Earthquake Geotechnical Engineering

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Lecture 47

Static Pressure on Retaining Walls

I welcome you all for this NPTEL online course on earthquake geotechnical engineering. Today we are going to have lecture number 47 and this is on slope stability in retaining walls that is under module 5. We already covered slope stability and we are discussing retaining walls, 5 lectures we already discussed for the slope stability and we will have another 6 lectures for retaining walls. So, one is already over 46 lecture which was introduction to retaining walls. Today in this lecture number 47 we are going to talk about static pressure on retaining walls that means, it will be the as for this module is concerned this will be chapter 5 for this module, chapter 6 from this module because 4 chapters was on slope stability and fifth one was on retaining wall's introduction and the sixth one is on static pressure on retaining walls. So, coming to after once we already introduced what types of retaining walls are there in the lecture number 46 and today, we are going to deal exclusively with static pressure because it is important before we go for dynamic loading or seismic loading that how static pressure on retaining walls acts.

When we talk about static pressures on retaining walls, popularly two theories, Rankine theory and Coulomb theories are mostly used. I think during your undergraduate courses you would have gone through both the theories which are of the particular those are from civil engineering background and the students who have taken as geotechnical engineering and soil mechanics they know which is normally the compulsory course. Coming to these two theories Rankine and Coulomb theory we will discuss one by one in detail and before that let me acknowledge that the major source of information in this lecture is from Kramer's book. When we talk about static pressure on retaining walls, the seismic behavior of retaining walls will depends on the total lateral earth pressure that developed during earthquake shaking.

When we say total lateral earth pressure that means it will include the static as well as dynamic. So, the total pressure includes both the static gravitational pressures that exist before an earthquake occurs and the transient dynamic pressures induced by the earthquake shaking. So, retaining wall is influenced by both static as well as dynamic component. Static earth pressures on retaining structures are strongly influenced by wall and soil movements. So, for example, one is what you called active earth pressure another is called passive earth pressure and you may be perhaps already aware about that active earth pressure developed on a retaining wall when the wall moves away from the soil or we say the moves away from the backfill. So, as a result it will induce extension lateral strain or we say the tensile strains in the soil because the wall is moving away from the backfill or soil. So, it will try to pull the soil components. When the soil movement is sufficient to fully mobilize the strength of the soil behind the wall then we say minimum active earth pressure act on the wall. So, there will be a minimum active earth pressure and this minimum active earth pressure will be coming the point where the wall is like that is enough to start the movement of the movement like fully mobilize the strength.

So, at that point we say this is active earth pressure maximum minimum active earth pressure. If we go beyond that then it will be already failed. So, we do not require more than that because very little wall movement is required to develop minimum active earth pressure. So, that is the difference you require small active pressure. So, that is why it is called minimum active earth pressure. On another side when we discuss that this passive pressure this will be other way. So, retaining walls are usually designed on the basis if you have the free-standing walls which retaining walls free standing retaining walls are basically the walls on the both sides of the wall there is no backfill it is kind of a you know that no more wall. So, in that case normally they are designed based on the minimum active earth pressure, but normally the retaining walls are not in the case where you have the free-standing wall. The second category you may be perhaps aware that it is called massive passive earth pressure and this develop on a retaining wall when the wall moves towards the soil or backfill thereby producing compressive lateral strength in the soil. So, when the active earth pressure produced like tensile stresses while the passive earth pressure produces the compression or compressive stresses and strength. When the strength of the soil is fully mobilized in this case maximum passive earth pressure act on the wall. So, in case of because here you can understand like this in case of tensile stresses only minimum strain stress will require because like basically you can understand it is like this the soil initially have some compressive stresses confining stresses and when the tension starts and it will be after minimum tension which is just crossing after 0 value then your wall will fail. But on the compression side is this is not the scenario rather on the compressive side soil is able to take some compression compressive load or compressive strength. But there will be a point which we can say the maximum which is related to maximum passive earth pressure where the stresses the soil is unable to carry any more than that value. So, it will so, then the stability of many free-standing retaining walls depends on the balance between the active pressure acting predominantly on one side of the wall and passive pressure acting on the other side.

