

Digital Land Surveying and Mapping
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Lecture – 28
Error Propagation and Survey Specifications

Welcome students. This is the 28th lecture on digital land surveying and mapping. Today I am going to discuss about the propagation of errors and specification of survey measurements. Now in the last class we have discussed the whenever we take measurements a specifically it is found it can be seen it can be observed when we take a set up observations or an observation repeated times, the measurements we find are not equal that itself indicates that the measurements of observations we have taking for the parameter are associated with the errors. Now as the quality of surveying work depends upon the quality of the measurement. So, we need to remove or minimize the errors. And in the last class we have found that standard error is in indicator which provides us about the quality of the measurement.

Now, in surveying we go for some primary measurement and from those measurements we do derive some data. Find to if you want to find out the distance between 2 points in case of plane surveying it is horizontal distance using total station we get the slope distance and from slope distance and vertical angle we get the horizontal distance. So, horizontal distance is a derive quantity from slope distance and vertical angle. So, like that many other parameters quantities of surveying which derived from fundamental measurements and if the measurements are associated with some error, while we will be computing or deriving the other parameters then those errors due to those measurements also will propagate to the derive quantity.

Now today's class we will like to discuss how this error gets propagated and finally, how we do find out the precision of the derived parameter and followed by manually go for surveying we need to maintain the quality of the surveying. So, how to maintain the quality for that we need to specify some requirements that is called specification for the survey work that we need to know what should be the specification of any surveying work.

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So, today I will discuss these 2 points and these will be talked under the following head that how the standard error which is an indicator of quality of measurement gets propagated, then how these standard errors get propagated in horizontal distance measurement or horizontal distance computation leveling work trigonometrical leveling in finding out the location and surveying specification.

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INTRODUCTION

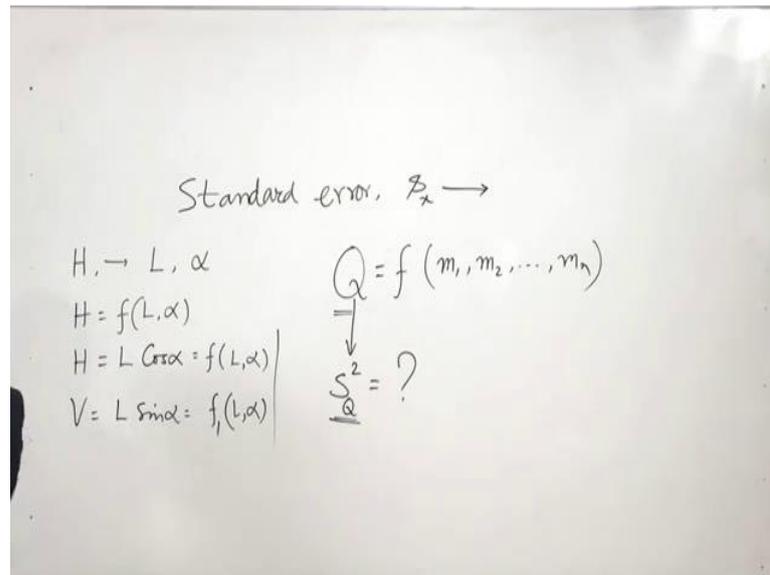
Error from individual measurement propagates to derived quantities. In order to determine the precision of computed parameters, it is required to understand how errors get propagated from measurements to computation of parameters. Since, original measurements contain randomly distributed errors, so also derived quantities errors.

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So, error from individual measurements propagates to derived quantities. In order to determine the precision of the computed parameters it is required to understand how

errors get propagated from measurements to computation of parameters. Since original measurements contain errors in random in nature or randomly distributed errors. So, we assume that the derived quantity will also contain the errors in randomly distributed; that means, random distribution means we will make use of the concept of normal distribution which we have discussed in the last class.

