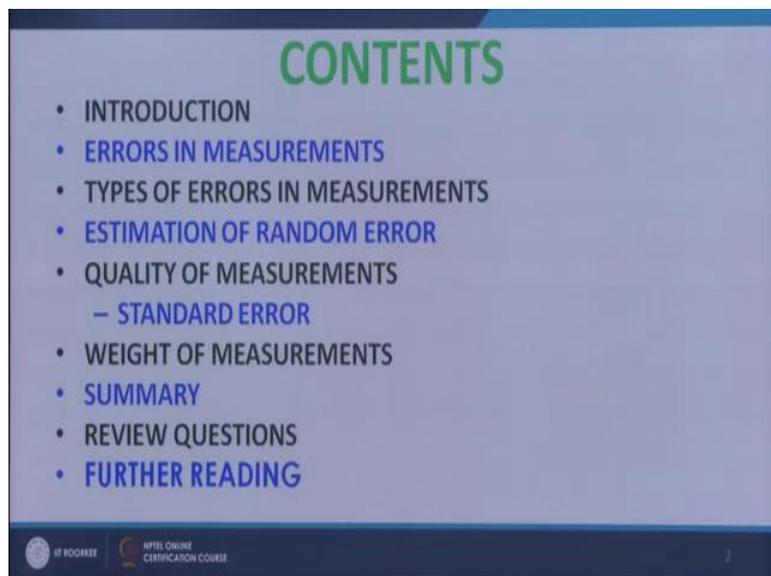


**Digital Land Surveying**  
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**Lecture - 27**  
**Errors and Quality of Surveying Measurements**

Good morning students today is a 27th class on digital land surveying and mapping. Today I will be talking on errors and quality of measurement in surveying.

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**CONTENTS**

- INTRODUCTION
- ERRORS IN MEASUREMENTS
- TYPES OF ERRORS IN MEASUREMENTS
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  - STANDARD ERROR
- WEIGHT OF MEASUREMENTS
- SUMMARY
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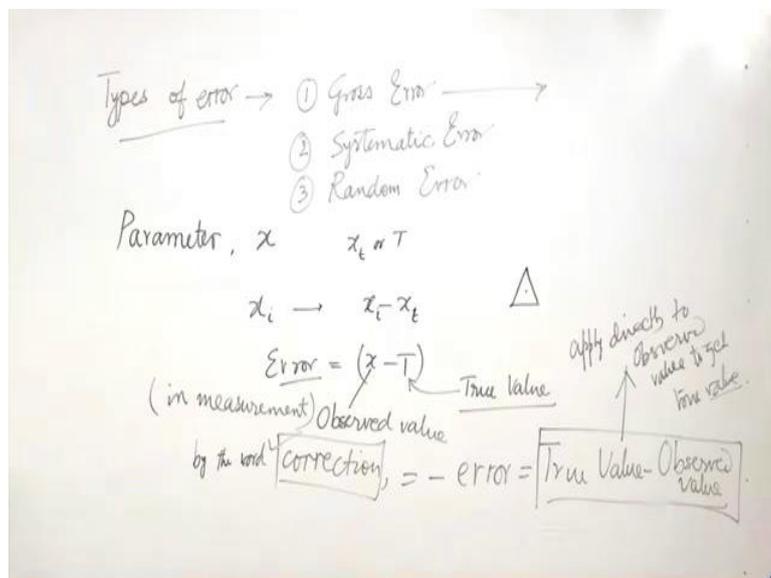
Now, these class will be discussed under following heads introduction followed by what are the different type what is error in measurement, what are the different types of errors in measurement, of this we will discuss at length about estimation of random error and followed by quality of measurement and weight of measurements, if we make a measurement of a parameter repeated number of times. Now we will see that every repetition may not give you or give us the same value, whether we will see and we defined that if we repeat the measurement of any parameter number of times, we do find measurements having different values. This clearly indicates that there is some errors that is associated with the observation of the measurements.

