

**Pavement Materials**  
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**Lecture: 27**  
**Rheological Properties of Bitumen (Part 1)**

Welcome back friends. Today, we are going to discuss about the rheological properties of bitumen. If you can recall from the last lecture that we have started our discussion on basic viscoelastic principles and viscoelastic properties related to bitumen and similar materials. In the last lecture, we have talked about the basic concepts related to linear viscoelasticity. We discussed about principles such as Boltzmann Superposition principle.

We discussed about Time Temperature Superposition principle and we also talked about the constitutive equations of different elements whose combinations are used to represent the response of a viscoelastic material. Before I begin today discussing about the rheological properties of bitumen, I think that there are few points which went missing in the last lecture which I will first discuss and then I will continue with the present topic on rheological properties of bitumen.

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**MISSED POINTS FROM PREVIOUS LECTURE**

- **Kelvin (Voigt) Model:** Spring and dashpot arranged in parallel
- **Total stress is the summation of stress in each element**
- For equilibrium, **strain should be same** in both the elements

$$\sigma = E\varepsilon + \eta\dot{\varepsilon}$$

$\varepsilon = \frac{1}{E}\sigma_1$

$\dot{\varepsilon} = \frac{1}{\eta}\sigma_2$

$\sigma = \sigma_1 + \sigma_2$

• **Creep:** At  $t=0$ , strain is 0, as dashpot will not allow the spring to stretch

•  $\varepsilon(t) = \frac{\sigma_0}{E} \left(1 - e^{-\left(\frac{E}{\eta}\right)t}\right)$

• When load is removed, at  $t = \tau$ ,

$$\varepsilon(t) = \frac{\sigma_0}{E} e^{-\left(\frac{E}{\eta}\right)t} \left(e^{\left(\frac{E}{\eta}\right)\tau} - 1\right)$$

↑ stress applied      ↑ stress removed

$\frac{\eta}{E}$  = retardation time

If you remember, this was a slide where we have discussed about the constitutive equation of Kelvin-Voigt model and we discussed about the response of this model when the element is subjected to creep and relaxation loading. Well, in this particular equation which we derived here, the ratio of  $\frac{\eta}{E}$  in contrast to the relaxation time of the Maxwell model which we discussed is called as the retardation time.

So, this is called as the retardation time which I forgot to mention in the last class and this time is basically a measure of the time taken for the creep strain to accumulate. So, shorter is the retardation time, the more rapid the creep straining will be. So, this again is an important parameter

you can say which describes about the retardation behavior of a particular material following Kelvin model. So, this was one point which was missing.

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**MISSED POINTS FROM PREVIOUS LECTURE**

$a_T = \frac{\omega_{Tr}}{\omega_T} = \frac{t_T}{t_{Tr}}$  - *Not known, keep say, find shift factor*  
*Approximation -  $b(T, T_r, G, R)$*

- Response at low temperature is similar to the response at higher frequency (fast loading rate)
- Response at higher temperature is similar to the response at lower frequency (slow loading rate)
- This correspondence between time and temperature is a characteristics of 'simple thermo-rheological materials' such as bitumen
- Can be used to construct master curve for any given rheological parameter

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Coming to this particular slide which we discussed regarding the time temperature superposition principle, I explained to you that this principle is applicable for linear viscoelastic material and we discussed that how the response at different temperatures can be superimposed to another reference temperature by multiplying the time with a shift factor.

So, there is a horizontal shift which we have to apply and for the horizontal shift, the time or the frequency, whatever the x axis is has to be multiplied with a shift factor in order to shift the curves from one temperature to any given reference temperature. But I never told you how mathematically this  $a_T$  is basically described.

So,  $a_T$  here is the ratio of time at any temperature  $\frac{t_T}{t_{Tr}}$  which means that the time at the reference temperature has to be multiplied with the shift factor in order to get the reduced time corresponding to the temperature which we are trying to shift to the reference temperature. I will describe this with an example in the next slide.

Before that, a question is like how do we determine for a given material the value of shift factors? There are various techniques that are available. For example, we have non-linear least square fitting techniques. We also have several standard models. For example, we have Williams–Landel–Ferry equation. We have several equations which can be used to find out the shift factor for any given reference temperature.

And these equations they rely on certain model constants which needs to be evaluated or sometimes are assumed for finding out the shift factor values. We also have another standard equation which is used and that is Arrhenius equation. If you see the equation later, you will see

that this Arrhenius equation is basically a function of the temperature which we are trying to shift, the reference temperature at which we are trying to shift the response.

It depends on the activation energy of the material and it also depends upon the universal gas constant. So, these are some model parameters you can say or equation parameters which are required to find out the value of the shift factor. Here in this equation which I have written here, if it is in the frequency domain, it will just be the opposite. So,  $a_T$  will be equal to  $\frac{\omega T_r}{\omega_T}$ . So, this is just inverse of the time scale.

So, as I say that it might be sometimes confusing to just remember the theory without actually doing a practical problem. So, I have just tried to put up a problem which you can solve using an Excel sheet. Of course, you can use various other methods, but this is just to explain how you can use the time temperature superposition principle and plot the master curve for the bitumen sample on which you are doing the measurements.