So, it is there in fact one side active pressure may act another side the passive pressure may act and accordingly it can be denied. Prediction of actual retaining walls forces and deformation is a complicated SSI problem. SSI means it is related to soil structure interaction problem where you have the soil is involved structure is your wall and so, that there is interaction between soil and structure like you know that wall. So, it is both a particular for dynamic loads. So, and it is not as easy problem.

Deformations are rarely considered explicit in the design. What is the typically for the design approach is to estimate the forces which are acting on a wall and then to design the wall to resist those forces with a factor of safety which is high enough to produce acceptably small deformation. So, rather than here point is this one that rather than designing the wall based on the deformation we estimate what is the forces which need to be registered by this wall. And once these forces are known then we consider for design purpose a factor of safety which is high enough. So, that we can expect that this when we keep high factor of safety that the deformation produced are small and they are within the permissible limit. So, it is this way that the we design based on the forces rather than on deformation. However, we check that deformations are within the limits. In number of simplified approaches are available to relate static loads and retaining walls and the most commonly used are Rankine theory, Coulomb theory. Then we have so, we will discuss these in detail both Rankine and Coulomb theory in this lecture itself. Then the third logarithmic spiral method and stress deformation analysis we will discuss in the next lecture that is lecture number 47 lecture number 48.

So, let us talk about the first one that is Rankine theory. In case of Rankine theory what is the assumption it was in give proposed by Rankine in 1857 and develop the simplest process for computing the minimum active and maximum passive earth pressure. By making the assumptions about the stress conditions and strength envelope of the soil behind a retaining wall that means the when we say behind the retaining wall that means backfill. Rankine was able to render the lateral earth pressure problem in a deterministic rather than because otherwise this problem is indeterministic and redundant, but with certain assumptions the problem can be deterministic and directly computing the static pressure acting on retaining walls. So, here the first of all in the Rankine one thing two assumptions are there. So, before I go ahead, I think two assumptions which are like it can be listed here. One assumption that wall is smooth that is there is no friction wall is smooth that is the assumption that means the friction between wall and soil is not considered in this case. Then the second assumption is wall is vertical. So, inclined wall is not considered that means when we say retaining walls it will be inclined, but not both sides. So, one side it will be inclined. So, this is like this. So, that means this is wall is vertical this side. So, this is vertical here is your backfill. So, it could be like this. So, this is vertical number one and then you have the delta which is angle of friction between delta equal to 0 between this.

So, with these assumptions good point with the Rankine theory it is able to deal with c phi soil able to deal with c phi soil that means it can be cohesive soils or it could be the this is sandy soil or like what is it or it will be combination both. So, let us talk about active pressure condition first then we will talk about pressure using the Rankine theory. For minimum active conditions Rankine expressed the pressure at a point on both on the back of retaining wall is given by this relation. It is the active earth pressure T A and what is K A? K A is a coefficient of minimum active earth pressure where c is called cohesion and sigma v you know that this is effective overburden pressure at a particular depth because effective overburden pressure will not be constant rather it will vary with the depth. So, as a result your P A this second quantity is constant this does not depend on the depth because c is cohesion is constant K A is depending on the phi.

$$p_A = K_A \sigma_v' - 2c\sqrt{K_A}$$

So, K A is a function of which we will discuss in the next slide is a function of angle of internal friction phi of the soil. So, for a given soil let us say if I consider the homogeneous strata then the second term will remain constant that is not going to change above along the depth of the along the depth. However, sigma v prime is normally gamma into h or gamma which is h is a height or I can say gamma into z. So, this will increase with the depth. So, as a result here there are two terms one term is constant in this equation second term while the first term increase with the depth.

