Earthquake Geotechnical Engineering

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Lecture 24

Two Dimensional

I welcome you again for this NPTEL lecture on earthquake geotechnical engineering. And today we are going to have the lecture number 24 which is under module 3 and module 3 consists of three chapters as we discussed and we are under ground response analysis that is the first chapter which consist of four lectures, three was on 1D GRA and the fourth one is two dimensional ground response analysis. So, we are going to talk about two dimensional ground response analysis. So, it is the last 2D ground response analysis which we are going to talk. Now the first issue is this why you need two dimensional ground response analysis. Is one dimensional ground response analysis not enough because one dimensional ground response analysis is simple.

But that 1D GRA is okay if you have a site for level or gently sloping sites with parallel boundaries and such conditions are not uncommon where one dimensional ground response analysis can be applied if you have layered site and there is no slopes, general slopes then you can use it. But there are problems of interest also where assumption of one dimensional wave propagations are not acceptable. And for the sloping or irregular ground motion surfaces the presence of, for example 2D ground response analysis will require when you have the sloping site, or you have irregular ground surface. In that case or if you have the presence of heavy structures or you have a stiff embedded structures or sometimes you may have wall and tunnels in that case you require either two dimensional analysis or even possibly three dimensional analysis may require.

Problems in which one dimensional analysis considerably greater than others like suppose one dimension is more, one dimension is going very long. In that case you can instead of using three dimensional analysis you can using the concept of what you call the plane strain conditions you can convert the problem instead of 3D into 2D. So, what are the issue like two dimensional ground response analysis for example, there you have the slope cantilever retaining wall in the first case. Now, in this case it is assumed you can use two dimensional analysis if the wall is continuously going in one direction with this uniform cross section, then you can use the cross section 2D analysis rather than consider third. Another problem could be a dam. In case of dam cross section will be in two dimensional, but if you go along axis then the third dimension it could be very long. Third could be for example, you have a tunnel which is the cross section of the tunnel circular here which appear to be in the figure 3 and this section cross section will remain uniform along the length of the tunnel. In that case using the plane strain condition two dimensional analysis can be carried out for the ground response analysis. Now, continue with 2D ground response analysis. Techniques for the solution of such problems have been developed using both frequency domain which is complex methods and time domain which is in the case of what we call the time domain method direct integration method.

So, you can have 2D and three dimensional dynamic response and soil structure interaction problems are most commonly solved using dynamic finite element analysis. For computational efficiency it is desirable to minimize the number of elements in a finite element analysis. So, naturally suppose in 2D analysis as we discussed you need to carry it out, this analysis will be carried out using finite element analysis. That means, it is not like in 1D case where you find out the transfer function and then you are carried out the closed form solution. So, wherever you use finite elements analysis then computational efficiency will increase. And another issue comes the maximum dimension of the elements are generally controlled by the wave propagation velocity and frequency range. So, it is saying maximum dimension of the elements what could be the because number of elements is one part, another part is the maximum size of the element. So, the maximum size of the element you cannot exceed after certain, you can have a smaller elements, but the maximum size of the element will be governed by what we call the wave propagation velocity or frequency range of interest. So, in two parameters it depends on that. So, when the frequency range of interest increases your size of the element decreases.

So, if you want to do the analysis for higher frequency then you need to take the smaller element. But similarly wave propagation velocity will also be governed. In fact, wave propagation velocity and frequency range both of these parameters can be clubbed in one parameter what is called wavelength. So, wavelength alpha or lambda let us say in shear like we have the shear velocity divided by frequency. So, wavelength now normally it is said that maximum dimension of the element which is maximum dimension is linked with this lambda and normally it is should be around lambda should be maximum dimension should be one-eighth of the lambda. But because many times you carried out the harmonic excitation, harmonic loading, so what I will suggest the size of the element it could be lambda by 4, lambda by 8, lambda by 12, but not lambda by 10. So, this lambda by 8 this should be in the multiple of 4 so that you do not miss the peak value. So, lambda by 4 is okay, lambda by 8 is okay, then you have lambda by 12, lambda by 16 and lambda by 20. So, if you use lambda by 20 let us say then your element become very small and for the non-linear analysis it is considered.