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The screenshot shows an Excel spreadsheet with the following data tables:

Frequency/Temperature	10	20	30	40	50	60	70	80
0.0159	1330000	2010000	3110000	4310000	5610000	7010000	8510000	10000000
0.0252	1330000	1780000	2430000	3180000	4030000	5080000	6330000	7780000
0.0384	1330000	1620000	2170000	2820000	3570000	4520000	5670000	7020000
0.1	1330000	1470000	1920000	2470000	3120000	3870000	4820000	5970000
0.159	1330000	1410000	1860000	2410000	3060000	3810000	4760000	5910000
0.252	1330000	1390000	1840000	2390000	3040000	3790000	4740000	5890000
0.384	1330000	1370000	1820000	2370000	3020000	3770000	4720000	5870000
1	1330000	1330000	1330000	1330000	1330000	1330000	1330000	1330000
1.59	1330000	1330000	1330000	1330000	1330000	1330000	1330000	1330000
2.52	1330000	1330000	1330000	1330000	1330000	1330000	1330000	1330000
3.84	1330000	1330000	1330000	1330000	1330000	1330000	1330000	1330000
10	1330000	1330000	1330000	1330000	1330000	1330000	1330000	1330000
15.9	1330000	1330000	1330000	1330000	1330000	1330000	1330000	1330000

Temperature	Shift Factors
10	1
20	1
30	1
40	1
50	1
60	1
70	1
80	1

The graph shows stiffness (S) on the y-axis and frequency (omega) on the x-axis. Four curves are plotted, labeled T1, T2, T3, and T4, representing different temperatures. The curves show that stiffness increases with frequency and decreases with temperature. The relationship is summarized as  $T_1 < T_2 < T_3 < T_4$ .

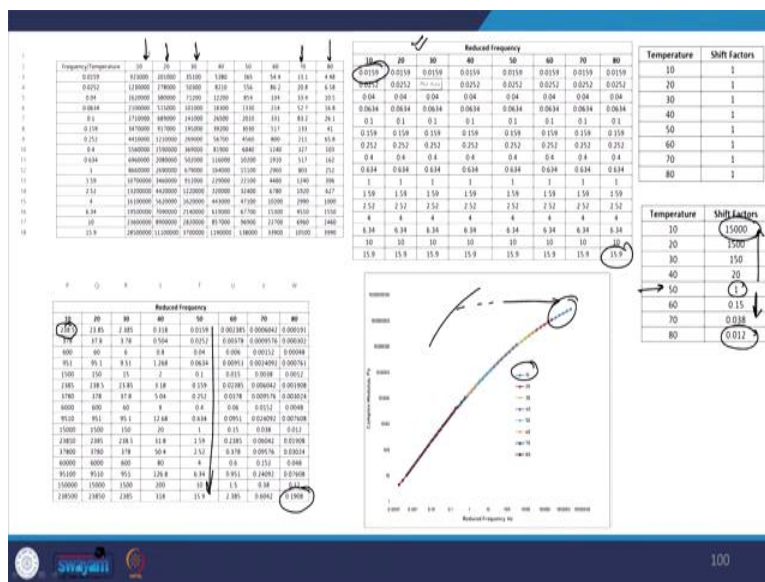
So, let us say that you have done an experiment, here in the first column we have the frequency values. So, I have done a frequency sweep test, at different frequencies we have measured the response. And what response have we measured? Let us say we have measured the stiffness of the bitumen sample or any linear viscoelastic material and at different temperature.

So, you can see we have 10 degrees, 20 degrees, 30 degrees, 40 degrees, 50 up to 80 degrees. So, at these temperatures we have done a frequency sweep test and the results which are shown in the table corresponds to the stiffness values. So, now, I have done here in the same excel sheet, I have created a separate set of cells at each temperature you see here, up to 80 degrees and I have written down the frequency values.

So, because at all the temperatures I am using the same frequency range, I have kept the same values in all the columns which you can see. Now, this I have done considering that the shift factor right now is 1. I am not doing any shifting. So, I am just seeing the individual response. So, if you will see it with respect to a curve. So, how this will look like? So, if we have the stiffness, if we have the frequency...

So, you will have these set of curves something like this. So, this can be a temperature T1, T2, T3, T4 and so on, where  $T1 < T2 < T3 < T4$ . So, we have the same frequency, but I have written it in different columns considering that I am multiplying each cell of a particular column with the respective shift factor which now I have considered as 1. So, I have considered the shift factor is 1.

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So, let us say that we are interested to develop the master curve at a given reference temperature. Let us say we are interested in reference temperature of 50 degree Celsius. So, as I explained you yesterday now what we are trying to do, these x axis values at different temperatures I will be multiplying with the corresponding shift factor and this shift factor is a function of the temperature which I am going to shift to 50 degree Celsius.

So, since I am interested in 50 degree Celsius. So, this shift factor at 50 degree Celsius is equal to 1. Now, at temperatures lower than this temperature, I will have to shift the curve to the right hand side. So, I have kept shift factor greater than 1. So, as we move from 50 degree Celsius down to 10 degree Celsius the shift factor will increase.

Similarly, above 50 degree Celsius that is 60, 70 and 80, the shift factor will reduce because I am doing an opposite shift here. So, once I multiply the shift factors with each cell for a respective column you will get these values. For example, you see here this 238.5 is basically equal to  $15000 \times 0.0159$ . Similarly, let us take another random point here, this 0.1908 is actually equal to the shift factor at 80 degree Celsius multiplied by the last value that is 15.9.

So, this is how I have recreated these columns by multiplying the, by multiplying the frequency at which I have done the measurement with the respective shift factors. And you will see that at 50 degree Celsius, since the shift factor is 1 the value remains the same with respect to this particular table. So, the shift factor does not change.