Now, what is K A here? K A is called the this is coefficient of active earth pressure and when the principal stress planes are vertical and horizontal as in the case of a smooth vertical retaining wall. So, two conditions have come here smooth vertical wall which retain a horizontal backfill. So, that means, the backfill return by the retaining wall is horizontal. So, this is the situation. So, this like backfill return is like this. So, it is horizontal. So, in that case the K A value is given by 1 minus sin phi 1 plus sin phi which can also be written 10 square 45 minus phi by 2. So, that means, where what is phi? Phi is angle of internal friction of soil. So, using the value of phi you can calculate the value of K A. For the case of a cohesionless backfill inclined at an angle beta with the horizontal if c if we assume c equal to 0 then and if you have inclined case inclined means if I inclined this backfill by at angle beta that means, this is with the horizontal. So, beta is an angle which max you backfill inclined which is going here in clockwise the anticlockwise direction with the horizontal.

$$K_A = \frac{1-\sin\phi}{1+\sin\phi} = \tan^2\left(45 - \frac{\phi}{2}\right)$$

$$K_A = \cos\beta \frac{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}$$

So, backfill if I increase here then the equation this equation will be applicable when beta is your beta should be less than phi where phi is angle of internal friction. In this case K A can be find out from this equation and this equation is applicable for the cohesionless backfill though here c is not coming in both the equation. Now, if I put beta equal to 0 in this equation then you see there when I put the beta equal to 0 in the second equation then you will get the top one because it will be 1 everywhere and it will be 1 minus cos square phi will be which is nothing but sin phi and root of that. So, here you get this equation the top equation for beta equal to 0. So, that means, the second equation K A can be used for both beta equal to 0 as well as when beta is not 0.

So, this way you can find what is called coefficient of effective earth pressure coefficient. Continue with the active earth pressure the pressure distribution on the back of the wall depends on the relative magnitude of the frictional and cohesive components of the backfill of the strength which we will discuss in detail. Cohesive soils are undesirable why as backfill material as a backfill for retaining structures due to creep stress relaxation and low permeability characteristics. However, sometime you may not have choice because you may have the natural material which may be cohesive or it may not be it may be cohesive, but that is not desirable as a backfill material and as the simple result is because it is creep stress reduction low permeability. So, for example, in Rankine theory active pressure case when the c equal to 0 in that case the active earth pressure K A is given from this relation 10 square 45 degree minus 5 by 2. This is the diagram which shows for the cohesion less soils and P A active because the first term if you recall that in the P A in general is given by K A sigma v minus 2 c root K A. If I put c equal to 0 in this equation then you will get this equation P A K A sigma v is nothing but gamma z it is replaced by gamma z here sigma v. So, this quantity is your effective overburden pressure where gamma z should be taken as gamma d for dry soil or it should be gamma submerged for the saturated soil so the sigma v. And so, P A and total pressure at the for this triangular this is the pressure active pressure which is acting which is varying with the z, but total pressure will be the area of this triangle which will be half K A gamma h square by 2 and where this P A will act this will act at one third height. So, this will act at from the base this will act at a height h by 3.

And the pattern the variation is here like triangular this is the case when c equal to 0 when there is no c. The failure surface will make an angle with the horizontal 45 degree plus 5 by 2. Here you mind it the this angle is plus while the negative sign is coming here in the K A. So, K A for calculating the value of K A it will be tan square 45 degree minus 5 by 2 because for when phi equal to not 0. So, as a result K A will be always less than 1 because tan 45 degree will be 1 or even using this expression also when phi equal to 0 sin phi equal to 0 K A will be 1, but when phi is not 0.

$$\mathbf{K_A = \tan^2(45^o - \frac{\phi}{2})}$$

$$\mathbf{p_A = K_A \gamma z}$$

$$\mathbf{P_A = K_A \gamma\, H^2/2}$$

So, as a result numerator will be less than 1 and denominator will be more than 1. So, as a result whatever you find for the real value of phi K A will be less than 1. So, that is coming from this equation or directly from this equation. So, that need to be understood. So, this was the case and then total pressure is given here and this soil is gamma phi and c equal to 0. Now, the second case when we say phi equal to 0 that means no angle of internal friction, but naturally when phi equal to 0 c cannot be 0. So, as a result in that case the equation of P A will be gamma z minus 2 c because in this case phi equal to 0 then K A will be 1 in this case as we discussed. So, K A will be 1 tan square 45 degree if I put phi equal to 0 it will be 1. Now, when K A is equal to 1 there your then your equation become gamma z minus 2 c because root K A and K A will be 1 here this will be here if z naught z naught is the depth where the P A becomes 0. So, to find the value of z you can find by putting this equation 0 and solve for z equal to z naught.

