

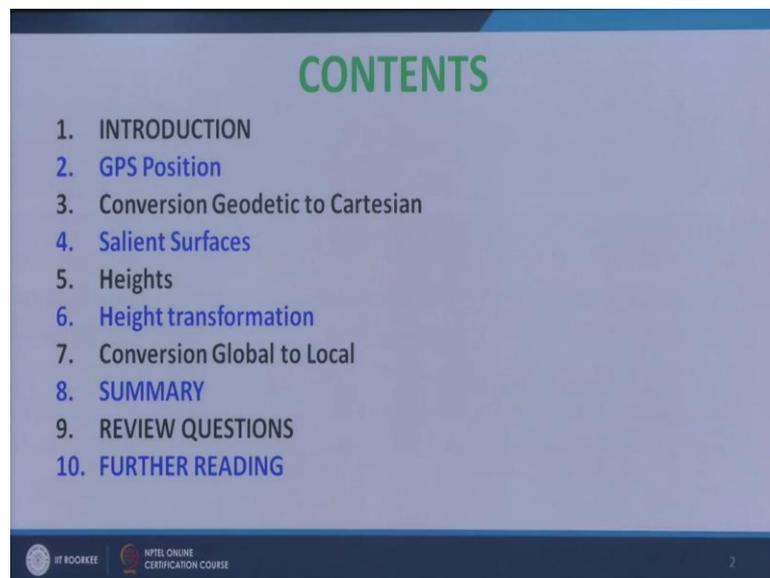
**Digital Land Surveying and Mapping (DLS and M)**  
**Dr. Jayanta Kumar Ghosh**  
**Department of Civil Engineering**  
**Indian Institute of Technology, Roorkee**

**Lecture - 09**  
**GPS Position**

Welcome students today. I will be discussing the lesson 9. The topic is GPS position. As you know we make use of GPS to find out the location of any points or station, but we need to know our in how this point is designated or how GPS represents these point. And in this class we will be discussing on that finally, we will make use of the location of the position of the point for making map.

So, we need to know how that position that has been given by the GPS need to be converted or transformed to some coordinate which is useful for mapping purpose. So, this lesson will be discussed under these following heads like first I will introduce to the topic than, what how GPS provides the position than we need to convert the position from one firm to other.

(Refer Slide Time: 01:36)



<b>CONTENTS</b>	
1.	INTRODUCTION
2.	GPS Position
3.	Conversion Geodetic to Cartesian
4.	Salient Surfaces
5.	Heights
6.	Height transformation
7.	Conversion Global to Local
8.	SUMMARY
9.	REVIEW QUESTIONS
10.	FURTHER READING

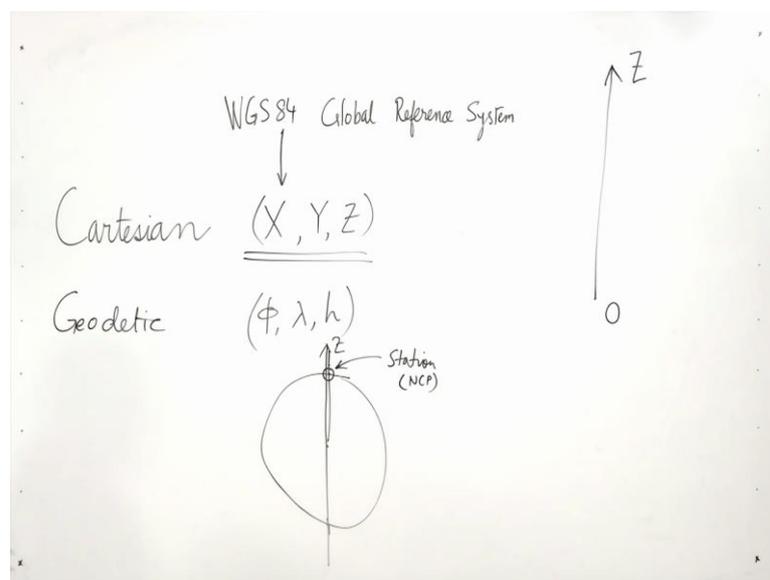
IIT ROORKEE | NITEL ONLINE CERTIFICATION COURSE

Now, to know more about this GPS position we need to know some more fundamental things like different salient surfaces of the earth different types of height their transformation than finally, global to local conversion. As we know as we have already learnt that GPS is used to determine the position of any point, but that point GPS

