Earthquake Geotechnical Engineering

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Lecture 10

Wave Propagation (Continue)

I welcome you again for the second lecture on wave propagation for this course. So, we are in the last chapter of the module 1 which is wave propagation and the last lecture. So, with this lecture, the module 1 will get completed. So, let us have recap of wave propagation that is lecture number 9, what we have talked introduction to wave propagation, 1D wave propagation in elastic load, 3D wave propagation in elastic infinite medium and then we talk about 3D wave propagation in elastic half space. So, the last part that is wave propagation in elastic half space semi-infinite elastic half space we will continue from there.

So, in last lecture last topic was partly covered and partly will be covered in this lecture. So, topics which we are going to talk is starting from wave propagation in semi-infinite elastic half space. Dispersions of surface waves will be covered, phase and group velocities for surface waves and attenuation of surface wave material and radiation damping. So, basically in this lecture in the last three topics we are going to talk much about the surface waves.

The first one is like how the surface waves are generated which we started in the last lecture and continue here. So, continue wave progression semi-infinite elastic half space. So, in practice if you consider infinite elastic body then you consider only two waves, but it has little significance why? Because ultimately your waves will travel from the earth and the wave need to come at the top ground level. So, when the wave needs to reach if suppose your wave is not reaching at the ground level and if it is travelling inside the earth only then we do not have much concern because our buildings our structures on the surface they are not going to be affected. So, if that would have been scenario that would have been better, but ultimately what happens the waves due to earthquake which are travelling inside the earth they ultimately come on the surface and they start damaging the structures.

So, that is the issue here. So, again we assume as discussed in the last lecture we assume that our medium is homogeneous isotropic and elastic. So, assuming that medium is homogeneous isotropic and elastic medium. So, here this is a wave propagation in semi-infinite half space you have x direction y direction. So, you see wave front there is a no change in y direction y direction it is same, but in x direction this amplitude may be different this may be different they are in this case amplitude may be varying along the x direction.

In the z direction it is again varying you have the maximum here and then decreasing here. So, variation in this case we are considering only in x and z direction, but not in y direction. So, that is a like for semi-infinite elastic half space. So, continue with this your third wave generated. So, we already discussed.

Now, the issue is this one there are three waves one is a v c another is v s and last one is v r. What is their wave velocity is related? So, this graph suggests you how the wave velocities are related and they are linked with the Poisson's ratio. So, there are three curves in this case one is p wave which is velocity when the p wave is in this case, we say v s v c some text writes v p. So, v p velocity in this case is same as v c compression wave in this we are considering v c and the velocity s wave is denoted as v s and r wave is denoted as v subscript capital R. Here v c and the ratio of v c or v s we already discussed that is nothing, but 2, 1 minus nu divided by 1 minus twice nu.

So, this is the ratio. So, if I put nu equal to 0 what will I will get root 2 for nu equal to. So, this value is nothing, but root 2 this value root 2 and as nu increases particular when nu becomes 0 what will happen when nu become 0.5. So, denominator will become in 0 and it go to infinity.

So, 0.5 this ratio v p by v s is going to infinity. While s will be throughout because v s by v s will be 1 only if we talk about Rayleigh wave velocity. See v r is here v capital R is Rayleigh wave velocity. So, Rayleigh wave velocity this is different than v r we use v small r this is velocity in the rod it is compression velocity in the road. So, when we use a small subscript r that will be treated as a velocity in the rod while capital R will be Rayleigh wave velocity.

So, Rayleigh wave velocity and we already discussed that your v r will be always less than v s it is though as a Poisson s ratio increases when Poisson s ratio around 0.5 you get almost same. So, both almost merge, but if 0 then you have little difference around 0.91 or so. So, this value is around 0.91. So, this is the relation between v c v s as a result v c is the highest value v s less than v c and v r is the least one. So, Rayleigh wave velocity is the smallest. So, as this graph is also saying and we discussed in the earlier lectures also because the p waves travel the fastest. So, the p waves are the first waves to arrive on any station when an earthquake come first p wave will come then shear wave v s will follow and finally, Rayleigh wave will come Rayleigh wave. However, this is the arrival time of the waves, but the amplitudes are different amplitude you see that the amplitude is the highest for surface wave which we are going to discuss later in the next slide.

So, I will skip this one. Now, one of the like topics which is we call dispersion of surface waves which is very important and this dispersion is basically applicable for surface waves only. This dispersion phenomena are not for body wave. So, this is we need to understand this is for surface wave. Even for surface wave for a homogeneous half space the Rayleigh wave velocity is related to a body wave velocity and they are related what we call the by Poisson's ratio.

