Earthquake Geotechnical Engineering

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Lecture 23

One-Dimensional (Conti.)

I welcome you again for this NPTEL lecture on earthquake geotechnical engineering. And we are under third module of this course and lecture number 23. In the third module as we discussed, we have three chapters. The first one is ground response analysis. So, we are discussing this, and this ground response analysis will have total four lectures. We already covered two lectures and today we are going to talk about third lecture. So, this is the last lecture on one dimensional ground response analysis. And in earlier two lectures, we talk about the transfer functions.

Firstly, we discuss about when you have undamped soil on a rigid rock, when you have then the damped soil on a rigid rock, and we also discuss damped soil on elastic rock. So, the about the transfer function we have discussed in detail. Now what has been covered so far is there, but today we are going to talk about the two topics. One is what we call the non-linear approach, another is comparison of 1D GRA. Like comparison in the sense this comparison will be between equivalent linear and non-linear approach.

Coming to the equivalent linear approximation of non-linear response, how we can use the equivalent linear method? Actually, the response of the soil during the earthquake or a strong ground motion is non-linear, but we will try to estimate this response using equivalent linear technique. And what is equivalent linear soil model? We have already discussed when we discuss the dynamic soil properties and if you recall that was a lecture number 18, 19 and 20. So, actually lecture number 18 and 19 are on equivalent linear and last 20, last year the lecture number 20 was on non-linear response. Coming to this how we can use this equivalent linear approximation for non-linear response, since thus non-linear soil behavior is well known which, we already discussed, the linear approach must be modified to provide reasonable estimate of ground response for practical problems of interest. And how we do it? The equivalent linears in the case here there are two parameters which we have discussed, one is called shear modulus G which is secant shear modulus normally and the equivalent linear damping ratio, which is epsilon as the damping ratio, so it is rather xi.

So, this G and xi will produce the same energy loss in a single cycle for the actual hysteresis loop. So, this is defined equivalent linear damping ratio which we already discussed during equivalent linear model that it will produce the same energy loss in a single cycle as was the case for actual hysteresis loop. Since the linear approach requires that G and this damping ratio be constant for each soil layer, the problem becomes one of determining the values that are consistent with that level of strain induced in each layer. So, what we do? We determine what is the level of shear strain in each layer and then accordingly the level of shear strain we revise the value of G and damping ratio. And naturally for that you need to use modulus reduction curve that is g by g max versus strain and another is damping ratio curve.

So given let us say modulus reduction and damping ratio curves are given using those curves you use this equivalent linear approximation. To address this problem an objective definition of strain level is needed and for this what we do equivalent of non-linear response that laboratories from which modulus reduction and damping ratio curves have been developed use simple harmonic loading and characterize the strain level by the peak shear strain amplitude. So, you have like normally in the laboratory what we do? We use most of the time in the laboratory simple harmonic loading or sinusoidal loading is used for the testing and but in case of real earthquake it is varying it is transient. So, the time history of the shear strain for a typical earthquake motion is typically highly irregular with a peak amplitude they may that may only be approached by a few spikes in the record for example it is here. So, what do you see? This is in this figure you have shear strain versus time which is strain time history.

In this strain time history this is harmonic excitation. So, this one is you have the harmonic. So, this harmonic loading or like basically it is sinusoidal wave while this one is due to the real earthquake, or this is a transient motion which is good. So, in harmonic loading as you know that you have the regular cycles after one cycle it repeat again so amplitude remain constant but in transient motion you get the peak value here and here but then after that it decreases. So, the peaks are at only few spikes in case of transient motion.