So, we can very well say that most of the measurements are associated with errors. Also how much error it is we need to test it by looking into it is true value or exact value. And then only we should know what is the quality of the measurement, the quality of surveying work depends on the quality of the measurement means, what is the amount of error associated with the measurements that define the quality of the measurement.

So, to improve the quality of the measurement as well as quality of the surveying work, we need to have our measurements free from errors all to have error in minimum extent and for that reason. We need to know what are the different types of errors, how these errors are associated with it and how these errors can be eliminated or minimize from these measurements. So, and based on that only we will say the what is the quality or reliability of the measurement actually now first we know you have to know what really error is.

Now, the if we know suppose we want to find out the parameter we want to measurement  $x$ .

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Now, suppose the 2 value of the parameter is  $x_t$  or  $t$ . And if we observe the parameter as  $x_i$  then or  $x$  then the difference between  $x_i$  and  $x_t$  or we can say  $x$  minus  $t$  is our error. So, the observed value minus observed value minus true value provides us the error in measurement error in measurement.

Now, in many literatures we do make we do define error by the word by the word correction which is the opposite of error; that means, if we subtract the observed value from true value then we will get the correction.

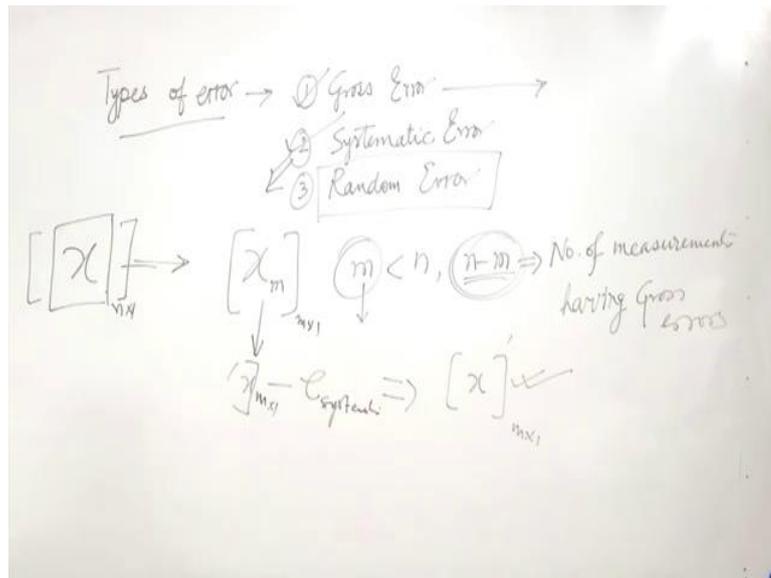
So, sometimes we define error in terms of true value minus observed value; that means, that is actually correction; that means, true value minus observed value whatever it is that you should apply directly we can apply directly to with to observed value observed value to obtain to get true value. So, this is what is the background behind error and it is measurement and how to make the adjustment for the error.

So, but now one point is that which is important before we go further about the error we need to know the types of errors. There are different types of error, in any measurement there will be 3 types of error one is that called gross error, gross error and now next one is systematic error and the third one is random error.

Now, the gross error actually it is these are the mistakes and these are very big in value. So, in any measurement if we take a set of measurements, and if we observe the set of measurements we will find that some of the measurements are very different from the most of the measurements. And those measurements we can very well say that are having the gross error. So, what you do we generally eliminate those errors though measurement from our computation.

So, gross errors are broadly the mistakes already we have discussed about this. With the with reference to total station I have discussed many of the gross errors that we can we do come across and how to overcome those gross errors also we have discussed. Now the systematic error the known itself tells systematic errors are the errors which are associated with the system.

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So, last few classes I had discussed at length about the different systematic errors that are associated with the total stations as well as previously we have discussed a lot of systematic errors that is associated with the GPS measurement. So, these systematic errors can be physically modeled, and has to be taken out from the measurements. So, and this is also taken out through physical model.

