

Earthquake Geotechnical Engineering

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Lecture 26

Soil-Structure Interaction (Conti.)

I welcome you again on this NPTEL online course on earthquake geotechnical engineering and we were discussing soil structure interaction . So, this is the second lecture on SSI. We are under module 3 that is the second chapter of module 3 which is on SSI and in this chapter, these are the topics which we are going to talk. We already discussed four of these topics and three last direct & substructure method, general steps for linear SSI analysis and recent advances in seismic soil structure interaction we are going to talk about that. So, let us talk about this first direct and sub-structure methods.

There are two way of carrying out the soil structure interaction analysis. The free field motion is applied at the fixed boundary at some distance from the structure. Both soil and structure are considered together in the direct method and advantage the nullity of the soil can be considered. While in the sub-structured method the two systems, foundation sub-system including soil and the structure sub-system they are considered separately.

So, it is here. So, direct and sub-structured method. The first one, this side, this is the direct method. So, the first one is basically direct method up to this one. Here you are considering the whole system together that means you are considering both structure as well as you are considering the foundation part together.

But in the sub-structure method you separate out the foundation part and the structure part. So, what is done here? You have the free field motion and the top of it normally what you do, you replace this free field motion like this with some spring and dashpot which is done here. So, this one plus this structure or in fact, because the structure is like completely here. So, this will be kind of a scattered motion where you have this part. But what we do for the analysis, we do not consider this part because it is difficult to deal.

We consider the free field which was the given in the first case. So, these are the directions of the structure method. Now, suppose you need to carry out the linear structure soil structure interaction analysis rather than non-linear, then it may be little simpler compared to non-linear analysis and how we carried out the linear soil structure interaction analysis has been discussed here. Here in the case, analysis can be in three steps, obtaining the motion of the foundation in the absence of the superstructure that is the first step. That

means, you consider the foundation on the top of it, there is no superstructure, and this foundation input motion includes what we call the translation as well as rotational components.

Determine the dynamic impedances, the second step. So, let me explain it here with the figure. Your complete problem is given in this case the whole system. In the whole system, we have let us say piles and the top of you may have the pile cap and then on that superstructure is there. So, when this system is subjected to seismic waves, how to analyze the system? We can divide this problem into different components and what we are doing different component that can be only for the linear case where you consider the soil as well as the structure properties are elastic, or no nonlinearities considered.

So, in this case, when you have the free field motion here, let us say first step is free field motion. In case of free field motion, seismic waves are coming from the ground and there is no foundation, no structure. And you consider the motion which is coming at the ground as a free field motion. So, let us say this is your input given at the base $u_g(t)$ equal to $u_g e^{i\omega t}$. While the output amplitude is u_{ff} .

Naturally $e^{i\omega t}$ for harmonic excitation will remain same of course there could be some phase lag. So, this is called free field motion or in the short we called it FFM free field motion. Now, if suppose I insert the piles or foundation inside the soil, then even for the same input motion which was the case earlier, output at the top of this foundation is different than you got without the presence of the pile. And as we discussed earlier, you get a rotational component even your input was only translation, but in output you get translation as well as rotation. And this is called what we call the kinematic interaction.

So, this effect of the presence of the foundation in the system is kinematic interaction. So, in this step what we have got? We have got the input is applied at the base of the structure and output you are getting at the top of the foundation this without considering the building or a structure. Now the third second step is this after computing first step, we find the impedance functions or what we call the dynamic impedances, which is basically dynamic stiffness of the system. And for that you apply the load on the top which may be translation rotation and you find the response at the same point. So, the load divided by the response will give the stiffness.

So, stiffness components are k_x, k_y, k_{xy} . So, these are the components here spring constants and these springs you find out for the case like when you apply the load from the top. Now you have motion at the top of here and you are defining these spring constant at the same point. So, now you have the whole system which was given here in this case, the first case the whole system is simply replaced by what we call the combination of spring and dashboards. So, you have the superstructure as it is $m_1 k_1$, where k_1 is the stiffness of the superstructure was the case, m_0 is the mass of this here.