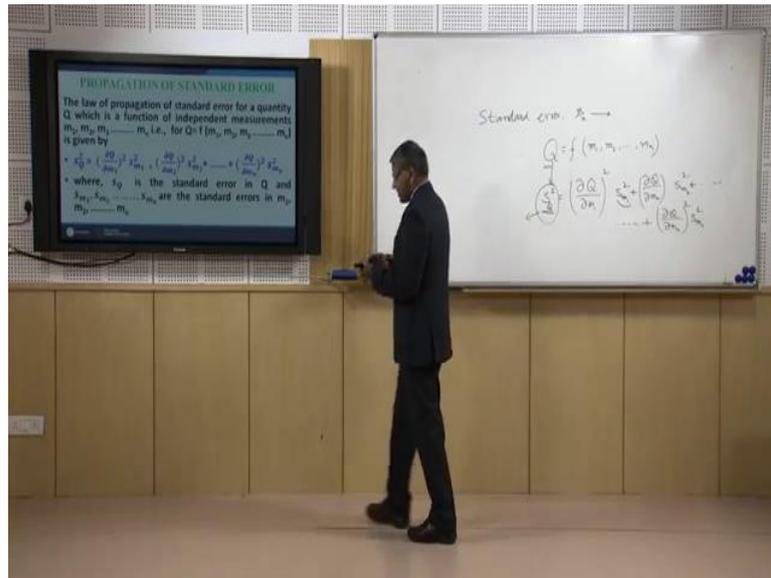
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Now, we know that the standard error standard error (Refer Time: 04:32) standard error S_x is the indicator of the quality of the measurements. So, we will like to know how the standard error gets propagated to the derived quantity. Suppose a quantity Q let us consider a quantity Q which is derived alter measurements m_1 m_2 m_3 up to m_n ; that means, n parameter it is a quantity (Refer Time: 05:01) n independent parameters. So, I can write the quantity Q and m_n x like for example, the horizontal distance we do get from the slope distance and the vertical angle.

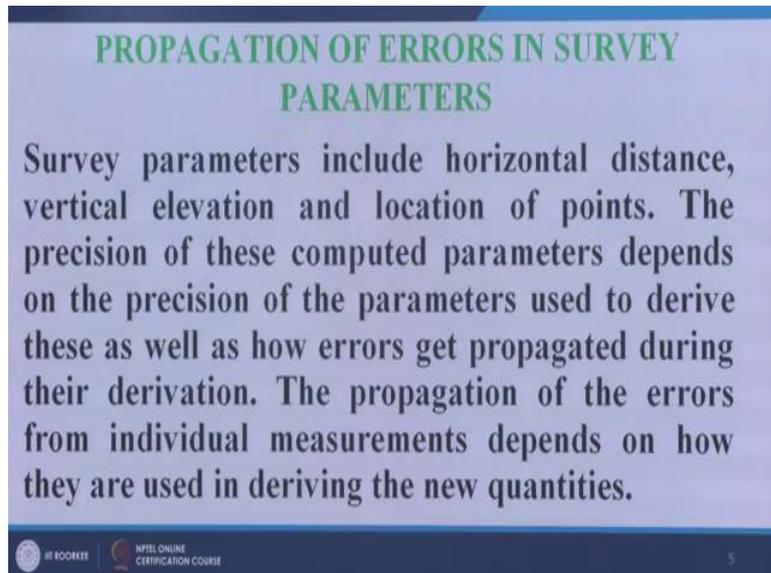
So, I can say is a function of L and α actually it is we know that horizontal distance equal to $S \cos \alpha$. So, which is a function of L and α the vertical distance is equal to $L \sin \alpha$. So, it is a function of again L and α , but this is an another function f_1 . So, a define function similar to that let us generalize this through like a quantity Q which is a function of measurements m_1 m_2 n_2 , then what will be the standard error standard error of the parameter x is equal to the what is the standard error.