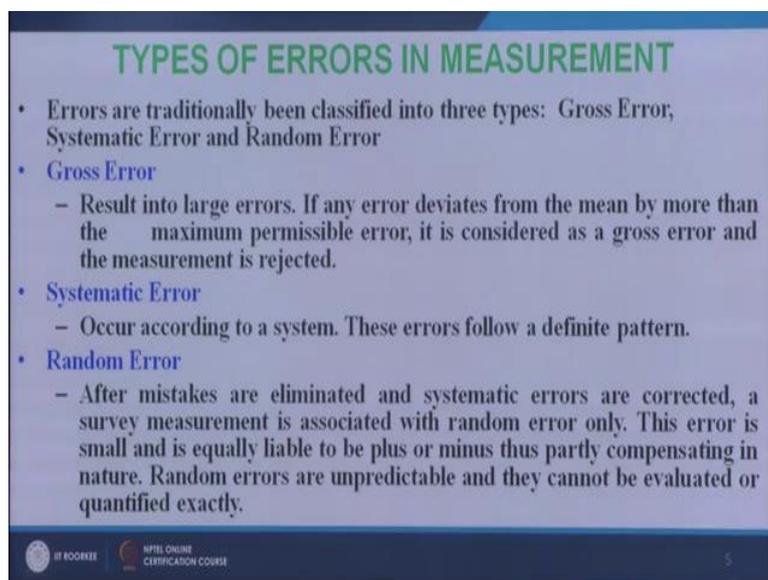
So, whenever we have some observation  $x$ , we will take out the suppose we have a set of measurement  $x$  first we will remark suppose  $x$   $n$ . So, we will take  $x$   $m$  value and  $m$  is less than  $n$ . So,  $n$  minus  $n$  minus  $m$  this numbers of measurements having gross errors, so these numbers of measurements we have found that out of the  $n$  numbers of measurement,  $n$  minus  $m$  number of measurement we or we can say  $m$  measures  $m$  are more or less similar, but  $n$  minus  $n$  number of measurement will be having a lots of variation. So, those have been taken out. So, we have left with  $x$   $m$  number of measurement now out of these from  $x$   $m$  measurements whether I should write in a metric form  $n$  in to 1  $m$  in to 1. So, of  $m$  in to 1 we will remark the systematic error.

So, we will get a measurement having some systematic. Now these measurements will be having another type of error that is called random error. Now these random errors by the name itself we can say see that it is random in nature. So, we does not follow any definite rule or law. So, we have to take out the random

error from the measurements by using some statistical analysis. So, we do make use of statistical analysis to eliminate or to minimize the random error from the observation.

So, random errors are the eliminated and systematic errors are corrected and are associated random error only, and this error is small and equally liable to be plus or minus thus partly compensating in nature random errors are unpredictable and they cannot be evaluated for quantified exactly.

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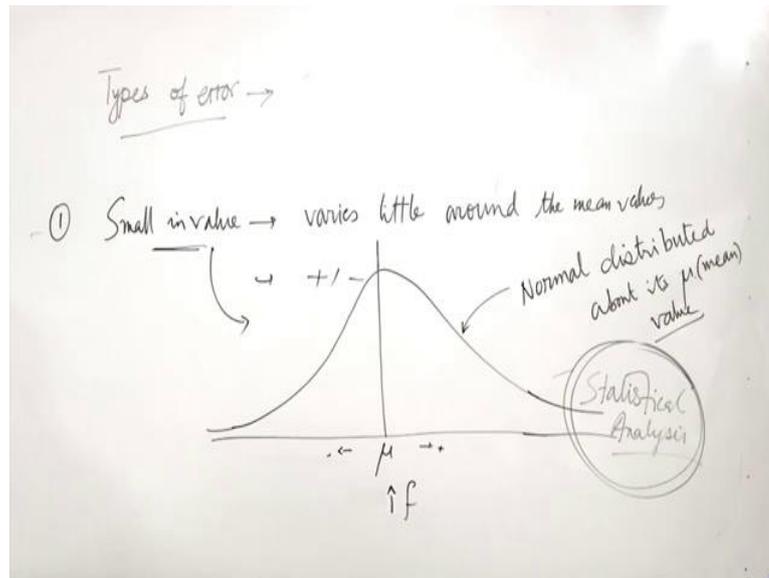
**TYPES OF ERRORS IN MEASUREMENT**