provides in a global reference system, and to understand the global reference system of GPS positioning we need to know different types of surfaces and heights and as we know that the engineering projects require local coordinate. So, we need to convert if we want to use make use of GPS observation for our engineering work, that has to be converted to local reference system. So, that conversion we will be learning in this class.

Now, GPS provides position in a global reference system that is known as WGS84 system. WGS84 global reference system the position of any point.

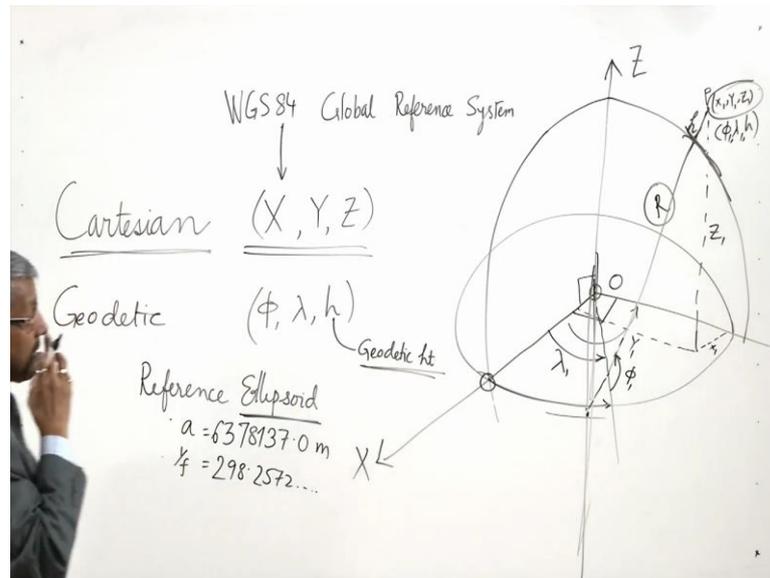
(Refer Slide Time: 03:03)



Under this reference system maybe 2 ways one is that Cartesian and as you know a Cartesian coordinate are defined by X Y Z, and it may be geodetic. So, it is latitude longitude and height above reference surface.

Now, in order to know really this, we need to define this axis. Now to define axis first we have to define the origin actually in case of WGS84, the origin of the earth has taken as the center of the coordinate system. And the line which represents the axis of rotation of the earth as the Z axis. Now we know that the earth is rotating about some axis, and that is actually an imaginary line. And, but on the surface of the earth that line intercepts at some other some point, and we have constructed some station here in the northern hemisphere it is the north in the north pole. And whenever station in the south pole and so long journey between these 2 is the Z axis.

(Refer Slide Time: 05:39)



Next. So, that Z axis is the most fundamental axis in our Cartesian coordinate system. Now if we imagine an infinite plane passing through this the primary meridian which is the meridian passing through Greenwich, we will find the line like this and an infinite plane which is perpendicular to the Z axis and passing through the center of mass of the earth that is the equatorial plane. So, if we take the intersection of the equatorial plane and the if we take the this is the equatorial plane, this is the plane which is this is the line of intersection of the surface of the earth will be equatorial plane. And if we take an infinite plane passing through the prime meridian, there will be an intersection like this with the surface of the earth.