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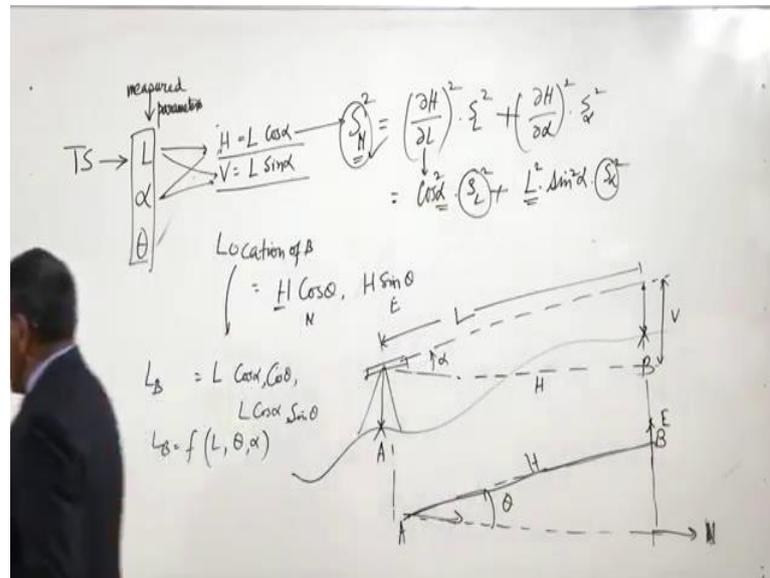
There is a law general law of propagation of error which states that the standard error of the derived parameter will be equal to the partial derivative of the with respect to the first parameter square, and the standard error of the first parameter like this. So, we can get this.

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Now, a with this background I will like to go for so propagation of error in engineering survey parameters.

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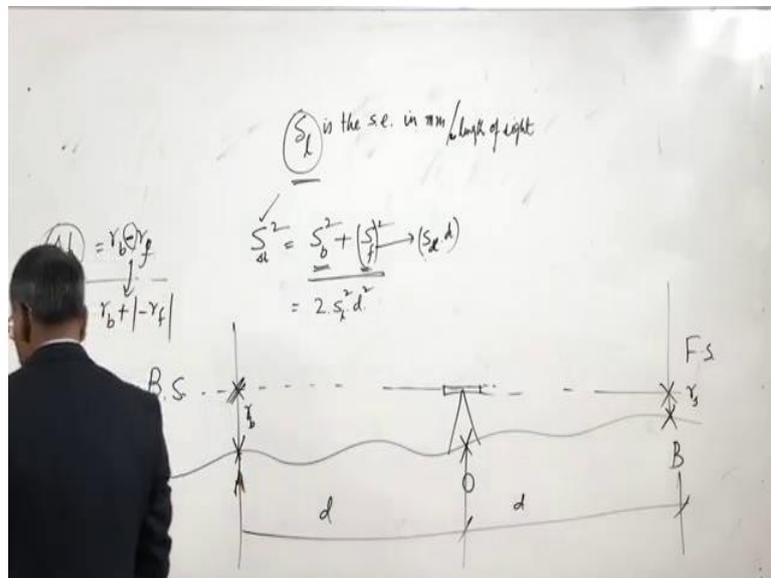
Now, in engineering survey generally we have the fundamental quantity in plane surveying, we do may go for measurement of in to using total station we go for measurement of slope distance and vertical angle and say horizontal angle. These are the parameters we measure this is the measured parameters and from there we do get horizontal distance the vertical distance; that means, horizontal distance and also we need to have the location. So, horizontal distance is $L \cos$ of alpha and vertical distance is $L \sin$ alpha. Now suppose I have a terrain like this. This is the station A and this is the station B now if I had a station total station here or a reflected here. So, I have the slope distance L and the vertical angle alpha and if we see from top then our A B must be along some line A B, and suppose this is the reference direction. So, this is our horizontal angle theta.

So, this is the horizontal distance and vertical distance will be V and this is H and this is H . So, if we know the if we want to know the coordinate location of B. So, we can get it down from H , $H \cos$ theta is the if it is our east direction north direction and it is our east direction generally local coordinates are in north and east. So, $H \cos$ theta is the north and $H \sin$ theta is the east. So, in this way you can see that location is a function of now H is a function of L , $L \cos$ of alpha and then \cos of theta and $L \cos$ of alpha and \sin theta. So, location of B is a function of L theta alpha this is our measured parameter and these are the derived parameter.