- Errors are traditionally been classified into three types: Gross Error, Systematic Error and Random Error
- **Gross Error**
  - Result into large errors. If any error deviates from the mean by more than the maximum permissible error, it is considered as a gross error and the measurement is rejected.
- **Systematic Error**
  - Occur according to a system. These errors follow a definite pattern.
- **Random Error**
  - After mistakes are eliminated and systematic errors are corrected, a survey measurement is associated with random error only. This error is small and is equally liable to be plus or minus thus partly compensating in nature. Random errors are unpredictable and they cannot be evaluated or quantified exactly.

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So, this is what is the random error now. So, as I told you that random errors are determined by using that statistical analysis. You have some assumption about the random error.

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First assumption is that that random errors are very small in value. And a small in value small in value and they varies little around their main values. And also they are may be either positive or negative; that means, it may be more than the mean value or it may be the less than main value.

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## RANDOM ERROR

- Random errors are determined through statistical analysis based on following assumptions :
  - Small variations from the mean value occur more frequently than large ones.
  - Positive and negative variations of the same size are about equal in frequency, rendering their distribution symmetrical about a mean value.
  - Very large variations seldom occur.
- random error in a measurement is eliminated by taking observations for repeated number of times. Normal or Gaussian distribution typifies the spread of samples of the measurement.

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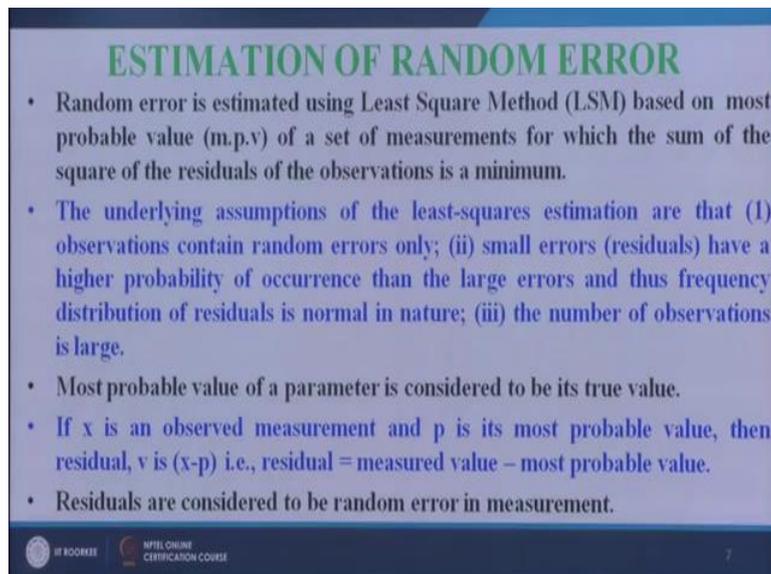
So, positive and negative variation of the same size about equal in frequency rendering and the and the number of measurements having positive random error

and number of measurements that is negative random error is assume to be of equal in number and have the symmetrical distribution about the mean value.

So, this is the assumptions one which we do this and then these assumptions satisfy that the random error are normally distributed; that means, if we say this is the mean of the observation than the random error and if we say the frequency this is the positive value positive negative values and the frequency will be like these. So, we can say that the random errors are distributed symmetrically about the mean and also positive and negative direction. So, this is the concept that random error are normally distributed about it is main value of the observations.