$$z_o = 2c/\gamma$$

$$p_A = \gamma z - 2c$$

$$P_A = \gamma H^2/2 - 2cH + \frac{2c^2}{\gamma}$$

So, when you solve then you get 2 c or gamma that is the depth where your active pressure becomes 0. Coming to the total pressure this is given by the first component gamma h square by 2 with K A equal to 1 minus 2 c h plus 2 c square. So, this is the integration if you do from this equation along the depth or you can say you find the net area this area of positive area minus the top area. So, this so the top area 2 c square y is this top area like this minus 2 c into h because 2 c or gamma 2 c or gamma should be multiplied by the z naught or 2 c or like we have z naught and this is the here a P A need to be calculated at the top at the top P A will be simply minus 2 c. So, what will happen if I find the minus 2 c will be at the top.

So, the area of this triangle will be minus 2 c the top triangle will be minus 2 c into height into z naught z naught is 2 c or gamma divided by 2. So, you will get here this will be minus 2 c square or gamma. So, this is this is the term of this like area which will be coming and then gamma h square by 2 minus 2 c h will be the area coming from the down. So, this way you find out the total pressure calculate the total pressure and because this minus is coming. So, you need to substrate minus because minus minus will be plus here because this need to be substrate.

$$z_o = \left(\frac{2c}{\gamma}\right) tan\,(45^o + \frac{\phi}{2})$$

$$p_A = \gamma z \tan^2\left(45^o - \frac{\phi}{2}\right) - 2c\, tan(45^o - \frac{\phi}{2})$$

$$P_A = (\gamma H^2/2) \tan^2\left(45^o - \frac{\phi}{2}\right) - 2cH\, tan(45^o - \frac{\phi}{2}) + \frac{2c^2}{\gamma}$$

Then you have here if c phi soil in case of c phi soil your equation become gamma z square tan square again you need to understand this quantity is nothing but your k and this quantity is root k is the same thing here. So, gamma z into sigma v naught into k and minus 2 c root

k. So, this is the pressure given at. Now you want to calculate the pressure at some depth z naught is the depth where your pressure becomes 0.

So, again put this equation 0. So, once you solve then you get 2 c, over gamma tan 45 because basically you get 1 over tan you need to understand 1 over tan 45 degree minus phi by 2 is same as tan 45 degree plus phi by 2 this these both are the same thing. So, you get the value of z naught z naught is the tension cut off the point where your active pressure becomes 0 net active pressure and once you solve then you get this equation this equation you solve p a total pressure gamma h square by tan square 45 degree minus phi by 2. So, this is for c phi soil. In this case again the failure surface will make an angle 45 degree plus phi by 2 when you have phi equal to 0 naturally this angle will become simply 45 degrees. So, in this case 45 degree plus phi by 2 45 degree and 45 by with the horizontal. So, this was the case. So, how to deal with the c phi soil using the Rankine theory. Continue with this active pressure case for dry homogeneous cohesion less backfill Rankine theory predicts a triangular active pressure distribution which is oriented parallel to the backfill surface the active pressure resultant act at a point which is located at h by 3. So, this is basically the case when you have the dry number 1. The second thing cohesively aspect fill that means, you have the c equal to 0 when c equal to 0 we already said the total active pressure can be the that is the area of the triangle p a k gamma h square because it is dry. So, gamma will be in this case simply gamma d if you have the dry case and k will be a function of phi and k active earth pressure coefficient will be like you know that is less than 1.