Now, we will like to I will like to first to say how these either horizontal or vertical distance it will be of similar nature. So, how the what will be the standard error in horizontal distance measurement. So, it will be equal to $d H$ by $d L$ whole square then $S L$ square plus $d H$ by $d L$ whole square in to S alpha square for $H \cos$ alpha we can get this thing. So, now, $d H$ by $d L$ mean derivative of $L H$ with respect to L , this is the equal to $\cos S$ square alpha \cos alpha. So, this is \cos alpha and square is \cos square alpha is L square alpha. And $d H$ by $d L$ and this $d H$ by d alpha this is the alpha.

$D H$ by d alpha will be equal to $L, L \sin$ alpha. So, L square \sin square alpha S alpha square. So, you can see now if we know the angle alpha, and the slope distance L as well as the standard error of slope length standard error of the vertical angle then we will be able to compute the standard error of the measurement in height. So, this is the way how we can find out the standard error of our $d L$ quantity, similarly we can do also for vertical distance on this \sin will be cosine will be \sin now in an another important thing which we do in case of surveying is that leveling.

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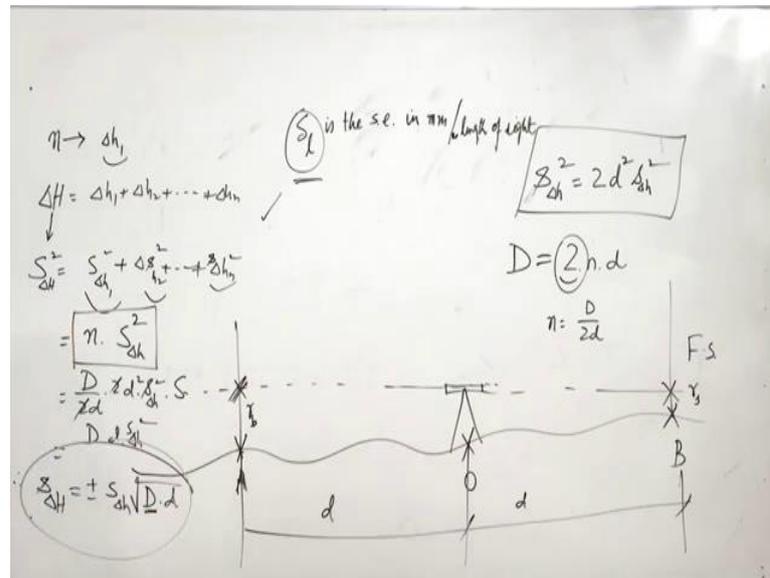
Now, in case of leveling, suppose I want to find out the difference in level between 2 stations A and B what is the difference in level. So, generally what we do we set up our instrument somewhere in between somewhere in between and preferably the distance horizontal distance between these 2 area equal area. So, suppose we want to find out the difference in elevation between A and B. So, we have set up the instrument here level

instrument here and we take our staff reading here. So, this is the staff reading we have taken and this is called back sight. So, first sight taking after setting of the instrument is called back sight. So, suppose we got a reading r_b and then we are taking a reading with staff held at B which is the fore sight and r_f is the staff reading.

So, the difference in level between A and B will be equal to r_b minus r_f . So, this is a very simple already all you know the difference in level between let me write dLH to now the what will be the standard error in measurement of difference in height. So, ΔH square, it will be equal to. Now if we take the first derivative here. So, it will be one. So, it will be $S_r d$ square plus S_f square now whether it is positive or negative one thing has to be marked otherwise we take this in this way, r_f the we do take the this as like this. And the standard error for r_b or minus of r_b will be same only it is opposite side. So, we do take this as positive and get this thing. So, this standard error are the difference in height will be the sum of the standard error of the 2 measurements that is involved in this which are related by an addition operator. Now if we consider the S_L is the standard error in millimeter per unit of length of sight.

