

**Artificial Intelligence**  
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**Lecture # 30**  
**Fuzzy Reasoning - I**

In the last lecture we discussed about different kinds of uncertainty and we have seen how probabilistic reasoning helps in reasoning under uncertain circumstances. Today we will discuss about a novel technique called fuzzy reasoning which is based on fuzzy set theory and consequently fuzzy logic. This notion of fuzzy set theory was proposed by Lotfi A. Zadeh.

We have already seen that in our day-to-day conversations and communications we often use terms, words which are rather imprecise in nature. It rained heavily, now what does the term heavily mean? The boy is very strong, how strong is heavy strong? The color of the curtain is more or less red or reddish. Now these terms do not find a direct mapping to any quantification like number. When I say the boy is good, the boy is very good then I obviously do not immediately imply that the boy is 7 good or 8 good or something like that.

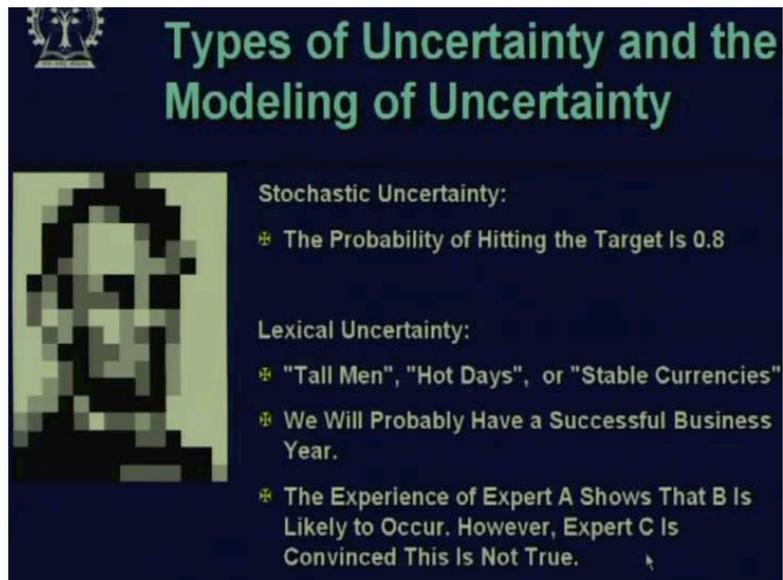
Now these sort of terms are very useful in our day-to-day communication but certainly poses a difficulty when you try to compute with these things. Fuzzy reasoning such as the fuzzy set theory and fuzzy logic deals exactly with this particular thing.

How to handle such imprecise scenario?

If you are shown this picture on the left side all of you will agree that it looks like Abraham Lincoln but it is more or less like Abraham Lincoln.

How is it that we can certainly identify that this is the figure of Abraham Lincoln? The complexity of the decisions from such subjective inputs the decision making that we do in our mind is really intriguing and often we do not really understand and often we cannot model them in quantified manner. Now we have already seen what is meant by stochastic uncertainty. Those are the cases which we handle with probability theory like the probability of hitting the target is 0.8. That means we have carried out a number of experiments and based on those experiments we have seen that in 80% of the cases we could hit the target properly. But there is another category of uncertainty which is called lexical uncertainty.

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## Types of Uncertainty and the Modeling of Uncertainty

**Stochastic Uncertainty:**

- ✦ The Probability of Hitting the Target Is 0.8

**Lexical Uncertainty:**

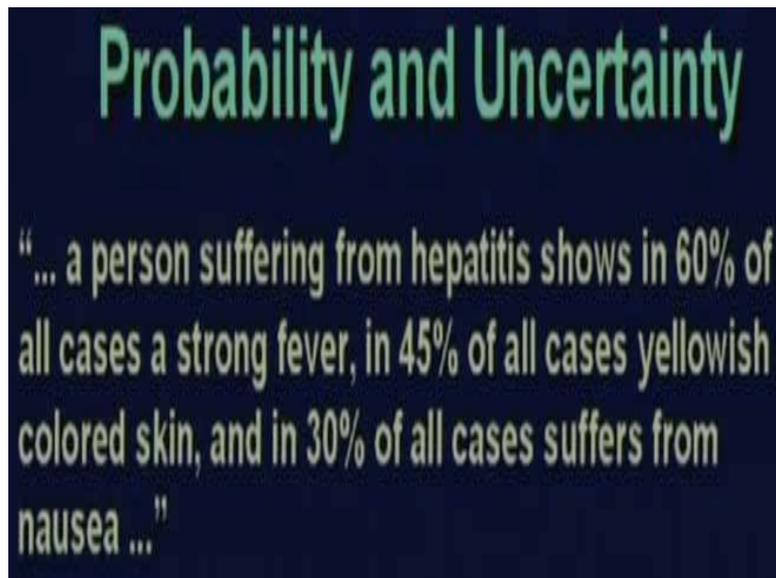
- ✦ "Tall Men", "Hot Days", or "Stable Currencies"
- ✦ We Will Probably Have a Successful Business Year.
- ✦ The Experience of Expert A Shows That B Is Likely to Occur. However, Expert C Is Convinced This Is Not True.

What is meant by lexical uncertainty?

Lexicon means the dictionary or the set of words. So the uncertainty that creeps in from our usage of day-to-day words those are subjective sometimes they are ambiguous and often they are pre- imprecise. So, for that a number different types of uncertainty creep in. For example, we use tall men, hot days, and stable currencies. Now you see everywhere tall, hot, stable etc so what do they really mean? We will probably have a successful business year. Now here the term that we have used probably is again not very stochastic in nature.

It is just our guess or feeling that we will have a successful business year. Now what do you mean by successful? When would you say that your business has been successful in this year? This word itself is very imprecise in nature. The experience of expert A shows that B is likely to occur. Now again B is likely to occur what degree of probability would you assign to the word likely? That is really imprecise. Expert C is convinced that this is not true. Now here you see convinced and it is likely. These are two different terms. Convinced means you are sure that the certainty is one. Now again in order to bring out the difference between probability and uncertainty in terms of the imprecise aspects such as the fuzzy weight aspects that we are talking about today.

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We can look at this sentence:

A person suffering from hepatitis shows in 60% of all cases a strong fever. That means in 60% of all cases so this is rather probabilistic. But what is imprecise? Strong fever, when do you call a particular fever or temperature to be strong. In 45% of all cases a yellowish colored skin.

What inch of the skin would you say yellow?  
Which tinge will you say to be surely yellow?  
Which one is more or less yellow?  
Which one is yellowish?