So, it depends on the level of non-linearity also. If you are going for the very high non-linear analysis then as a step size required is small. So, this is one side. Now you have fixed up, you have size of the element is governed from some other factors which is the factor is wave propagation velocity and frequency range. But within the size of the element, we have decided this should be the maximum size of the element.

Now the number of elements will depends on the size of your domain of discretized region. If you have a smaller domain, then number of size will be small. If your large domain with the same size of the elements the number of elements will be more. So, objective is now in 2D analysis will be to reduce the size of the domain and which can be done using what we call the boundary conditions which we are going to discuss. For many dynamic response and soil structure interaction problems, raised or near-raised boundaries such as bedrock are located at considerable distance particularly in the horizontal direction from the region of interest.

As a result, waves energy that travels from the region of interest may effectively be permanently removed from that region. So, like some of because when the waves travel far away they may not come back and so that energy is removed which we have discussed also in the last lecture. In a dynamic finite element analysis it is important to simulate the type of radiation damping behavior. What is radiation damping? What is material damping? That is also we already discussed. Now in case of two dimensional ground response analysis one of the most important issue is the boundary condition.

It will come in two dimensional analysis as well as in 3D. Boundary condition may not be important in 1D ground response analysis because in 1D ground response analysis your layer is going infinity and most of the time you have the bedrock at the base. So, the boundary condition does not like you know that comes except that at the surface the shear stress must vanish which we have discussed. But in 2D or 3D analysis you need to apply because you cannot consider the whole reason you will discretize like you know you will cut down your domain and for cutting down this domain for example you are carrying out the analysis you cannot go for infinity rather we cut like this. So, this even bedrock is also there so analysis carried out with this.

So, in this region you need to apply at the end some boundary condition. And what are the boundary conditions? Three types of boundary conditions are used elementary, local boundaries and consistent boundary. So, in this figure all three are given. The first A is called for the elementary boundary in which zero displacements are specified. So, what you have in this case? In 2D case you have rollers. So, along this roller that suppose in this here it can move horizontally but it cannot move vertically. While on the vertical face it can move vertically it cannot move horizontally. So, this is restraining the displacement of motion in one direction. Then you have dashboards. In the dashboards they are basically the absorber, they absorb the energy.

But in the third case which is consistent boundary you have used lumped mass parameter. So, here you have a combination of spring, dashboard and mass. So, all three are there spring, dashboard and mass and that is applied on each node even in 2D case for example this node is common. So, this at this node you applied in both the direction. So, these are the three types of boundaries which are used in two dimensional ground response analysis.

Now, we are going to discuss in detail about these three boundaries. So, condition of zero displacement or zero stress are specified at elementary boundaries. Elementary boundaries can be used to model the ground surface accurately as a free or zero stress boundary. So, here in the elementary boundary is the simplest boundary where we apply the condition either of zero displacement or zero stress at that node in particular direction.

And they can be used to model the ground surface accurately as a free or zero stress boundary. So, lateral or lower boundaries how the perfect reflection characteristic of elementary boundaries can trap energy in the mass that in reality would radiate boundaries away from the region of interest. So, this is the limitation of this boundary. What happens? When you put these conditions some restrictions like some fixed condition then they becomes like you are restricting that means it should not go beyond that. You put a condition that whatever energy is coming then it will sink there at this point it will not go beyond but that is not the actual in the actual scenario that is not the case. And in actual case when the waves travel, and it intersects some interface it may go up it may go deep and then it may not come back. So, it is called the resulting box effect which is due to the elementary boundary. It can produce serious errors in a ground response of or solution traction analysis. So, as a result naturally the elementary boundaries are not good.

They are simple. In fact, you need to put restriction only you do not use any other element. So, but if these elementary boundaries are placed far enough from the region of interest, then reflected waves may not come back or if they come, they may be damped out sufficiently to neglect their influence. So, if you are using the elementary boundary then you need to use the large size. So, as a result number of elements will increase and computation may increase though. So, the boundary is simple, but the size domain size required is large.