Let us say this is the standard error, generally in case of an instrument these standard errors in measurement of height is real in terms of in this way standard error in the measurement of level means height is in millimeter with respect to some per meter length of per meter length of sight. So, the if now if you assume that this back sight distance and fore sight distance are equal. Then the distance is equal then these 2 error will be same as well as. So, I can write it $2 S_L d$. So, $2 S_L d$ and then our it will be squaring of this. So, this is S_B or S_f will be equal to S_d in to S_L in to d . So, it will be squaring this H this error will be like this. Now if these an instead of having only one set up, sometimes when the bridge mark has to be has to be taken far off from far off then we will be need to they for n numbers of set up now let us consider. So, when there will be n numbers of set up then there will be n numbers of defines in height.

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So, the total height difference ΔH will equal to $\Delta H_1 + \Delta H_2 + \dots + \Delta H_n$ now. So, the error associated with ΔH will be equal to ΔH^2 will be equal to error associated with ΔH_1 error associated is the ΔH_2 like that $H L$ square.

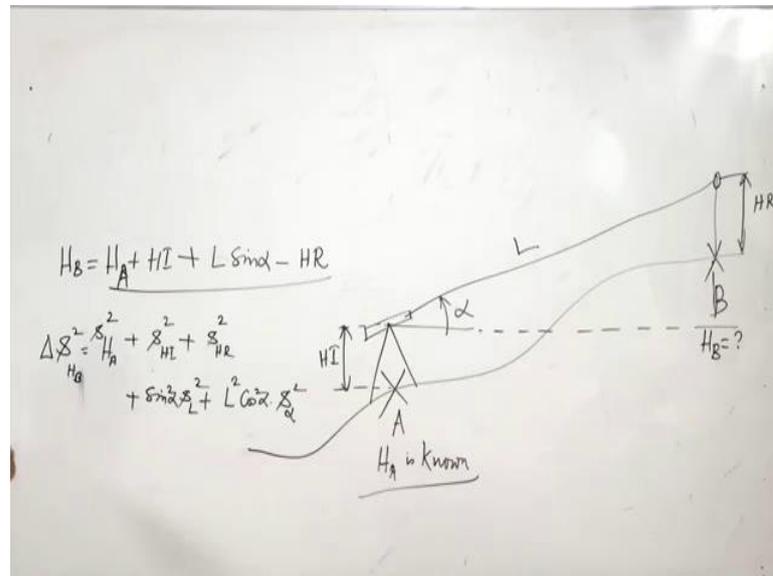
Now, since this we have considered if we see say that all these back sight and fore sights are equal, then all these value will be equal, then we can write it will be in standard error of any one of this and this. So, now, already we have to know that ΔH^2 equal to $2d^2 S_{dh}^2$ this is already we have derived. So, we can substitute the same thing here, then we will get now one more thing if the total distance is D then our total distance is 2 in to n in to D . So, now, because there are back sight and fore sight show this 2 will come. So, our total distance if the distance is d actually then it will be 2 and D .

So, from here we can substitute n equal to D by $2D$. So, if you substitute this n equal to D by $2D$ and S is equal to $2D^2 S_{dh}^2$. So, from here we can get this is called this and $D S_{dh}^2$. So, a standard error for the height will be equal to plus minus standard error of the single measurement square root of D in to D . So, this is the standard error for taking the setting out the difference in level between 2 way stations having distance D which is observed by taking a setting of the instrument at an interval of distance D . So, the back sight the D is the distance for back sight and fore sight for each set of the instrument and which is considered to be equal for all set up. So, in that another condition we will we will able to get the error in measurement in height the

standard error in measurement of height will be equal to this. So, in these way we can compute the error associated with the difference in level.

Now, the next another important thing which we do go for in go for in measurement of height especially when we go for measurement with using total station another thing we do in case of total station we do measurement we do define measure of the height difference by trigonometrical leveling.

