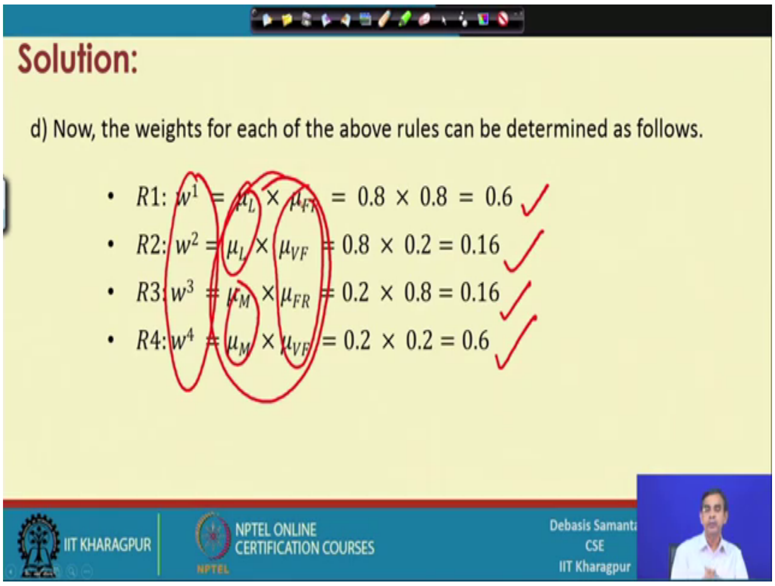
So, this are the 4 different values that can be calculated from the fuzzy linguistics state description and using the similarity of principal that we have already discussed about in Mamdani approach.

Now, so these values are known to us now. Now, we will use these values and then the output values for each can be calculated and so this is the method that can we calculated here. Now, here you can see, again just like Mamdani approach there are 20 rules, here 9 rules. Now, out of the 9 rules we have to select those rules, which are basically fireable. Fireable means, those rules are relevant in the context of current input. Now, as you see the, for the I 1 equals to 6.0 and I 2 equals to 2.2 and when I 1 is 6.0 it belongs to the fuzzy state L and M. Similarly, when I 2 is 2.2, it belongs to the fuzzy state FR and VF.

So, putting all this things together, there are therefore 4 rules, the 4 rules are discussed here. So, if I 1 is L and I 2 is FR, then it basically gives y 1, 1 rule. Similarly r 2 gives you y 2 and this one gives you y 3 and this gives you y 4. So, the 4 rules and related to the 4 rules, the 4 different output can be obtained. So, the task, the object, the next task is basically for each rule we have to calculate w 1. These are w 1 that is the weighted value of the rule, for this rule also 2, for this rule 3 and for this rule 4.

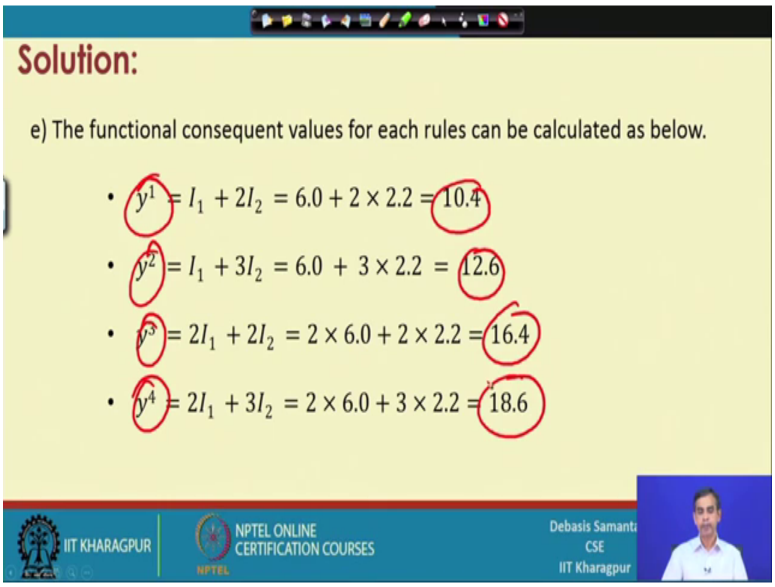
So, we have to calculate these are the values, so far the output is concerned and these are the values, so far the rules strength is concerned or weight is concerned. Once, we have these values, then we can calculate w 1 and y 1, where i equals to 1 to 4, is a numerator and then the summation of w i, i equals to 1 to 4. So, this will give you the final output according the Takagi Sugeno approach. Now, let us see in the current context, how the results can be obtained for the 2 things are there. We have considered the different values of a 1 and b 1 in the context of this example.

(Refer Slide Time: 21:00)



Now here, so this basically shows the computation of the o h value, as I told you it is a simple product of the membership value is there. So, for the rule 1 the value that can be obtained is 0.6, for the rule the value that can be obtained 0.16 and like this one. So, these values can be obtained based on the different values of the membership values in the context of current input. Current input here or here so far for the I 1 is concerned and these are the I 2 is concerned. So, w 1 can be calculated. Now, once the w 1 is known, then we will see how the y 1 for each rule, R 1 and R 2 can be calculated.

