Introduction to Soft Computing
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Lecture - 10
Defuzzification techniques (Part - 1) (Contd.)

We are discussing about defuzzification technique. Defuzzification technique is required in order to find a crisp value for whether it is a fuzzy set or it is a fuzzy relation or it is a fuzzy rule. So, in the last lecture we have studied lambda curve method to find a crisp value for a fuzzy set and fuzzy relation fuzzy rule. And also we have discussed about how a fuzzy rule can be graphically displayed and there are certain graphical way to find the output of the fuzzy rule. And then today we will discuss about if a fuzzy rule is portrayed in the form a graph, then how we can obtain the corresponding crisp value .

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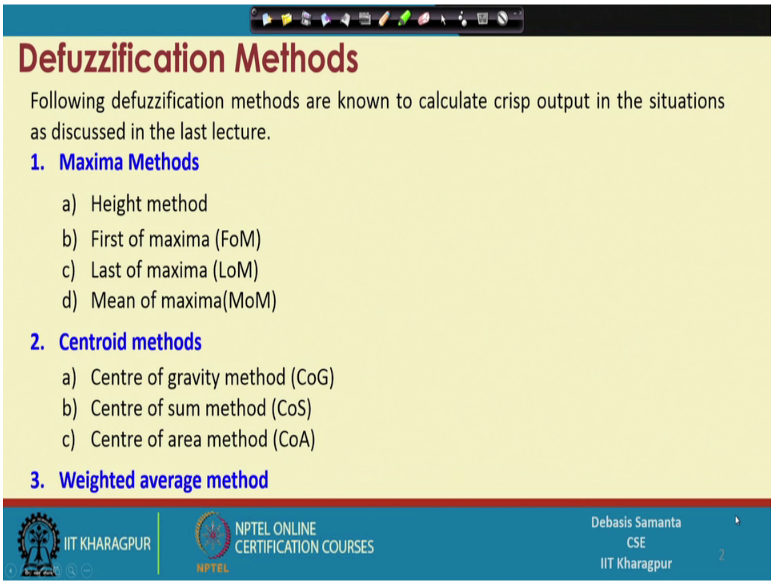

The additional advantage is that by proper choice of the aromatic group, fuzzifiaction defuzzification technique there are many methods other than the lambda curve method that we have discussed. So, all the method can be categorised in to three broad categories.

They are maxima methods, centroid methods and weighted average method. So, so for the maxima methods are concerned, there are again many methods like height method, first of maxima, last of maxima and mean of maxima method. Again so for the centroid method is concerned, there are three popular methods; centre of gravity method, centre of sum method and centre of area method weighted average method is only one approach. Now although we will discussed in the form of a graph, but actually the all methods belongs to this categories can be obtained is in numerically also. So, you will get an idea about how numerically the methods can be applied, but we will initially learn the graphical oh approach and then I will give an idea about the how the numerically the same thing can be obtained.

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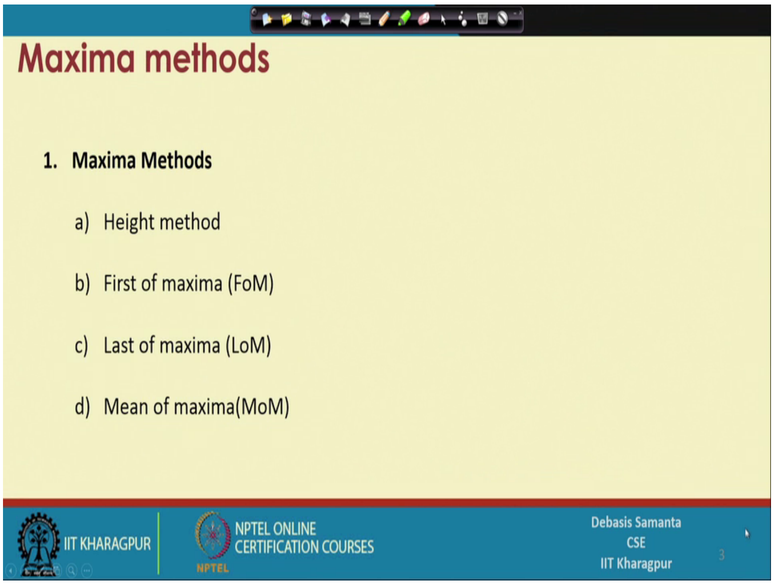

So, let us first discuss about the maxima method. As I told you belong to this method there are four different approaches; one is height method, first of maxima method, last of maxima method and mean of maxima method.

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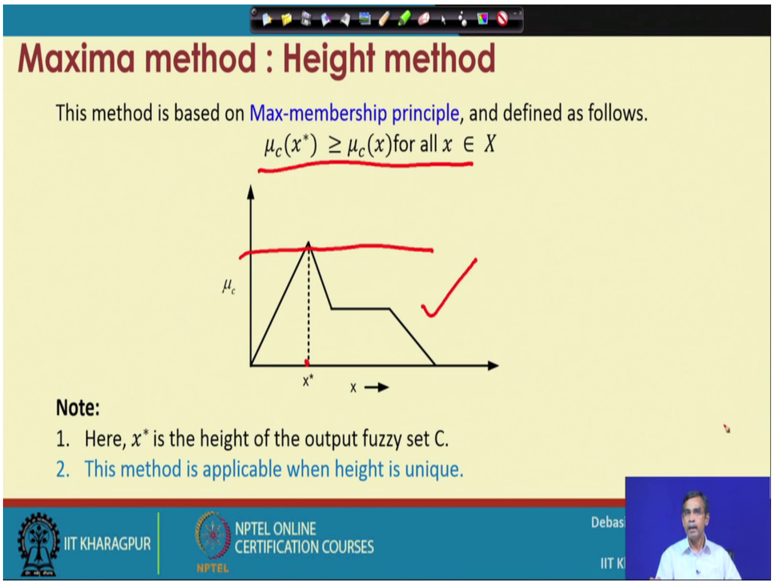

So, let us first discuss about the first method the height method. So, these slides show how the height method will look like this; this method in fact, based on the max membership principle. So, max membership principles can be expressed within this expression. So, it basically find what is the maximum values of membership for an element and the element which has the maximum value it becomes a crisp value for that. As an example, suppose this is the one graphical display of the fuzzy sets and if we see for the different elements. So, there is an element x star for which the membership value is high.

So, this means the crisp value for this set will be x star. So, here in fact, you can see x star having the highest membership value is become the height of the fuzzy set. So obviously, we can observe that this method is applicable only when a unique height is applicable. So, if there are more than one height. So, we have to follow some other method.