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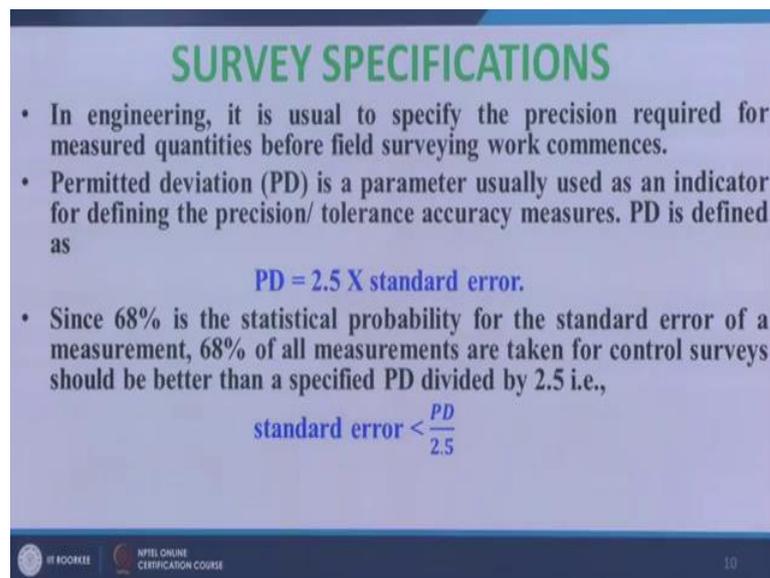


Now, what is that already we have discuss suppose a and B of the 2 stations whose difference in elevation we want to find out. So, who is want to find out the height of B what is the height of B and height of A is known. So, what we do we do set up the instrument here, and we do hold the reflector and we do take an observation to this reflector and we get the slope length and vertical angle. Now also we do take the height of the instrument and also we know that will be height of the reflector height of the reflector. So, to find out the height of B can be find out from height of A plus height of the instrument plus $L \sin \alpha$ minus height of the reflector. So, this is the way how we do measure the.

Now, what will be the standard error in height of B, that will be height of B will be equal to the height of sorry height of A this is A then height of standard error of the height of A because it is we need. So, standard error of the height of instrument minus standard error plus standard error height of the height of the reflector plus for this sin square alpha, and

then this one and then L square cosine square alpha S square. So, this is the I had derived it for directly from the law of propagation. Similarly, also we can do the propagation of error while we will be finding out the location of any point. So, already I had given you the totals how to find out the location and we can do it. So, if these I like to go for next point the surveying specification. Because we want to maintain the quality of surveying work.

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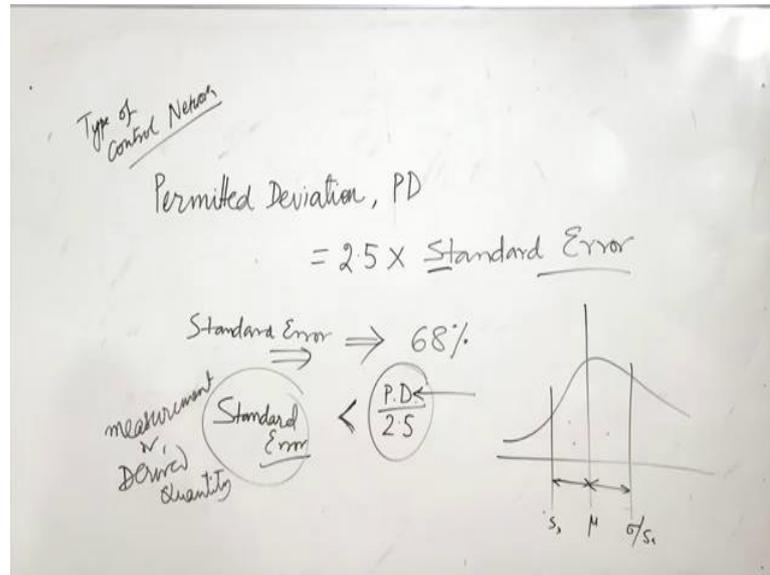
SURVEY SPECIFICATIONS