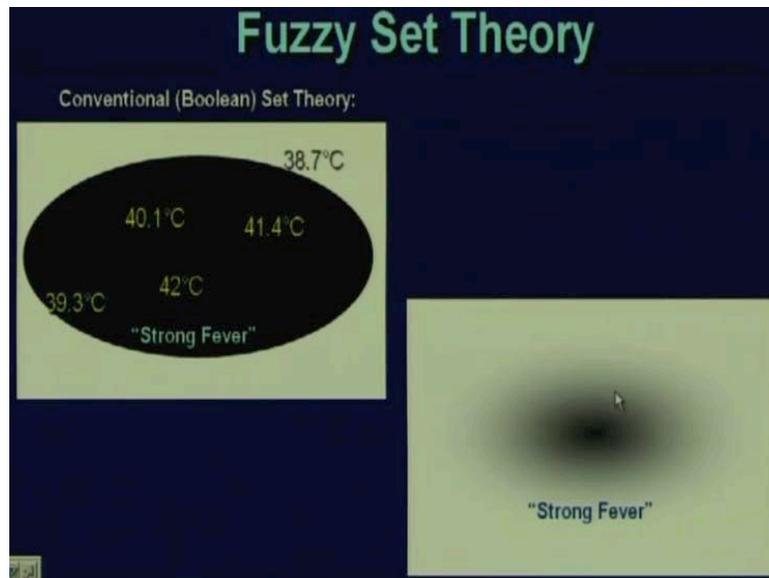
This term yellowish is again imprecise and vague and in 30% of all cases they suffer from nausea. Now 30% of all cases they suffer from nausea so the symptom nausea is present in 30% of all cases so this part is certainly probabilistic. Whereas this first part, a person suffering from hepatitis shows in sixty percent of all cases. Now this is probabilistic, strong fever is again imprecise so that is fuzzy. In 45% of all cases it is yellowish colored skin. This is also a mix of probabilistic and fuzzy. So here we will show you a conventional Boolean set that is a conventional set.

Now this slide has been taken from another source so these measures of temperatures are not like the ones that we do in India. But in west they measure the body temperature in degree Celsius. So, if we have strong fever as a set then here you see 40.1 degree Celsius is certainly strong fever, 41.4 degree is strong fever, 42 is strong fever and 39.3 is strong fever, and 38.7 goes absolutely out of this set so this is not a strong fever. But the question remains that when I say 39.3 to be strong fever why should not I consider 39.2 to be strong fever as well? And if I consider 39.2 to be strong fever why should not I consider 39.1 or 39 or 38.9 and in that way 38.7 also to be strong fever? Can I make such a crisp boundary?

In many cases where we use such terms it is very difficult to make such a crisp boundary. On the other hand, in this diagram you see that the boundary is not very crisp it is gradually fading out so at the core it is certainly strong fever and as I go out

it is becoming gradually faded so it is becoming less and less of an element of strong fever. This is a critical term that I have used. It becomes less and less of an element.

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Gradually its belonging to the set is becoming less. Here either an element belongs to the set or it does not belong to the set. It is a binary decision yes or no. On the other hand, here we can see that 40.1 degree is inside the set, 42 is inside the set, 41.4 is also inside the set, 39.3 is slightly in this shaded region and 38.7 is even farther. So, in order to capture this scenario fuzzy set theory was proposed.

Always there is a very close correspondence between a set and a and logic because in logic we use set. So when we say a particular set let us say we take any particular set a set X in which there are elements a, b, c then obviously we can say that a belongs to X and suppose d is outside the set d does not belong to X. So this can also be expressed as predicate that belongs to (a, X), does not belong (d, X). Might be d is belonging to another set Y so belongs (d, Y). So this belongs is a predicate and therefore this belongingness of a set or definition of a set immediately maps to predicate. So, from set we can always come to predicate logic.

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**Fuzzy Logic**

Fuzzy Logic : Reasoning with qualitative information

This is more realistic than predicate calculus, because in real life we need to deal with qualitative statements.

Examples:

In process control: Chemical plant:

Rule : If the temperature is moderately high & the pressure is medium, then turn the knob slightly right.

Now we say that fuzzy logic deals with qualitative information, the information that is qualitative, we always do not have qualitative figures to deal with it. This is more realistic than predicate calculus because in real life we need to deal with qualitative statements. Here is an example from a chemical plant. We often use rules like this: If the temperature is moderately high and the pressure is medium then turn the knob slightly right. Now, if the temperature is moderately high that is this part, earlier we had dealt with predicates like; if age of Tom is 50 and weight of Tom is 65 here you see everything was precise. But here we are using, if temperature is moderately high. That means what if I write in this way? **If- is temperature** and then write moderately high. Now this is no longer a precise term. So here we are facing a difficulty but it is nevertheless more realistic and the pressure is medium.

Which pressure is medium?

Turn the knob slightly.

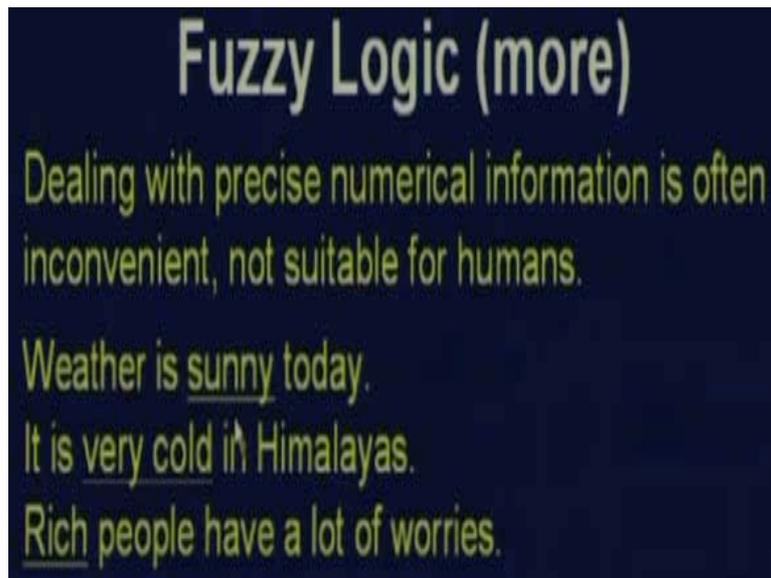
What angle would make it slightly right or slightly left?

So we use this sort of terminologies. So dealing with precise numerical information is often inconvenient because it is not very suitable for human beings.

If you ask the expert to say what exactly the pressure should be, he will not say that because he has got a feeling that if the pressure is medium turn the knob slightly right, exactly how much will not be told like 42.7 degree or 40.56 degree not to that precision that is more realistic and so here are some more examples which deal with such imprecise statements and these impreciseness are coming from the power of language, linguistic variables these are called as.

Natural language allows us to express in a much more comfortable way. And in order to model it in the quantitative world often we have to translate that into a quantitative statement and that translation is often artificial. So we are trying to be as natural as possible in this case.