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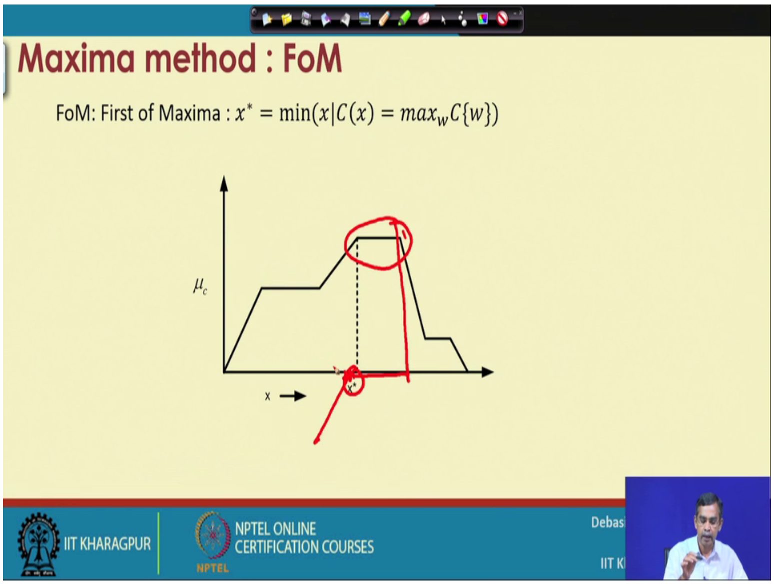

So, method is basically the first method in this line is called the first of maxima method. It is the method by which if we have more than one values having the highest membership values, then we have to take that a element which first highest value. So, again if we draw this graph, you can see within this portion, it has the highest membership value is this one.

So, the first element this is the x star which has the highest membership is become the crisp value. So, in this case the crisp value will be obtained here for this fuzzy set. So, this is the first maxima method and this method can be mathematically expressed this one. Now let us consider another method it is called the last of maxima, it is just opposite to the previous method. So, in this method the element this is the largest element which has the highest values of membership value.

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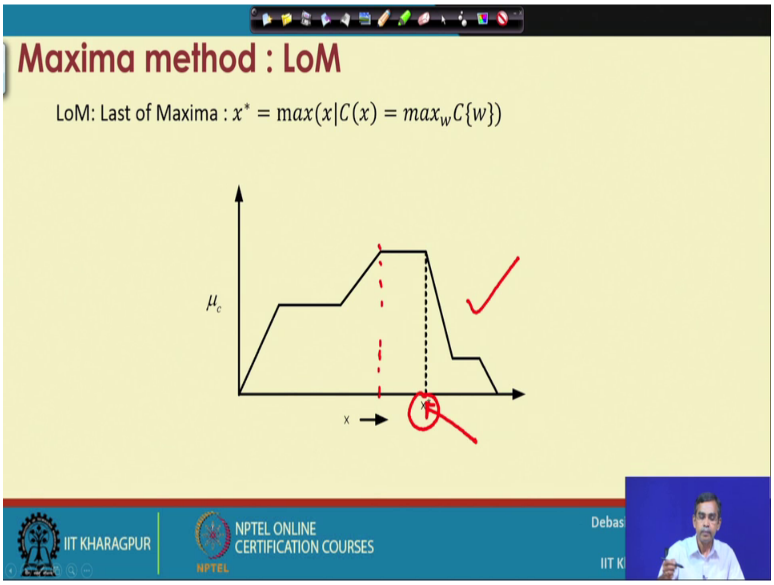

So, for example, again the same graph if you see. So, here the these are the different elements which has the highest value and the largest elements this one. So, these become the crisp value for this fuzzy set.

Now, one thing we can note that different method if we follow, so far we have discussed about the height method. Height method and faster maxima method is basically same if it is a unique anyway. So, all the methods if we follow they give the different result for the same input in fact. So, this means that result can vary from one approach to another, but all results are acceptable.

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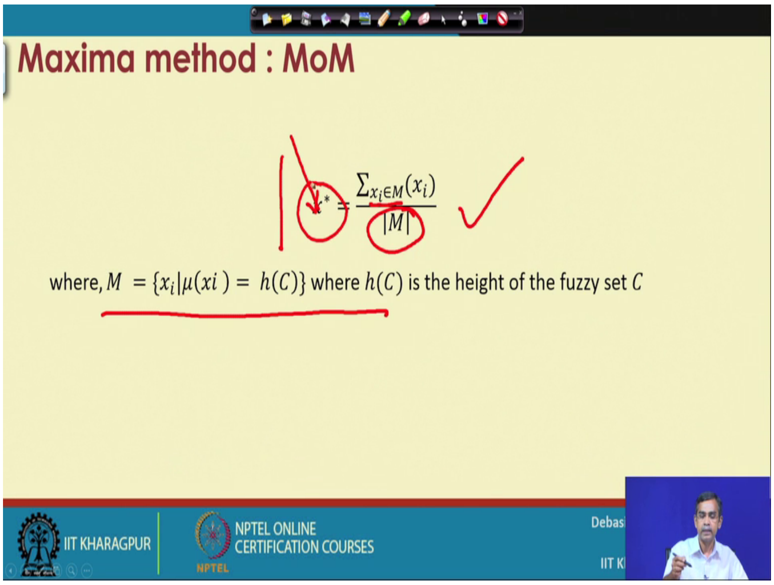

Now so this is the another method is called the mean of maxima method. So, in this case if we have more than one element having the highest value of their maxima then which will be the as crisp value. So, it basically takes the average of all the values that is their for which the height is more than 1. So, as an example this is basically expressed in this form where M; M is the set of all elements which has the membership value its same as the height of the fuzzy set. And then for all element x i that is M, we have to take the summation.

So, it basically this way we can take the mean of the maximum. Here M is basically size of the set n which has the membership value same as the height of the fuzzy set. So, this is a simple formula although it can be displaying the graph, but in the numerically if we have a fuzzy set and then we can use this expression to calculate the crisp value where x star is the crisp value for the fuzzy set.

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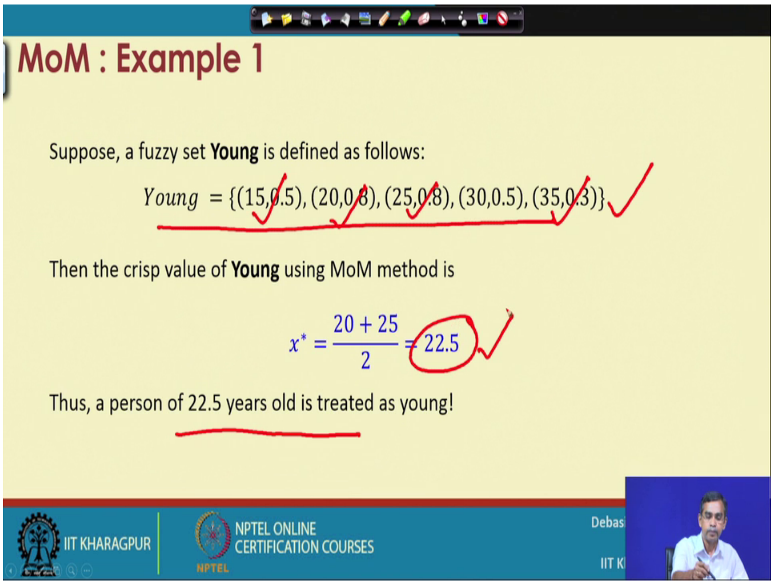

I would like to give some examples so, for the min or maxima is concern one example.

Let us consider this is the fuzzy set, and if we follow the first of maxima method then this is the crisp value. If we follow the last of maxima method then this is the crisp value. And if we follow the mean of maxima method, then we see in these sets these are the 2 element which have the highest heights or have the membership values same as the height. So, taking the average of the 2 values and we can use we can get it that this is the crisp value for this fuzzy set. So, a fuzzy set is like this then the crisp value for this fuzzy set can be obtain as 22.5. For example, if this fuzzy set denotes the young as a fuzzy set then the person of the year 22.5 years old is treated as young.