$$P_A = \frac{1}{2} K_A \gamma H^2$$

So, this was all about active pressure. In case of passive pressure under maximum passive pressure condition Rankine theory predicts that that passive pressure at any depth is calculated. So, here one two changes have occurred compared to active pressure. Instead of k you have k p now that is and instead of plus sign you have instead of minus sign you get here the plus sign. So, this sign have changed here where k p is the coefficient of maximum passive earth pressure for a smooth vertical wall retaining horizontal backfill this is given by 1 plus sin phi and this form is itself is saying when phi is not 0 when phi equal to 0 k p will be 1.

$$p_p = K_p \sigma_v' + 2c\sqrt{K_p}$$

$$K_p = \frac{1+\sin\phi}{1-\sin\phi} = \tan^2\left(45 + \frac{\phi}{2}\right)$$

$$K_P = \cos\beta \frac{\cos\beta + \sqrt{\cos^2\beta - \cos^2\phi}}{\cos\beta - \sqrt{\cos^2\beta - \cos^2\phi}}$$

So, however, k p will be always greater than 1. So, basically you can say that you can prove that for phi equal to 0 both k and k p are same and the value is also fixed that is 1, but when phi is greater than 1 then k a will be less than 1 and k p will be more than 1. In fact, for any value phi k k p you can prove that they are opposite to each other 1 over k. So, or the product of k a and k p will give you 1. So, k p can be calculated also once you find the value of k a it is opposite of k a. So, this was the case when you have a smooth retaining horizontal backfill, but if you are the backfill is inclined at an angle beta that means this is the case as we said that is the backfill inclined backfill for a retaining wall and this angle is beta where is the backfill in that case k p is given by this relation.

$$P_P = \frac{1}{2} K_P \gamma H^2$$

Again, if I put beta equal to 0 in this relation then you get the top equation for k p. Coming continue with this passive earth pressure distribution for various fractals as we will discuss the same thing and total passive pressure for the case when c equal to 0 this is the case this equation will be applicable when c equal to 0 is the similar form as we discussed for active earth pressure. There is when c equal to 0 in that case thus these things become easy for cohesion less soil and this will act at h by 3 above the base which is a similar case in case of active earth pressure. But different cases when c equal to 0 then your k p is it will be given by tan square 45 degree plus phi by 2 instead of minus now you have plus. So, as a result your k p will be you need to understand that more than 1 k was less than 1 and the passive pressure at any depth because z gamma z z is varying so p p will increase with the z with the depth.

$$p_p = \gamma z \tan^2\left(45^o + \frac{\phi}{2}\right) + 2c \tan\left(45^o + \frac{\phi}{2}\right)$$

$$P_A = (\gamma H^2/2) \tan^2\left(45^o + \frac{\phi}{2}\right) + 2cH\, tan\left(45^o + \frac{\phi}{2}\right)$$

So, here p p into k p into gamma z and total pressure is k p gamma h square by 2. So, you have failure surface here and the failure surface will make in this case an angle 45 degree minus phi by 2 it was 45 degrees plus phi by 2 in case of active earth pressure but now it is minus phi by 2. And this total pressure p p in this case will act at a height h by 3. So, this will be the this distance will be in this case this total h is there. So, this will be h by 3 and the p p is given from this relation which we have discussed.

Continue when phi equal to 0 in that case this was the case when c equal to 0 but in case of when c is not 0 phi 0 then phi 0 that means you need to understand that nothing but k p will become 1. So, that is the meaning of this and the equation can be written simplify gamma into z plus 2 c. Now in this case you will not get any tension cut off because both are plus. So, even at the top when z equal to 0 at the z equal to 0 also you get p p equal to 2 c it is positive it is not negative like earlier case. So, and the angle which makes failure surface will be 45 degrees for phi equal to 0 and p p in this case will be given by gamma h square by 2 plus 2 c h.