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**Fuzzy Logic (more)**

Dealing with precise numerical information is often inconvenient, not suitable for humans.

Weather is sunny today.

It is very cold in Himalayas.

Rich people have a lot of worries.

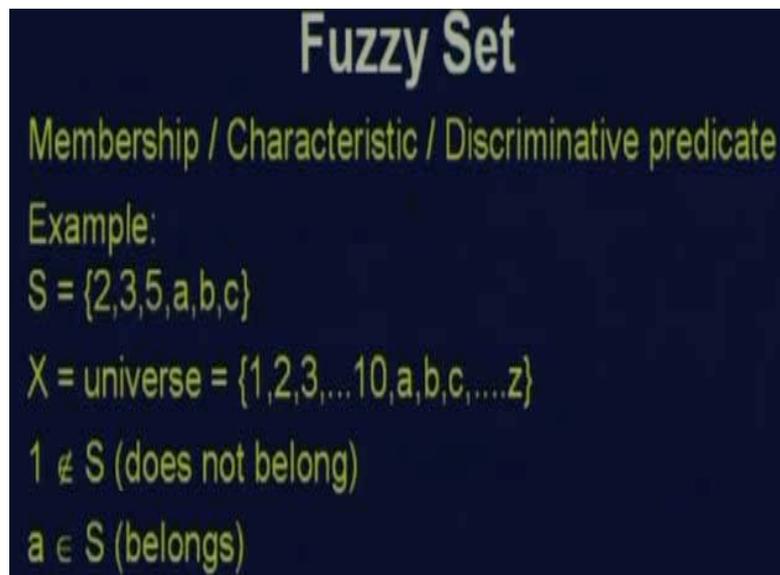
The weather is sunny today, what do we mean by sunny?  
It is very cold in the Himalayas.

Rich people have a lot of worries.

Now, when would you call a person to be rich? When would you call temperature to be very cold?

All these things are imprecise.

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**Fuzzy Set**

Membership / Characteristic / Discriminative predicate

Example:

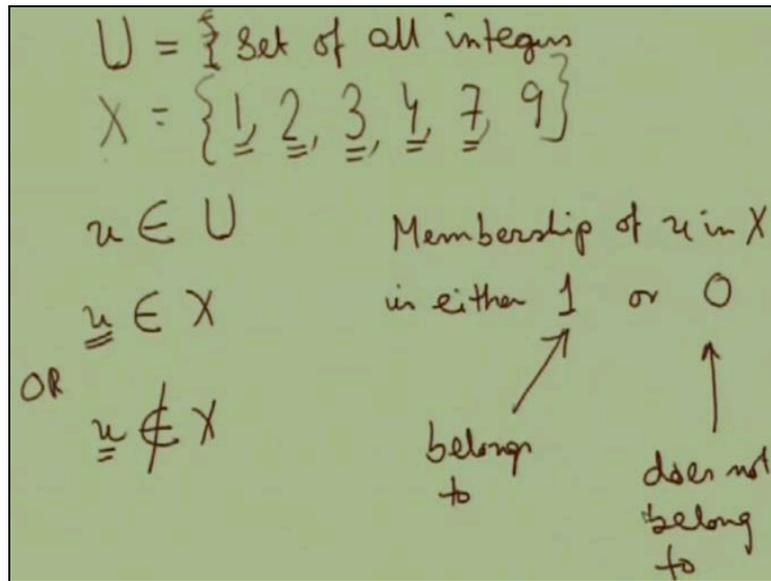
$$S = \{2,3,5,a,b,c\}$$
$$X = \text{universe} = \{1,2,3,\dots,10,a,b,c,\dots,z\}$$

$1 \notin S$  (does not belong)

$a \in S$  (belongs)

A fuzzy set differs from crisp set in terms of membership. In the case of a crisp set whenever we look at a crisp set any particular set X which consists of 1, 2, 3, 4, 7, 9 then any of these elements either belongs to this set or it does not belong to the set.

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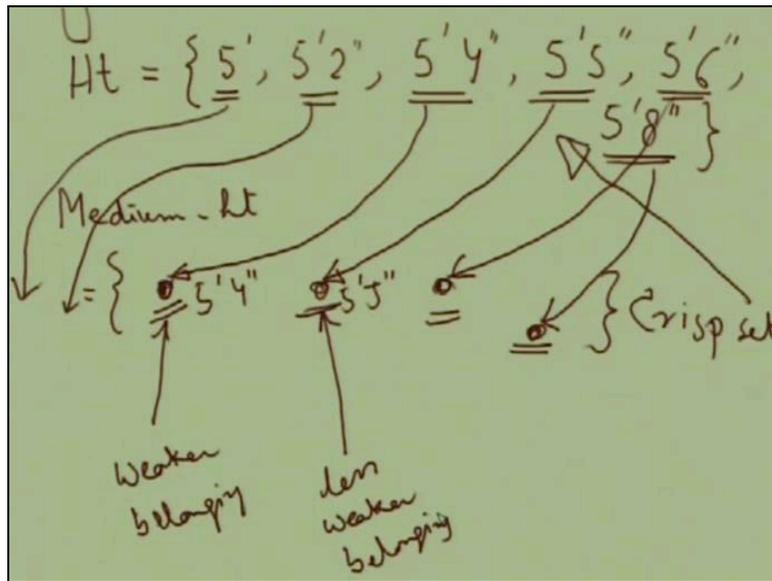


So, if I take a super set a universal set that is a set of all integers now given any  $u$  belonging to this. So if I take any integer either that will belong to this  $X$  or it will not belong to this  $X$ . So the membership of this in the case of a crisp set, membership of  $u$  in  $X$  is either 1 or 0. That means belongs to or does not belong to and there is nothing in-between. Whereas in the case of fuzzy set the thing is different.

We will use the same notion of membership but if I now change this set to a fuzzy set then let us see how can we talk about it. Suppose now I say that it is a set height which is 5 ft, 5 ft 2 inches, 5 ft 4 inches, 5 ft 5 inches, 5 ft 6 inches, 5 ft 8 inches etc and I define a set medium height. Now this is my universal set  $U$  and this set is a conventional crisp set. For example, 6 ft is not in this set and in this discourse, 5 ft 5 inches is there.

Now, what would that set medium height be? Will 5 ft 4 inches be in medium height? Suppose you say it is there then 5 ft 5 inches you think is also in medium height? 5 ft 6 inches will also be in medium height, 5 ft 8 inches will also be in medium height, and you say that these two will not be there. These are outside the set so I have taken these four. But the question is, if you consider 5 ft 4 inches is it belonging to medium height as strong as 5 ft 8 inches or 5 ft 6 inches? Probably it is not. This 5 ft 4 inches has got a weaker belonging and this 5 ft 5 inches has less weaker belonging. Whereas 5 ft 6 and 5 ft 8 certainly belong to this set. So the degree of belonging of these elements requires to be specified. But this was not the case when we are discussing about crisp set.