So, as it is (Refer Time: 13:05) that now random error is measurement is eliminated by taking observation for (Refer Time: 13:10) repeated number of times normal or Gaussian distribution typifies as the spread of samples in the measurement. So, these are the samples are this is the frequency and maximum frequency of the mean and gradually it will be reducing towards both positive and negative direction. So, this is what is the assumptions we do during analysis of random errors present in measurement.

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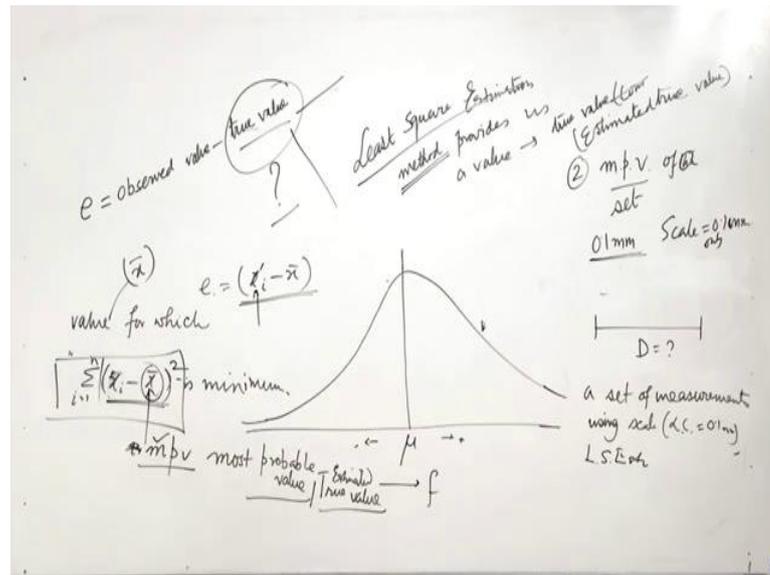
**ESTIMATION OF RANDOM ERROR**

- Random error is estimated using Least Square Method (LSM) based on most probable value (m.p.v) of a set of measurements for which the sum of the square of the residuals of the observations is a minimum.
- The underlying assumptions of the least-squares estimation are that (i) observations contain random errors only; (ii) small errors (residuals) have a higher probability of occurrence than the large errors and thus frequency distribution of residuals is normal in nature; (iii) the number of observations is large.
- Most probable value of a parameter is considered to be its true value.
- If  $x$  is an observed measurement and  $p$  is its most probable value, then residual,  $v$  is  $(x-p)$  i.e., residual = measured value – most probable value.
- Residuals are considered to be random error in measurement.

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Now, how to estimate the random error, to estimate the random as the random error is again to be normally distributed. So, we do make use of the concept of normal distribution in finding out the random error.

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As we as we have told that the error is equal to our observed value o minus true value.

Now, why now there is a very this is a very ambiguous time. What do you mean by true value of a parameter? Actually trend ways it can be I have explained or and implemented generally whenever we will measure a parameter, suppose we want to measure the distance between 2 points. So, what is the distance variable. So, what will be the true value of these? So, one school of thought or sometimes what we do suppose we want to measure the distance between to (Refer Time: 14:49) with a scale having a least count 1 millimeter, 0.1 millimeter. Then the true distance of these will be considered if we take a scale having least count less than 0.1 millimeter.

So, if we measure the may distance using a scale whose least count is less than 0.1 millimeter we will get a value and that value may be least count less than 0.1 millimeter. And these value can be considered as true value for measuring the distance with a scale having least count least count greater than 0.1 millimeter. So, this is one similar of thought another similar thought is that we can find out the most probable value of a set of measurements. Suppose I have we do not have any scale or having least count 0.1 millimeter. So, and we will be using a scale having scale having least count 0.1 millimeter only we do not have less than that.

So, what we will do? What we will do, we will take a set of measurements you are using a scale having least count 1 millimeter and then we will apply the method of least square least square estimation method. So, least square estimation method provides us a value which may be which is considered to be the true value, considered true value or estimated true value estimated true value. So, this is what is estimated least square.