- In engineering, it is usual to specify the precision required for measured quantities before field surveying work commences.
- Permitted deviation (PD) is a parameter usually used as an indicator for defining the precision/ tolerance accuracy measures. PD is defined as
$$PD = 2.5 \times \text{standard error.}$$
- Since 68% is the statistical probability for the standard error of a measurement, 68% of all measurements are taken for control surveys should be better than a specified PD divided by 2.5 i.e.,
$$\text{standard error} < \frac{PD}{2.5}$$

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So, whenever we want to maintain the quality of surveying work we should go for some specify the specification according to which we should carry out surveying work, specification has to be defined with respect to some indicated.

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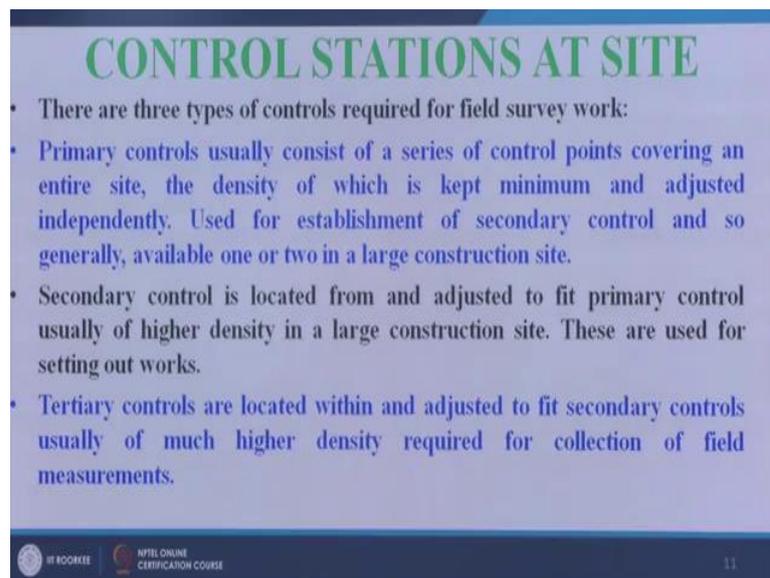
So, we do make use of permitted deviation as the indicator for quality or surveying or the as an indicator for a specifying the specification of surveying.

Now, what is permitted deviation. It is considered to be 2.5 times the standard error. So, already we know what is standard error, how to find out the standard error for different for basic measurements as well as from derived manometer. So, on the basis of the standard error if you multiply it by 2.5 that will be the value of the permitted deviation and depending upon the permitted deviation value we have to a specify what should be the requirement for particular specification. And we know that the standard error standard error means that the parameter is 68 percent since 68 percent if we say that the standard error if we remember the normal chart. So, while their $\mu + \sigma$ $\mu - \sigma$ this is the μ this is the σ or the $S \times$. So, this is; that means, if any measurement false within this then the chance of that measurement having the true value is 68 percent. So, that is the way how we do define the 68 similarity and so our standard error should be less than less than what about permitted deviation will be defined given divided by 2.5.

So, a standard error of our standard error of a measurement or the derived quantity derived quantity should be less than this and now this will depend upon the specification that has been given to you for meeting to a particular quality. Now there has many specifications available and this specification are actually based on the type of control

network type of control network on which it falls control network or control. So, if I there are 3 types of control generally we go for in carrying out the surveying. One is called primary control and secondary control and tertiary control. Primary controls are the most stringent bond and it is rarely available however to have our work in in case of a very big construction work at present at least we need you have one primary control point.

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CONTROL STATIONS AT SITE

- There are three types of controls required for field survey work:
- Primary controls usually consist of a series of control points covering an entire site, the density of which is kept minimum and adjusted independently. Used for establishment of secondary control and so generally, available one or two in a large construction site.
- Secondary control is located from and adjusted to fit primary control usually of higher density in a large construction site. These are used for setting out works.
- Tertiary controls are located within and adjusted to fit secondary controls usually of much higher density required for collection of field measurements.