Now, this intersection point if we imagine a line through that point that is our X axis in this way we define the X axis, Z axis is the primary axis first we define the origin then we define the Z axis and with reference to Z axis when infinite olden type plane passing through the center of mass of the earth perpendicular to the Z axis, that is the equatorial plane and another plane infinite plane passing through the Z axis as well as through the center of mass and perpendicular to the equatorial plane. And passing through the prime meridian that is an intersection of that to and will be a point, through this if we pass this line that is the X axis. Now if we imagine another line which is perpendicular to the X axis in the equatorial plane than in the right hand direction. So, that is the Y axis. So, in this way X Y Z axis is being defined for WGS84 coordinate system

So, now if we imagine any point on the surface of the earth any point. So, that may give here by if we drop a line perpendicular to this in parallel to Z axis. And if you take a line parallel to Y axis and a line parallel to X axis this is the X coordinate of this point, this is the Y coordinate of this point and this is the Z coordinate of this point. Or  $X_1 Y_1 Z_1$ . So,  $X_1 Y_1 Z_1$ . Now this is how this is where how Cartesian coordinates is being defined and we can get the position of any point.

Another system in which the GPS position we get that is the geodetic coordinate for that first, we have to define a reference surface which is called reference ellipsoid reference ellipsoid.

(Refer Slide Time: 09:02)

## GPS Position

Cartesian coordinates (X,Y,Z);

- Origin : Centre of mass of the Earth including oceans and atmosphere.
- Z-Axis : The direction of IERS Reference Pole (IRP) – IRP directed towards CTP (Conventional Terrestrial Pole) as defined by BIH (the Bureau International de l'Heure) with an uncertainty of 0.005".
- X-axis : Intersection of IERS Reference Meridian (IRM) and the plane passing through the and normal to the Z-axis. (IRM coincides with BIH zero meridian (epoch 1984.0) with an uncertainty of 0.005").
- Y-axis : A right-handed, Earth Centered Earth Fixed (ECEF) orthogonal coordinate system, measured in the plane of the CTP Equator.

Towards IRP  
IRM  
Earth's Center of Mass  
Z<sub>ECEF</sub>  
X<sub>ECEF</sub> Y<sub>ECEF</sub>

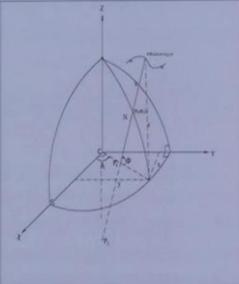
IIT ROOKEE    NPTEL ONLINE CERTIFICATION COURSE

(Refer Slide Time: 09:04)

### WGS 84....

**Reference Ellipsoid**  
A geocentric ellipsoid of revolution with center of mass of the earth as origin and the Z-axis of the WGS 84 Coordinate System as the rotational minor axis having semi-major axis (a) : 6378137.0 m and reciprocal of flattening, 1/f : 298.257223563.

**Geodetic Coordinates ( $\phi, \lambda, h$ )**  
Geodetic latitude ( $\phi$ ) of any location on the surface of the earth is the angle measured between the plane of the WGS84 ellipsoidal equator and the normal to the ellipsoid which extends to the earth's location above the WGS84 ellipsoid. Geodetic longitude ( $\lambda$ ) of any location on the surface of the earth is measured in the plane of the WGS84 ellipsoidal equator between the IRM (International Reference Meridian) and the meridian passing through the position. Geodetic height (h) is the distance of the point along normal from the WGS84 ellipsoid.



IT ROOKEE    NPTEL ONLINE CERTIFICATION COURSE

So, in this figure you can see this is the reference ellipsoid because it is a sectional view. So, it looks like this now an ellipsoid is an ellipse which rotates about the minor axis and it is defined by the semi major axis and in the reciprocal of flattening. So, this is the 2 parameters how we do define the ellipsoid.

Now, in case of WGS84 it is the semi major axis is given by 6378137 meter. So, you can say 6378.137 kilometer or and the reciprocal is 10 2 90.2 5 7 2 like that 2 5 7 2 like. That now this is the reference surface reference ellipsoid above which the height is measured in case of WGS84 coordinate system.

Now, suppose this is the same point if we take that will be defined by a latitude longitude and height. Now for the height latitude means now, it is if we you see here latitude is defined by if we drop a perpendicular actually this is the reference surface, suppose this is the reference surface. So, if we drop a normal on it will reach to the equatorial plane at some point like this now if we draw like this. So, the from this equatorial plane the angle above this that is the latitude. And the this is the reference meridian; that means, this one; that means, this one is the longitude of the place or point and since this is the reference surface reference ellipsoid this is the height, and this is called also geodetic height geodetic height above reference ellipsoid.