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We can define with the example of a set 2, 3, 5 a, b, c and suppose we have got a universe of discourse now 1 does not belong to S and a belongs to S so that is a crisp set.

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### Basics of Fuzzy Set Theory

Generalization of crisp set theory.  
Fundamental observation:  $\mu_S(x)$  is no longer 0/1. Rather  $\mu_S(x)$  is between  $[0,1]$ , both included.

Example:  
Crisp Set,  $S_1 = \{2,4,6,8,10\}$   
 $\mu_{S_1}(x)$  is a predicate which denotes x to be an even number less than or equal to 10.  
Given any 'a' which is a number, the  $\mu_{S_1}(x)$  question produces 0/1.

But if we try to generalize it in the case of fuzzy set, in the crisp set the membership is either 1 or 0. But in the case of fuzzy set the membership of an element X in a set S is no longer 0 or 1 rather  $\mu_X$  is between 0 and 1 and both are included. Now **this  $\mu_X$**  requires a little bit of explanation. I take the example of medium height here. Now I have got an element x belonging to a universal set X.

For example, some element x belonging to the set height which was a crisp set. Now I am defining a fuzzy set medium height or medium which is a subset of this universal set height or this thing X, these two are the same thing. Now, when I define medium I

can say that 5 ft 4 inches has got a belonging of point three in this set. And 5 ft 5 inches has a belonging of 0.5, 5 ft 6 inches has a belonging of 0.8. Now these are rather subjective and are coming from the users' notion. It is your So in this set what we have actually done is, along with every element of X. I have own notion of medium height. And 5 ft 10 inches is certainly medium height. put in a membership function mu of this particular element x in the fuzzy set medium and that is followed by that particular value of x.

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$x \in X$        $x \in \text{Height}$   
 Fuzzy set     $\text{Medium} \subseteq X$   
 $\text{Medium} = \left\{ \frac{0.3}{5'4''}, \frac{0.5}{5'5''}, \frac{0.8}{5'6''}, \frac{1.0}{5'10''} \right\}$   
 $= \left\{ \frac{\mu(x)}{x} \right\}$        $\mu(x)$   
 $\mu_m = 0/1$  for crisp      = Membership of  
 $\mu_m = [0, 1]$                        $x$  in  $M$

Just as in the case of crisp set this  $\mu(x)$  in a fuzzy set medium would be either 0 or 1 in the case of crisp set but for fuzzy set it will be in the interval 0 and 1. It can be 0, 1 or any number in-between. So this is  $\mu(x)$  in the set M where M is a fuzzy set which means membership of x in M. This is also known as degree of membership. So this is a generalization of the conventional set theory. Here you can see that mu in a crisp set  $S_1$  is a predicate which denotes x to be an even number.

So here is the final definition that I was trying to explain. Boolean or crisp set A is a mapping for the elements of S to the set 0, 1. That is, A is mapping elements of S to 0 and 1. So a characteristic function for a crisp set is 1 if x is an element of the set A, 0 if x is not an element of the set A.

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## Fuzzy sets

- Boolean/Crisp set  $A$  is a mapping for the elements of  $S$  to the set  $\{0, 1\}$ , i.e.,  $A: S \rightarrow \{0, 1\}$
- Characteristic function:
- $\mu_A(x) = \begin{cases} 1 & \text{if } x \text{ is an element of set } A \\ 0 & \text{if } x \text{ is not an element of set } A \end{cases}$

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**Fuzzy set  $F$**  is a mapping for the elements of  $S$  to the interval  $[0, 1]$ , i.e.,  $F: S \rightarrow [0, 1]$

Characteristic function:  $0 \leq \mu_F(x) \leq 1$

1 means full membership, 0 means no membership and anything in between, e.g., 0.5 is called **graded membership**

On the other hand, fuzzy set  $F$  is a mapping for the elements of  $S$  to the interval  $0, 1$ . So, characteristic function  $S$  a membership function of  $x$  belonging to  $F$  will lie between 0 and 1 where 1 means full membership, 0 means no membership and anything in-between for example 0.5 is a graded membership that is telling us the degree of its belonging. So we can say that  $\mu(x)$ . Again  $x$  is an element of the crisp set  $S$  and this is also a predicate, it is a membership predicate. Now let us give some examples.

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## Membership Predicate

$\mu_S(x)$   
where  $x \in S$ ,  $S$  is a crisp set.

In the reverse way,  
Given any predicate, we can describe a set, trivially.

Given  $P(x_1, x_2, x_3, \dots, x_n)$   
the set that is defined with  $P$  is the set of tuples  $\langle x_1, x_2, x_3, \dots, x_n \rangle$  for which  $P$  is true.

Set  $\longleftrightarrow$  Logic

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### Example: Crisp set Tall

- Fuzzy sets and concepts are commonly used in natural language
  - John is tall*
  - Ravi is smart*
  - Sajni is happy*
  - The room is hot*
- E.g., the crisp set *Tall* can be defined as
- $\{x \mid \text{height } x > 1.8 \text{ meters}\}$ 
  - But what about a person with a height = 1.79 meters?
  - What about 1.78 meters?
  - What about 1.52 meters?

Fuzzy sets and concepts are commonly used like for example, John is tall, Ravi is smart, Sajni is happy, and the room is hot. Now, for example, the crisp set tall can be defined as  $x$  where from  $x$  is coming?

$x$  is coming from the set height and we say that all the  $x$  which are greater than 1.8 meters will belong to the set tall. So tall is a set consisting of all  $x$  where  $x$  is greater than 1.8 meters.

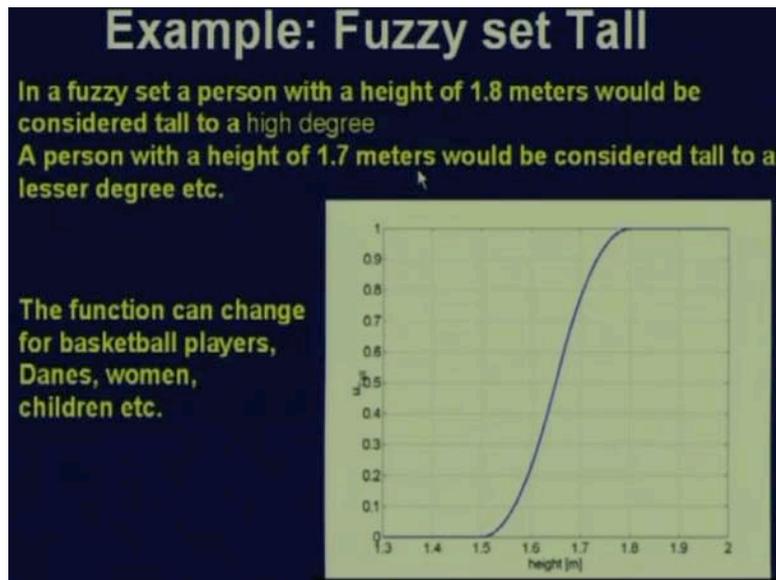
What sort of set is this? A crisp set or a fuzzy set?