So, the two shear strain time histories are there with identical peak shear strains the maximum value same here for both the cases for harmonic and other things but the difference is that in case of harmonic it is repeating after when a cycle get complete it repeat again but this is not the case in case of for the transient motion. So now the issue with this one how we can equate like suppose in a real earthquake scenario you have this transient motion, and you want to find the equivalent harmonic loading so how we can go. We cannot like you know that in this case naturally this what we have here both harmonic as typically done in the laboratory test and transient motion as typically earthquake that have the same peak cyclic shear strain. The harmonic loading will represent a more severe loading condition than the transient record although the peaks values are identical why because the peak was only 1-2 in case of transient loading but in harmonic loading the same value of peak is getting repeated after one's completion of the cycle so that we need to understand. As a result, what is done it is common to characterize the strain level of the transient record in terms of an effective shear strain which has been empirically found to vary between about 50 and 70 percent of the maximum shear strain.

So, you have maximum shear strain so like you have peak value but peak value in the transient strain time history is not repeating. So, what we do we have 50 to 70 percent of the maximum shear strain that is considered to be equivalent. So, what has been done from the past experience the computed response is not particularly sensitive to the percentage however effective shear strain is often taken as 65 percent of the peak strain. So, this 65 percent is the equivalent which is taken as the effective shear strain. So, what is done you get the peak value from that transient and then you multiply by a factor of 0.65 and 0.65 then you get the kind of equivalent harmonic loading for that peak which can be considered to be irregular or harmonic loading. Similarly, with this since the computed strain level depends on the values of the equivalent linear properties an iterative process is required to ensure the property used in the analysis are compatible with the computed strain level in all layers. So, this is important what happens initially we assume some property and then we calculate what is this level of strain is coming and this level of strain which you are getting then you back calculate using modulus reduction and damping ratio curves the properties related to those strain it will be different it is not the same as. So, we keep doing this iterative process until there is convergence that means whatever the strain you assume and corresponding the properties are same. So, this is the case here for example, in this slide what has been explained that how to carry out the iterative process for the like you have modulus reduction curve that is shear modulus varying with the shear strain.

So, it is though not g by g max so, it is directly, and it is not on log scale it is g on y axis and shear strain on x axis. Similarly, you have damping ratio versus strain. So, let us say that in the first iteration what is done in most of the time to start your calculation you assume the in the values which are at very low strain and low for very low strain let us say I assume that at 0 strain shear strain. So, at 0 shear strain the value of g is g1 while the value of damping ratio is this xi 1. So, we know that assuming this value g and xi 1 we start calculation and once you compute and after computation, we will get the shear strain because to find out the shear value of shear strain you require the material property shear modulus and damping ratio.

So, in the first iteration and then you find out answer of the shear strain have come effective shear strain have come gamma effective 1 here. Now what we do we find out the value of g and damping ratio corresponding to the shear strain. So, I draw a vertical line and this horizontal g2 is the value of shear modulus corresponding to the system.

Similarly, on the damping side this xi 2 will be give you the value of damping ratio corresponding to the system. Now you have find out so you now you need to update the value of shear modulus and damping ratio in your system and again you calculate the strain and using the shear modulus and damping ratio you find the strain you find out strain and that corresponding to that strain because strain will be changed now strain will not be the same as gamma effective one.

So, corresponding to the strain corresponding to point 2 you find out the shear modulus. So, it is at point 3 and then using the shear modulus and then corresponding to this strain shear modulus you find out that will be your final value because here your level of strain is same. So ultimately when you keep iterating then in the last iteration again if you do the basically idea is you keep iterating until there is no change you reach to a point after which if you are still carrying out the iteration but there will be no update in the value of shear modulus and damping ratio at that point we will left. So, here corresponding to this strain you find shear modulus G3 and corresponding to the same strain here strain remains same you find out the damping ratio. So, this was the and normally depending on how much non-linearity you have you may get the convergence in few cycles maybe 3 cycles, 5 cycles or maybe 10 cycles you may not require 100 cycles or like this for conversion.