So, for the same point t we can GPS provides us position in 2 ways one is that Cartesian coordinates  $X_1 Y_1 Z_1$  another is that latitude longitude and height. So, this is the most

fundamental way how the GPS provides us the position at any point, and this is with respect to a global reference system now many times we need to convert between the geodetic coordinate and Cartesian coordinate.

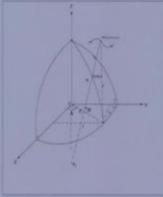
(Refer Slide Time: 12:45)

**Conversion : Geodetic TO Cartesian**

The Cartesian coordinates(x, y, z) can be obtained from its (in the same coordinate system) geodetic coordinates (φ, λ, h), using the relation:

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} (R+h) \cos \phi \cos \lambda \\ (R+h) \cos \phi \sin \lambda \\ \left(R \frac{b^2}{a^2} + h\right) \sin \phi \end{bmatrix}$$

where R is the radius of curvature of the prime vertical at P<sub>2</sub> passing through P and can be obtained from  $R = \frac{a^2}{(a^2 \cos^2 \phi + b^2 \sin^2 \phi)^{1/2}}$



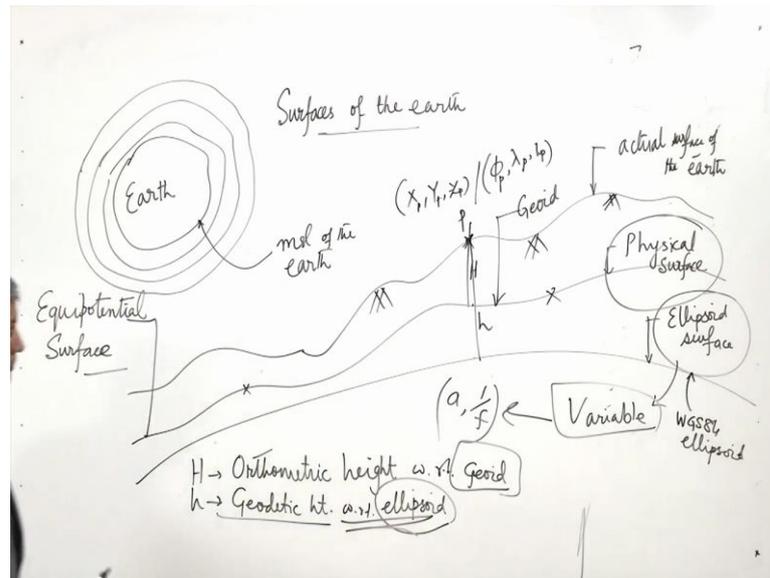
IT FOOTEKKE    NPTEL ONLINE CERTIFICATION COURSE

So, if we know the geodetic coordinate phi lambda and small h then the Cartesian coordinate can be determined by using this relation. Where r is given by this r means it is the distance from the reference ellipsoid to the point where this if we extend this Z axis it will cut at sub point. So, now, this is the radius of curvature at this point this is called r is the radius of curvature at the point and the reference ellipsoid at which the normal from the point is intercepting.

So, and the r can be determined by using the semi major axis as well as semi minor axis b is the semi minor axis. So, and also it depends on the latitude of that place. So, the radius of curvature at any point on the surface of an ellipsoid depends on 3 factors semi major axis semi minor axis as well as latitude of the place.

Now, and it is given by the relation a square divided by a square cos square phi plus b square sin square phi to the power half. So, using this relation we can convert the geodetic coordinate to Cartesian coordinate. And inverse by inversing, we can get the Cartesian coordinate from Cartesian coordinate we can get also the geodetic coordinate.