So, the Rayleigh wave velocity is linked with the Poisson's ratio. The Rayleigh wave velocity in a homogeneous half space is independent of the frequency. So, when you have and here one thing important what we say homogeneous if your medium is homogeneous Rayleigh wave is a surface wave, but still, they will be they will not be dependent on frequency. The contrary to this the velocity of the love wave on other hand varies with frequency between upper and lower limit. So, the issue is here Rayleigh wave will not be varying with the frequency in homogeneous medium, but love waves whether it is homogeneous or heterogeneous medium their velocity will be changing with the frequency.

And we defined dispersion is a phenomenon in which waves of different frequency or different wavelength propagate at different velocity. So, the velocity will be depended on your frequency on what frequency or wave is travelling. So, at different frequency the waves will travel at different velocity and this is what we about surface wave not both for body waves. So, as a result this phenomenon is called dispersion. Dispersion means wave velocity changing with the frequency, if wave velocity is not changing with frequency, then we say it is non dispersive.

So, as a result your Rayleigh waves love waves are always dispersive whether your medium is you know homogeneous or heterogeneous. However, Rayleigh waves in a homogeneous half space are non dispersive. If you have homogeneous then it is non dispersive. Same thing for Rayleigh wave they will be dispersive Rayleigh wave will be dispersive for what they will be dispersive so or I can write it here. So, Rayleigh waves they will be dispersive for heterogeneous material.

So, in the sense you have a condition for Rayleigh wave whether it will be dispersive or non-dispersive that will depend on your medium. If medium is homogeneous non dispersive medium is heterogeneous dispersive, but love wave will be always dispersive irrespective of your type of medium. Again the Rayleigh waves if you have long wavelength that means low frequency then they will travel fastly and if you have the short wavelength that is high frequency then they travel slow. So, that means if you have this surface waves which is traveling very fast at a very fast rate in the medium in a particular medium then you can say they are traveling at a very low frequency. But if they are in the same medium if they are traveling at a slow speed then we can say they are traveling at a like you know the at high frequencies.

So, it is opposite. So, at what frequency they are traveling. Now since the velocities of both relevant and decrease with increasing frequency. So, as a result the low frequency components produce can be expected to arrive first at the site. So, first low frequency component will come then higher frequency components this thing we already discussed. So, the tendency to spread the seismic energy over time as in important effect of dispersion. So, after the dispersion the next issue is phase and group velocities and again these issues are for the surface waves only not for body wave. The solution for the Rayleigh wave velocity vr and love wave velocity vl they are based on the assumption of the harmonic loading which produces in infinite wave train. The velocities which are like this vr and vl they are nothing but they are called phase velocity. So, phase velocity is the velocity of the wave and love wave. But what happens they way these waves prefer to travel in a group.

So, as a result when they are traveling in the group their behavior is different than the waves individual wave is traveling. So, as a transient may produce a packet of waves with similar frequencies. If you have a packet of waves and these packets of waves, they should travel with the similar frequency then only we club together and this packet of wave travels at a speed what we call the group velocity. So, group velocity means the velocity of a group of waves is traveling together and naturally when they are traveling together then they need to talk to each other they are not independent. So, group velocity is given by this relation where C is called phase velocity which could be either vr or vl.

$$\mathbf{c_g = c + k\frac{dc}{dk}}$$

So, C may be if you have relative wave then vr if you have love wave then vl. So, C is known to you and dc by dk is the rate of this. So, rate of traveling and here k is called wave number and this wave number is nothing but omega by vr if you have relative wave or omega by vl. So, this is the wave number. So, now here the value of because dc by dk it is negative this quantity dc by dk is negative.

So, this is dc by dk this is negative and this is negative and once you have negative that means your Cg is going to be less than C. So, because as dc by dk is 0 if this is 0 then what will happen Cg will be same as a C that means group velocity and phase velocity are same. But in general, because dc by dk is less than 0 as a result group velocity is lower than the phase velocity that means your Cg will be less than C. So, this is the condition here. So, this was about phase and group velocity.

Now one of the important issues for the surface waves attenuation of course, it is applicable for body waves also, but we are going to discuss for the surface wave. As for attenuation is concerned attenuation literal meaning you understand what is the meaning of attenuation. So, this always like hitting here attenuation. Attenuation means reduction rate reduction in something attenuate even suppose if I see this light and if I go away from this light what will happen if I go far away then its intensity will decrease that is basically attenuation. So, if you are near to the source then you get maximum intensity.