If I define in this way then it is certainly a crisp set because any number any height that is greater than 1.8 meters will certainly belong to the set tall. And if it is less than 1.8 then it will not belong to that set. But such definitions of tall would put us in trouble because what about a person of height 1.79 meters? What about 1.78 meters? What about 1.52 meters?

Obviously, logically you will think that 1.8 is in the set tall. We should not be so much cruel to 1.79. We should consider that to be also tall but not certainly, 1.78 should also be in the list tall but then how would I distinguish between somebody who is 1.8 and somebody who is 1.78? So the crisp set notion does not allow us to do that.

On the other hand, in a fuzzy set a person of height of 1.8 meters would be considered to be tall to a high degree and a person with a height of 1.7 meters would be considered tall to a lesser degree. Therefore I can define a function to describe the fuzzy set. For example, here is a set which is a typical graph.

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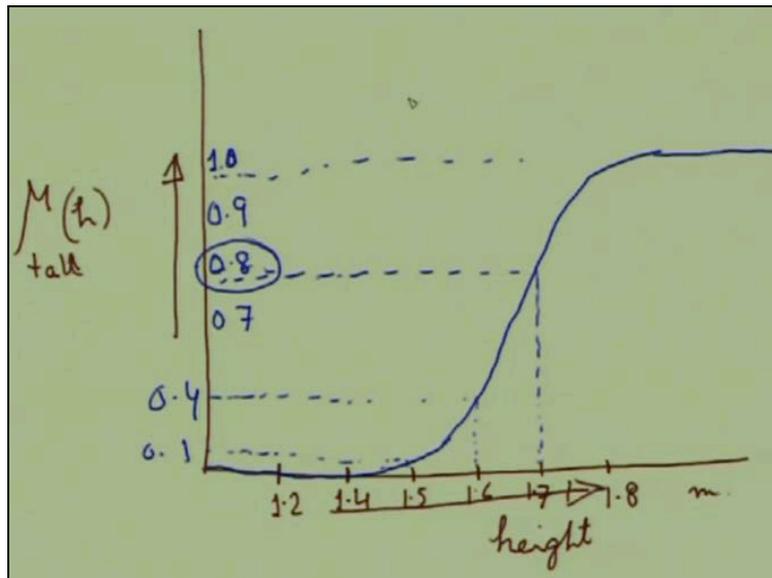


What is the x axis of this graph?

On this side of the graph we have got height and on this side we have got the membership. We are having two axes. On this axis I plot the height in meters and this height is coming from the crisp set that is the universe of discourse. Now it may be 1.2 meter, 1.4 meter, 1.5 meter, 1.6 meter, 1.7 meter, 1.8 meter in meters. On this axis I am plotting the membership of a particular height  $h$  in the set tall because I am going to define that set tall. So, in tall the graph may be something like this. Over here up to this I am not considering them to be tall but then it is going up and here it is saturating to 1 so this membership is 1.0 and here it is 0.9, 0.8, 0.7 and so on. Therefore here this graph is mentioning my subjective interpretation of the concept tall.

Now if I come across some person who has got a height 1.7 then simply I will project this over here and also give that a membership and place in tall and but with a membership of 0.8. If somebody comes with 1.6 then I will also give him a place maybe with a degree 0.4. If 1.5 comes in he is also not deprived and probably get .1 and below that everybody is getting 0 or something. Now this is my one particular definition of tall. But again now let us say I am trying to select a basketball team and I have been told to select only tall people. In that case my notion of this fuzzy set I may like to change because I may like to have again the same axis and on this side I will have height again like 1.8, 1.7 where all the same figures are here and on this side-side I am putting in  $\mu(\text{tall})$  or anything maybe  $h$ .

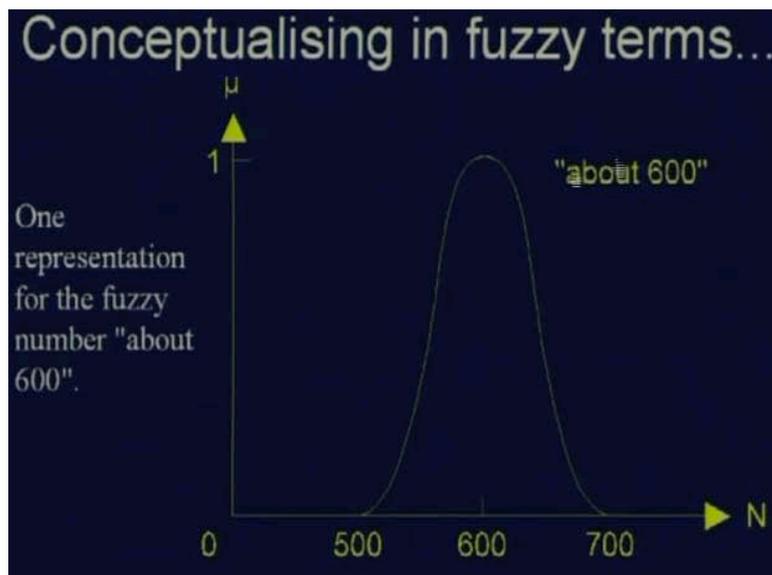
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Now I can redefine my fuzzy set in a different way because I am rather strict about the height so maybe I will make my curve something like this. I will make it very slow and make it 1. So somebody who is 1.8 will certainly get a membership 1. Somebody is 1.7 will get a very low membership.

The earlier curve was a little different it was a little flattened something like this. So, there 1.8 also got a membership, 1.7 got a quite some membership but when my purpose is to select a basketball team I will go for a steeper curve like this. That is what my notion of tall is when I select the basketball team. That is how the fuzzy sets are defined.

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Here some numbers and I want to say give me a number which is above 600, what do you mean by that? How much money would you need?