So, once you do that you will get a smooth curve where you see that 10 degree Celsius is the extreme right one which has been shifted. So, earlier maybe it was here and now we have shifted it by multiplying with a shift factor to this particular location, but we are not changing the value of the y here. So, this is again you have to remember that the value of stiffness remains the same. What we are doing we are multiplying the x axis with the shift factor.

So, we are changing the axis here not the vertical values. Sometimes for viscosity related functions we also have to apply vertical shift factor, but generally in the study of bitumen if we want to draw a master curve a horizontal shifting is sufficient. So, we are not talking about vertical shift factor here. So, I hope that this example it is clear.

And if you are given a similar example, so using an excel program you will be easily be able to draw the master curve. And just again one more point that this shift factors can be arrived using different process, sometimes people do manual shift which means you keep on changing manually until you obtain a smooth curve.

You can also use non-linear least square fitting techniques where you are minimizing the error between the points until you get a smooth curve or you can use equations such as WLF equation or Arrhenius equation and again there are various other equations that are available in the literature. One more point which remained was with respect to our discussion of Burgers model.

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**MISSED POINTS FROM PREVIOUS LECTURE**

- Generalized Model:** Burgers model with  $n$ -number of Kelvin (Voigt) elements
- Quantitatively, a single Kelvin model is insufficient to describe the retarded strain over longer period

**Creep:**  $\varepsilon(t) = \sigma_0 \left( \frac{1}{\eta_M} t + \frac{1}{E_M} \right) + \sum_{i=1}^N \frac{\sigma_0}{E_{Ki}} \left( 1 - e^{-\frac{E_{Ki}}{\eta_{Ki}} t} \right)$

**Recovery:**  $\varepsilon(t) = \frac{\sigma_0}{\eta_M} \tau + \sum_{i=1}^N \frac{\sigma_0}{E_{Ki}} e^{-\frac{E_{Ki}}{\eta_{Ki}} t} \left( e^{-\frac{E_{Ki}}{\eta_{Ki}} \tau} - 1 \right)$

$$J(t) = \frac{1}{E_M} + \sum_{i=1}^N \frac{1}{E_{Ki}} \left( 1 - e^{-\frac{E_{Ki}}{\eta_{Ki}} t} \right)$$

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So, many a times students ask that why we are doing so many calculations I mean if we are doing the measurement let us say we are interested to see the creep and recovery response of the bitumen we are measuring it in the lab using let us say a dynamic shear rheometer. Then why, since we already have the values why are we interested to do the modelling.

So, this is just to clarify that these techniques helps us to get more information about the property, the response, the behavior of the material or the fundamental aspects related to the response of the material which may not be visible within the measured domain. You might be you are doing an experiment within the limited measuring domain, but these measuring domain will not tell you much in a quantitative way about the actual viscoelastic response or behavior of the material.

So, if you are given two or three different bitumen, just by visualizing how the curves are changing from the experimental values you would not be able to fundamentally describe these changes or the behavior. So, these modelling techniques it helps us, because you see these modelling techniques involve parameters. For example, if we talk about the elements we are using we have spring elements corresponding to Maxwell model, we have spring element corresponding to Kelvin model.

We have dashpot element related to Maxwell model and Kelvin model. So, these parameters they have some definite meaning and the values of these parameters, once we compare we will be able to know more about the behavior of the material. Additionally these modeling techniques also helps us to do prediction in case you have only limited number of data or let us say you do not have an actual data, but you have some supplementary data.

Let us say you are interested to see the responses at 50 degree Celsius, but you have the response at 70 degree Celsius. So, using some additional techniques, even if you have the data at 70 degree Celsius you will be able to predict or tell about the response of the material at 50 degree Celsius. So, this is actually the importance of these modelling techniques and we have preliminary discussed about simple models here.

Let us say the spring and dashpot element, there are other complicated elements, we have fractional dashpot elements, we have a combination of spring and fractional dashpot elements. So, there are several other models which exist and you may definitely look at them if you are interested. The main purpose of this lecture was to describe in a simple way how the viscoelastic behaviour of bitumen can be expected to be.

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## WHAT ARE WE GOING TO LEARN?

- BITUMEN- A BINDING AGENT
- PRODUCTION OF BITUMEN
- CHEMISTRY OF BITUMEN
- PHYSICAL PROPERTIES
- INTRODUCTION TO VISCOELASTICITY
- **RHEOLOGICAL PROPERTIES**
- GRADING OF BITUMEN
- MODIFIED BITUMEN
- BITUMEN EMULSION
- CUTBACK BITUMEN



So, now, with this revision or some missing points from the previous lecture let us start discussing about the rheological properties of bitumen.

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### DEFINITION

- Rheology is the study of flow and deformation characteristics
- Rheological characteristics of bitumen is generally determined using a **dynamic shear rheometer (DSR)**
- DSR facilitates measurement of wide range of properties at varying loading conditions in short test time

$\omega = 2\pi f$

1 Cycle - A-B-C-A  
frequency = no. of cycles (or) Sine Oscillations in 1 second  
10 rad/sec 1.59 Hz

Response can be measured in **controlled stress**, as well as **controlled strain** mode

So, what actually is rheology? So, in very simple terms if I want to define rheology it is nothing but the study of flow and deformation characteristics of any material. So, for us the material of interest is bitumen. So, we are trying to study the flow and deformation characteristics of bitumen. Well, there are various techniques as we discussed, there are transient techniques such as creep experiments, relaxation experiments, which can be performed to understand these rheological characteristics.