The same thing happens with the earthquake also if you are near to epicenter if suppose an earthquake originate in the Himalaya and if you are in the Himalayan region only then it is expected that if we do not count local site effect then thus this maximum intensity will be the near the source. But if you go away from the source then what will happen intensity decreases and that is basically attenuation. So, when the elastic waves travel through the real medium then they attenuate with the distance. So, decrease with the distance intensity decrease with the distance and this attenuation can be attributed to two sources one of which involve the what is called the material through which the waves travel and another is called geometry of the wave propagation. So, the first one is called material damping.

What is damping? You know what is damping? I think we discussed earlier damping is nothing, but dissipation of energy, dissipation of energy that is the meaning of the damping. If any system has the damping so, it will dissipate energy. So, these dissipations are done in two form one is called material damping in another is called radiation damping. So, in the next slides we are going to discuss these both in detail. So, first thing is material damping in the real materials part of elastic energy of trailing waves is always converted to heat there is a loss or in fact, what I say you can understand when the waves is passing through a material then this material is trying to absorb some of the energy.

So, like you know that if you have springs below this seat of a cycle then you ride then spring like will be kind of a shocker. So, that will eat up some of the energy. So, that will try to reduce the jerk. So, similarly when the waves elastic waves pass through the material then the material like some of the energy converted into heat and this conversion decrease in the amplitude of the wave. So, what will happen it will go in the form as a when the wave travel further then there will be decrease in the amplitude.

And there is one thing what is called viscous damping which is easy to understand mathematically convenience. So, that is normally used for dissipation of elastic energy and when we consider the viscous damping then one of the models which is used like is called Kelvin-Vogat solids. In Kelvin-Voigot solids you have one spring and another dashboard Kelvin-Voigot solids. And so, it is this can be explained from this next slide here. So, this is a Kelvin-Voigot solid. Here you have two layers. In these two layers you have one spring which is represented by shear modulus G and you have damper which is represented by viscosity. So, here G represent your elastic shear modulus. G is nothing but shear modulus. And what is in this figure eta, eta is simply viscosity.

$$\tau = G\gamma + \eta \frac{\partial \gamma}{\partial t}$$

So, G will represent your strength while viscosity will come damping properties. So, let us say if I have this element which get deformed in x direction with an amount du. u is particle displacement in x direction and so, you have dz and dx. Thin element of Kelvin-Voigot solid which is subjected to horizontal shearing. Total resistance to shearing deformation will be given by the sum of elastic component as well as viscous components. So, how it is done? It is here. So, total shear stress is G into gamma and eta into del gamma by delta t. What is gamma here? Gamma is nothing but shear strain. So, the first component is proportional to shear strain while the second component is proportional to the rate of change of shear strain with time. So, this way supposes if you do not have viscosity, then your relationship is simplified as tau into G into gamma. But because the materials have the damping or so in the viscous form or other forms, then we need to use the second term also in this equation.

$$\gamma = \gamma_0 sin\omega t \qquad \text{(2a)}$$

$$\tau = G\gamma_0 sin\omega t + \omega\eta\gamma_0 cos\omega t \qquad \text{(2b)}$$

So, continue with this, so gamma can be represented in harmonic excitation, gamma equal to gamma naught sin omega t and as a result, the equation number this tau will be G into gamma naught sin omega t and the second term is omega eta into gamma 0 cos omega t. So, that means it will consist of the shear stress will be coming from both elastic component as well as viscosity component. So, they both together Kelvin elements. So, what we happen like if you draw stress and strain relation, so let us work on the stress strain relation that means tau and u.

This is called hysteresis loop. Either you will consider F or tau. So, both are not considered. If you consider force, then we consider displacement. If we consider stress, then we consider strain. So, let us consider as a stress strain loop. So, time being for simplicity I can remove that we are not considering force displacement. Right now, we are considering a stress and strain relationship. In this stress strain relationship, what you have? You have a loop. Area of this loop, the area of this loop which is shaded is given as a del W. What is W? W is the area of this triangle and this ratio del W by W divided by 4 pi is called nothing but what we call is damping ratio.

It is given del W is here. So, this is damping ratio. What is Jhae? Jhae is nothing but damping ratio. And this can be also derived and once we write it here, W is a total energy which is half g gamma naught square. What is g shear modulus? What is gamma naught? Gamma naught will be the maximum value here, this corresponding to this point gamma naught, this will be gamma naught. So, the strain, the total energy stored in the one cycle will be this.

$$W = \frac{1}{2} G {\gamma_0}^2$$

$$\xi = \frac{1}{4\pi} \frac{\Delta W}{W} = \frac{\eta \omega}{2G}$$

Damping ratio is given by 1 4 del W or W. So, as you ultimately end up n omega over 2g. What is n? n eta, sorry eta is viscosity. This eta is we already discussed nothing but viscosity. Equation 4 indicates that dissipate has energy proportional to the frequency of loading. So, what happens here damping ratio which you are getting is Jhae is proportional to omega.