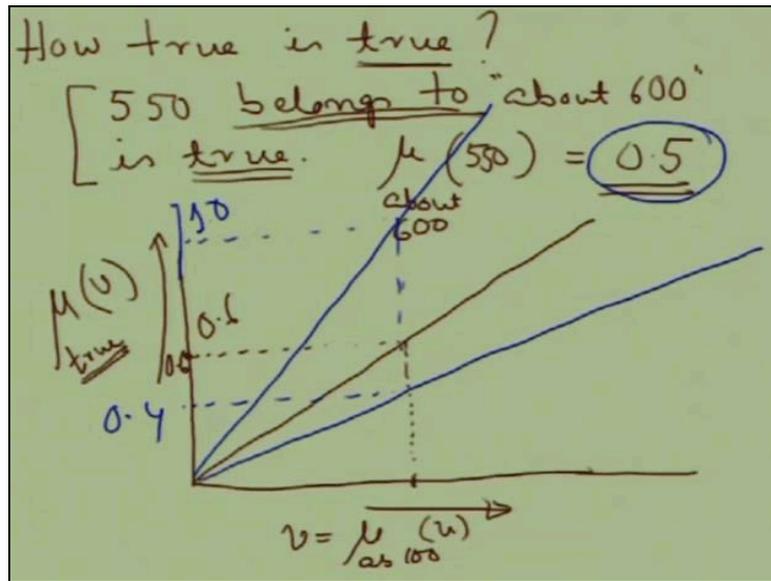
I would need about 600 rupees. Now what do you mean by about 600 rupees?  
Now if I want to conceptualize this notion about 600 then I have to define this about 600 to be a fuzzy set over a crisp set of these numbers 500, 600, 700 etc and the shape of the curve will be like this because for 600 certainly it is the membership of 1, 600 has the membership of 1. In the fuzzy set about 600 whereas 599 will have a very high membership maybe 0.97 or something here, 550 is it about 600?  
Well, by looking here we can say it is around 0.5, it is about 600, its membership in the fuzzy set 600 is about 0.5. And for 520 its membership is around 0.2, what about 650? Its membership is again 0.5. So here this would be a very natural curve of distribution of the membership to the fuzzy set about 600. Here is another example about conceptualization. I am defining a set.

Here is a rather philosophical question, how true is true?  
That means I say well what you said is true or I say 550 belongs to the fuzzy set about 600 this statement is true. In predicate logic we were saying that a predicate evaluates to true or false. There the concept of true was 1 and the concept of false was 0. Now how true is true? 550 belongs to about 600 is true, I make this statement.

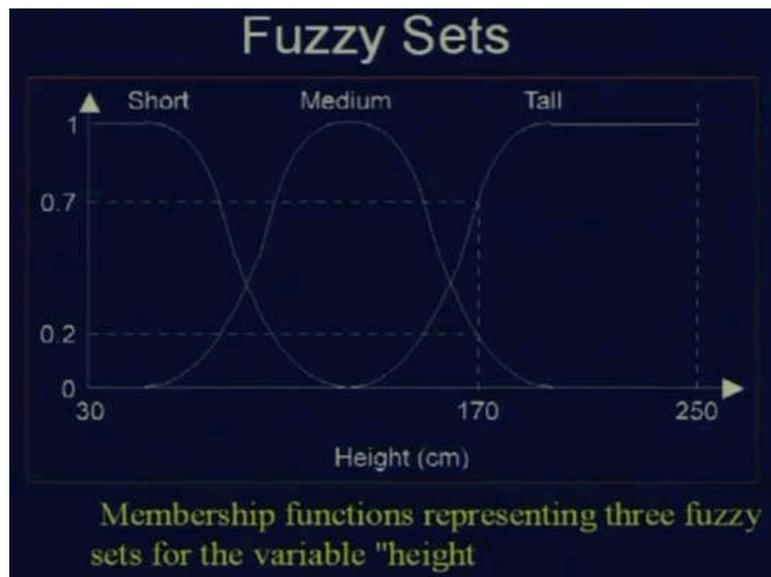
Now about this truth should I give 1 or what?  
In the earlier diagram we have seen that 550 has got a membership  $\mu$  of 550 in about 600 in that fuzzy set was 0.5. Now membership of 0.5 means its belonging was to the degree 0.5 then it belongs to about 600 is it true?  
Now I can define truth in fuzzy term where here I put in  $\mu$  of about 600 and the different values of this thing. And here is the membership in a fuzzy set I define true to be a fuzzy set  $\mu$  true of  $v$ . Now this 0.5 was somewhere here. If I define the truth fuzzy set to be something like this and this curve is such that it is something like 0.6.  
But you may be even stricter than me and say that this is not my notion of truth, my notion of truth is something different. My notion of truth is this. My notion of truth is very steep and this is dilated so your notion of truth is this.

In that case in your notion of truth the statement 550 belongs to about 600 how true it would be?  
It will map here and its membership will come to 0.4. Whereas another person may be a bit more lenient because he feels that any membership that is more than 0.2 then I can say it belongs to it a particular set. So we will go for this curve and the same membership of 550 in about 600 is point five and that will be taken up here and it will evaluate to might be 1. So in this way the notion of truth can also be modeled in fuzzy set.

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Here is another description of a fuzzy set. On the x axis we have plotted height, on the y axis we have plotted the membership of the different heights in different fuzzy sets. Here we have defined three fuzzy sets; one is tall, one is medium, one is short. Now, if you follow the curve for short it is coming like this that is, height of 30 centimeter is certainly short but height of around this 0.50 centimeter in short but is up with membership 1 that is somewhere here, height of 80 centimeter its membership in short is 0.7, height of 100 centimeter its membership in short is 0.2 etc whereas in medium height the distribution is something like this whereas for tall the distribution is something like this. So we capture the different notions.

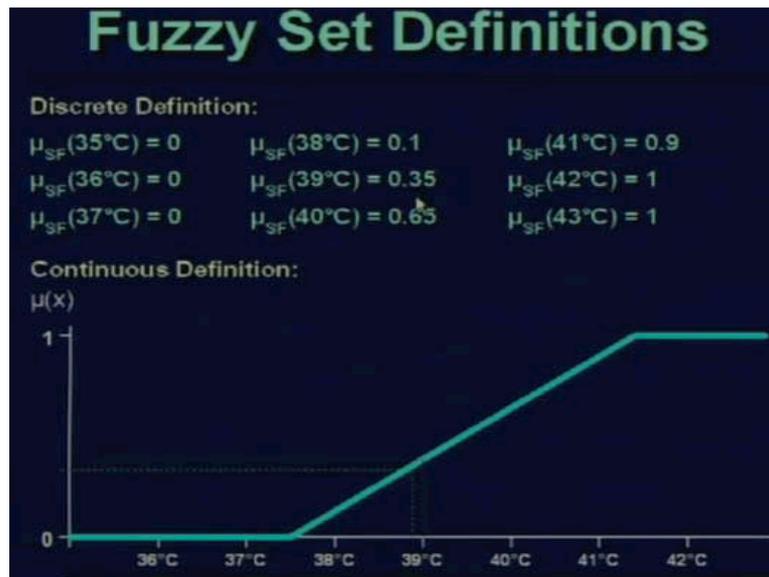
Now you can ask that where from am I getting these figures?