The rheological characteristics of bitumen they are popularly or you can say generally determined using a dynamic shear rheometer. So, the advantage of using a dynamic shear rheometer or a

dynamic technique is that it facilitates us to get more information about the characteristics or response of the bitumen at varying loading condition, at varying temperature conditions in a relatively shorter period of time.

So, this gives us more information and that is why it is more popularly used to study the characteristics of bitumen. Now, what actually we do in a DSR? You can see as it is shown here we have a bitumen sample which is sandwiched between two plates. So, you can see that there is a top plate here there is a bottom plate here. So, this bitumen sample is sandwiched between these two plates.

And one part, usually the top part of the plate, it rotates or oscillates and the bottom part is fixed. Now, we can apply different form of loadings, as I said we can have an oscillation where we are giving a sinusoidal load or we can have a rotation, for example, if you are interested to see the, study shear state behaviour or study shear properties of the material, so we can also do a rotational test.

But usually we apply a sinusoidal load while studying the properties of the bitumen, so that we can have a better representation of what is actually happening in the field. So, we are trying to induce a loading condition similar to the field condition and that is why an auxiliary load is generally applied and then the response of the bitumen is observed.

So, here what we do when we apply the torque, so we are applying a torque here from the top plate which is moving and bottom plate is fixed. So, when we apply the torque to the oscillating plate, it starts say from point A, so it starts from point A it goes to point B. From point B the plate moves back and goes to C and from C it again goes to A. So, this is basically called as one cycle of oscillation.

So, what is one cycle of oscillation from A to B to C to A. So, this is one cycle of oscillation. If two such oscillations or two times this happens that is two times this happens in one second. So, if there is two oscillation in one second, which means that the frequency of oscillation which we are giving is 2 hertz or two cycles per second. So, one cycle is basically equal to A to B to C to A and how we are defining the frequency?

This is the number of cycles of such oscillation in one second. So, if it happens two times this movement from A to B to C to A, if it happens two times in one second, the frequency is two cycles per second or 2 hertz. And frequency of oscillation it can also be expressed in terms of the circumferential movement, let us say we are talking about radian. So, we have  $\omega = 2\pi f$ .

So, if we have the circumferential movement in radian it can be expressed as the distance travelled from A to B to C to A. So, this is, it can also be expressed in terms of radian. So, we will see that when we talk about specifications related to measurement of rheological properties of bitumen, for example, what we do in case of superpave PG grading, we do the standard technique or standard test at a fixed frequency of 10 rad/sec.

We will discuss about that in later in our presentation, it is believed that this 10 rad/sec or 1.59 hertz of frequency is a representation of the speed of the vehicle which is around 80kmph. But of



course, we will discuss about some debatable aspects related to this assumption, later when we will discuss about the grading of the bitumen.

Just for now we have to understand what is the meaning of oscillation, what is the meaning of one cycle, what is the meaning of frequency and how this particular oscillation is described in case of DSR? Well in the DSR when we give this auxiliary load it can be given in different forms. So, we can have a controlled stress experiment, we can have a controlled strain experiment. So, what is a controlled stress experiment?

In the controlled stress experiment we are basically fixing the torque of the spindle which is moving. So, the torque we are fixing and we are trying to change the distance which the spindle will move. In other terms, I will tell you later in the slide, in other terms we are basically changing the value of theta which is the rotation or the movement of this particular spindle. So, in control stress torque is fixed and the distance of movement is variable.

In case of controlled strain test the distance of movement which is from A to B is fixed and to keep this movement constant the value of torque keeps on changing. So, a DSR can be operated both in control stress and control strain mode and later we will discuss that usually the superpave tests or the, in the superpave grading they are done in constant stress mode and we will see how.

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**Responses**


- The response of a **purely elastic** material will be 'in-phase' with the input load ( $\delta=0^\circ$ )
- The response of a **purely viscous** material will be 'out of phase' with the input load ( $\delta=90^\circ$ )
- **Viscoelastic response** of bitumen lies between these extreme responses ( $0^\circ < \delta < 90^\circ$ )

$\epsilon = \epsilon_0 \sin(\omega t)$

Elastic  $\rightarrow \epsilon(t) = \frac{1}{E} \sigma(t)$       Viscous  $\rightarrow \dot{\epsilon}(t) = \frac{1}{\eta} \sigma(t)$

$\sigma(t) = E \epsilon_0 \sin(\omega t)$        $\sigma(t) = \eta \epsilon_0 \omega \cos(\omega t)$  *phase lag*

$\sigma(t) = E \epsilon_0 \sin(\omega t + 0)$        $\sigma(t) = \eta \epsilon_0 \omega \sin(\omega t + 90)$



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So, now let us move forward and try to understand more about the rheological properties in combination or linking it with the viscoelastic characteristics. So, when we are doing an auxiliary testing you will see that for a purely elastic material. So, let us say we are measuring the properties of a purely elastic material using the DSR the response will be in phase with the input load.

So, if you are giving a stress, so the response will be in terms of strain. So, strain will be in phase with the input load and there the phase lag will be equal to 0. I will tell you how. I will also tell you what this phase lag means. Moving forward if the response is purely viscous, let us say purely

viscous material, the response will be out of phase with the input load and here this phase lag will be equal to 90 degree.

