Earthquake Geotechnical Engineering

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Lecture 43

Slope Stability Analysis (Conti.)

I welcome you again in this NPTEL online course on earthquake geotechnical engineering. This is lecture number 43 and we are under the module 5th of this course that is on slope stability and retaining walls. We are on the first part that is slope stability and then we will talk about the second part retaining walls. So, the first one the introduction to earthquake induced landslides we already covered. Then in the slope stability analysis it was in two part one is static slope stability analysis and other is seismic slope stability analysis.

So, static slope stability is already over in fact, we have started seismic slope stability and we will continue with that in this lecture. In the seismic slope stability, we already talk about pseudo static method and today we are going to talk one of the new methods. So, Newmark sliding and block analysis we are going to talk today and in this influence of yield acceleration on slope displacements will be also discussed. So, Newmark sliding block analysis first what is the difference between this pseudo static and Newmark sliding block analysis that will be discussed and most of the stuff for this lecture is from this Kramer's book on geotechnical earthquake engineering. So, it is acknowledged. Here when we talk about why we need the Newmark sliding block analysis because when you carried out the pseudo static method which is basically a static analysis you repeatedly do the static analysis pseudo static method. So, the pseudo static method or analysis provide you a factor of safety against slope stability, but it does not provide any information on deformation which is associated with slope failure. Now, why you need these deformations? When we talk about the surface stability of a slope after an earthquake it will be controlled not only by the stability, but also with the deformations and the analysis that predicts slope displacement provide more useful indication of seismic slope stability. So, one part is factor of safety another part is on information of the deformation.

So, we get both when we carried out the Newmark sliding block analysis. And since earthquake induced acceleration will vary with the time the pseudo static factor of safety will also vary throughout an earthquake. So, the good point with the Newmark sliding block analysis that because if you go for the regression analysis then that may require finite element methods and that may be dealt using some software. However, Newmark sliding block analysis can be carried out analytically it can be carried out in the if for the simplest case it can be carried out even using the hand calculator or maybe excel sheet. So, we are going to come on this one and this Newmark sliding block analysis give you the factor of safety as well as this deformations information on the deformation.

Now, if the inertial forces which act on a potential failure mass is large enough that the total, total means static plus dynamic, dynamic which is due to earthquake or seismic driving forces exceed the available resisting forces in that case factor of safety will be less than 1. So, if the total stresses exceed this resistance which is available that means demand have increased beyond capacity in that case the factor of safety will decrease less than 1 and that the once factor of safety will be less than 1 your slope is not going to be stable. So, this is one case. So, Newmark in 1965 considered the behavior of a slope under such conditions where factor of safety was less than 1 the potential of failure mass is no longer in equilibrium and it will be accelerated by the unbalanced force. In that case it because once factor of safety is less than then it will not be in equilibrium.

So, here what was the analogy given by this like you have a slide which is for the potential landslide in the figure a is shown and you have the sliding mass and then you have the failure surface which could be the circular or it could be assumed some different shape as we discussed in the last lecture. So, this potential sliding mass can be analogue with a block on an inclined plane in the second case. So, you have inclined plane or sliding block. Suppose in a static case this sliding block and inclined plane they are may be sitting on each other and we can in case of static case for we can find the factor of safety. So, for static equilibrium in the inclined plane under static conditions equilibrium of the block required that the available static resisting force which is R as exceeds the static driving force dr. So, you have one side resistance of the block another side is driving forces. So, if driving forces exceed the resisting force, then the factor of safety is less than 1 and how for the pure like if you have the pure sand and the resistance is due to purely frictional. So, in this case it is c equal to 0 and this condition you know that this is for pure sand in that case assume c equal to 0 the factor of safety can be given by this R s it can be like you know work out like this you have w let us see this is a case here. In this figure what you have the w? w is the weight of this block. Now this weight of this block can be having two components one is along the plane and another is perpendicular.