Now, we have another point least square estimation method. What is that now least square estimation method being assumes that the value for which sum of the square of the errors sum of the square of the errors let us say values suppose  $\bar{x}$  is the true value. Now if we take  $\bar{x} - x_i$  observed value sum of the  $I$  equal to 1 to  $n$  suppose we have taken  $n$  numbers of sum of the square of the other this is the error. So, because if we take a square whether it actually it should be error is  $\bar{x} - x_i$  sorry  $x_i - \bar{x}$ .

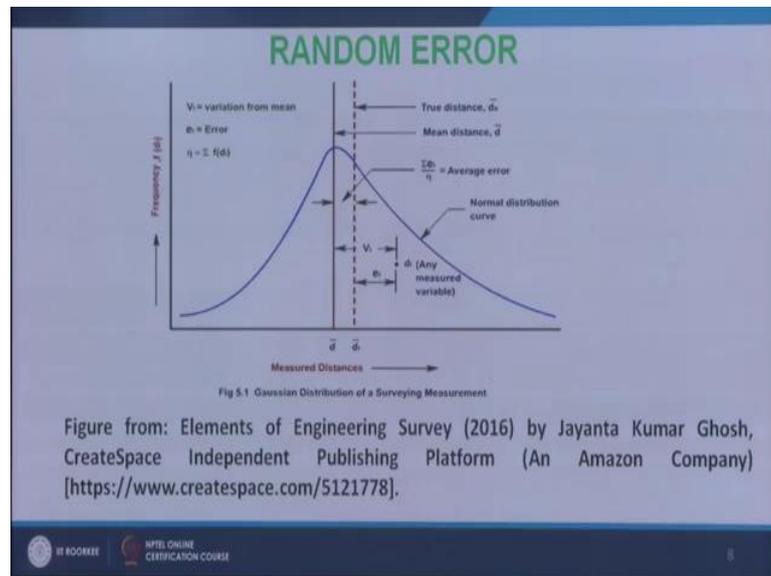
So, whether it is  $x_i - \bar{x}$  or  $\bar{x} - x_i$ . So, hardly (Refer Time: 18:34) because it is a square. So,  $x_i - \bar{x}$  sum of the square of the error minimum is minimum. So, this is what is called least square method. Now then in these case for the value for which this is minimum for that value is called the true value or in statistical term that is called most probable value and most probable value also that is what is considered to be the estimated true value. So, estimated true value. So, by least square method of analysis least square estimation method, we can find out the most probable value and these most probable value is considered to be the estimated true value. This is what is our concept behind measurement or to finding out the most probable value.

Now, if we find out the most probable value then for further observation we can find out if we know then this will give the error for  $x_i$  dashed many other observations other than  $x_i$ . So, there is what is written here random error is estimated using the least square method of analysis based on most probable value. So, this is the most probable value which can be determined by least square estimation method what is their estimation method, if say is that the sum of the least square of the error this is the error square sum of the least square of the error is minimum.

Now, for these underlying assumptions now for to in order to find out these we have to do some assumptions least square method of analysis or estimation assumes has some assumption like least that observation contained random errors only; that means, these contains only random errors; that means, systematic error gross error has been taken out from original observation a small errors small errors; that means, now see this is the most probable value  $\mu$  or  $\bar{x}$  I have written here. So, I should write it now, this  $\bar{x}$  or we can write it  $\mu$ , now we can write it  $\mu$ .

So, small errors have higher probability of occurrence now you see here this is a this is a the whatever is the observation frequency more here frequency is less here. So, the probability of occurrence of small error is more that; that means, that means frequency is more and the number of observation is large. And the most probable value is considered to be the estimated true value. Now if  $x$  is the observed measurement  $p$  is the most probable value than actually these measured value  $x_i - \mu$  that is called residuals. Residuals are nothing, but random errors residuals are nothing, but random errors.