So, I will shortly tell you what do you mean by this  $\delta$  value, what I am saying that the phase is it is in phase and out of phase I am describing the phase corresponding to a terminology or an expression like delta, which I have told is equal to 0 in case of elastic material and 90 degree in case of viscous material. Actually our bitumen which we are discussing about here is a viscoelastic material.

So, the response of the bitumen will definitely be somewhere in between these extreme responses which is elastic response and viscous response. Therefore, the value of phase lag should be between 0 degree and 90 degree. Now, let us see what do you mean by this 0 degree and 90 degree? So, let us say that you are giving a sinusoidal load and you are giving a strain to the material and the strain is described as  $\varepsilon = \varepsilon_0 \sin \omega t$ . So, you are giving a sinusoidal load to the material.

And of course, we are trying to see here the response in terms of stress. So, if it is a purely elastic material. now from our last lecture if you remember that the constitutive equation for an spring element, which is a purely elastic material, is  $\varepsilon(t) = \frac{1}{E} \sigma(t)$ . So, if I want to find out  $\sigma(t)$  it will be equal to what  $E \times \varepsilon(t)$  and  $\varepsilon = \varepsilon_0 \sin \omega t$ .

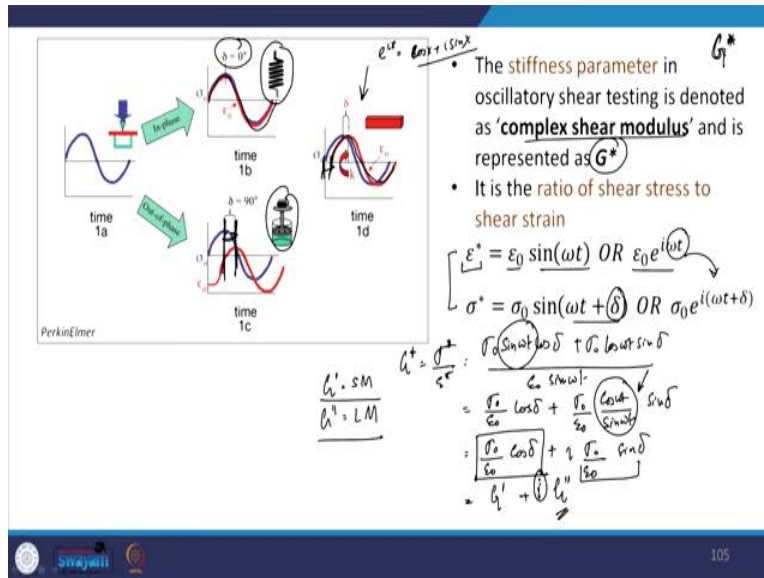
So, sigma here becomes equal to  $E \times \varepsilon_0 \sin \omega t$ , is not it. Now, the same expression I can write in this way that  $\sigma(t) = E \times \varepsilon_0 \sin \omega(t + 0)$ . So, this 0 indicates actually that the response is in phase, what is in phase, what do I mean by in phase that the response was in terms of a sinusoidal curve with a frequency of  $\omega t$  and the similar response I am getting with respect to strain in terms of  $\sin \omega t$ .

So, nothing is changing here. So, therefore, this is  $\omega t + 0$ . Now, let us see what, if it is a viscous material. So, again from our last discussion you can recall that this was the constitutive equation of a purely viscous material, where we say that the viscosity is the ratio of stress divided by strain rate. So, if I want to find  $\sigma(t) = \eta \times \dot{\varepsilon}(t)$ . What is strain rate?  $\frac{d\varepsilon}{dt}$ .

So, we are differentiating the strain is not it. So,  $\sigma(t)$  can be written as if you differentiate this strain it will be  $\varepsilon_0 \omega \cos \omega t$ , is not it. So, we have  $\varepsilon_0 \omega \cos \omega t$ . Now,  $\cos \omega t$ , I can also write it  $\sin \omega(t + 90)$ , is not it. So, you see now here the response is out of the phase by a degree of or a phase lag of 90 degree. What I am calling it a phase lag.

This shows that a purely elastic material will have a 0 degree phase lag, whereas a purely viscous material will have a 90 degree phase lag. Therefore, the response of a viscoelastic material will be somewhere between 0 degrees and 90 degrees.

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So, this is again a figure just to explain what we have discussed that if you are giving a loading to the material and you are trying to see the response, let us say you have given a stress and you are trying to see the strain response. So, if it is a purely elastic material you see the strain is moving in line with the stress. So, there is no change in the geometry of the curve. So, the point matches with each other, the peaks matches with each other in the x scale. So, therefore, delta is equal to 0 degree.

In case of a purely viscous material you see that the x axis at the peak is separated by a distance which is phase lag and delta is equal to 90 degree here. In case of a viscoelastic material you have the phase lag between 0 degree to 90 degrees and you can see here that how the strain response has a difference in x which is somewhere between 0 and 90 degrees.

So, this you can also imagine, now this behavior of the material you can also imagine with respect to a let us say a rubber ball. So, if you take a rubber ball which is let us say a polymeric type of material and if you drop it in the ground. So, it will jump back, but of course, it will not jump back to the same position from where you actually dropped it.

So, basically what is happening here that this energy which is in the ball a part of the energy is recovered, a part of the response is recovered and there is some dissipation of the energy, so when it is hitting the ground there will be some frictional forces there will be some damping which will happen because of which the rebound height of the ball is not same as the height from where I dropped it.