(Refer Slide Time: 21:54)

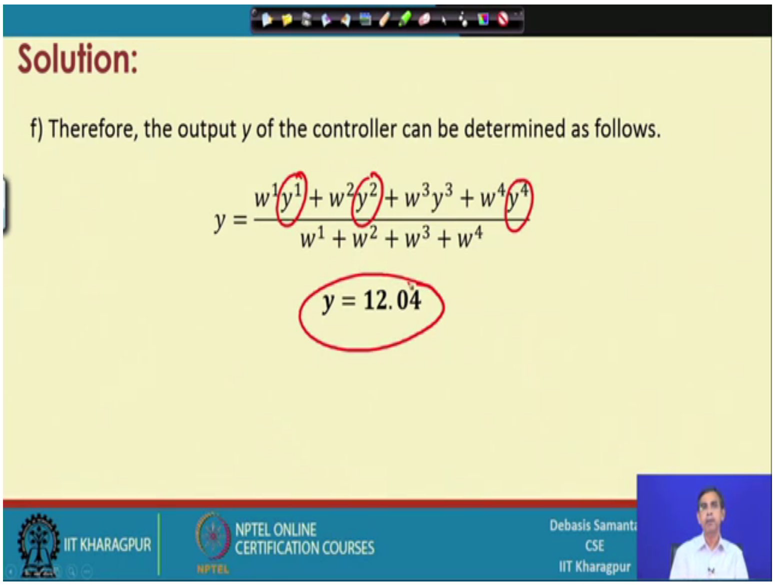


So, here is the method by which the y 1 that means output value can be calculated. I told you that, we will consider the different coefficients. For the rule 1, we take a 1 is 1 and b 1 is 2 and for others, a 2 is 3; a 2 is 2 and b 2 is 3. So, these are the values of rule can be calculated like so for the different values. So, here basically a 1 is 1 and b 1 is 2 and in this case b 2 is 3 and a 1 is 1.

So, these are the, basically the rule mathematical representation of the different rules are there and then we can obtain the values of the outputs using this. This is basically the mathematical representation of the rule 1 and this is a mathematical representation of the rule 2 and mathematical rule 3 and rule 4. So, I have given this as an example, but actually the, these are the tricks by which this rule can be decided or defined in the system and once this rule is defined according the system, it is a straight forward to calculate in terms of the different value. For example here, I 1 is 6.0 and 2.2 and taking the v values 2 1. So, this rule I 1 can be calculated this 10.4.

So, this is the, I one; y 1 the value is this one and for the, I 2 the value will be there and I 3 the value will be there and y 4 the value will be there. So, 4 rules related to I 1 is 6.0 and I 2 is 2.2 can be calculated, their corresponding output values. We have calculated the weights of all these rules also. So, these values then can be used finally to calculate the final output.

(Refer Slide Time: 23:51)



So, the final output is the formula according the Takagi Sugeno approach. That says that, it is basically product of the weight, weighted output values. So, it is basically, these are the sum of the product of weighted output values, divided by the sum of the weights and if we follow the calculation as we have obtained in the last calculations, then finally the y value that can be obtained as this one.

So, that straight forward way from the rule base we can select the relevant rules and from the relevant rules we can calculate the weights of each rules and then output values of each rules and using this expression according the Takagi Sugeno approach we will be able to calculate the output value and once the output value is known then will we can take a decision and you can note that this is the output that we have obtained is crisps. As I told you, no need to do any, follow any defuzzification method in this case state way.

Here, basically state way actually, this is because state way for each rule we have taken the crisp value. All these y 1, y 2 are all these are the crisp value for each rule. That is why, state way we can obtain the crisp value as the final output. So, this is the method that is followed there in Takagi Sugeno approach and so this is the way that the 2 controllers, the Mamdani approach and the Takagi Sugeno approach can be designed and 2 approaches have their own advantage as well as the disadvantage and as I told you, so Mamdani approach is easy to interpret, easy to interpret means rules are interpretable. On the other hand, the Takagi Sugeno approach the rules are difficult to interpret, so less interpretability. Difficult to interpret because, it is expressed in terms of some mathematical formula, which needs some coefficients to be design decided.

Now, deciding the coefficient it basically is a task for the fuzzy engineer or fuzzy designer. If the fuzzy engineer can decide the coefficient correctly or accurately, then it will give the accurate result. So, accuracy of the fuzzy logic controller according to the Takagi Sugeno approach solely depends on the performance or experience or prudence of the fuzzy designer. That is the only thing; otherwise, fuzzy logic controller according to the Takagi Sugeno approach is fast and more accurate, compared to the Mamdani approach.

So, it is up to the fuzzy designer, which approach he wants to follow, if it is a critical application, where accuracy and then the speed of the controller is very important, then they should follow Takagi Sugeno approach. Otherwise, both the approach are equally applicable to design any fuzzy logic controller. So, this is the 2 controller we have discussed and then using this the concept, we can follow any fuzzy system can be designed from the fuzzy logic. So, this is end of the fuzzy logic discussion and we will study about some case studies that will be given as a some special problem for your practice.

Thank you.