(Refer Slide Time: 14:46)



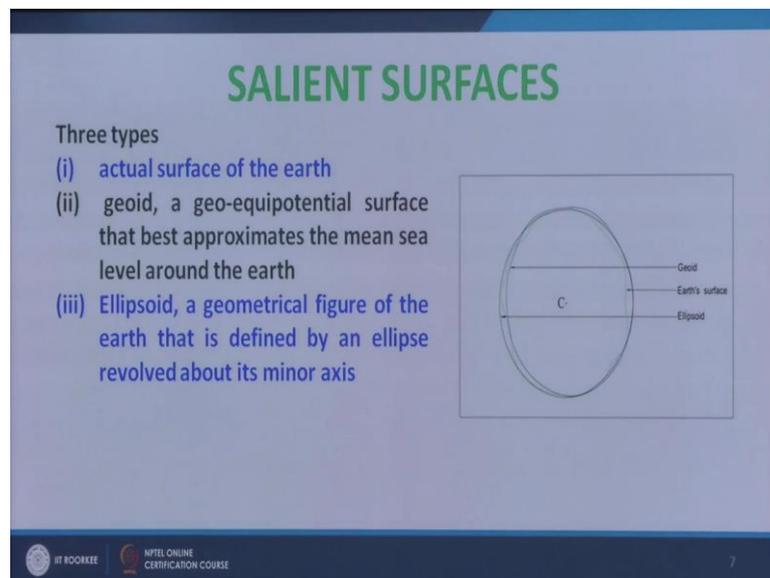
To understand the GPS positioning better, we need to know some fundamental things like the surfaces of the earth. Actually fundamentally it is the surface of the earth means the place or the surface on which we people are living, that is the actual surface of the earth actual surface of the earth, but it is this surface is very undulating and you can see that any point may change any time, because we are going for cutting filling and we are doing. So, many activities on it. So, it is very difficult to know or model this type of surface.

Now, as we know that the earth is about ellipsoid in shape. So, geometrically earth is mathematically or geometrically earth is defined as ellipsoid surface. Earth is defined as a soli ellipsoid surface, mathematically, but geometrically, but it really does not represent the earth or different activities that we different engineering projects that we do carry on, we will have a very we will not have we or we will not be able to do by considering the ellipsoid surface of the earth as the reference surface. So, we need to have another surface which is called which is called geoid surface or which is called physical surface of the earth, physical surface of the earth actually this is a surface which represents an equipotential it is an equipotential surface; that means, if we consider one point here another point this than we if we there will not be any energy required to go for this point to that point because these points are on equi potential surface. And most of the engineering activity depends upon the gravity energy and all this thing.

So, we represent the earth through this surface and this is called physical surface of the earth and to take make it a reference actually, if we consider the earth something like this we may get infinite numbers of equipotential surfaces around the earth this is the earth we can get infinite numbers of surfaces around the earth so, but which we will consider as a reference. So, scientist have considered the that physical that equipotential surface of the earth will be considered as reference which best approximates the mean sea level, mean sea level of the earth. Because as we know that the earth surface is mostly covered by water 70 percent of the surface of the earth is water. So, the equipotential surface that best approximates the mean sea level of the earth is considered as the reference surface and that is known as geoid.

So, geoid is considered as the reference physical surface and this actual surface of the earth is whatever is available and the ellipsoid surface this is a variable, because it will depend upon the value of a and 1 by f. So, depending upon our requirement we can vary this value of a and f and we can take this difference ellipsoid surface and for WGS84 already I have shown you that the value is taken as a and f and with value of a and f the WGS84 has been decided.

(Refer Slide Time: 19:44)