So, this difference again represent the, a typical behavior which we are talking about that the response is neither purely elastic, if it would have been elastic it would have jumped back, neither purely viscous it would have stayed on the ground, but it is somewhere in the between. So, there is an elastic response and there is some viscous response which has been dissipated.

With this let us move forward and here in case of rheological experiments when we have the response when we have the input load it can be a strain or stress and the corresponding response

in terms of stress or strain you can also define a stiffness parameter. So, here the stiffness parameter in auxiliary shear testing is denoted as complex shear modulus and is represented by this term  $G^*$ .

Here  $*$  represents that there is a complex behavior in the material and  $G$  represent that we are talking about the shear modulus. So, this is complex shear modulus. So, how do you define a complex shear modulus? It is the ratio of the stress to the shear strain. Let us say that we are giving an input strain to the material I am denoting the strain as  $\varepsilon^*$ , this star again is for the complex notation because I am describing the complex behavior here.

So, this is equal to  $\varepsilon_0 \sin \omega t$ . Now this can also be written in a Euler form, which is  $\varepsilon_0 e^{i\omega t}$ . So, I think all of us know that  $e^{ix}$  can be written as  $\cos x + i \sin x$ . It can be because this is complex form which we are trying to describe here for this material. So, it can also be written as  $\varepsilon_0 e^{i\omega t}$ .

Now let us see what is the stress response. So, the stress response will be  $\sigma_0 \sin(\omega t + \delta)$  and this  $\delta$  is neither 0 nor 90, it is somewhere between 0 and 90 or if I want to write it in the Euler form I can write as  $\sigma_0 e^{i\omega(t+\delta)}$ . So, I can also represent the stress in this form. Now as I said that complex modulus basically is the ratio of the stress to strain.

So, if I take the first two form. So,  $G^* = \frac{\sigma}{\varepsilon^*}$ . So, this becomes equal  $G^* = \frac{\sigma_0 \sin(\omega t + \delta)}{\varepsilon_0 \sin \omega t} = \frac{\sigma_0 \sin \omega t \cos \delta + \sigma_0 \cos \omega t \sin \delta}{\varepsilon_0 \sin \omega t}$ , So, if I just separate it out I will get that this is equal to  $\frac{\sigma_0 \cos \delta}{\varepsilon_0} + \frac{\sigma_0 \cos \omega t \sin \delta}{\varepsilon_0 \sin \omega t}$ .

Now this part because you see  $\cos \omega t$  is nothing, but  $\sin(\omega t + 90)$  and this is the out of phase component I can describe this form in complex notation that this is  $\frac{\sigma_0 \cos \delta}{\varepsilon_0} + i \frac{\sigma_0 \sin \delta}{\varepsilon_0}$ . So, this here, this is the in phase component because you see this was the in phase component and this is the out phase component.

So, this is basically denoted as  $G'$  which is the storage modulus. So, complex modulus can be distributed in two forms. So, one is the storage modulus plus  $i$  times, this is the out of phase component and it is obvious because it is with  $i$ . So, this is  $G''$ . So,  $G''$  denotes here the loss modulus. So,  $G^* = G' + G''$ .

Or you can say that  $G'$  has a part of elastic response of the bitumen,  $G''$  has a part of viscous response of the bitumen. Now sometimes students are confused that  $G'$  entirely represents the elastic response and  $G''$  entirely represents the viscous response. This is not the case because in a viscoelastic material like bitumen there are many a times some retarded elastic response which means that the elastic component keeps on building on as we give more time to the material.

So, this delayed elastic response is also a part of these modulus and elasticity is a part of storage modulus and viscosity is a part of the, or the viscous response is a part of the loss modulus. So, this should not be confused exactly as elastic modulus and viscous modulus. So, well you get this expression. Similarly, if you use the second form you will get the same.

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- The stiffness parameter in oscillatory shear testing is denoted as 'complex shear modulus' and is represented as  $G^*$
- It is the ratio of shear stress to shear strain

$$\varepsilon^* = \varepsilon_0 \sin(\omega t) \text{ OR } \varepsilon_0 e^{i\omega t}$$

$$\sigma^* = \sigma_0 \sin(\omega t + \delta) \text{ OR } \sigma_0 e^{i(\omega t + \delta)}$$

$$G^* = \frac{\sigma^*}{\varepsilon^*} = \frac{\sigma_0}{\varepsilon_0} \frac{e^{i(\omega t + \delta)}}{e^{i\omega t}} = \frac{\sigma_0}{\varepsilon_0} e^{i\delta} = \frac{\sigma_0}{\varepsilon_0} [\cos\delta + i \sin\delta]$$