So, one of the components which is in this direction and another is in this direction. So, this will try to have driving force. So, this will try to destabilize this will try to move, but when this component will generate a reaction at the base which is equal to w cos beta. w cos beta if I multiply by tan phi then you will get this frictional force between the block and plane. So, the friction force is w cos beta multiplied by tan phi. So, this whole term will be R s and this w sine beta will be driving force. Here w cos beta is reaction which is like you know that which is acting perpendicular to the plane. So, in the perpendicular direction to the plane tan phi is friction angle between the block and plane. So, in this case what happens because when c equal to 0 the equations is very much simplified w cancelled out and you will left out the factor of safety tan phi or tan beta. So, simply in this case f will be greater than 1 if phi is greater than beta.

$$F = \frac{R_S}{D_S} = \frac{W \cos\beta \tan\phi'}{W \sin\beta} = \frac{\tan\phi'}{\tan\beta}$$

So, as long as friction angle is more than the angle of inclination of this plane beta in that case the factor of safety is going to be more than 1. So, no issue, but if it is less if phi is less than beta then factor of safety will decrease and unstability will come. So, this was all about static case. What happens in the dynamic case in addition to the weight of this block some horizontal force also act and this horizontal force act weight always act in the vertical direction. And this horizontal force this is basically horizontal seismic force and if you recall this was like a horizontal seismic force you can say that this is nothing, but horizontal seismic force.

And this force act in the horizontal direction. Now, this horizontal force will add a component. So, now, in this direction you will have two component one is due to w which will be w cos beta you will have this one component which will be w sin beta and another will be plus k h into t into w cos beta. So, this will be acting in this direction. And this will create your driving force which is coming down here.

$$F(t) = \frac{R_d(t)}{D_d(t)} = \frac{[\cos\beta - k_h(t)\sin\beta]\tan\phi'}{\sin\beta + k_h(t)\cos\beta}$$

And on the top, you will have this component w was acting this side its component another component will act upward. So, you will have two components one is this side and another this side perpendicular. So, that as a result resisting force will decrease. So, in this dynamic case when the factor of safety equations is converted here cos beta minus k h t into sin beta tan phi and sin beta. So, here what you could see when we consider the effect of initial forces which transmit the block by horizontal vibration and with an acceleration a h t that is what is a h t? Acceleration horizontal in the horizontal direction which is k h t into g k h t is a coefficient which is varying with the time it is not constant because k h t is not dimensionless.

However, it is varying with the time. So, as a result your horizontal acceleration will also vary with the time. So, here if this equation is written by neglecting the effect of vertical acceleration. So, at a particular time horizontal acceleration of the block will induce a horizontal inertial force which is k h w when inertial force is acting in the down slope direction as was the case here where it is acting in the downward direction then the factor of safety is equation becomes here. Now, if you analyze this equation, you see the effect of k h t if I put the k h t equal to 0 then the numerator will increase.

So, the effect of k h t this effect it is negative in the numerator and positive in the denominator. So, as a result the factor of safety which you find will be less than the static factor of safety. So, as a result this effect of this horizontal force is to decrease the factor of safety. Now, suppose if you have like factor of safety in static case is more than 1. Then if you increase the value of k h t then what will happen your factor of safety will start decreasing and then it points will come where a factor of safety becomes 1.

And if I put factor of safety equal to 1 in this equation and solve this equation then I get a very simple relation which is for k h what is k y? k y is nothing but k y is the value of k h t for when the factor of safety becomes 1. And in this equation last equation if you put 1 that means numerator and denominator should be equal and replace k h t with k y then you solve then once you solve then you find tan phi minus tan beta which is basically nothing but tan phi minus tan beta 1 plus tan phi tan beta tan phi tan beta. So, this way you find and this k y is called yield coefficient. Dynamic factor of safety decreases so the dynamic factor of safety will decrease in this equation when k h increases and there will be a value of like you know when how much we should increase the value of k h. If we increase the value of k h to a point where your factor of safety becomes 1 in that case this coefficient will be termed as the yield coefficient k y and this yield coefficient k y will be correspond to the yield acceleration which is given by a y is nothing but k y multiplied by g.

This is a minimum pseudo-static acceleration which is required to produce instability of the block for sliding in the downstream direction downslope direction. So, k y will be tan phi minus tan phi minus beta. Here in this figure, you see that there is slope with beta equal to 20 degrees. Now in this case the factor of safety where what is the value of k h where the factor of safety becomes 1. For example, like this curve is for 20 degrees when phi equal to 20 degrees.