Now this was using so far whatever was discussing equivalent linear approximation continue with this. So, the steps are here with these steps I already explained you. So, for the first step initial estimate of shear modulus and damping ratio are made for each soil layer. The initially estimated values usually correspond to the same strain, but the low strain values are often used for the initial estimate. Then estimates shear modulus and damping ratio values are used to compute the ground response including time series of shear strain for each layer. Then effective shear strain in each layer is determined from the maximum shear strain in the computed shear strain time series for layer J. So, suppose if you have number of layers then for J layer the effective shear strain is calculated using the relation which is given here. In this relation what you have gamma effective J subscript is saying is it is for JTH layer. So, this is subscript and here is and gamma max is the maximum value, and this is for I superscript saying is that I is the superscript is denoting as a iteration number you keep iterating for the same layer. So, what we do we keep applying iteration for the same layer until you get the convergence and once convergence is reached then we find.

$$\gamma_{eff\,j}^{(i)} = R_\gamma \gamma_{max\,j}^{(i)}$$

What is R gamma here? R gamma is naturally the ratio of effective strain to the maximum shear strain which could be like 0.65 or whatever the like. Here the value of R y has been linked with the earthquake magnitude and it can be estimated using m minus 1 by 10. For example, if I have a 7 magnitude earthquake then it will be R gamma will be 0.6 but for m if you have m equal to 7.5 which is the standard R so R gamma will be 0.65. From this effective shear strain new equivalent linear values that is new values in that is I plus 1 TH iteration shear modulus g I plus 1 and damping ratio are chosen for the next iteration. So, step 2 to 4 are repeated until difference between the computed shear modulus and damping ratio values in two successive iterations far below some predetermined value in all layers. Predetermined value could be very less it could be 10 to power minus 3 or even if you have 10 to power minus 6 also.

$$R_\gamma = \frac{M-1}{10}$$

So, 10 to power minus 3 may be enough for that difference. Although convergence is not actually the difference of less than 5 to 10 percent are usually achieved in 3 to 5 iterations. That is you may not be want to get exact convergence it may take time but within the 5 percent limit you can easily achieve the convergence. Continuing with this, even though the process of iteration towards strain compatible soil properties allow non-linear soil behavior to be approximated but it need to be noted that a complex is still a linear method of analysis. Because what we do in the equivalent linear method, we do the same thing as for the linear method except that we are finding the value shear modulus and damping ratio for given value of strain.

The strain compatible soil properties are constant throughout the duration of the earthquake regardless of whether the strain at a particular time are small or large. The method is incapable of representing the in soil stiffness that actually occur during the earthquake. The equivalent linear approach to one dimensional ground response analysis of layered site has been coded into a widely used computer program called SHAKE. That is this SHAKE program was authored by Schnabel et al., 1972 that is more than 50 years back. Here this program SHAKE this was these authors was at University of California UC Berkeley.

So, UC Berkeley, some of the and then some others they have created. In the SHAKE program equivalent linear approach has been used to carry out the 1D ground response analysis 1D GRA has been carried out. Now, as we discuss in very much detail about equivalent linear method, but as we discuss that the behavior of the soil is truly non-linear and although equivalent linear approach is computationally convenient, and it is efficient, and it provides a reasonable results for many practical problems. However, this is still will be an approximate analysis of to the actual non-linear process of seismic ground response. Therefore, an alternative approach is to analyze the actual non-linear using direct numerical in the time domain has been carried out.

So, this is by integrating the equation of motion in a small time steps, any linear or non-linear stress system model or advanced constitutive model can be used. So, what do we do? We integrate the equation of motion in small steps and using integration a linear or non-linear stress strain advanced constitutive model can be used. At the beginning of each time step the stress-strain relationship is referred to obtain the appropriate soil properties to be used in the time step and by this method a non-linear inelastic stress-strain relationship can be followed. In a set of small incrementally linear steps, so what we do? You have a complete time step. In this complete time step you select some window or the time step where you apply this non-linear response and this need to be done like when we carried out you know that the equivalent linear approximation then we find out the strain time step for the whole time step too, but here it is done step by step.