So, for the crisp value is concerned, but according to the fuzzy 15 is also young 35 is also young 20 is also young 25, but according to the crisp the 22.5 is the young.

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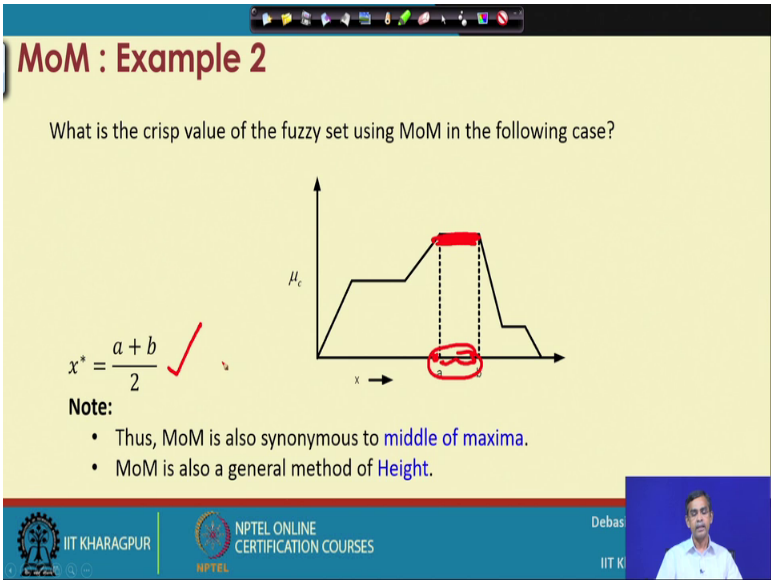

Now this is the another example, here if it is basically if you see the is a is a continuous sets right. So, these are the so many values which belongs to this are basically having the membership value same as the height. Then the average of this according to the mean of the method can be obtain as a plus b by 2. So, this is the a and b and this is basically average of all the values which belongs to in this range, then this is the crisp value. So, crisp value. So, crisp value in this case if we say that this is the a and this is b then crisp value.

That can be obtained as in the min of the method is this one. Now here actually this becomes a middle of maxima; because a plus b by 2 is basically middle of this one. So, sometimes min of max also it is called the middle of maxima method and in fact, this is the one method it is a generalise a generalised method, far the maxima method is concerned.

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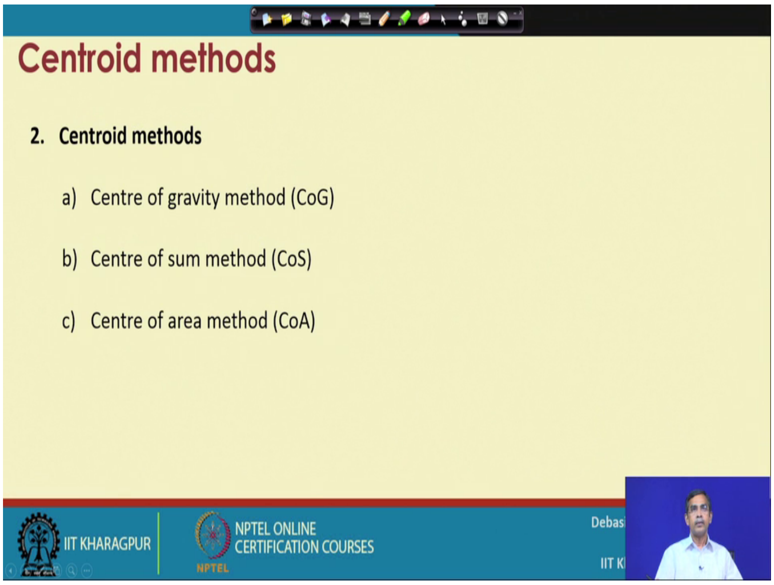

Now, let us discuss about the another approach it is called the centroid methods. Belong to this category there are methods centre of gravity method, centre of sum methods and centre of area methods, we will discuss each method one by one this method; however, compared to maxima method is computationally much expensive; however, it gives more result better result than the maxima method .

Now, first we will discuss about centre of gravity method.

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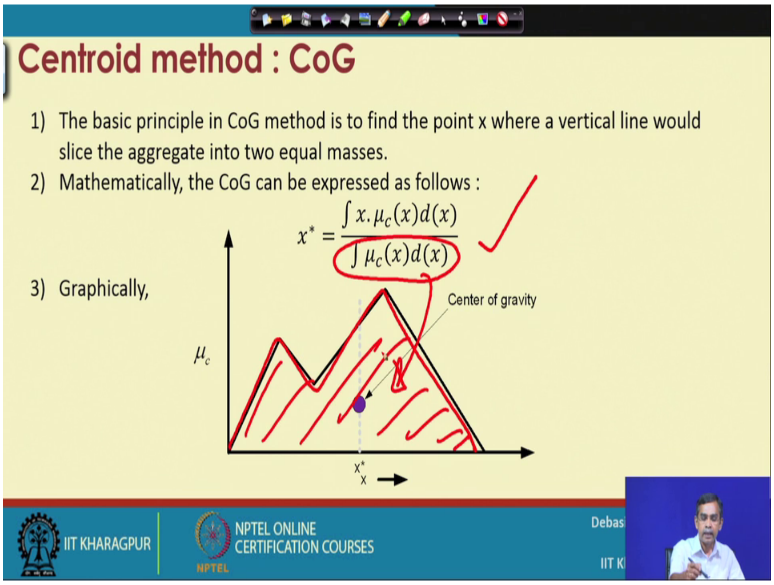

This method is popularly called CoG method. It is a short form of the centre of gravity method. Now we know exactly for a any for any geometrical object what exactly a centre of gravity means, it is basically similar to the centre of mass calculation of an object a geometrical object whether it is A 2 dimensional object or three dimensional object whatever it is there. So, it is basically centre is a centre where a vertical line can segregate the things into A 2 equal size of masses. So, that is a general concept of centre of gravity; that means, the entire mass will process through this point sort of thing.

Now so for the geometrical object is concern, the same thing looks like this. So, if it is an object then centre of centre is basically this one. So, this is basically centre of gravity sort of thing. Now the same concept it is applicable here, now in this graph this is a graph of a fuzzy set and this graph is look like this and the centre of mass suppose it lies here. Now if we draw a vertical line from this here on the x axis, which cut at this point this point is basically called the centre of mass point; now having this centre of mass point this basically the crisp value for this fuzzy set. Now so for the computation is concern how this can be calculated. So, there is an expression for calculation of doing this thing assuming that.

These x varies over a continuous range of values, and the graph of the membership values for this fuzzy set is like this, then the method by which the CoG of this set or a graphical things can be calculated within this formula. So, it is basically x mu C x d x divided by mu C x into d x. You can note that these basically represents area of the curve under the curve mu c. So, if this is a mu c. So, these basically area of the these portion. So, it basically represent this one and the numerate the this portion is basically is a weighted. So, x into a particular area of this portion or x into corresponding mu cdx, and then next x into another mu cdx and this one. So, if you take the instigation of all over the things it will gave the coverage of all the entire portion.