Naturally when phi and beta are equal then factor of safety will become 1 even when k h equal to 0. So, the factor of safety 1 when k h equal to 0. But if you increase the value of phi to 30 degrees of 40 degree when k h equal to 0 your basically k h equal to 0 means this is for basically this belong to static case. This is static factor of safety. So, the static factor of safety will be 1 here when phi equal to 30 degree it will be more than 1.5 when 40 degree it will be more than 2.5 and when the k h increases the factor of safety start decreasing. So, here in phi equal to 20 degree it will start decreasing and become less than 1. But for phi equal to 30 degree there will be a point this line is for factor of safety equal to 1 where the factor of safety becomes 1 and this this point is nothing but 0.17. So, this value is 0.17 and this is for phi 30 degree. Similarly for phi equal to 40 degree this is 0.36. So, naturally the value of k h will depend on two things one is the angle of inclination beta and another is your phi which is angle of internal friction. So, if phi is higher then you can have the k h higher value where the factor of safety will be 1. If it is low then the in that case at low value of k h the factor of safety will becomes 1. So, this was all about that. When a block on inclined plane is subjected to a pulse of acceleration that exceeds the yield acceleration then in that case the block will move relative to the plane. And how it the moment let us consider the case in which an inclined plane is subjected to a single rectangular acceleration pulse of amplitude A and duration delta t which is shown in the slide and we will be using the slide frequently to explain. What is in this case let us discuss the acceleration part only right now. Acceleration rather than a times, we are using a rectangular pulse. In the rectangular pulse maximum value of the acceleration is capital A while duration is this is t naught and this duration is simply here to here is this duration is delta t. So, this pulse has total duration delta t and, in this case, because A this maximum value of A is greater than A y that is why yield acceleration because in that case the permanent displacement will develop. Here single rectangular pulse A and duration delta t and the condition is here A is greater than A y yield acceleration more than yield acceleration. If it is less than yield acceleration in that case there will be no moment because the condition is that A should be greater than A y.

Now in this case yield acceleration A y or other way we say the A y is less than A the acceleration of the block relative to the plane during the period from t 0 to t 0 plus delta t during this interval you have a relativity which is given by A b t what is A b t? A b t is nothing but base acceleration which is varying with the time and A y is yield acceleration. The maximum value of A b t is constant here during this interval during this delta t period your acceleration is not changing it is constant at the A capital A. So, the difference will be A minus A y and this is applicable only in this period t naught to this one. So, what we do if suppose this is an excess acceleration so this you can treat like this one the difference of the acceleration maximum value minus this yield acceleration. So, this is excess acceleration and if we integrate this one then you will find the velocity if you integrate once more then you will find the displacement.

So, where the relative moment or block can be obtained integrating the relative acceleration twice so it has been done here. So, whatever we find out in the last slide so if I integrate it by once with the time then I get the relative velocity with the time. So, this give you again if I put in the relative velocity here integrate once again then you find the relative displacement and here both relative density velocity and relative displacement both are function of time that means they are varying with the time. Now, what will happen as a result relative velocity because acceleration is constant here as shown here in the slide as a result your velocity will be linear be increasing from this time to this time because and the velocity will become maximum at t equal to t naught plus delta t why because after that acceleration stops and it becomes 0. So, up to this point velocity will reach to maximum point.

So, what you can do, you can find velocity by integration and you can find the displacement up to this point. So, this has been done another issue after this you have acceleration stops there is no acceleration after this point in this case, but there is yield acceleration though. So, the relative will be minus a y the difference between then this one it was positive when this a becomes 0 here and this is then it difference will become negative. So, that we are going to work on that. So, using that this deceleration so between from this point to this point there is deceleration.

So, the velocity first will reach to the maximum value and after that because then there is a deceleration rather than 0 acceleration. So, as a result velocity will decrease and will reach to a value 0 which the velocity becomes 0 at time t equal to t 1 and then you integrate then again you find the displacement here and the displacement become constant after this point this maximum value of this. So, how it is done? So, we already find the values at this, this is the equation for velocity at time t if I put t minus t naught equal to delta t then I can find the velocity at this point and so this is the delta t times this is given here this is the velocity at that there and the displacement. So, these values represent the values at this point here as well as here. Now, after this there will be deceleration as we discussed after time t equal to later and this deceleration will be base acceleration minus a y.

So, it will be 0 minus a y so that means, it will be negative acceleration minus a y and with this negative acceleration t 0 plus delta t and this will be applicable in a time period after t 0 plus delta t, but less than t 1. How we can find the value of t 1? t 1 can be found out t 1 is the point where your velocity becomes 0. So, here you have got the maximum value and the maximum value after time with the given acceleration where your velocity becomes 0. So, using this can be found out the relative velocity will decrease with time according to this and if in using this expression if on the left-hand side, I put velocity equal to 0 and solve this equation then t whatever you get t from this equation will be t 1. So, you can obtain the value of t 1 from this one.