Now coming to the second boundary which consists of dashboard and that is called local boundaries. In this case a viscous dashboard is used to simulate a semi-infinite region for the case of normally incident body waves. So, if you have this viscous dashboard is okay if you have vertically propagating waves or normally incident body waves. That means the wave strike normally or perpendicular to the plane in that case they are good and it has been shown for example, Wolf and other researchers that the value of the dashboard coefficient necessary for perfect energy absorption depends on the angle of incidence of the impinging wave. So, if impinging wave is vertical normally then there is no issue it will absorb most of the energy.

But if your wave strike at some angle then these boundaries are not good. Since waves are likely to strike the boundary at different angles of incidence a local boundary with specific dashboard curve will always reflect some of the incident wave energy. So, it may not be able to absorb completely that wave energy. Still these are better than the elementary boundary and you may face some additional difficulties with local boundaries when dispersive surface wave reach a local boundary since their phase velocity depends on frequency. A frequency dependent dashboard would be required to absorb all their energy.

The effects of reflection from local boundary can be reduced by increasing the distance between the boundary and the region of interest. So, if you increase again it is similar to what we have discussed for the elementary boundary. If you use larger size of the like region then when these wave strike to this boundary then already, they may be damped out rather than so, then in that case your dashboard may be more effective. Now, the last one that is the third type of boundary consistent boundary. Boundaries that can absorb all types of body waves and surface wave at all angles of incidence and all frequencies are called consistent boundary.

So, there is no limitation. It could be if deal with the body wave, it can deal with the surface wave, it may be your wave may strike normally that is perpendicular or even it may be like the angle of incidence may be something different than 90 degree. So, even if it is striking at some angle then also it can absorb. The consistent boundaries can be represented by frequency dependent boundary stiffness matrices obtained from boundary integral equations of the boundary element method. So, in the boundary element method is in the short called BEM. So, like you have FEM like you also have BEM.

And in 1991, Wolf developed a lumped parameter model which consists of an assemblage of discrete three things, spring, dashboard and mass. And using those, the behavior can be approximately carried out. So, for example, this is the last one is consistent boundary and we discussed the boundary consistent consist of spring, dashboard and the mass. All three are there in case of consistent boundary. Now, similarly when we did 1D ground response analysis, then we use the equivalent linear analysis for GRA and non-linear approach also.

So, for this 2D GRA also, we are going to discuss for the application of equivalent linear as well as non-linear and then comparison between two. The two dimensional equivalent linear approach is very similar to the one dimensional approach as we discussed. A soil structure system is represented by two dimensional finite element method model. The input motion which may be due to an earthquake is represented by Fourier series that means you have in time history, normally acceleration time history, then using FFT you find out Fourier's spectrum. In case of Fourier spectra, you get frequency on x axis and Fourier amplitude on y axis.

And the equation of motions are solved for each frequency of the series with the results and then finally, you sum up the results to obtain the total response and then again use the inverse FFT. Now, how 2D ground response analysis can be applied for the real field problems which is given in this case. Suppose you have a dam or like say this is the dam and for simplicity, it is assumed in a river let us say or in the channel you have a dam. Now in this dam, if you go along the axis of the dam, then you will have quite long like you know the length of the dam. But if it is uniform and if I pick up a cross section of the dam, let us say the center here cross section of the dam which is given, and this cross section is reproduced in part b.

So, and then this cross section can be analyzed here with the what we call the plane strain condition. Plane strain condition will be applied that in the second direction beside the cross section you have a dimension very long. Practical situation where two dimensional ground response analysis are used. So, this is called plane strain conditions which can be assumed at the center of long dam along center section of dam to be modeled in two dimensions. Now, 2D GRA, so equivalent linear approach, continue with this.