Though there are different ways of doing it these figures are representing pure conceptualization of a particular term. You say that the room is hot. You are saying it

from a feeling and your feeling may differ from mine. So up to what room temperature would you really tolerate and after that you would say it is becoming hot and the AC should be turned on?

That is your notion. So based on your notion of the term you have to find a particular distribution of the membership values and that will define the fuzzy set. Here again we are revisiting that strong fever case. In conventional Boolean set theory it was like this and in fuzzy set it is represented in this way.

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Now we can define discrete definitions we can say strong fever. If I say strong fever is 35 degree these are zeros and these are discrete definitions and I can have a continuous definition like this. Discrete definition is, 43 degree centigrade membership in strong fever is 1, 42 degree centigrade membership is 1, 41 degree centigrade membership is 0.9, 40 degree centigrade is 0.65 and so on and so forth. So I have discretely taken up all the values but in this way if I want to tabulate it will be difficult because since it is a continuous domain I have to write for 350.5 degree, 35.4 degree and everything else. If I select some discrete values I can do it in this form and I can represent it in the form of a table. Whereas this is a continuous definition here, this is a curve going up so we can take any particular value and come to this point and find out the membership. Therefore 41 degree membership is somewhere here which is 0.9, 39 degree membership is somewhere here and 38.9 degree membership is somewhere here.

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**Describing a set**  
A set is derived in one of the two ways.  
(a) By extension  
(b) By intension  
(a) requires 'listing'.  $S_1$  is  $\{2, 4, 6, 8, 10\}$  -- needs finiteness.  
(b) needs a closed form expression related to properties.  
 $S_1 = \{x \mid x \bmod 2 = 0 \text{ and } x \leq 10\}$   
 $S_2 = \text{set of tall people}$   
 $\mu_{S_2}(x) \in [0, 1]$   
What is  $x$ ?  
 $x$  is called the underlying numerical quantity.  
 $x = \text{height}$

Since we have to work with computers how can we describe a set?

A set can be described in two ways. One is by extension **or** by intension. Even in the case of a conventional set we are familiar with these notions that I can either extensively enumerate that my set  $X$  consists of 1, 5, 3, 7 in that way I can go on and this is one sort of definition.

Another sort of definition is I can define a set  $Y$  to be all  $x$  such that  $x$  is greater than 5. That means I am defining a set 6 onwards,  $x$  is greater than 5 if I say only integers 6, 7 etc. This is a definition by extension and this is an implicit definition. Now this is also the case in case of fuzzy sets. So we can define  $S_2$  to be a set of tall people and we can put in a membership to this.

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**Other representations of fuzzy sets**

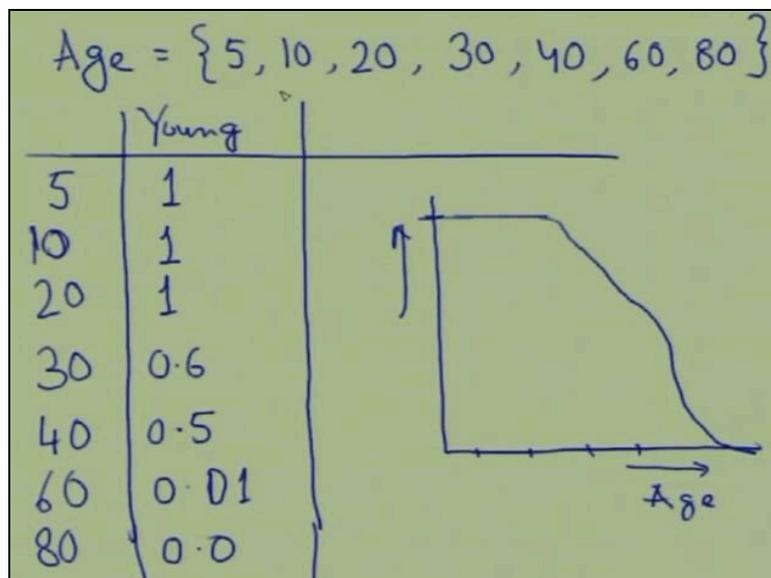
- A finite set of elements:  
$$F = \mu_1/x_1 + \mu_2/x_2 + \dots + \mu_n/x_n$$
  
+ means (Boolean) set union
- For example:  
$$\text{TALL} = \{0/1.0, 0/1.2, 0/1.4, 0.2/1.6, 0.8/1.7, 1.0/1.8\}$$

Now in the case of fuzzy set we can define it in this form; tall 1.0 its membership in tall is 0, 1.2 its membership is 0, 1.4 its membership is 0, in that way I go on. Now in some books you will find that this comma is replaced by this plus. Now this plus is not addition this means union. So I am talking of a set where this element union this element union this element so all these things are together.

Now how do we represent a fuzzy set in a computer?

I am defining a set, for example I have got a set age where age is 5, 10, 20, 30, 40, 60, 80. Now I can define different fuzzy sets and also represent them in the form of table. For example, I define one fuzzy set young and here I put in 5, 10, 20, 30, 40, 60, 80. Now I put in the memberships that 5 is certainly young, 10 is certainly young, 20 is certainly young, 30 is .6, 40 is 0.5, 60 is .1 or .01 and 80 is 0 so I can also draw the same thing.

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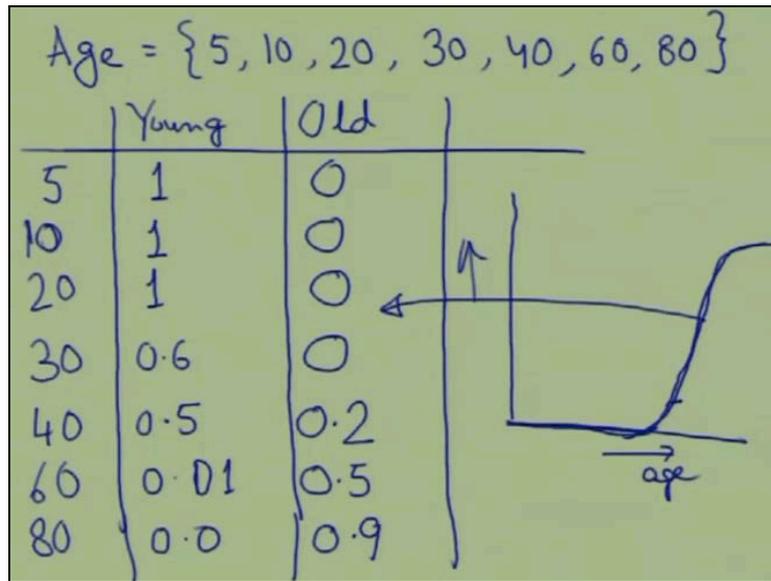
This is my definition of (.....)young.