So, t 1 is obtained this is answer here t 1 is obtained. So, now, you know everything here this point is known this time is known and then we can do this once more this once more the integration of this equation between this time period and the integration will give you the displacement. So, we find the displacement and this displacement is between t 0 plus delta t and t 1. At time t 1 the block and inclined plane now at this time t 1 what is the condition at time t 1? You do not have any acceleration you get a stopped here and no deacceleration nothing as a result the block and this plane they will be moving together there will be no relative displacement no relative the block and inclined plane move together during the total period of time between t equal to t naught and t equal to t 1 the relative moment of block is shown as here. So, this was about so, this way we can find ultimately what is our objective use doing this all this we find this value because this is a permanent displacement. So, this value this becomes constant from here to here and your objective is to find this value for and which is the maximum value of the displacement and this displacement maximum value of displacement is used further in design. So, between t 0 the relative velocity increases linearly and between t 0 and t 0 plus delta t thus and the relative displacement quadratically at t 0 plus delta t has reached maximum value which we already discussed and these values at t equal to t 1 thus this is given. So, this give you this using this equation at t 1 you find the displacement which is the maximum value. So, this displacement is at this point. So, this equation gives and naturally this will not be depended on time because this time is fixed t 1 is fixed.

So, as a result this value is also fixed. So, this way you can find the maximum value of displacement using this expression and this depends on naturally this depends on what? A y is yield acceleration which depends on mere material property which we already discussed how to find the value of A y? A y can be calculated simply k y into g what is k y? k y is nothing but tan phi minus beta. So, what is phi? Phi is angle of internal friction what is beta? Beta is this slope angle. So, that means we know the A y that depends on your geometry and the material property. Now, this relative displacement will depend on other two factors one factor is A and another factor is delta t. Delta t is a time interval of your pulse that will depend on your frequency of excitation and A is amplitude.

So, the total relative displacement strongly depends on the amount which is the length of time during which the yield acceleration is exceeded number one that is delta t and the second thing is amplitude. Relative displacement caused by single pulse should be related to both the amplitude and frequency content this is the case. Thus, so this was regarding Newmark sliding block analysis. Now, before we will discuss one example on Newmark sliding block analysis later but before that what is the influence of yield acceleration on slope displacement? First, we will discuss using the Newmark sliding block method and then later with some other method. The sliding block model will predict zero permanent slope displacement if earthquake induced acceleration never exceed the yield acceleration. So, if your A y or A max is greater than 1 that means your yield acceleration is greater than A max. A max is the maximum value or you can say p g which you may expect during an earthquakes time history. If your yield acceleration is more than that then there will be no permanent slope displacement and you do not require any further analysis. However, if your A y yield acceleration is less than A max then you require. Since the permanent displacement is obtained by double integration the computed displacement for a slope of a relatively low yield acceleration.

So, as much the difference between A y and A max if difference is large, you can expect that permanent displacement will be more. If this is less in case of because let us say in a given earthquake, I fix the value of A max. So, if my A y is low yield acceleration compared to A max in that case you will get more higher values of permanent displacement but in case of your A y is higher than you will get less so which has been explained here. So, what do you see here in the first figure A you have maximum value which is just touching A y that means difference between A i and A max is 0.

So, no permanent displacement will be developed. Then so in the second case value of A y decreases yield acceleration is here the this remains as it is. So, this will create because in during this interval your acceleration have exceeded the yield acceleration and it will create the permanent displacement. Further A y decreases and you get these peaks here as well as here. So, that means permanent displacement was 0 in case A then it increases and then in case of C it further increases. Now coming to the Newmark also demonstrate that maximum permanent displacement which we say d max which is produced by an earthquake motion is given by the relation d max in this case in this relation v max square divided by 2 A y A max for and in this case this expression which give you the maximum value of displacement using this expression will be valid provide that the value of A y by A max is equal to or greater than 0.17. So, before we use this expression, we need to make sure that the value of this ratio yield acceleration 2 is greater than 0.17 and naturally when E y is greater than 0.17 that means even it is not greater than 1 that means in this case yield acceleration is less than the maximum PGA. So, in that case factor of safety will be more than 1, but what happens like you have yielding is started then still it will create some thus maximum permanent displacement during over the time of earthquake. So, where A max and v max are the and what is v max and A max maximum acceleration peak base velocity due to earthquake excitation respectively A y is the yield acceleration.

$$d_{\max} = \frac{v_{\max}^2}{2a_y} \cdot \frac{a_{\max}}{a_y} \quad for \quad \frac{a_y}{a_{\max}} \geq 0.17$$