So, naturally when phi equal to 0 when you have this case so this will increase the c will increase the value of p p. For the c phi soil equations is given by here and in both the equations the first term is nothing but k p and the second term is root k p. So, this was the pressure acting at any depth z because z is varying so this first component will vary with the z but the second component does not depend on the z this is fixed. So, the second component is shown on the top even when the z equal to 0 tau c turn 45 degree plus phi by 2 so this is the value and the first component will increase with the z and the maximum will be when z equal to h. At total pressure can be find out by the area of this ways which is given by gamma h square by 2 tan square 45 degree plus phi by 2 for the first term and the second term will be 2 c tan 45 degree. In fact, you can say you can to find out this area of this total you can say like this way this is one rectangular another triangle from here. This rectangle I have will have this 2 c tan 45 degree 45 plus phi by 2 into h height and this p p will act somewhere between h by 3 to h by 6 it will not be now at h by 3 but little on that away. So, this will is this distance is going to be more than h by 3 now so that that can be work out on. In fact, how you can work out this rectangular will be middle of this rectangular area will act here while the triangular will act at h by 3.

So, the average will act somewhere between h by 3 to h by 2. So, this is the case. Now, in case of Rankine theory what happens when the there is a presence of water the presence of water in the backfill behind a retaining wall influence the effective stresses and hence the lateral earth pressure which act on the wall. And you know when the backfill is present the effective stresses will decrease and so for wall design the hydrostatic pressure due to the water must be added to the lateral earth pressure. So, hydrostatic pressure need to be added to the lateral earth pressure because the total lateral thrust on a wall retaining a saturated backfill is considerably greater than that of a wall retaining dry backfill. You have saturated backfill naturally gamma saturated will be always more than the gamma dry. So, if you saturated gamma saturated will be high. So, what will happen the wall need to protect the more weight on the lateral direction. As a result the provision of backfill drainage is an important part of the retaining wall design. So, because the earth will apply more pressure on the wall when it is saturated compared to dry. So, in that case we need to either provide some drainage system to avoid this water problem otherwise we need to design thicker wall.

So, this was all about Rankine theory. Now, we quickly discuss Coulomb theory which is in Coulomb theory what is done. Coulomb in 1776 was the first to study the problem of lateral earth pressure on retaining structures. By assuming the force acting on the back of a retaining wall is resulted from the weight of ways of the soil above a planar failure surface. Coulomb used force equilibrium to determine the magnitude of the soil thrust which is acting on the wall for both minimum active and passive maximum passive condition.

First of all, Coulomb theory one difference compared to Rankine. Here we will assume that c equal to 0 that means this theory we are going to discuss only for cohesion less soils not cohesive soils. So, compared to this Rankine this is the limitation of this. However, what is the positive point for this theory? In the Rankine theory we assume that wall is smooth and vertical. In this case it is not necessary that your wall should be vertical as well as smooth. Since the problem is in number of potential failure surface must be analyzed to identify the critical failure surface that is the what is the critical failure surface that is the surface that produce the greatest active thrust on the smallest passive thrust.

So, here in active pressure condition this is a scenario in the Coulomb theory. This is here under minimum active earth pressure condition. The active thrust on a wall with the geometry which is shown from the force this geometry is obtained from the force equilibrium. Here what you have this is the forces acting on the wall. First of all, compared to the Rankine theory understand that this wall is not vertical on this side backward side rather it met an angle theta with the vertical one thing.

Second thing there is an angle of internal friction between wall and soil this is wall and soil here it is delta. So, this angle is not 0 which was considered earlier delta. So, and the failure surface will make an angle alpha with respect to this alpha angle a will be with respect to horizontal. What is w? w is the weight of the this ways and this will act always w always act vertically in downward direction. Now when you have and this is the total force F which is acting on this way, we will make an angle phi with respect to the normal to this surface.

And alpha is the angle which your failure surface this failure surface will make with the horizontal we do not know value of alpha a which can be determined. Other things theta delta phi beta all other things are known. Beta is the angle which the surcharge is inclined with respect to horizontal. So, everything is known in this case except alpha a and this p a. p a is the active earth pressure w will act in the downward direction F is acting along this direction if I join from this point to this point this is p a direction of p a which is acting here.