The soil and foundation and piles are replaced here by springs, which is k_x in the horizontal direction, k_{ry} in the rotation and cross coupling. So, you have one displacement and rotation. Here these are the spring constants. In this case, a spring constant is only spring, but you can also consider the dashboards. And at the end of these springs, the motion applied is the same, which has been obtained in the first step, which is kinematic displacement and kinematic rotation.

So, these springs are subjected to the kinematic displacement and rotation and then we analyze this system, and you know that analyzing the system is much much easier compared to what we do. This can be done because it is a discrete system, which consists of the springs and but because the superposition is assumed, this technique will be applicable only for linear analysis. Now, for the linear analysis, depending on the type of damping which you consider material damping, it could be either viscous damping or a static damping, then we can define dynamic stiffness or dynamic impedance function, which is because it is a complex number. So, it is denoted by k^* , star is denoting that is a complex number. And this can be easily proved how this equation have come.

This equation is coming directly if I write equation of motion, let me write equation of motion here. Equation of motion is $m\ddot{x} + c\dot{x} + kx = I \sin \omega t$. If I represent x equal to $x \sin \omega t$, then from both sides $\sin \omega t$ will be cancelled out and I will left that k^* is $k - M\omega^2 + i\omega C$ where k^* is nothing but in this case will be \sin amplitude of the force divided by the amplitude of the motion. So, k^* is simply defined this and you can find the value of this stiffness $k^* - m\omega^2$. You see in this equation for dynamic stiffness, you get both real part as well as imaginary part.

a) For Viscous Damping:

$$K^* = K_{st} - M\omega^2 + i\omega C$$

b) For Hysteretic Damping:

$$K^* = K_{st}(1 + 2i\zeta) - M\omega^2$$

This was the case when the form of the damping considered in the system is viscous. So, the unit of viscous damping if I consider a static damping, in that case my equation because this damping this you will have minus this inertial force and then you have the spring constant including the effect of damping. So, effect of damping is club with static stiffness. Now because both the equations are like coming in one case, we have viscous damping and a static damping. Using both the equations you can find out what is relation between c and ζ .

Here c is damping coefficient while ζ here is damping ratio static damping ratio. So, if you compare then comparing these two terms because other terms are same. So, just you need to compare this term with this term here and this comparison give you $k^* - m\omega^2$. So, c is

equal to twice zeta omega kst. So, that means damping coefficient here in this case is static damping ratio and static stiffness.

$$iK_{st} 2\zeta = iC\omega \quad \text{means} \quad C = \frac{2\zeta}{\omega} K_{st}$$

It will depend not only on damping ratio, but it will depend on static stiffness as well as your omega. Omega is frequency of excitation. So, the value of c will keep changing with the frequency because it is coming in this equation. In general, dynamic impedances k star may be represented in the form k star equal to k1 kst k1 plus i k2 where k1 and k2 are nothing but real and imaginary parts of the real and imaginary parts of the stiffness. Significant advancement has been made in SSI studies in last three decades and what are the advances made in because there are advances in computer technology.

Now, it is possible to model the analysis SSI problems more rigorously. Most of these advances are on two major and very important issues. What are those issues? One is related to basically modelling of the soil and another is related to considering the non-linearity of the soil. So, the two issues which are considered are listed here. Though the soil is an unbounded domain, precise modelling is still possible because computer like you know advancements are there and the modelling part is taken care.