So, this is the formula that will be used to calculate the centre of gravity and the same formula can be also extended in case of discrete value also.

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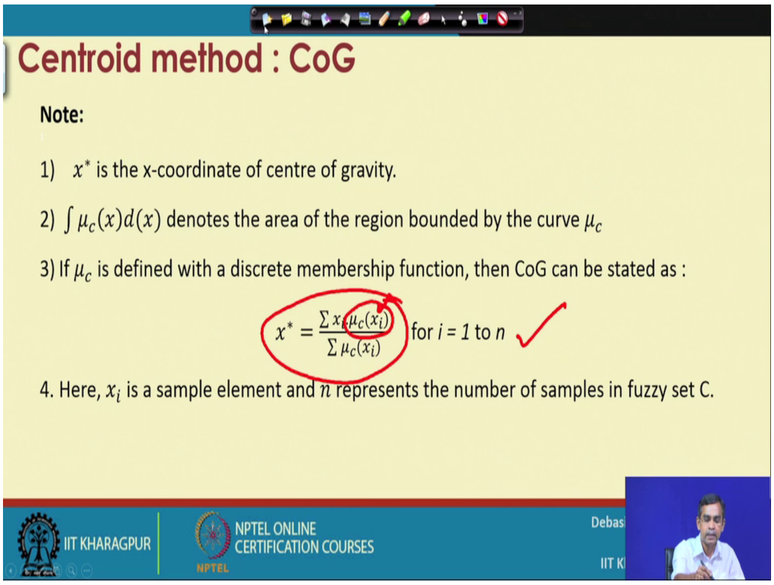

So, in case of discrete value the formula is like this. So, instead of integration we can use a summation formula if we know the value of mu C x i for different x i mathematically, then mathematically this can be calculated easily. So, this is applicable for if the fuzzy set has the discrete set of elements and the previous example that we have discussed if it has the value for the continuous elements.

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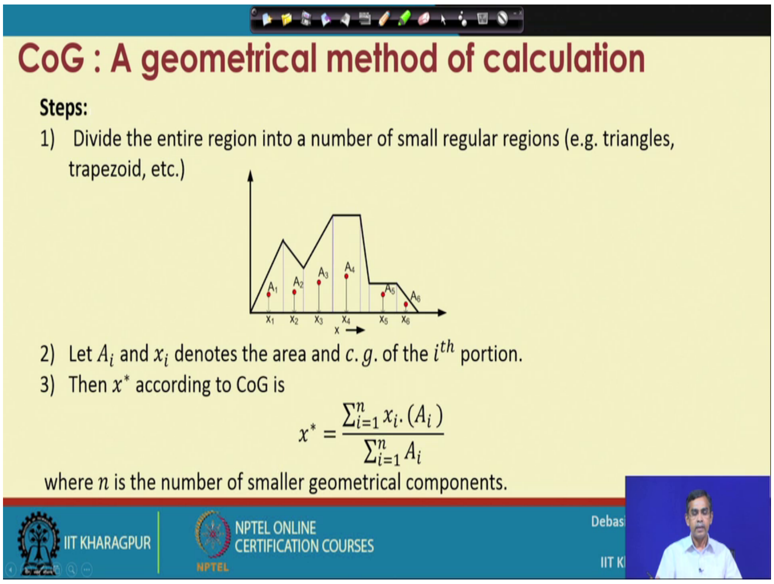

Now, this is one example hoe the CoG method can be applied to calculate manually, and I will give another example have the same method can be calculated numerically.

Now suppose, this is the one fuzzy set that is shown here and. So, the in this fuzzy set, we can do one thing we can have some segmented area, so that the area of each segments can be easily calculated. Now for example, if the this is the entire we can find one area of this one and then next area this one and then next area this one and so one so on. So, different portion of the area if we can indentify manually, then for each area applying a same method CoG formula, we can calculate its centre of gravity and then taking the sum of each then we can obtain this one . So, alternatively the method also can be like this one if we say this is the area of a ith segments.

And x i is its centre of gravity, then taking the some for all the centre gravity and their corresponding area and dividing by the total area of the curve, the we will get the crisp value for this fuzzy set. So, this is the one geometrical method by which the centre of CoG can be calculated and hence the crisp value of this can be calculated. So, this is a graphical method, now for each graphical segment this is just simply using are area of triangle we can calculate, and this is suppose using area of a trapezium we can calculate. So, calculation will not be tedious only the thing is a number of more calculation is involved, because there are n number segments then we have to calculate n areas and then there is a product and then average divided by the total area like.

Now, I can give an example of the same method but say is in numerically.

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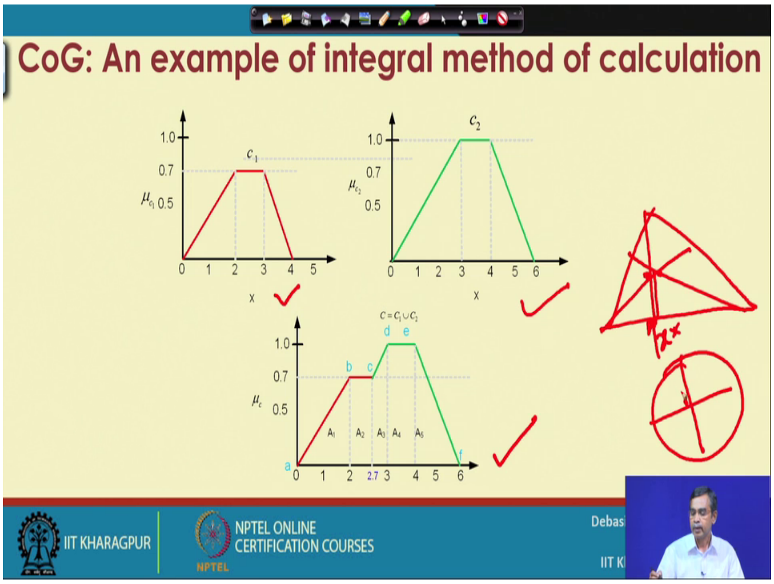

Now let us see these graph again here. So, suppose this is the one fuzzy set, this is a another fuzzy set and this is another output of the fuzzy set. So, output of the fuzzy set is basically taking the union of the 2 and. So, if we take the union of the 2 and plot on the same graph it will give this one. Now so for the geometrical method is concern, again we can do the segmentation. So, it has A1, A 2 A3, A4 and A5. So, the five segments and for each segments, we can calculate how the area of the each area of each segment can be calculated we have an idea and also the centre of gravity.

One thing if this is the one triangle I just forgot to mention it, then how you can calculate the point here which is the centre of gravity and therefore, we take the this one as the what is call the centre of gravity point say x star like. So, idea is that if we take some geometrical method of this one and then this one who is basically point we intersect this one. So, it basically the centre of gravity for the circle it is very simple. So, it will. So, this one. So, this is the centre of gravity and like this one.

So, there is a geometrical formula by which the centre of gravity can be calculated area can be calculated accordingly. Now let us consider the same, but with the help of some numerical calculation. Now so we can calculate the area if we know what is the equation of this curve.