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$$G^* = \frac{\sigma^*}{\varepsilon^*} = \frac{\sigma_0}{\varepsilon_0} \cos(\delta) + i \frac{\sigma_0}{\varepsilon_0} \sin(\delta)$$

$$G^* = G' + iG''$$

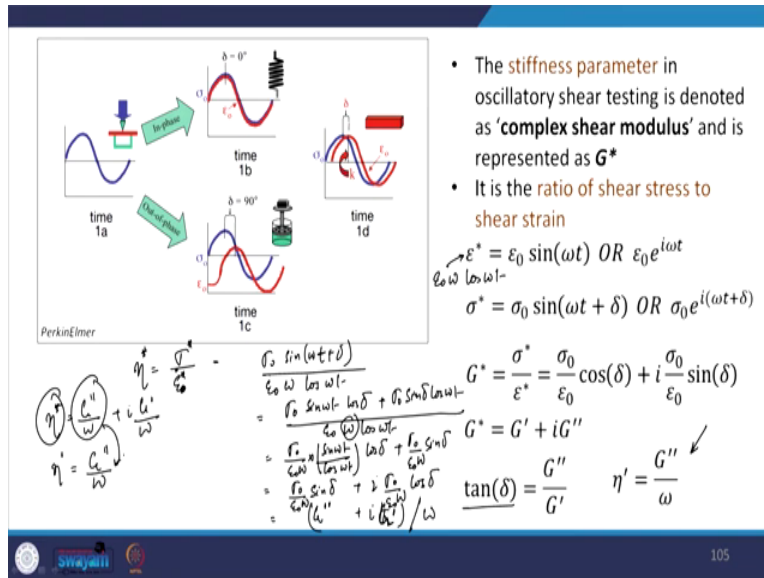
$$|G^*| = \sqrt{G'^2 + G''^2}$$

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So, if you use this form, it is again,  $G^* = \frac{\sigma^*}{\varepsilon^*}$ . So,  $G^* = \frac{\sigma_0 e^{i(\omega t + \delta - \omega t)}}{\varepsilon_0}$ . So, this becomes  $G^* = \frac{\sigma_0 e^{i\delta}}{\varepsilon_0}$ . So, this you can write it  $\frac{\sigma_0(\cos\delta + i\sin\delta)}{\varepsilon_0}$ . So, again this gives you the same equation for  $G^*$ .

So, this is what we have discussed, now this is how you describe  $G^*$  and this is how you write  $G^*$  in form of storage modulus and loss modulus and absolute value of  $|G^*| = \sqrt{G'^2 + G''^2}$ . So, this is again something which you can remember for calculations.

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There is another term which is called as the loss tangent or this is the damping component of the material, it describes the damping or the tendency or describes how well the material can absorb or lose energy. So, this is called as loss tangent  $\tan\delta$  which is described as the  $\tan\delta = \frac{G''}{G'}$ .

Additionally using the complex form of viscosity we can also find out the absolute viscosity using the loss modulus and frequency. So, how this expression comes? Let us say we know that eta is basically ratio of stress to strain rate, is not it, strain rate. So, if you are seeing complex behaviour, so let me put a star here. So,  $\sigma^*$  here if you see is equal to  $\sigma_0 \sin(\omega t + \delta)$ .

And if you take the derivative of this equation you get as  $\epsilon_0 \cos\omega t$ . So,  $\epsilon_0 \cos\omega t$ . So, if again I open up this equation what do I get? This is  $\frac{\sigma_0 \sin\omega t \cos\delta + \sigma_0 \cos\omega t \sin\delta}{\epsilon_0 \omega \cos\omega t}$ . Now, here this becomes  $\frac{\sigma_0 \sin\omega t \cos\delta}{\epsilon_0 \omega \cos\omega t} + \frac{\sigma_0 \sin\delta}{\epsilon_0 \omega}$ .

So, which means that this can be written as  $\frac{\sigma_0 \sin\delta}{\omega \epsilon_0} + i \frac{\sigma_0 \cos\delta}{\omega \epsilon_0}$ . And I know that  $\frac{\sigma_0 \sin\delta}{\epsilon_0} = G''$  and  $\frac{\sigma_0 \cos\delta}{\epsilon_0} = G'$ . So, therefore, what we get here that  $\eta^*$  and of course, there is one thing which is missing that the entire thing has an  $\omega$  with it because there is this  $\omega$ .

So, there is this  $\omega$  here. So, which means this expression is actually divided by  $\omega$ . So, what we can write that  $\eta^* = \frac{G''}{\omega} + i \frac{G'}{\omega}$ . So, if I want to break  $\eta^*$  in two forms, the real form and the viscous form, so the real form is the dynamic viscosity or the absolute form of viscosity. So, this can be actually written as  $\frac{G''}{\omega}$ . So, this is again something which is an important expression to remember.

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PerkinElmer

time 1a      time 1b      time 1c      time 1d

$\delta = 0^\circ$

$\delta = 90^\circ$

Stiffness alone cannot be used to describe the response of asphalt binder

Viscous Behavior

Elastic Behavior

both viscous and elastic behavior

- The stiffness parameter in oscillatory shear testing is denoted as 'complex shear modulus' and is represented as  $G^*$
- It is the ratio of shear stress to shear strain

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$$\sigma^* = \sigma_0 \sin(\omega t + \delta) \text{ OR } \sigma_0 e^{i(\omega t + \delta)}$$

$$G^* = \frac{\sigma^*}{\varepsilon^*} = \frac{\sigma_0}{\varepsilon_0} \cos(\delta) + i \frac{\sigma_0}{\varepsilon_0} \sin(\delta)$$

$$G^* = G' + iG''$$

$$\tan(\delta) = \frac{G''}{G'} \quad \eta' = \frac{G''}{\omega}$$

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Here in this graph what you see is two materials which can have the same value of  $G^*$ . So, you have  $G_1^*$  and  $G_2^*$  whose magnitude can be same, which means  $\sqrt{G_1'^2 + G_1''^2} = \sqrt{G_2'^2 + G_2''^2}$ . However, you can see that these two materials are entirely different. So, what differentiates between these two materials?