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Now, secondary control point actually a based on primary control point. And secondary control points are primarily used for setting out works and within a tertiary control if I have secondary control point tertiary control points are established as I have discussed in the first few classes that the principle of working od surveying is to work for whole 2 part. So, first we have to establish the primary control point with in the primary control point, we should go for secondary control point which should take primary control point as the basis and we should feed it. And within the primary control the secondary control point tertiary control points again come up and tertiary control points are basically used for collecting our further data. So, tertiary control point actually depend on means really serves the purpose for collecting the data and secondary control points are used for setting out go. Now the with the with this I we should go for some specification and there are many specifications available.

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Permitted Deviations for different types of Control, as per ICE (Institution of Civil Engineering Surveyors)

Control		Distance (mm)	Angle (Degrees)	Height
Primary	First Order	$\pm 0.5\sqrt{L}$	$\pm \frac{0.025}{\sqrt{L}}$	$\pm 5\text{mm}$ (inter distance < 250m)
	Second Order	$\pm 0.75\sqrt{L}$	$\pm \frac{0.032}{\sqrt{L}}$	$\pm 12\sqrt{K}$ (inter distance > 250m & K in KM)
Secondary		$\pm 0.15\sqrt{L}$	$\pm \frac{0.09}{\sqrt{L}}$	$\pm 5\text{mm}$ (between adjacent secondary control points)
Tertiary	Structures	$\pm 15\sqrt{L}$	$\pm \frac{0.09}{\sqrt{L}}$	$\pm 3\text{mm}$
	Roadways	$\pm 5.0\sqrt{L}$	$\pm \frac{0.15}{\sqrt{L}}$	$\pm 5\text{mm}$
	Drainage Works	$\pm 7.5\sqrt{L}$	$\pm \frac{0.20}{\sqrt{L}}$	$\pm 20\text{mm}$
	Earthworks	$\pm 10.0\sqrt{L}$	$\pm \frac{0.30}{\sqrt{L}}$	$\pm 30\text{mm}$

L is the distance between points (inter distance) being checked in meters.

If there is from place to place country to country and from work to work one of the specification know I have shown you here, just as an indicative this is the in specification that institution of civil engineering surveyors the adapt adapted. So, we can see primary control point having the first order and second order then the distance angle and height. These are the 3 primary measurement parameters which we need to have or we generally we should get and from there we can derive the derived quantities.

So, these are the specification that has been defined and you can see here these values well first order primary control point. Then distance should be having a standard error 0.5 is square root of L when a this whole of this is in millimeter, but L is in meter. So, this is the criteria it is it satisfy when you should go for primary first order distance measurement. Primary first order angle measurement should satisfy these thing and primary first order height measurement should be. So, satisfy this thing if the distance between 2 points is 250 meter. So, in that way there are difference specifications given which has to be satisfied and an on the other way depending upon what value we touch it we can say that is the quality of the surveying. So, with these I like to conclude summarize today's class precision of most surveys can be determined by analyzing the way in which error propagates through them, from the standard error of the primary measurements we can find out the standard error of precision of the computed value. So, So, for all types of survey precision of outcomes should be done.

Now, while we will be going for computation of propagation of errors, if there is some involvement of measurement angle measured then those angles have to be converted to radian. This is one thing which we have to be care full that degree should be converted to radian. Now well of we know that 2π equal to 360 degrees. So, a 360 degree equal to 2π from there we can convert any number of degree unit second to radian. So, that is and quality control surveying should follow 10 multi specifications, and specifically surveying may be related to surveying measurements as well as work.

So, tertiary specifications as I show it is based on work. So, with this I like to conclude today's class and next class I will go for another topic of this that is the mapping part. So, first we have discussed the surveying control by (Refer Time: 32:03) establishment then we have discussed the total station which in which we do really go for collection of field data. And now that measurement by this over now we will go for mapping fundamentals we land how the mapping work has to be done what are different considerations to be taken to make use of the measurements to convert to survey map.

Thank you.