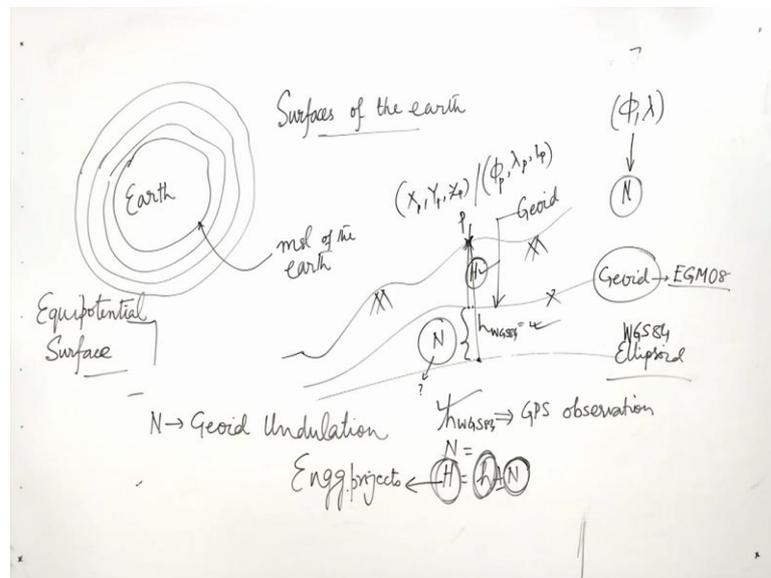


Now, if we say a point suppose again  $p$  whose coordinate is  $X_p$ ,  $Y_p$  and  $Z_p$  or a square  $\phi_p$  and this is.

Now, this point p will have may have different types of height depending upon from which reference we are considering. Now if we consider the height of this point above the physical surface of the earth than this height is called as orthometric height. So, h is the height ortho metric height, and if we consider the height of this point with respect to ellipsoid than it is called geodetic height.

Now, orthometric height is with respect to geoid and now this geoid is constant for the earth, but geodetic height is with respect to with respect to ellipsoid now as I told you this ellipsoid may vary from your consideration your different project to different project or different system. So, this ellipsoid height may vary from place to place for the same point if we take different ellipsoid we can have the different ellipsoid at different height juridic height, but in case of GPS we take the ellipsoid the WGS84 ellipsoid. So, so in case of GPS this height is with respect to WGS84. So, we should write WGS84; that means, we are taking the WGS84 ellipsoid as the reference surface this is the orthometric height.

(Refer Slide Time: 22:23)

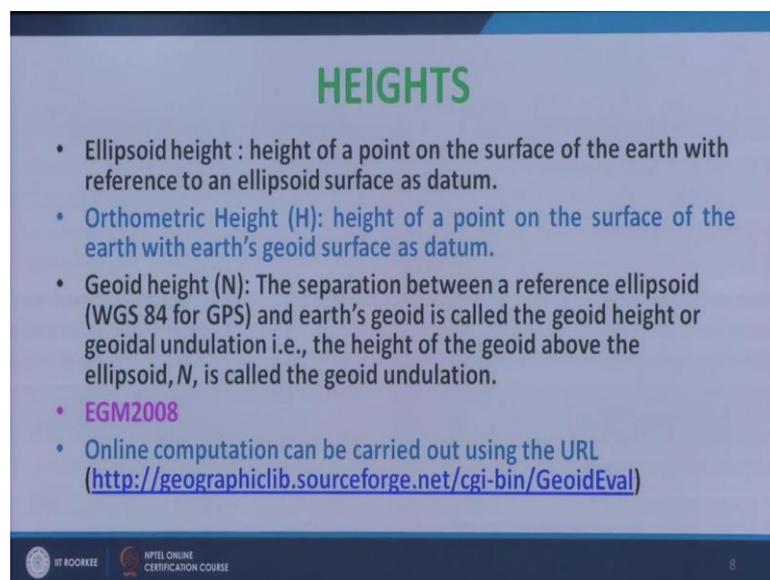


Now, one more point now from GPS now from GPS observation we will be getting this value, now how to get the orthometric height. So, now, you can see from here if we know this separation than if we add this with if we subtract this value from this, then we will be able to get this and this separation which is the separation between geoid and ellipsoid that is called geoid height, or geoid undulation and represented by N geoid

undulation now in this case, if we this  $h$  WGS84 we can get it form GPS observation and if we know what is the value of  $N$  which is the geoid undulation, we will able to get  $h$  equal to  $h$  minus  $N$ .

Now, in some case in many cases this surface may be below this or in some cases it will be above this. So, generally we write it like this. So, by knowing the geoid undulation and geoid and from GPS observation, we will be able to find out the height orthometric height and this is the height which we need for our engineering projects.