Considering the problem of an earth dam which is shown in the figure, the last figure. Assuming that the axis of the dam is long, so we already discussed it, relative to its height the response of the center portion can be assumed to be two dimensional. So, two dimensional analysis can be carried out if the axis of the dam is long. So, in that case we apply the plane strain condition. Similar to 1D case, we also find out what we call the transfer function vector which is expressed in a transfer function if you recall in the 1D analysis that was a function of frequency.

When you change the frequency, the value of transfer function changes. So, H omega is a transfer function which is a function of frequency omega and on the right hand side you have two matrices, one is mass matrix M, another is k star which is complex stiffness matrix and omega is nothing but your frequency of excitation or harmonic loading component. So, the transfer function can be calculated. Once the transfer function vector has been obtained, computation of the response follows the same processor as we discussed for one dimensional complex response analysis. And what is the processor? You need to carry it out for equilinear approach, you need to carry out the iterations.

$$\{H(\omega)\} = \frac{[M]}{\omega^2[M] - [K^*]}$$

You assume some values of shear modulus and damping ratio at very low strain and corresponding to very low strain value, you find out the shear strain. Once shear strain is known, then you recalculate the shear modulus and damping ratio and whatever the value of shear modulus and damping ratio you get, again using the updated value of shear modulus and damping ratio you do calculation and again find the value of shear strain. So, if the shear strain which is find out now is same, no much difference than the previous cycle then it is okay, otherwise you continue iterations. So, in this case in 2D analysis, the primary compression of factor is evaluation of the transfer functions which we already discussed. For large problems, the mass and stiffness matrices are large, and evaluation of the transfer function can be quite time consuming.

So, as a result, we need to use some software rather than doing the hand calculation. For computational efficiency, the transfer functions are often explicitly related at only a limited number of frequencies with values at intermediate frequencies which are obtained by interpolation. Interaction towards strain compatible material properties can be incorporated on element by element basis. So, you have strain compatible material property because material property keep changing with the level of strain in both equivalent linear model as well as non-linear model. So, we need to update the value of material properties depending on the level of strain.

In case of two dimensional non-linear analysis, they can be used to estimate what is called the permanent displacement of slopes retaining structures and other constructed facilities. Two dimensional non-linear dynamic response analysis are performed by writing the global equations of motion from a finite element idealization in incremental form and integrating these in finite domain. So, you have this equation and this equation is, here in this equation, this is what is called the equation of motion, but damping is not considered. You have mass multiplied by and k star is complex number and the damping is considered in the part of k star. So, you have k, k is stiffness here, it is not the what we called the wave number.

$$[\boldsymbol{M}]\{\ddot{\boldsymbol{u}}\} + [\boldsymbol{K}^*]\{\ddot{\boldsymbol{u}}\} = -[\boldsymbol{M}][\boldsymbol{1}]\ddot{\boldsymbol{u}}_{\boldsymbol{b}}(\boldsymbol{t})$$

So, normally it is k into 1 plus iota twice, this way the damping is considered by considering the complex number rather than real number for stiffness. There is one called the shear beam approach which is much used for the dams. One of the earliest approach to the equilibrium of two dimensional ground was the shear beam analysis. And this was mostly applied for the analysis of the earth dams. However, even since then this is verified and expected to cover other varieties of problems and particularly explored by the Gazetas (1987).

The shear beam approach is based on the assumption that a dam deforms in simple shear thereby producing only horizontal displacements. So, here when the dam deforms, only the horizontal displacements are reproduced rather than the vertical displacements because like due to its weight or. So, here is the concept of the 2D shear beam approach. In this case, what is considered a homogeneous infinitely long dam as shown in this figure is considered. Assuming horizontal displacement to be constant at a given depth, so that means here it is assumed that your displacement may be different at different depths, but within the same depth these displacements are not varying in the horizontal direction.