The behavior of soil during strong excitations is highly non-linear and for saturated loose sands even the liquefaction may also occur. However, still today no commercial software such as SASSI, FLUSH, FLEX, CLASSI, FLAC3D, DYNA4, PLAXIS3D adequately deals with these important issues and fortunately our research group like including my PhD students, Emani, Sarkar, Syed and later on Firoz work extensively in the area of SSI and soil pile destruction. The journey, my journey for SSI started from 1997 from like from PhD thesis which was submitted in 1997 where in this as for SSI material and geometrical nonlinearity in the theory by Kaynia and Kausel which is a publication of 1982 for pile dynamics was introduced. Then later in one of the postdoctoral report that is Maheshwari 2003, Kelvin elements was used to model the boundary and HiSS soil model to deal with nonlinearity. Then at IIT Roorkee my PhD Emani carried out nonlinear SSI analysis using what we call the CIFECM.

CIFECM stands for consistent infinitesimal finite element cell method for boundary and hybrid methods. Then Sarkar considered liquefaction for SSI analysis. In 2014, Syed modelled the boundary using what you call SBFEM scale boundary finite element method and nonlinearity of soil using his model, soil model has been considered. Recently Firoz 2023 who submitted like his final thesis in March this year only carried out three-dimensional nonlinear soil-structure interaction analysis for combined pile raft nuclear power plants. So, these are the work which has been done recently by our research group on soil-structure interaction analysis.

I like to share with you some of the work that may not be possible in this lecture to share all the work but few of the simple work. First when we talk about modeling of boundary, it could be approximate modeling and it could be rigorous modeling and I am able to work for both approximate as well as rigorous modeling. These boundaries are local in space and time. So, they could be the elementary boundaries, the viscous boundary, Kelvin elements, transient transmitting boundary, infinite elements, there are so many. So, the work which has been done using Kelvin elements are shown here.

For structures which are supported on pile foundations, then problem of SSI can be reframed, and it can be treated as a soil pile structure interaction that is SPSI and response of an SPSI system that is soil pile interaction system can be again considered like sum of the kinematic interaction plus inertial interaction. And naturally this kind of treatment can be carried out only for linear analysis where you are summing up two responses. For SPSI system, SPSI means soil pile structure system, two types of interactions effect need to be considered. One is soil pile interaction where you are considering the pile group that is pile-soil-pile for soil pile interaction for a single pile while pile-soil-pile for a group piles and foundation structure interaction need to be also considered. One of the techniques which we have initially developed in 2004 and which is published in Soil Dynamics and Earthquake Engineering Journal long back in 2004 is given here.

Though it is linear analysis but using a technique what we call the successive coupling scheme, we can carried out the non-linear analysis also using this scheme. In this scheme what we do, you have soil pile structure system and in this soil pile structure system you have the super structure here and at the base you have pile foundations. Now one way for direct methods of soil structure interaction, this whole system need to be considered together and that is not a difficult task today because now computational facilities have developed like anything, now we are talking about 5G or so. But at the time the computer's speed was not so much and carrying out the soil structure interaction analysis considering the whole structure and the foundation was difficult. So, what has been done, we divide the system in two part, one is called foundation subsystem, and another is structure subsystem.

And with this division we consider one system at a time for the analysis but there is a continuous feedback from one system to another system. For example, for the time domain analysis for when t time equal to Δt that is the first step, you analyze the foundation subsystem, you apply some input motion at the base of the foundation subsystem and then so for what is the output at the top of the piles that is $U_p \Delta t$ for this case. Now when you consider the structure subsystem, then this is subjected to the motion which is due to the $U_p \Delta t$ which you have found in the first step and this is also subjected to your motion coming. So, we analyze this system. Now in the second step when you go for the pile foundations, then it is subjected to both input motion which is coming due to earthquake and the initial load which is same $F_p \Delta t$ find out using the in the structural analysis or the analysis of the building in the first step.

So, the forces applied in the second step are earthquake motion as well as the forces which is found at the base of the piles at the base of the columns or the top of the piles and then for the secondary step, we carried out the for the buildings also and this keep continue. Now the success of this scheme will depends on how small you consider delta t. So, for harmonic excitation we consider the delta t should be the less than T_p by 20 where T_p was a time period and for time history analysis delta t may be already given to you. Normally for example, delta t for many earthquakes is 0.02 second.