However, when for the real soil when experiments were conducted, then it has been found that damping ratio is independent of the frequency. So, to make it independent of frequency another form is used which is called like a stress damping. So, the damping ratio is independent of frequency, it has been observed and to make this damping ratio independent of frequency one thing is used what is called a static damping. So, I think this is missing that is called static damping. In case of static damping, static damping, this damping ratio is not dependent on frequency.

So, for the soil it is more applicable than the viscous damping. Viscous damping is more for the concrete and steel. So, this was all about material damping the first part. Now, what happens? The part of the energy which get attenuate is due to the spreading of the waves when the waves get spread then what happens? So, some since material damping absorbs some of the elastic energy of a stress wave, the specific energy that is what is the specific energy? That is energy per unit volume decreases as the waves travel through a material and the specific energy can also decrease by another common fin mechanism which is pure geometric origin and this decrease in a specific can occurs at the area of the rod increases. So, when the rod area is increases what will happen? Suppose this is my rod, if your area is constant then it is okay, but if I increase the area of the rod, so what will happen? Because the waves are traveling initially like this then they start spreading and then when the spark is spreading then the energy will spread out in the environment.

So, that is called radiation damping and as a result decrease in specific energy. So, this is called radiation damping, this is also called geometric damping or geometric attenuation. So, three names are same. So, radiation damping is same as geometric damping or it is also called geometric attenuation and this is due to geometry that is and it is not related to material property of the medium rather than the waves are traveling far away then this is, so this is called disprecipitated by viscous or static or other mechanism.

So, now continue with this. So, what happens when earthquake waves is released from a fault which is below the ground surface, body waves travel away from the source in all directions, body waves will travel altogether, body waves will go up and this will go up, this will go down, this side or like in 3D case this side down, this side, this side, this side, all the six directions it will be going. So, this is the case here. But in case of geometric attenuation there is a geometric attenuation causes the amplitude of the body waves to decrease at a rate of 1 by R. So, you can say this is linked, this second point is linked with the body wave.

So, this 1 by R is related to body waves. So, this is rate of attenuation. While for the surface wave the rate of attenuation is 1 by root R, geometric attenuation, 1 by root R, geometric attenuation. As a result, we can say because R is greater than 1, so for the body waves rate of attenuation is faster than the surface wave. So, as a result when you go away from this epicenter, let us say an earthquake have come if I say in the Himalaya near Badrinath. So, when the waves will travel from here to in Roorkee, so first wave will come P wave, then you will get S wave, then you will get this relative waves or surface wave. So, the body waves attenuate fastly and the signal of body wave which is coming here will be very diminished. But while surface waves attenuate is a slower rate, so their amplitude is more at some distance away from the epicenter. So, this explains the greater proportion of surface of motion that is commonly observed at large epicenter distance. That means away from the epicenter you still feel there is an amplitude is good. So, this explains that why we should use the surface wave magnitude rather than body wave magnitude for characterization of distant earthquakes.

Suppose an earthquake have occurred long distance, so better if we record the surface wave then we can get some amplitude in the body waves it is difficult. But another issue is here because it still has some amplitude, so this will also explain why more damage is caused by surface waves or damage caused by surface waves is greater. Because rate of attenuation for surface wave is slower than the body wave. For problems in which energy is released from a finite source which is ranging from the large scale for rupture along an earthquake fault to the smaller scale case of vibrating foundation, radiation damping can be extremely important. And in fact, in such cases the effect of radiation damping often dominate those of material damping.

So, you have radiation damping and on another side is material damping. So, total damping before I end up this lecture, so you can say total damping will consist of material damping plus radiation damping. Now which one will dominate it will depend on your frequency of like you know that at what frequency. So, if you have the low frequency normally material damping dominate at higher frequency normally this radiation damping dominates. So, this completes this lecture and thank you very much for your kind attention.

So, with this lecture we completed the module 1. So, today this was the 10th lecture, so that means we completed 5 chapter. All 5 topics here starting from introduction of this course to the wave propagation we finished. Now then the next module we are going to start one of the most important topics of geotechnical earth cooking meaning which is called dynamic properties of soil or what we call the properties of soil for dynamic loads. So, that we will discuss and what are the components we will discuss in the next module 2. So, thank you very much for your kind attention. Thank you.