And most currently available non-linear one-dimensional ground response analysis computer program characterize the stress-strain behavior that is what we call the constitutive model of the soil by cyclic stress-strain models and those models, cyclic stress-strain models are some of the models hyperbolic model which we already discussed when we talk about modified hyperbolic model, Ramberg-Osgood model, Hardin-Drnevich-Cundall Pyke (HDCP) model, Martin-Davidenkov model, and Iwan-type model. So, there are a number of models available, and these models are basically for to carry out non-linear analysis. And there are other models which are based on further advanced constitutive model such as the nested yield surface model. So, then there are other models also. Here some of the most commonly used computer programs for non-linear one-dimensional ground response analysis are given here.

So, computer program for non-linear one-dimensional ground response analysis, you have soil model and reference here. So, the program name is listed here, you have CHAR SOIL, DESRA-2, DYNA 1D, MASH, NONLI13, TESS1 and the name of the soil model used in these programs are also given for Ramberg-Osgood model, hyperbolic, nested yield surface, Martin-Davidenkov, then Iwan Type, HDCP and references the authors who have invented those models are given in the last column. So, some, a number of techniques can be used to integrate the equations of motion of these the explicit finite difference technique is most commonly easily explained. Now, in case of non-linearity, consider the soil deposit of infinite lateral extent, which is shown in next slide. So, here in this slide what you see, you have a soil layer and in the soil layer, the properties are let us say rho s, if we have single layer, if you have uniform soil deposit of infinite lateral extent overlying bedrock.

So, in that case, you have a single layer then the properties mass density is rho s and Vss is the shear velocity for the soil layer. And it is, the soil layer is lying over a backdrop and this backdrop is mass density is rho r and Vsr is the shear velocity of this rho. Then with this, we can divide this into a number of like no layers, if the properties are varying. If properties are varying, then we have from here to here, like you can have the first layer, on the top of the first layer is node number 1, the top of the second layer is node number 2, so you have 1 to n plus 1. So, in between these nodes, you will have n layers, first layer will be between 1 and 2 and 2 and 3 and like so on.

So, you have n number of layers, which has been discretized for the nonlinear analysis and these programs are able to do, deal with when you have the layered soils. One of the program which is based on so equivalent linear model is deep soil, which we discussed earlier. Now, comparison of this one dimensional ground response analysis, this comparison has been done from two aspects that suppose you carried out the analysis using equivalent linear method and the same analysis is carried out considering some nonlinear soil model, what are the difference in the results which you may obtain, so that has been discussed in these slides. First of all, the inherent linearity of equivalent linear analysis can lead to spurious resonances, that is high levels of amplification that results from coincidence of a strong component of input motion with one of the natural frequency of the equivalent linear soil properties deposit. If suppose you have, we have discussed what is the natural frequency of the soil layer. If suppose you have a soil layer, which is fixed at the base and thickness of the soil layer is h and v s is the shear velocity, in this case, the fundamental frequency is simply given by v s by 4h. Suppose your input motion have a similar frequency near to this motion, then what will happen? You will get the peak there and that is basically you get the high peak there, so that is the resonance condition. So, if a high level of amplification will be result, if some component of a strong motion, input motion coincide with the natural frequency of the equivalent linear soil deposits. And since the stiffness of an actual non-linear soil will not be constant, rather it will be changing over the duration of a large earthquake, such high amplification level will not develop in the field. So, normally it does not develop, so that is why that means you are on the conservative side.

So, the use of an effective shear strength in an equivalent linear analysis can lead to an over-softened or over-damped system when the peak shear strength is much larger than the remainder of the shear strength. So, if you get peak shear strength at very high value compared to other peaks, in that case, you may get over-softened or over-damped system. But on another side, you will get an under-softened or under-damped system when the shear strength amplitude is nearly uniform that is in kind of harmonic loading. So, in case of peak value, your one peak is going very high and other peaks are very low, then over-softened means it could be like that this stiffness will decrease very fastly and so that will happen. But if you have a uniform case like a harmonic loading, then it could be a case of under-softened or under-damped system.