Because it is basically mu x into d x formula if the equation is defined by this one now this is suppose a straight line. So, the equation of this line can be easily obtain if we now the slope of this line. And similarly this is the area, area can be obtained if we know this one and this is the line. So, this is the straight line into the d x this one. So, eventually the idea is that if we know the different portion and they are corresponding equation mathematically, then taking the simple numerical method of equation integration we can find the area of each pieces each piece and then the area of the entire curve and then the CoG can be can be roughly can be taken as the middle of this point then.

So, it is a basically little bit not so, much accurate whether inaccurate calculation, but this can be useful for it. Now let see detailed example about that how taking this into this using this information how we can calculate numerically.

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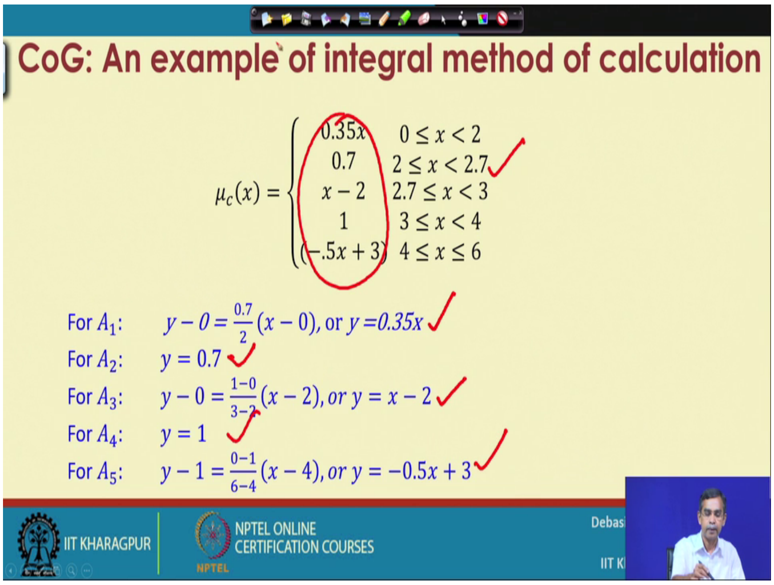

So, here is the idea the mu C that is the membership function of the output function can be expressed using this expression. This can be obtain readily for A 1 for example, as it is passes through 0 0 and having the slope this one. So, we can this is the slope of the line and it passes through 0 0 the first line rather. So, again we can see it. So, this is the area of the curve A 1.

Similarly, area of the curve way to is like this which has the equation this one and area of the curve A 3 A 4 and A 5 can be calculated having these are the equations of the membership function it is there. So, this is basically can be. So, this this way area of the each parts can be calculated using some numerical form.

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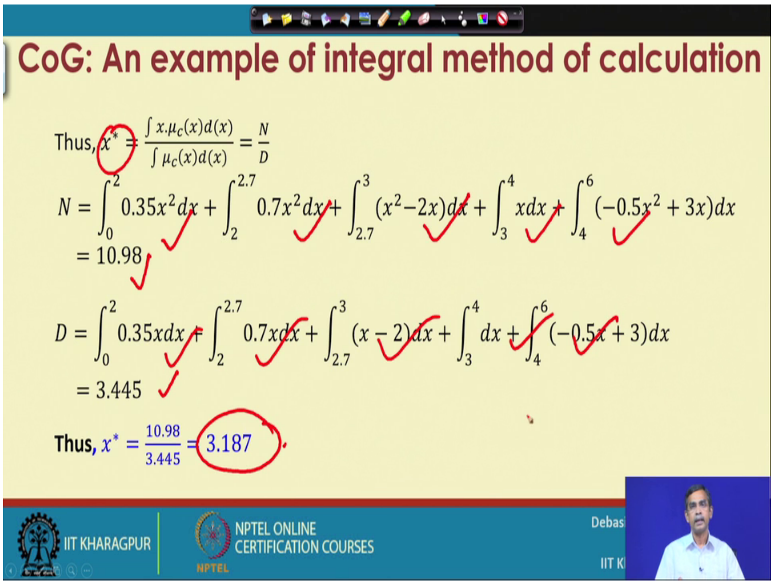

Now having this is the area then we will be able to calculate the CoG value for this fuzzy set, and this is the formula that we have already discussed now you can have shown here separately now the numerical component and then denominator component can be calculated. So, for the numerator component it as the different parts, for each piece actually. So, this is for the A 1 this is for the A 2, this is for the A 3 this is for the a four and this is for A 5. So, the numerical result that can be obtained using this integration method is this one likewise for the denominator for the five parts, which is shown here the value can be obtain this one, and therefore, CoG x star can be calculated as this one this one. So, this means that the output fuzzy set for the output fuzzy set it has the corresponding crisp value according CoG method is like this.

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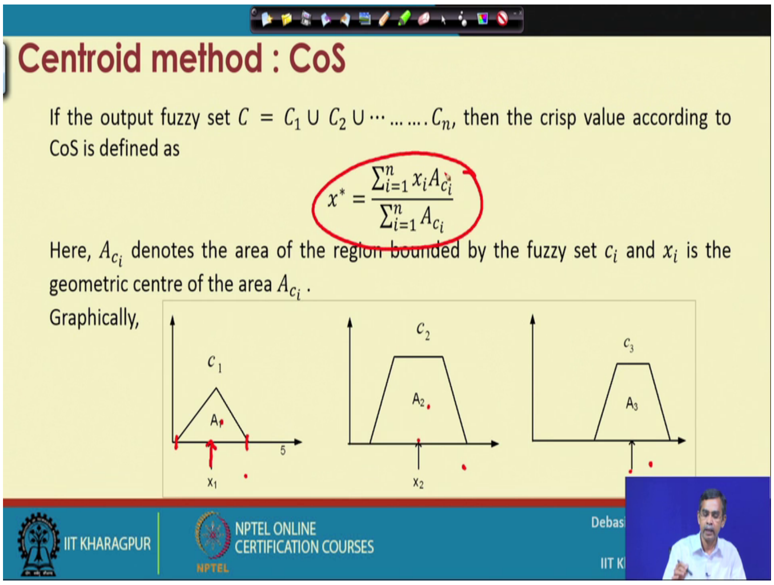

Now. So, this is an example how using an integral method, the CoG can be calculated. Now I want to give another example for the another method which belongs to the centred method it is call the centre of sum method. |It is relatively computationally very easy compared to the previous method CoG .

Now this method can be better explained if we consider let C the output fuzzy set obtained from the n number of fuzzy tests C 1, C 2, C n etcetera. Then according to this method according to method the fuzzy crisp value the crisp value for the fuzzy set C can be obtain using this formula, where x i is basically the middle value of the fuzzy set and A C i is basically the area of the fuzzy sets C i and this is basically the sum of all areas. Now as an example suppose this is the C 1 and this is C 2 and this is C 3 and C is the output fuzzy sets. So, in CoG method we have plotted the three graphs together, but here we do not have to do these things rather we can take individually one by one.

So, the first one we can calculate the area A 1 it can be calculated either within geometrical method or within some numerical method, and X 1 is basically middle of the two; that means, this is the middle . So, X 1 into A 1 and for this X 2 into A 2 and X 3 into A 3 is the numerator component and A 1 plus A 2 plus A 3 is the denominator component then the co method CoS method will give you the crisp value for this fuzzy set. So, this is similar that of the CoG method, but in case of CoG method we have to plot all the graphs together and then taking the resultant graph and then for the resultant graph we have to calculate CoG, but here we do not have to do.

