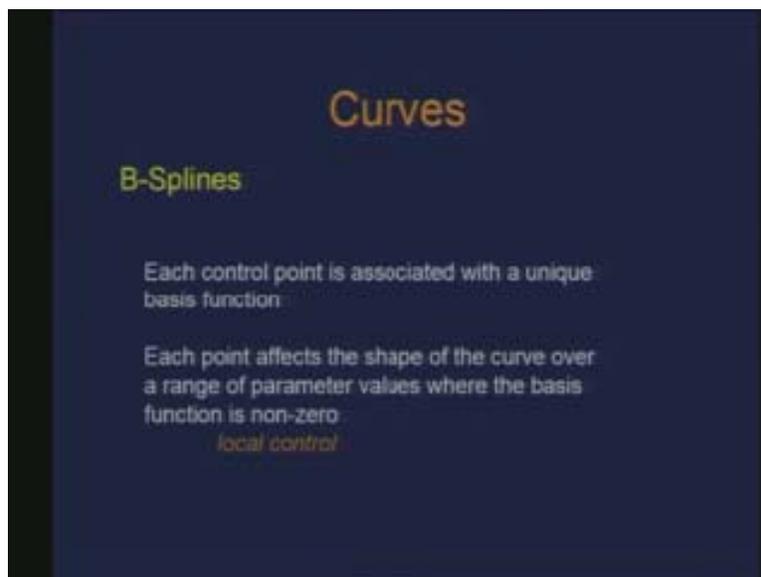


Introduction to Computer Graphics
Dr. Prem Kalra
Department of Computer Science and Engineering
Indian Institute of Technology, Delhi
Lecture - 15
Curves (Contd....)

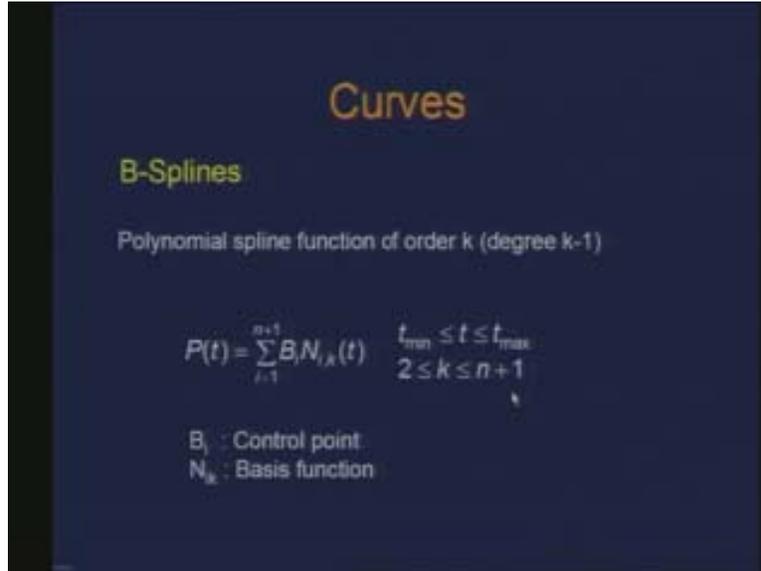
We have been talking about B-Spline curves. Here is a recapitulation of some aspects of B-Splines. What we observed in the case of B-Splines is that B-Splines enable us to define an associated unique basis function for each control point.

(Refer Slide Time: 01:31)



In turn what happens is that the influence, the way, the point on the control polygon affects the shape of the curve. It is actually in a range of the parameter values, it is not the entire range of the parameter values but it is a sub range of the parameter values. And in turn we get the local control which is not the case in the case of Bezier curves. Also, we observed that these B-Splines help us decoupling the order of the curve or the degree of the curve and the number of control points you specify for the B-Spline polygon. So the polynomial spline function which is used as the basis for B-Splines is basically defined as these $N_{i,k}$'s where k is the order of the curve.

(Refer Slide Time: 03:00)



Therefore it ranges from 2 to $n + 1$ where $n + 1$ is the number of control points you specify. So the maximum order of the curve which you can obtain is $n + 1$ and the parameter t ranges from some t_{\min} to t_{\max} and these B_i s are nothing but the control points. This is a very standard way of defining the parametric curves as some sort of a blending of the geometric information specified through the control points. And if we see how these $N_{i,k}$'s are computed they are computed in a recursive manner where the base cases for $N_{i,1}$ s which is nothing but some sort of a step function which you find as 1 or 0. So it is 1 when the parameter t is in the knot values x_i to $x_i + 1$. So there is also a notion of knot vector which is in some way the junction points for the parameter domain for the curves which are defined. So x_i and $x_i + 1$ are nothing but the pair of knots.

(Refer Slide Time: 03:44)

The slide is titled "Curves" in orange. Below it, "B-Splines" is written in yellow. Underneath, "Cox-de Boor Recursive Formula" is written in light blue. The slide contains two mathematical formulas for B-spline basis functions. The first is a piecewise function:
$$N_{i,1}(t) = \begin{cases} 1 & x_i \leq t < x_{i+1} \\ 0 & \text{otherwise} \end{cases}$$
 The second is a recursive formula:
$$N_{i,k}(t) = \frac{(t - x_i)W_{i,k-1}(t)}{x_{i+k-1} - x_i} + \frac{(x_{i+k} - t)W_{i+1,k-1}(t)}{x_{i+k} - x_{i+1}}$$
 At the bottom, it states: x_i 's are the knot values $x_i < x_{i+1}$.

Then the $N_{i,k}$ is just defined in a recursive way using the $N_{i,k}$ plus 1 and $N_{i,k}$ minus 1. These are some combination of these. And the x_i is the knot values and there is a property to these knot values that they are monotonically increasing. In particular we observed that there could be a number of ways in which we can define or give the x values or the knot vector x .

(Refer Slide Time: 05:25)

The slide is titled "Curves" in orange. Below it, "B-Splines" is written in yellow. Underneath, "Knot vector X can be:" is written in light blue. Below this, three options are listed: "Uniform (periodic)", "Open-Uniform", and "Non-Uniform".

So there could be uniform x or uniform knot vector or periodic knot vector. It is periodic because the $N_{i,k}$ s which are computed using these knots turn out to be periodic in nature.

Or it could be open uniform so there we have the multiplicity of the knots at the two ends. So the knot is repeated k times that both the.....