So here it is age and my values are here 5, 10, 20, 30 like that.

What are these curves then?

Here is its membership so for 5, 10, 20 the value is 1 and after that it has come to 0.6 and after that it has come to 0.5 and then there is a short fall. This is the graph that is representing young and the same thing can be represented in the form of a table. That is one way we can represent the fuzzy set in a computer. Now similarly I can define another fuzzy set say old where obviously 5 is not old, 10 is not old, 20 is not old, 30 is not old here you say 0.2, 60 is 0.5 and 80 is 0.9, 90 might be 1. So if I describe this fuzzy set then also I have actually mapped a particular curve.

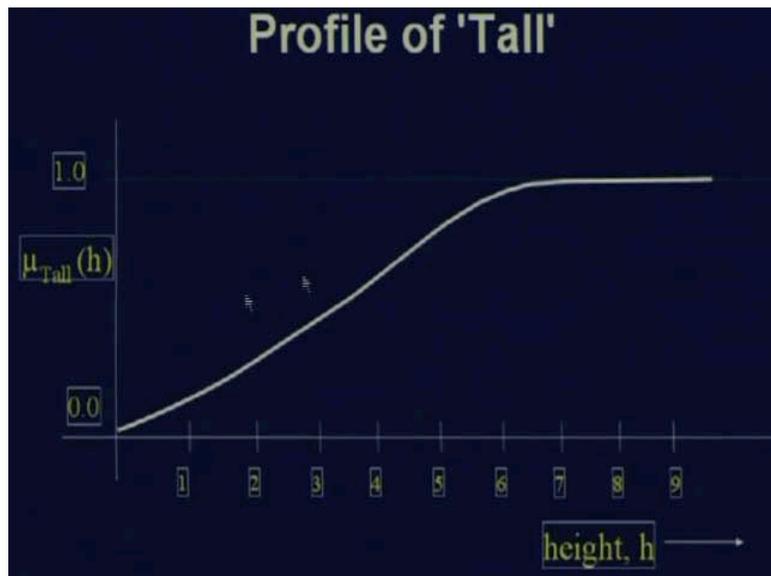
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What is this curve?

If this be the age here and the membership on this side these are all zeros and after 40 it is 0.5 and then 0.9 and something of this sort. Now this curve can be mapped to this form of a table. So the curves that we are showing you till now can be represented in the form of a table. Therefore it is amenable to be represented in a computer. Here define a fuzzy set adult. Just put in the membership values here and that will define your notion of adult. This is one way in which fuzzy sets can be represented in a computer. So we always describe these things in terms of some profiles and these profiles reflect our conceptualization of the fuzzy term like tall.

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Here is a profile of different heights and this is my notion of tall and what are my axes here?

This is the membership one, this is the membership of h where h is any variable on this, membership of h is in the fuzzy set tall this is the profile of tall you must have realized this by now.

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## Shapes of Profiles

Shapes of profiles are obtained from experiments and expert judgement.

Statistically obtained by % count (sometimes).

Profile itself is somewhat "vague".

Now there are different shapes. How do we get the shapes of profiles?  
 Shapes of profiles are obtained from experiments and expert judgment. We sometimes carry out some statistical experiments like I fill up a questionnaire and go to 1000 people and say would you consider 6 ft to be tall, would you consider 5 ft 8 inches to be tall and then we see how many people have really said that 5 ft 8 inches should belong to tall and based on that we try to put in some measure of membership. But however this profiling all said and done is some somewhat vague and this has to be done by the expert or the user who wants his or her notion of fuzzy set to be captured.

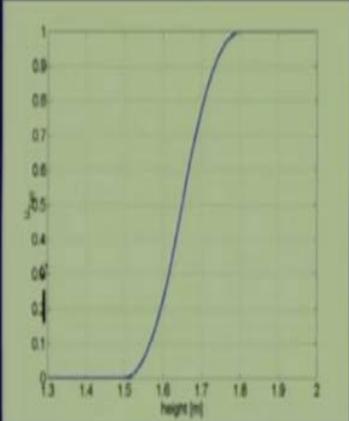
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## Membership functions: S-function

The S-function can be used to define fuzzy sets

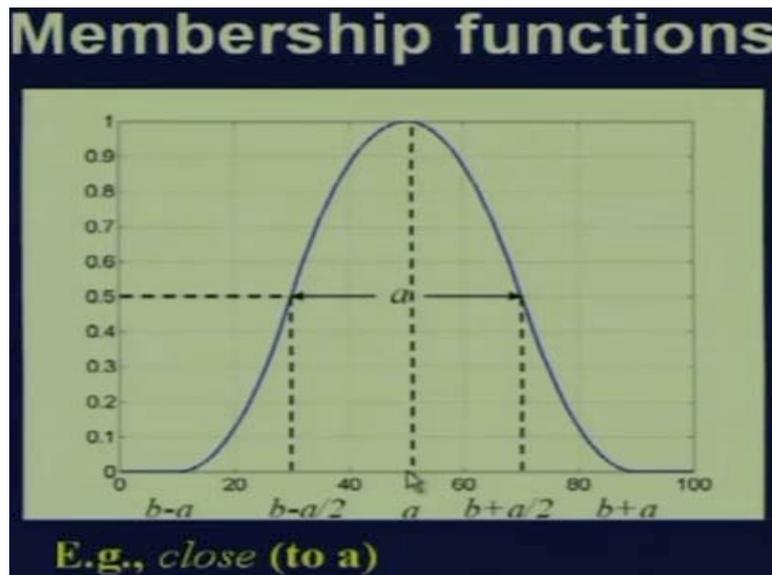
$S(x, a, b, c) =$

- 0 for  $x \leq a$
- $2(x-a/c-a)^2$  for  $a \leq x \leq b$
- $1 - 2(x-c/c-a)^2$  for  $b \leq x \leq c$
- 1 for  $x \geq c$



So there are different types of functions. Here is another type of function which is called the S function. This function this curve can also be expressed as for any  $x$  less than a particular value it will be 0. Between two points  $a$  and  $b$  it will be computed by some function. Between two other points it will be computed by some other function and after that it is 1. This is one type of profile. So this profile can be represented as a table or can be represented as this function which can be computed using a procedure in the computer.

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Similarly here is a very interesting thing. I say that a number which is close to  $a$ , now conceptually it is something like this;  $a$  is certainly close to membership 1, something which is in-between these two points if it is at  $b$  either plus on the positive side or negative side to 0.5 then for anything in-between we will have a membership and in that way we can have a bell type of curve. This is another way in which the membership functions can be represented. I would encourage you to find out a mathematical function and expression to represent this sort of a curve close to  $a$  so that we can also use it. We can of course tabulate it but we can also use it for computation as a function as a procedure.