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So, now in these diagram we can show you with these diagram now you can see here that the measurements are frequency versus measurement, measurement value. And you can see this is the maximum frequency and this is the main value.

In case of normal distribution, we will find that at the maximum frequency you will we will have the maximum frequency we will have the at the mean value

Now, suppose the (Refer Time: 22:51) this is the true value, then these difference true value minus the mean value that is the average error. Now suppose for any year measurement this one  $d_i$  is the measured value. So, from that the  $e_i$  will be the true error whereas, the  $v_i$  will be the residuals because residuals will be computed from the most probable value which is (Refer Time: 23:21). So, short these diagram is like this, and this is now quality of measurement now because, as I told you the quality of surveying measurement depends on the amount of errors associated with it.

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**QUALITY OF MEASUREMENT**

- Quality of a measurement having a set of repeated observations is being tested by using indicators Accuracy, Precision and Relative Precision
- **ACCURACY**
  - Overall estimate of the errors present in measurements.
  - amount of closeness the mean of observations to the true value.
- **PRECISION**
  - Represents repeatability of measurements and concerns only with random errors.
  - Precision pertains to the degree of closeness of observations among each other in a set of repeated observations of a measurement
- **RELATIVE PRECISION**
  - is the ratio of the precision of a given measurement and the value of the measurement itself. It is expressed as percentage or a fractional ratio such as 1/500 or by parts per million (ppm).
- **WEIGHT**
  - Weight of a measurement may be taken as inversely proportional to its standard error.

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Now, what is the what is the amount of error what is the nature of the error this has to be indicated. Now there are parameters like accuracy precision relative precision and weight these are the parameters which actually defines the quality of measurement already I had discussed about for accuracy actually accuracy provides us the overall estimate of errors present in the measurement. So, accuracy should be very high precision only it represents the random error present in the measurement.

So, once the systematic and gross errors have been taken out then only we can talk about precision and precision should be good and relative precision is the

ratio of the precision of the given measurement divided by the value of the measurement itself. So, it is also a kind of precision, but this we will give us the comparison between 2 measurements and what is the relative precision and another the relative precision and the weight of a measurement is taken is mostly proportional to the standard error these are the 2 parameter relative precision and weight which represents the reliability of the measurement and the we can compare 2 things also using these.

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**STANDARD ERROR**

- Surveying involves small set of observation
- To compare one set of observations with another, the spread of normal distribution is assessed using standard error.
- Standard error provides a measure of the dispersion of each individual measurement in a series.
- Standard error is given by

$$s_x = \pm \sqrt{\frac{\sum_{i=1}^n v_i^2}{(n-1)}} = \pm \sqrt{\frac{\sum_{i=1}^n (\mu - x_i)^2}{(n-1)}}$$

- Precision of measurements i.e., of mpv is given by standard error of the mpv.

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Now, what is standard error actually standard error is used to compare the quality of the or to define the amount of accuracy or precision. Standard error is given by your summation of I should say mu minus x i whole square i equal to 1 to n divided by n minus 1 is square root this is the standard error plus minus.

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The image shows handwritten mathematical formulas and a diagram on a whiteboard. At the top left, the formula for the standard error of the mean is written as  $\bar{S}_x = \frac{S_x}{\sqrt{n}}$ , with a note "S.E. with mpv." below it. Below this, the formula for the standard deviation is written as  $S_x = \pm \sqrt{\frac{\sum_{i=1}^n (\mu - x_i)^2}{n-1}}$ . To the right of the formulas is a diagram of a normal distribution curve. The mean  $\bar{S}_x$  is marked on the curve. A single measurement  $(S_x)$  is shown as a point on the curve, with a double-headed arrow indicating its distance from the mean. The text "single measurement" is written below the diagram.

So, from these most probable value what is the deviation of the observed value square of that gives the standard error. And another parameter this is the standard error actually associated with the with the single, measurement single measurement. Now if we want to know what is the standard error of the whole set of measurement then we have to find out the standard error of most probable value and that is given by  $S_x$  by square root of  $n$ , and  $n$  is a number of observation.