Similarly with this comparison, equivalent linear analysis can be much more efficient than non-linear analysis. So, this comparison is between equivalent linear and non-linear analysis. So, these equivalent linear analysis because they have simplicity, their models are simple, so as a result, these analysis are more efficient. They are like the simplicity, and they may not be so accurate like non-linear, but they are very efficient in working. Particularly when the input motion can be characterized with acceptable accuracy by a small number of terms in Fourier series.

As the power, speed and accessibility of computers have increased in recently, so the difference between whether you use linear analysis, equivalent linear or non-linear analysis that is not like decreased. Earlier you do not have choice, you need to carry out equivalent linear analysis because the computer program was not available or the like the speed of the computer was not so much to carry out the non-linear analysis. But nowadays carrying out the non-linear analysis is not an issue; it can be carried out easily. Still non-linear methods can be formulated in terms of effective stresses to allow modeling of the generation, reduction, eventually dissipation of excess pore pressure. So, this is one of the limitations of the equivalent linear model.

So, if you use non-linear model, then you can model excess pore pressure which is important for liquefaction analysis. So, if you need to carry out the study for liquefaction analysis, then you need to select a non-linear model rather than equivalent linear model. So, this because equivalent linear methods, they will not be able to capture the dissipation of excess pore pressure. So, if you go from this point, then non-linear model, it goes in the favor of non-linear models rather than equivalent linear model. The fifth point, non-linear methods require a reliable stress strain or constitutive models.

So, important issue is this one, non-linear model can give you better result, you can consider the pore pressure, but in these models, you will require a constitutive relationship, which is stable or reliable. Many times, the parameters that describe such models are not well established as of those equivalent linear models and even it is difficult to find out because for those parameters, you need to carry out specific field and laboratory test and that may require to calculate the parameters of the non-linear models. So, to evaluate non-linear model parameters. So, issue is here, some points are good in equivalent linear model, some points are good in the non-linear models.

Now, it depends the requirements. Again, continue difference between the results of, now when we talk about the how much is the difference between the results of equivalent linear and non-linear analysis, that will depend on the degree of nullity of in the actual soil response for problem where strain levels remain low. So, suppose, see, normally what happens, the non-linear models are required when the level of strain is high. Equivalent linear models are good when the level of strain is low. But suppose if your level of strain is low, then you use equivalent linear model or non-linear model, the difference in the results will not be large, it will not be significant. However, at higher strain, there may be difference between the results of equivalent linear models and non-linear models.

So, for the low strain problem, you may consider to go with the equivalent linear models. But for high strain problems, for example, including the liquefaction, then one need to go to consider rather than non-linear model, one need to consider the, rather than equivalent linear model for high strain problems, one need to consider the non-linear analysis and that is expected to provide reasonable results. So, in summary, both equivalent linear and non-linear techniques can and have been used successfully for one-dimensional ground response analysis. The use and interpretation of each requires knowledge of their underlying assumptions, understanding of their operations and recognizing of their limitations. So, once we know what is the assumptions which has been assumed for the approach, one approach is equivalent linear, another approach is non-linear.

So, before using those both the approaches, we need to understand what the assumptions for both the cases are, how they operates and how the results coming out these need to be interpreted. Neither of these approach can be considered mathematically regress or precise, though normally these non-linear models are considered to be more regress compared to the equivalent linear model. Yet their accuracy is not in consideration with the variability of soil conditions. So, accuracy will depends on the soil conditions, uncertainty in soil properties and whatever the scatter you have in the experimental data through which you determine the input parameters for particularly for the non-linear model. So, thank you very much for your kind attention. Thank you.