So, this is parallel to each other. How the p a is given here? p a is given by half k gamma h square this is the similar equation exactly similar equation as we discussed this is for quasi-low soil first of all this is c equal to 0. So, the equation is same as we discussed in Rankine theory. But what is basic difference? k in this case is given by this relation cos square phi minus theta cos square theta. So, here earlier in Rankine theory only phi was involved but here you have theta delta and theta two more parameter have come. Earlier beta was there also but two more parameter that is delta and theta have come in picture here where delta and theta we already explained what is delta and theta.

$$P_A = \frac{1}{2} K_A \gamma H^2$$

Where

$$K_A = \frac{\cos^2(\phi - \theta)}{\cos^2\theta \cos(\delta + \theta)\left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi - \beta)}{\cos(\delta + \theta)\cos(\beta - \theta)}}\right]^2}$$

Delta is this angle which makes and theta the wall is not vertical. So, theta could be 0 if your wall is vertical and delta could also be if you consider the smooth wall. So, this was for active pressure. Here in this case alpha a is given from this relation phi plus tan square tan phi minus beta plus c 1 and c 2 and there is long equation for c 1 and c 2 which are again function of phi, beta, theta, delta all four angles. Four angles in this problem involved delta, theta, beta and phi but all four are known and this alpha is unknown which can be found out from this equation.

$$\alpha_A = \phi + \tan^{-1}\left[\frac{\tan(\phi - \beta) + C_1}{C_2}\right]$$

$$C_1 = \sqrt{\tan(\phi - \beta)\,[\tan(\phi - \beta) + \cot(\phi - \theta)][1 + \tan(\delta + \theta)\cot(\phi - \theta)]}$$

$$C_2 = 1 + (\tan(\delta + \theta)\{\tan(\phi - \beta) + \cot(\phi - \theta)\})$$

This was active pressure. On the same line you have active pressure, passive pressure also for Coulomb theory and this is again for cohesion less backfill that means when c equal to 0. In this case this equation is also similar to what we discussed for Rankine theory but k p will be given from this relation and the angle which it makes alpha p with the horizontal can be find out in this case using this relation phi plus tan phi plus beta c 3 c 4 where c 3 and c 4 are given from these equations which are function of again four angles phi, beta when you have theta and delta. So, all are known when you can calculate the value of alpha. Here in this case for passive pressure the waves will be like this w will act and again the f will act with angle phi similar to that but p p earlier was acting in the downward here but now the direction of p p is changed.

$$P_P = \frac{1}{2} K_P \gamma H^2$$

Where

$$K_P = \frac{\cos^2(\phi + \theta)}{\cos^2\theta \cos(\delta - \theta)\left[1 + \sqrt{\frac{\sin(\delta + \phi)\sin(\phi + \beta)}{\cos(\delta - \theta)\cos(\beta - \theta)}}\right]^2}$$

$$\alpha_P = -\phi + \tan^{-1}\left[\frac{\tan(\phi+\beta) + C_3}{C_4}\right]$$

$$C_3 = \sqrt{\tan(\phi+\beta)\,[\tan(\phi+\beta) + \cot(\phi+\theta)][1 + \tan(\delta-\theta)\cot(\phi+\theta)]}$$

$$C_4 = 1 + (\tan(\delta-\theta)\,[\tan(\phi+\beta) + \cot(\phi+\theta)])$$

So, this angle will be in this case this angle will be delta. So, if I draw the force polygon then w will always act vertically it was same and active as passive pressure and f is acting along the direction f and then from I join from this point to this will be the direction of p p that is passive earth pressure. So, this was all about Rankine theory and Coulomb theory. We have come across the value of delta in these equations which is a friction angle between wall and soil. The delta depends on your material if you have concrete massive concrete of the wall 17 to frame concrete. So, range is 17 to 26 steel sheet pile it will reduce because piles will be still will be smooth so 11 to 17 degree so accordingly we can select the value. Thank you very much for your kind attention. Thank you.