The elastic and viscous component which is described by the phase lag  $\delta$ . In case of the first material you see you have a higher  $\delta$ . What do you mean by higher  $\delta$ ? More towards 90 degree which means higher viscous response. In case of second material you have lower value of  $\delta$ , more towards elastic behavior or spring type behavior, so more elastic response and less viscous response.

So, lower is the value of  $\delta$  more will be the elastic response in the material, higher is the value of  $\delta$  more will be the viscous response in the material. So, we have to remember that stiffness alone or magnitude of complex modulus alone cannot be used to describe the response of the asphalt binder. The use of phase lag is equally important to see the entire behavior.

Now, we move on and we will try to discuss about the different parameters for bitumen testing. One of these parameter is describing the linear viscoelastic regime. You see in the last class we have discussed that for most of the purpose the response of the bitumen is described or the properties of the bitumen are measured under small strains.

And this strain should be within the linear viscoelastic regime. Just to recall what do I mean by a linear viscoelastic material whose response is a function of time and temperature only, whereas in non-linear viscoelastic materials the response will be a function of time temperature as well as the magnitude of the load which we are giving to the material.

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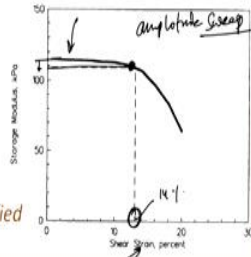
## DSR Parameters for Bitumen Testing

- **Linear Viscoelastic (LVE) Regime:** Generally standard DSR measurements are made within the linear viscoelastic regime to ensure application of TTSP and related theories
- As per SHRP, LVE is the point at which the value of  $G^*$  drops to **95% of its initial value**
- LVE limits are function of both **time (frequency)** and **temperature**, and can be evaluated using **amplitude sweep tests**
- LVE is stiffness dependent

$$\gamma_{LVE} = \frac{12}{|G^*|^{0.24}}$$

$$\sigma_{LVE} = \frac{0.12}{|G^*|^{0.71}}$$

From SHRP applicable to unmodified bitumen at 10 rad/sec



Generally the standard DSR measurements as I said are made within the linear viscoelastic regime. When we are applying time temperature superposition principle, when we are applying several other theories to explain the behavior of the material, these principles and theories necessitates that the material should be within the linear viscoelastic regime. So, therefore, it is important to find out the linear viscoelastic regime. And how do we define it?

As per strategic highway research program the LVE or the linear viscoelastic regime is the point at which the  $G^*$  value drops to 95 percent of its initial value. So, this is an assumption to describe the linear viscoelastic regime. So, you can see that here you have the value of say  $G'$  or  $G^*$  and here you have strain. So, this is an amplitude sweep test.

So, we are seeing how the modulus of the material is changing with the increase in strain value. So, if you remember from our discussion that we saw that when the materials are under small strains, we have the same graph for modulus or stiffness is not it, and then we had other curves which were on the lower side, but with higher strains. So, individual curves we had if you remember from the last presentation.

So, therefore, what we are expecting that within the linear viscoelastic regime the modulus does not change with change in strain amplitude. So, that is what we are trying to find here and how it is described? It is described up to a point where the  $G^*$  drops to 95 percent of its initial value. So, this 5 percent margin is kept to describe the linear viscoelastic regime.

So, you have to do a amplitude sweep test or a strain sweep test and then plot the variation of stiffness with respect to the strain, it can be a stress also and then you find out the point at which the value drops to 95 percent of its initial value and that point which means this x point, you can see here this is 10, 12, 14, somewhere around 14 percent for this particular figure.

Which means that whenever we are going to do the test on this material at the respective temperature and frequency we have to ensure that the testing is done at a strain level lower than 14 percent. LV limits are function of both time or frequency and temperature and can be evaluated



using amplitude sweep test. Now, this is an important point because our material is a time and temperature dependent material.

So, let us say you are testing at 10 rad/sec at 40 degrees Celsius. So, at this particular combination of frequency and temperature you will have one linear viscoelastic strain value or stress value whatever it is. If at 40 degrees Celsius if I change the frequency, let us say I am doing the test now at 1 rad/sec then my LV limit will again change.

So, the LV limit essentially is a function of both time and temperature. So, it is important that when doing this testing we have to ensure that at that combination of time and temperature the strain or stress which we are selecting are within the linear viscoelastic regime. Under the strategic highway research program when they were working to develop this PG grade specification a lot of testing were done on unmodified bitumen.

Now, this is a caution here that these equations which they state is mostly applicable for unmodified bitumen. They found that the LV limit can be related to the stiffness of the material and I think it is obvious because stiffness is a parameter which depends on temperature and time and so with respect to stiffness we can define the LV limits for the strain and stresses.

So, the strain LV limit is given by this equation which is  $\frac{12}{|G^*|^{0.29}}$ . Here the LV limit is in terms of strain the value of  $G^*$  is in terms of Pascal. Similarly, we can have a stress based LV limit it is equal to  $\frac{12}{|G^*|^{0.71}}$ . Here again the  $G^*$  is in terms of Pascal  $\sigma$  is also in terms of Pascal.

So, this equation can also be used and one more important point here that this equation we have developed corresponding to a frequency of 10 rad/sec. So, this is fixed here because most of the testing under the PG grading. In fact, the testing under the PG grading they are performed at a fixed frequency of 10 rad/sec.

So, we will stop here in today's lecture and we will continue our discussion from this particular lecture in the next class. Thank you very much.