We have to take on an individual output and then take the summation of all those things and then average, and then the result can be obtained. So, result definitely will be different than the CoG method if course, but it is the computationally less expensive.

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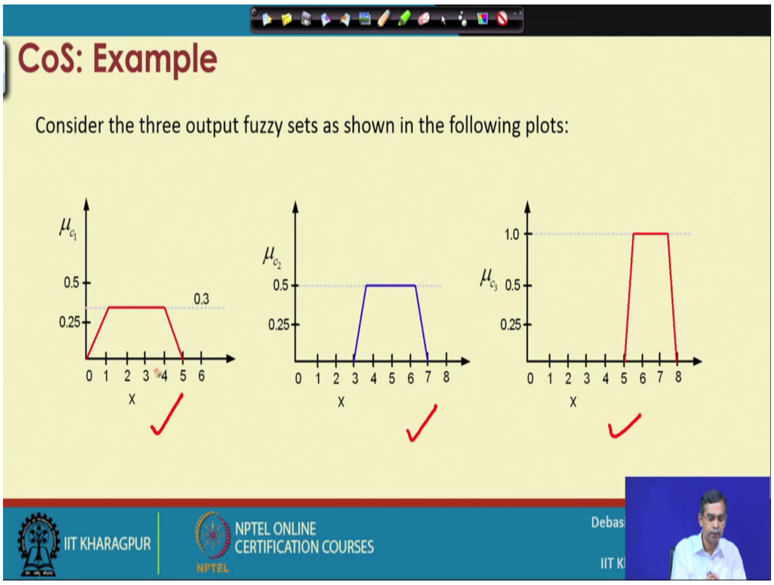

Now, so this is the ca method and this is an example we can again exercise. So, this is the one curve C 1 this is the C 2 and C 3. So, for this C 1 we can easily calculate the area of this one and this one let see, what is the area of the 3 components here and we can calculate.

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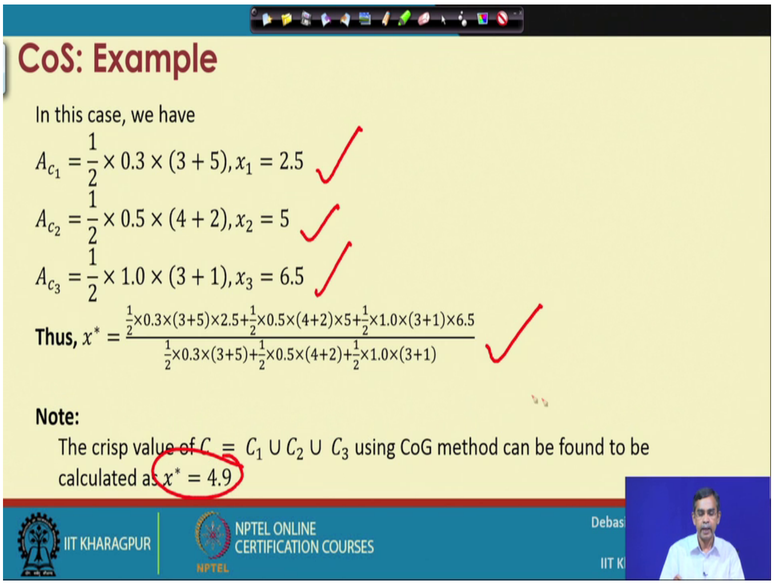

So, we can calculate here for example, for the first fuzzy set C 1 this is the area second and second and using the formula we can obtain the crisp value for this, and this is the result this one. Now again if we apply if we apply the CoG method to the same graph; obviously, the result will be different it can be observe that result that the CoS method will give little bit higher values than the CoG method, because it will take area for the 2 curves more than twice whereas, the same area will taken only into one in case of CoG method. So, these are the CoS method and there is another is the simplest method it is called the centre of largest area.

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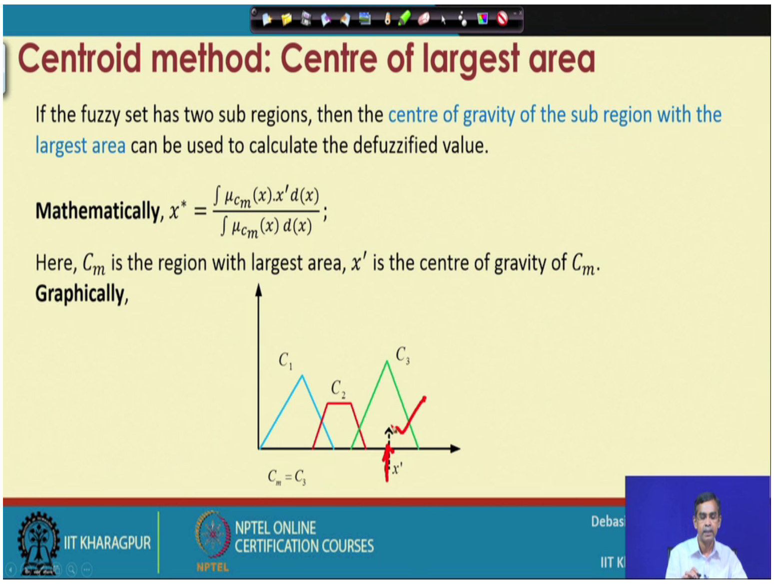

It is just one another what is called the simplification of the centre of sum method rather it will rather consider only the one fuzzy sets which having a largest area. So, if this is the fuzzy set having a largest area then it will take only that fuzzy sets and then it has the area of this one and this one divided by x. So, it basically gives this is the crisp value of this fuzzy set. So, this method is very simplified form of the previous method hardly it is used, but the mostly used method is CoG and then the co then the CoS method is preferable yeah.

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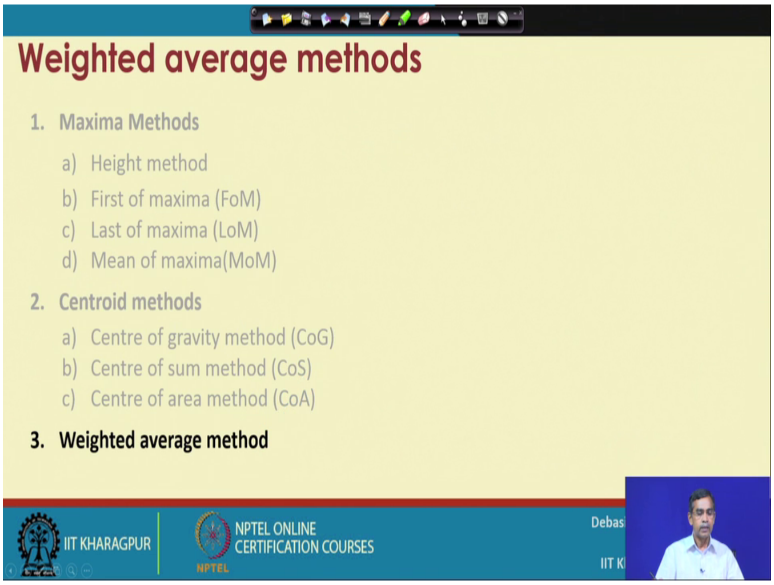

So, this the different method that we have discussed about and then weighted average method I will just discuss quickly.