(Refer Slide Time: 24:04)



**HEIGHTS**

- Ellipsoid height : height of a point on the surface of the earth with reference to an ellipsoid surface as datum.
- Orthometric Height ( $H$ ): height of a point on the surface of the earth with earth's geoid surface as datum.
- Geoid height ( $N$ ): The separation between a reference ellipsoid (WGS 84 for GPS) and earth's geoid is called the geoid height or geoidal undulation i.e., the height of the geoid above the ellipsoid,  $N$ , is called the geoid undulation.
- **EGM2008**
- Online computation can be carried out using the URL (<http://geographiclib.sourceforge.net/cgi-bin/GeoidEval>)

IT ROORKEE | NPTEL ONLINE CERTIFICATION COURSE | 8

So, this is what it is written here in this slide, ellipsoidal height, geoid, height orthometric height this is the figure what I have shown you in a.

(Refer Slide Time: 24:15)

## HEIGHT TRANSFORMATION

- Depending on the position of the geoid and ellipsoid surfaces, the relation between orthometric height ( $H$ ) and its geodetic height ( $h$ ) is given by  $H = h \pm N$

The diagram illustrates three surfaces of the Earth: the actual surface, the geoid, and the ellipsoid. A point on the actual surface is shown with its orthometric height  $H$  measured from the geoid. The geoid height  $h$  is measured from the ellipsoid. The geoid undulation  $N$  is the vertical distance between the geoid and the ellipsoid. The relationship is given by  $H = h \pm N$ .

Figure 2. Three types of surfaces of earth

IT ROOKEE NITEL ONLINE CERTIFICATION COURSE 9

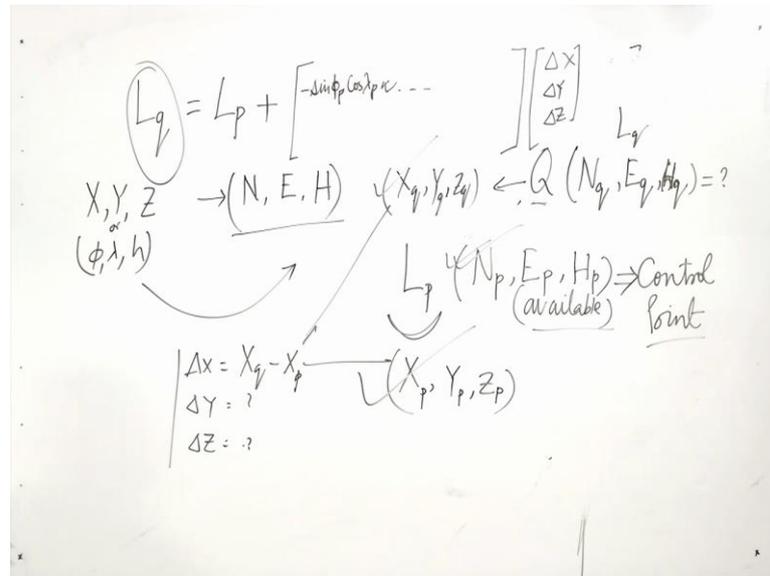
So, depending on the position of the geoid and ellipsoid suppose the relation between orthometric a height and geoid height, it can be given by  $h$  plus minus  $N$  small  $h$  plus minus  $N$  is equal to capital  $H$ .

Now, the problem is how to get this. Now in case of GPS observation, we know we will be able to get this value. And we know this reference is the WGS84 ellipsoid WGS84 ellipsoid. And WGS84 coordinate WGS84 reference system also provides ask a mathematical model, which represent the geoid that is called earth gravitational model 2008. So, it is denoted by e 008 this is a mathematical model which represents the geoid surface of the earth and it is available in the internet. So, by making using of this we can directly compute the if we provide the latitude and longitude of a place, then we will get the value of  $N$  at that of that place. So, this is one of the ways if we which we go to this link and it will a software will come, in that software you have to give the latitude and longitude of the place, and that software will provide you the geoid undulation with respect to WGS84 ellipsoid we have to keep it in mind.