So, using this you find out the value of this useful and find out the response of the system. Now using the successive coupling scheme, we try to generate some results which is already published one was for case for single pile another was for case for 2 by 2 pile group. In case of single pile and 2 by 2 pile group both are situated or like you know inside the soil you have a pile here single pile and in this case 2 by 2 pile group. In the first case the structure is directly assumed as a lumped mass on the top of the pile. In the second case you have 2 soil columns which are scattered like away from both the piles but ultimately load will be transferred on the pile cap.

So, you have 2 by 2 pile group and the spacing between piles is important. So, the spacing considered between the piles is s by d equal to 5 where s is the spacing centre to centre spacing between the piles and d is the diameter of the pile. So, the dimensions are given here because it was a linear analysis and the load applied is was also symmetrical. So, as a result we were able to use only the quarter model of the soil. So, rather than completing considering the full finite element model which is computationally expensive.

So, we consider one fourth of the model and for consideration of this when you cut the model then you need to consider what we call the axis of symmetry and axis of anti-symmetry. So, you need to apply the conditions axis of symmetry and axis of anti-symmetry which is given here. For example, if I draw the plan of these models it is plan here and the front elevation in this case. So, let us see in the plan this is the boundary condition where rollers are applied, this is the boundary condition applied using axis of symmetry concept and this is axis for anti-symmetry. Once you apply this rollers that means in 2D case you can move in the horizontal direction but not in the vertical direction.

But in this boundary at the sides again you can move in the horizontal direction but not in the vertical direction. So, these are the two boundaries axis of symmetry and axis of anti-symmetry. So, when cutting another boundary, you put the Kelvin elements which consists of spring and dashpot. So, in the elevation the location of the Kelvin elements is also shown, and you have the bedrock, and this is also cut down. So, this is axis of anti-symmetry which you can expect here.

So, this work is already published in Canadian Geotechnical Journal in 2004. So, now what are the results? Let us discuss some of the results using effect of SSI on frequency

response. So, the results are segregated in different steps. First what is the effect on the free field response? Then what is the effect of response of single pile that is soil pile interaction? Then response of pile group and finally soil pile structure interaction that is the response of the complete system where you are considering the soil pile as well as structure.

Here is effect of soil on free field response. You could see that the free field response is getting peak value at some particular frequency, but after that it is decreasing. So, in this case on y axis you have the normalized free field response. One thing is important here to note that this normalized free field response is not a constant rather it is changing with the frequency, and you get peak value at some particular frequency the peak is coming and then after that it is started decreasing which could be seen in this case. So, this is the effect of SSI on free field response that means there is no pile or the foundation on the system inside the soil. Now if I insert the pile group inside the soil then the response which I get at the top will be considered as kinematic interaction and in this case the effect of soil pile interaction is considered and, in this case, there is a transfer function that is the response at the pile head divided by the input motion.

So, u_z . So, you have the input motion here in this case and that this difference in this case will be the difference in this case is nothing, but this was input motion was u_{naught} and u_z was the normalized free field response. So, in this case y axis is normalized with respect to the amplitude of the input motion u_{naught} while in the second case this has been normalized with the u_z where u_z was nothing but u_z was found out in this step first step. So, u_z is known. So, in the second step then we have defined the transfer function u_p divided by u_z where u_p and u_z are find out that the same point, but difference is that in case of u_p piles was present in case of u_z it is free field motion. So, you could see that this transfer function is not unity it is start from unit value then it goes some peak value and then again come down.

So, again there are certain frequencies where the response is peak. Now, we also carried out the analysis for the complete system that is soil pile reduction for a group and for a group of piles you can see that the effect of s by d that is a spacing ratio is quite large. So, for example, in this result you have the real part and imaginary part for four types of cases one is single pile then in another case you have 2 by 2 pile group but varying spacing between the pile group. So, you could see the single pile response is more or less flat which is almost near to 1 and even for the damping case also. But as you go increase the number of piles then it will depends on the spacing between piles.