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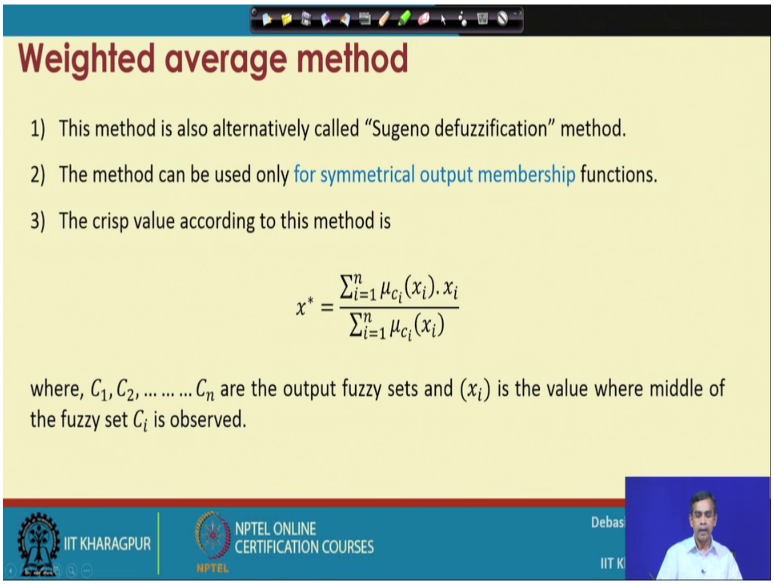

So, it is very simple to know and. So, the weighted average method it is similar to that one. So, similar to the this one this method also called popularly called sugeno defuzzification method and; however, this method is a simplification of the previous centroid method, but it is it is only applicable.

For the symmetrical output membership; that means, if a fuzzy set has the symmetric in shape then only we can apply this method symmetric means so, these are the symmetric method.

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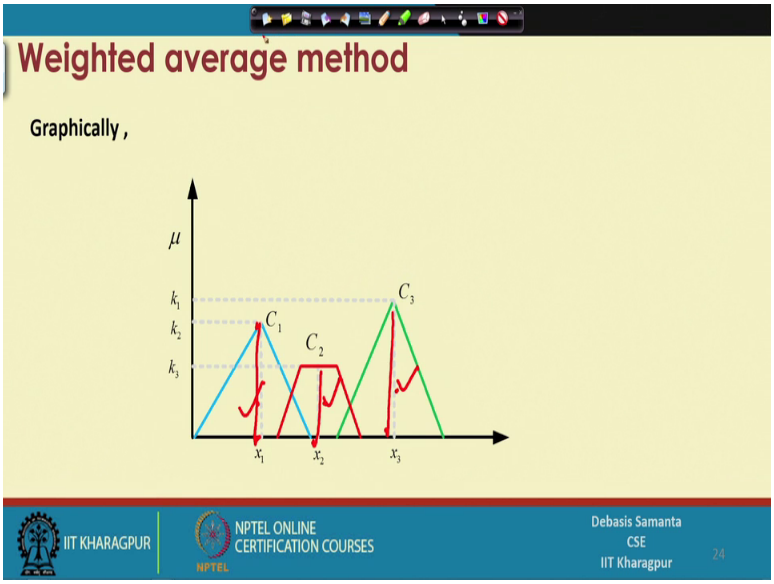

And then we can have it this is a symmetric, this is all the curve this is also symmetric this is also symmetric and this is also symmetric. Then for this symmetric method we can have the middle value of this x 1 and this x 2 and this is x 3 they taken individually and then area of this one area of this, one area of this one divided by this one it is there. So, it is basically same as the CoS method or CoG method we can say in some extent, but it is applicable only for CoG if it is for if we can use it for the symmetrical fuzzy set then it gives a better result that is why if we know that the fuzzy sets are symmetric, then without any second thought we can use for this method and then we can get it. Now in the last few slides I have plan few examples so that you can understand it.

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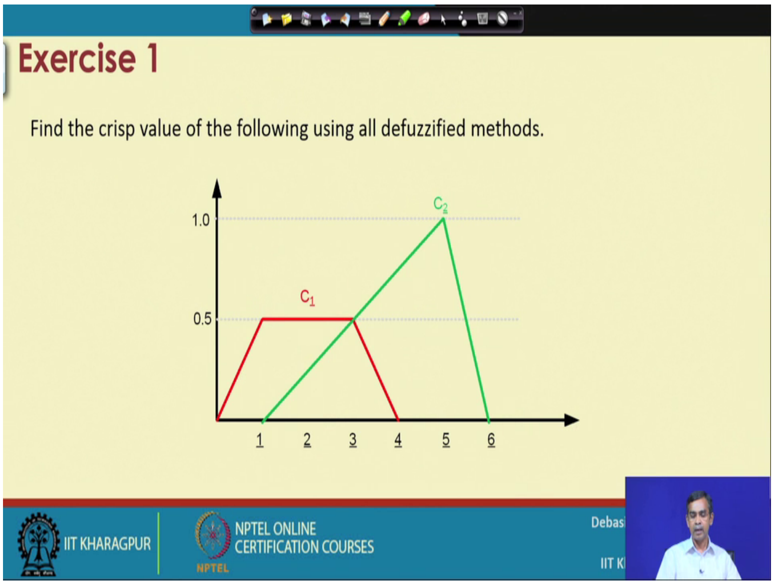

For example, this is the one output having the 2 fuzzy sets C 1 and C 2, we can easily calculate the either using maxima method or CoG method or CoS method. So, you should try using the different method, how the crisp value can be obtain and you can compare the results easily.

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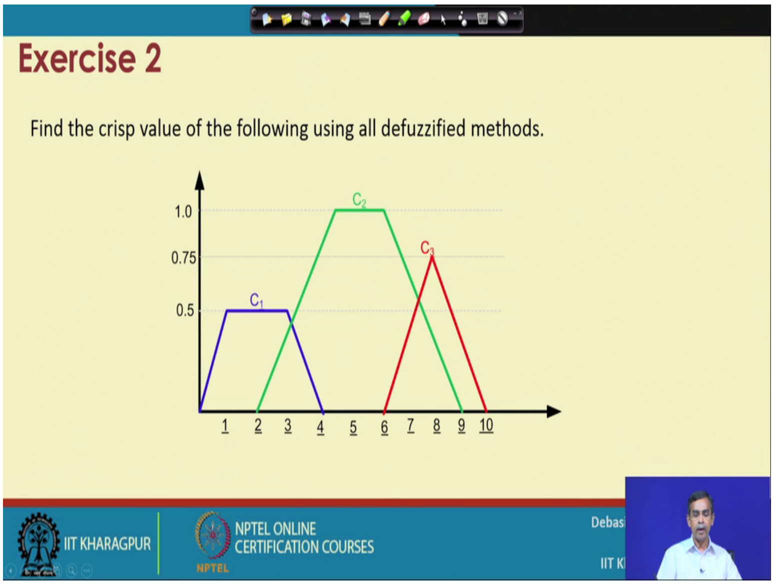

And this is the another example. So, we can find defuzzified method following either maxima method or CoG method or CoS method and then weighted method.