And once you know this value automatically from GPS only you know this thing. So, you will be able to calculate orthometric height. Further as I told you that engineering projects makes use of local coordinates for it is all activities. So, on the other hand GPS provides us global coordinates either in Cartesian coordinates or in geodetic coordinate. So, to make GPS observation useful for our engineering works or mapping purpose, we

need to convert the global coordinates to local coordinate. And generally the local coordinates we do represent by a system called north east and orthometrical height or vertical height vertical height also can be told as orthometric height.

(Refer Slide Time: 27:07)



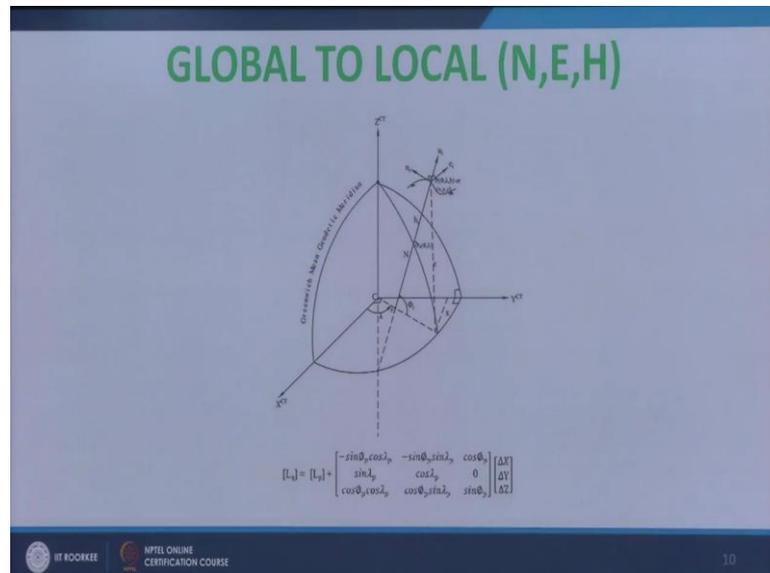
So, we need to convert our Cartesian coordinate or geodetic coordinate to north eastern height. Now for this if you want to do this thing we need to have few more thing. Suppose I want to find out the total coordinate of any point Q for suppose the local coordinate is represented by N q north east, and I of the point Q this is the local coordinate I can represent it by a matrix called this.

For in order to know the local coordinate of the point Q we need to know the local coordinate of the point p; that means, we need to know N p E p H p; that means, this is the control point where w is a point. So, 1 point p will be behaving as a control point control point means a point whose position is known and it is position in terms of local coordinate should be known. Apart from that we have to take observation of p point p and we will have the Cartesian coordinate using GPS like this, as well as we need to see the local coordinate we need to take the GPS observation at station q X q Y q Z q.

So, this is known from GPS observation this is known from GPS observation and this should be available it should be available before. So, by now using this we can go for now del X means this is the changing coordinate means X q minus X p like that del Y del Z these are the 3 things we can determine by using this and this. After having this thing

by using this relation  $l_Q$  can we can get  $l_P$  plus a matrix del X del Y del Z. This matrix is based like this minus  $\phi$  sin minus of sin cosine of like this.

(Refer Slide Time: 30:09)



So, by multiplying this matrix with this matrix we will get some value which has to be added with this, where we can get the local coordinate of the point Q. In this way we can go for conversion from global Cartesian coordinate from GPS observation to local north east and height coordinate to be useful for all engineering works.

With this I like to complete today's lesson, but before then let me summarize GPS provides position in global reference system, either or both we can get Cartesian coordinates X Y Z or geodetic coordinates lambda phi h latitude longitude and height. There are 3 surfaces types of surfaces which we consider for geodetic study purpose or GPS positioning purpose, the actual surface of the earth all the activities are going on the ellipsoid surface which is the geometrical surface which we do for our mathematical work, and the physical surface of the earth that is the equi potential surface and geoid as the reference which is really reference surface for all our engineering works. And for all our engineering works actually we need to have the local coordinates in terms of north eastern orthometric height, which we can get from GPS coordinate through conversion with this I like to conclude. And next class I will be talking on GPS principles and observables.

Thank you.