So, black line was for single pile and other three are for 2 by 2 pile group with varying spacing. So, for the low spacing your behavior is quite different that means after certain point it come like you will have the negative values and that is there. While in case of s by d equal to 5 and 10 the peak values are happening at very different frequencies. So, that means what is the peak value that will depends on case by case whether this negative

stiffness may come when you have s by d equal to 2. Similarly, this was the case for stiffness same thing is for damping which is given in the part b which shows the imaginary part, and you could see again the black line which is for the single pile which is remain same more or less as before.

So, this is work which is already as mentioned published in Canadian Geotechnical Journal. When we consider the total system soil, piles, the piles like a foundation and then superstructure building what is the effect of SSI which is given in the slide. You could see that there are lines which are this line red line is without interaction and then these two are with interaction. That means without interaction means you are not considering the effect of soil as SSI. In that case your response acceleration factor shoots up like this which you could see here.

But when you consider the effect of interaction then these values first of all come down from very large value to a finite value which is coming here and then start decreasing with the period or time frequency. So, this is the more real case then. So, effect of SSI coupling on the response of pile soil dynamics and earthquake engineering it has been published. Now modal of boundary we already discussed there are different types of boundaries. First one is the elementary boundary where you need to put rest and only.

Then local boundary where you use the viscous dampers, and the coefficient of these viscous dampers can be found out using these equations here and this is for first one is for P wave another one is the for S wave. So, this should be CP P wave. So, using this equation you can find the damping coefficient. Then you have consistent boundary where in case of consistent boundary spring and viscous dashboards in parallel is used to simulate a semi-infinite region. For example, in this case dynamic stiffness is given by this relation g divided by $2\pi r$ naught s_1 plus $i s_2$.

In this case you have different conditions which we already discussed one during dynamic soil property. Here you have the rollers that is rest and has been put. So, this will be treated the first one will be treated elementary only. Then the second one is local boundaries.

Third one is called consistent boundary. So, in this case the coefficient dynamic stiffness can be find out using these relations which is for the springs. You have in this equation s_1 plus s_2 they are two dimensionless parameter. They are dimensionless parameter. So, in this case modelling of regress boundary has been done rigorously also using the concept of CFECM it has been done by Emani and Maheshwari published in SDEE 2009. So, what has been done? Here the interface is considered very near to the base of the structure and in this case, this is called CFECM, Consistent Infinite Simul Finite Element Cell method.

This is an alternative to the BEM. BEM we already discussed that is boundary element method which applies analytical solution to incorporate radiation damping. A cloning algorithm based on fully FEM formulation have been suggested by Dasgupta and Emani

and Maheshwari use the CFECM boundary for the SSI problem and its pile foundation. So, here is a modelling of boundary again for the sub structure system. What has been done? Here whole total system is divided into two part.

One part where you do not have the piles. At the location of piles some exploration has been done. But in the second part which is for the near field you consider that pile and pile cap together part of the soil and we analyze the near fields this case the second case and then find out the spring and dashpot coefficients which are used in the design for the far field case the first case. Then some work has also been done for considering solution reduction in liquefiable soils. Initial work was published by Finn and Fujita (2002) which proposed the design and analysis of pile foundations in liquefiable soils. Later on, Liyanapathirana and Poulos (2004) investigated the effect of nature of earthquakes and the assessment of liquefaction of soil deposit during earthquakes loading is carried out here. A significant parametric study on the influence of soil liquefaction on single degree of freedom system structure was performed. Sarkar investigated the three dimensional soil pile behaviour under dynamic load condition for the soil with liquefaction. This is reported in this publications Maheshwari & Sarkar (2011, 2012), Sarkar and Maheshwari (2012a and 2012b). So that you may go through if you are interested. These are the references which are the key references which are used in the preparation of this presentation and thank you very much for your kind attention. Thank you.