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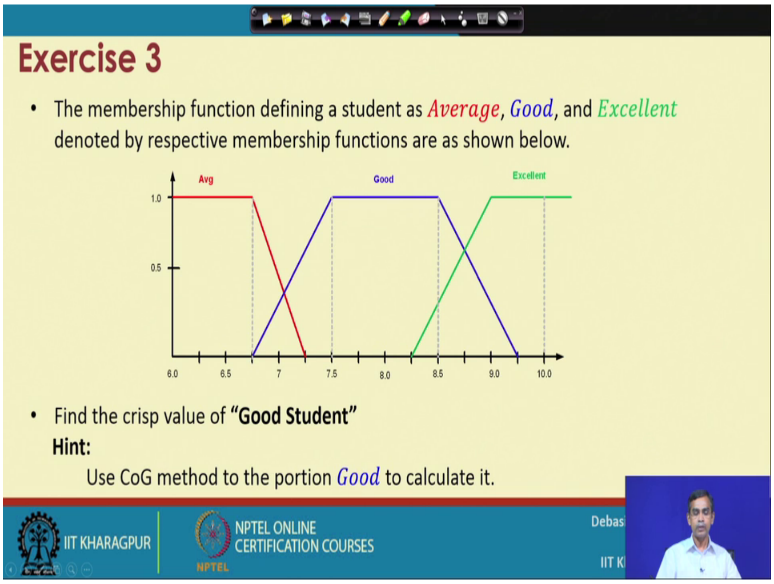

Now if I ask you how the crisp value for the good student whether this is a fuzzy set for the three set is given, but as the good is the only our objective. So, we can limit our fuzzification to this portion only and then we can calculate. Again the same method either centroid method or maxima method or weighted method can be applied to calculate it easily then you can understand what is the crisp value corresponding to this fuzzy set.

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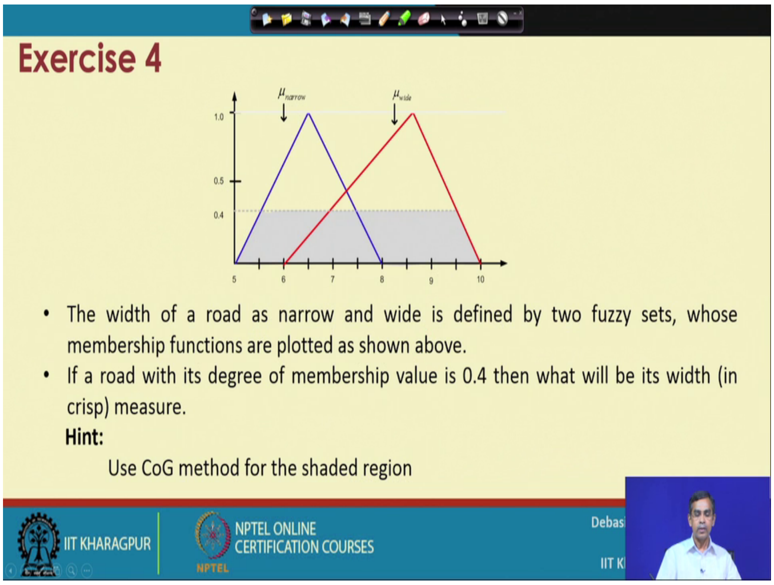

Now, this is an another example. So, here the 2 fuzzy sets namely the narrow of a road and wideness of a road is given. So, the fuzzy sets are described for the narrow this one and then wide fuzzy set is this one now here. So, suppose here actually the width of a fuzzy set having different what is called the degree is known to you, then we have to calculate what is the road if its degree of membership value is 0.4.

So, we can the simplest is that the 2 graph can be plotted on the same plots and then taking the common area corresponding the qualified value of the membership value. For example, in this case the qualified value for the membership is 0.4. So, if we take this curve 0.4 and then this is a common area and so, we have to take the fuzzified value of this is the fuzzified value of this result. Now we can take the crisp value taking again CoG method or CoS method or maxima method or some weighted method and then we can calculate the crisp value. Crisp value basically if the road is narrow and wide is defined by some fuzzy sets then for a particular road having some width.

And its degree of membership 0.4 then the crisp value can be obtained. So, the area and then corresponding the crisp value gives that if the road is having what is call the width say some value and its degree of membership 0.4, then this basically gives you the a crisp value.

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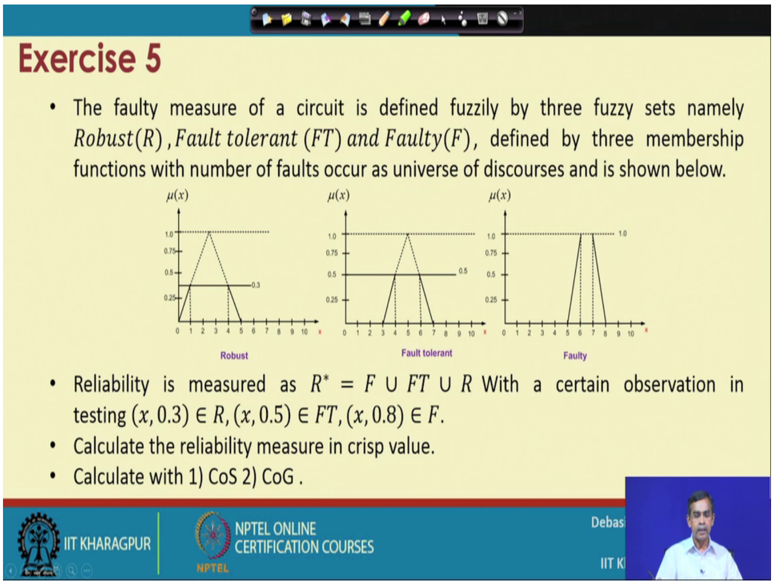

Now there is another example that is more interesting to note; say suppose here the faulty measure of a circuit that is that can be defined fuzzically fuzzily by three different fuzzy sets. There are three fuzzy sets namely robust fault tolerant and faulty. So, these are the three fuzzy sets and corresponding the membership value is defined for this robust and fault tolerant and then faulty.

Now, suppose reliability is measured this is basically reliability whether it is a faulty or fault tolerant or a robust. So, it basically reliability of a system is measured by this formula. Now if we define all these are the fuzzily then their resultant value is also can be obtain fuzzily. So, union of the three fuzzy sets can give you the reliability of the fuzzy sets.

Now the same thing can be plotted on the same graph and then we can have the reliability measure. As an particular instance say suppose one circuit is tested with some x value and then degree of membership is 0.3 that is basically belongs to the robust and then x is the number of circuit fault that is obtained with degree of membership 0.5 and this is the belongs to with the fault tolerant, and there is a x is the number of test performed with degree of membership which basically gives the faultiness.

So, we can obtain its crisp value; that means, crisp for the reliability if we take this is the output for the first component; that means, robust and this is for the second fault tolerant and this is the area occupied by mu equal to 8.0. So, it is 8.0. So, this is the area and then we can take either CoS method or CoG method and then we will be able to calculate the crisp value and that basically the crisp value for the reliability of the circuit. So, these are the few example that we have discussed and so this way the defuzzification method for the different according to the different techniques can be obtained. Now in the next lecture we will apply this defuzzification technique in more general sense whenever we will discuss about fuzzy system design. So, our next topics will be how we can design fuzzy sets using the different concepts that we have learn so far. So.

Thank you we will meet again in the next lecture