So, with this we continue. Now there are some other researchers also who try to estimate permanent displacement using the value of yield acceleration A y by A max and one of them Ambraseys and Menu in 1988 suggested that permanent displacement in centimeter this equation is not dimensionless if you find out from this then you will find the displacement because this term is coming 0.90 and it carries some unit. So, the answer which you find from this u which will be finding it is log scale and log u. Then you whatever you find u it will come in centimeter. So, by actual ground motion for a small and this will be applicable when A y by A max is small if it is quite large then it will not be applicable and this equation have a standard deviation sigma log u equal to 0.30. So, in this equation you have A y by A max and this powers and the 0.9 and this equation is will be applicable as we said for the in the range A y by A max should lie between 0.1 to 0.9 to application for this then this equation is applicable for the magnitude surface magnitude of earthquake between 6.6 to 7.3 and A y is computed using residual solstice time this was by Ambraseys and Menu.

$$\log u = 0.90 + \log[\left(1 - \frac{a_y}{a_{max}}\right)^{2.53}\left(\frac{a_y}{a_{max}}\right)^{-1.09}] \qquad \sigma_{\log u} = 0.30$$

Similarly, Yegian et al. in 1991 developed the following expression for the which also in this last equation you have only A y by A max 1 parameter. But in this case, you have any Q which is number of cycles of earthquake and time period also involved here. And in this case, you have this in the but the issue is in this case is in this equation u is coming on the right-hand side also. So, you need to do the you need to work iterate here in this is log u star.

$$\log u^{*} = \log\left(\frac{u}{a_{max}N_{eq}T^{2}}\right) = 0.22 - 10.12\left(\frac{a_y}{a_{max}}\right) + 16.38\left(\frac{a_y}{a_{max}}\right)^{2} - 11.48\left(\frac{a_y}{a_{max}}\right)^{3}$$

So, u star is a dimensionless which is u divided by A max and n e q t square. So, on the right-hand side you have A y by A max and again here because this unit is coming. So, this is also in centimeter. So, this equation will be also not dimensionless rather it is having some unit and the standard deviation is given here. So, what do you do use this equation you find on the right-hand side but here because this is dimensionless because this you find ultimately this ratio.

So, once you find this ratio then you find out this value and once this value is known then you can find out the u value of u u equal to A max n e q into t square where n e q is an equivalent number of cycles and t is the predominant period of the input motion. Jibson also suggested correlated sliding block displacement with the areas intensity and using the areas intensity which is in I a in meter per second and A y in g and in that case using this expression the permanent displacement can be find the answer comes which you get is u is in centimeter. Two aspects of seismic slope visibility are clearly as registered by these studies first is earthquake induced slope displacements are very sensitive to the value of the yield acceleration. Consequently, small difference in the yield acceleration can produce large variations in predicted slope displacement. So, if you have a small variation then there will be displacement there will be large variation in the because it is ultimately difference between your maximum value of acceleration minus the yield acceleration.

$$\log u = 1.460 \log I_a - 6.642 a_y + 1.546 \qquad \sigma_{\log u} = 0.409$$

So, if difference is large then you will get the large displacement. The second the great variability of distribution of acceleration pulse amplitude between different ground motion produce great variability in predicted slope displacement. So, if you have the large variability then the variation in the slope displacement will be also large. Even ground motions with similar amplitudes frequency content and duration can produce significantly different predicted slope displacement. The uncertainty must be recognized in the prediction of earthquake induced slope deformations.

Coming to this one example a small example or Newmark sliding block analysis. Here the peak values peak acceleration a max is given to you as a 0.442 g and peak velocity v max is 33.7 centimeter per second. And one more information is given in the question which is yield acceleration 0.112 g. So, first thing what you do you find the ratio of Ay by A max.

Ay is this one 0.1124 and A max is 0.442 and this ratio comes out to be 0.2543. So, this is more than 0.17. So, this is okay as a result we can use the equation which is given by the Newmark here v max. So, just what we do we put the numbers here Ay is given you yield acceleration 0.1124 g and the value of Ay if I put the value of g 98 1 centimeter per second square and A max by A. So, all the numbers are put and then finally d max you should find is about 20.25 centimeter is the displacement which is maximum permanent displacement upper bound estimate of the permanent displacement using Newmark sliding block analysis.

So, this was all about Newmark sliding block analysis as well as the influence of the input parameter. Thank you very much for your kind attention. Thank you.