For example, I consider a strip of thickness dz at a depth z from the top. So, it is assumed that the displacement in this strip may vary that will be constant along this horizontal direction. That means because it is a particular depth, so they will not be changing in the horizontal direction, but if you change the depth, then it will be different. So, in the vertical direction there could be variation, but not in the horizontal direction which is similar to what we have assumed this case for what is 1D ground response analysis. And it is assumed that it is homogeneous infinitely long dam and in that case the displacement at any point will be function of depth z and time t, but it will not be function of the horizontal distance from one point to another point, it is assumed that it is same in the horizontal direction.

In this case, what you have the shear stress acting on the top of this strip toe, but at the bottom of the strip toe plus del toe dz dz and once we have the total thickness of the dam is small h and the total height is capital H which is given here. So, these are the stresses which act in this case shown here. Then continue with this with the shear beam approach. And the shear beam approach is an excellent example of processor that through the judicious use of appropriate assumptions greatly simplify the important class of. So, here the good point was the shear beam approach that because some assumptions are made, but with those assumptions the problem gets very much simplified and one of the assumption is that there is no variation displacement in the horizontal direction.

The shear beam approach allows rapid estimation of many important response parameters and can be used to check the reasonableness of the results of mirror. So, what is done, whatever the results you get from shear beam analysis may not be exactly accurate but for the preliminary check on that then it may provide you quick solution and then you can compare whatever you got result from the shear beam analysis and the results which you obtain from the software. The shear beam transfer functions can be used in equivalent linear analysis or non-linear in elastic stress behavior can be assumed in an increment and non-linear analysis. So, while using the shear beam you may have equivalent linear and you can also have non-linear. Continue with 2D ground response, difference in the underlying assumptions and formulations of two-dimensional dynamic response analysis is normally lead to differences in their results.

So, you may have two different analysis, the results may be different but most of the time difference is that what are the assumptions in the first analysis, what are the assumptions in the second analysis that need to be understood. The proper use of these analysis require understanding of these differences. The two-dimensional equivalent linear method can suffer from the spurious resonances and difficulties which are associated with effective strain determination described for 1D case. So, for effective strain determination we have whatever the, for the equivalent linear models one of the limitation is that pore pressure is not generated. In addition, the different methods of vibrations associated with the extra degrees of freedom in the two-dimensional case completed the computation of the maximum shear strain and it require the use of another material parameters.

For example, Poisson's ratio but finding the Poisson's ratio is not a difficult. In addition to the shear modulus and produce more complicated stress paths. The equivalent linear approach is restricted to total stress analysis rather than what you call the effective stress analysis because this pore pressure will not generate in case of this equivalent linear approach but you can go for cyclic non-linear models or advanced constitutive models for modeling the pore pressure. Continuing with the comparison, two-dimensional non-linear methods have the enormously beneficial capacity of computing pore pressures, hence effective stresses and permanent deformation. The accuracy with which they can be computed depends on the accuracy of the constitutive models on which they are based.

So, on what constitutive model you are using for your analysis that also is important. The shear beam models are fundamentally different from the equivalent linear and non-linear finite element models and they restrict particle movement to the horizontal plane only. Finite element analysis which are capable of modeling an actual stem's tendency to respond vertically as well as horizontally, whether your load is applied vertically or horizontally but they can be modeled using the finite element. But the shear beam model forces all of the elastic wave energy to produce horizontal deformations rather than vertical deformations. So, it is there but in that case the finite element can do with the horizontal as well as vertical.

In fact, you can apply the load in all three directions together. As a result, shear beam models generally overestimate the fundamental frequency of most stems by about 5 percent and higher natural frequency by increasingly greater amounts. Shear beam displacements are compared well within about 10 percent with those computed by finite element analysis. But shear beam test, beam crest acceleration for flexible dams can be up to 50 percent or greater than those from the finite element analysis. So, one side you have the finite element analysis, another is the size of the shear beam crest acceleration.

So, there are some discrepancies. And this discrepancy is related to the whiplash effect produced by the higher shear beam models. For steep dams where these higher modes are associated with frequencies greater than those associated with earthquake motion, the computed accelerations match much more closely. So, whatever you compute using the given data and that is matching closely, that is very good. So, thank you very much for your kind attention. I stop it here. Thank you.