So, this is the standard error associated with most probable value; that means, if decides about the set of measurements together. So, if you had a one set of measurement another set of measurement if we want to compare between that then we should go for  $S_x$  bar, but if we have a set of measurement here and if we want to compare among these measurements then we should go for  $S_x$ . So, this is what is about this.

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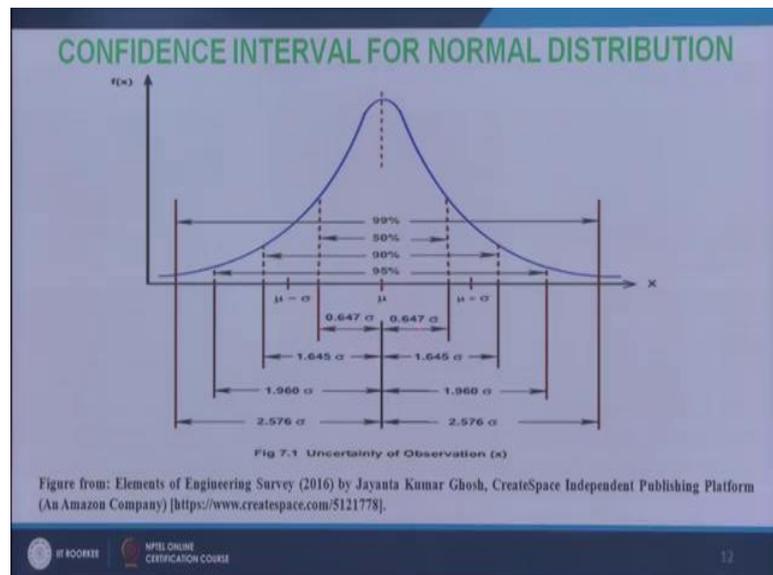
## CONFIDENCE INTERVALS

- Standard error of mpv indicates the chance that the true value for the measurements lies within a certain range of the sample mean (mpv), an indicator of reliability of observation.
- If an observation falls within  $(\mu \pm \sigma)$  [where  $\mu$  represents the mean and  $\sigma$  the standard error of the mpv of the set of the observations] then the observation has 68.3% chance to be true value. Thus, 68.3% is designated as the uncertainty of the sample.
- To ascertain the reliability of an observation, it is required to find within what percentage of error a particular observation lies. This defines the uncertainty of the observation and consequently its reliability. Lower the percentage of error within which a particular observation lies, lower is its uncertainty and thus greater is its reliability.
- For example, an observation is 50% uncertain if it lies within  $(\mu - 0.6745 \sigma)$  and  $(\mu + 0.6745 \sigma)$ ; 90% uncertain if lies in  $(\mu - 1.645 \sigma)$  and  $(\mu + 1.645 \sigma)$ ; 95% uncertain if it is within  $(\mu - 1.960 \sigma)$  and  $(\mu + 1.960 \sigma)$  and 99% uncertain for observation lying within  $(\mu - 2.576 \sigma)$  and  $(\mu + 2.576 \sigma)$ .
- [Figure 7.1](#) represents the uncertainty involved in an observation.
- Weight of a measurement may be taken as inversely proportional to its standard error.

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And another important thing now, what is the reliability or what is the chance that is given by the by a factor. Standard error of most probable value as I told you indicates the chance of that the true value of the measurement lies within a certain range of the sample mean.

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It an indicator of reliability of measurement; suppose a measurement lies within this for this point to this point. So, this is I can say I can see 0.6747 sigma. If we say maintain a range this one within that range if I get a measurement, then I we

will be able to say that these measurements will have 50 percent chance to become the true value. Or we can say other way that the uncertainty of this measurement is 50 percent.

Now, if we take a range between  $\mu + \sigma$  and  $\mu - \sigma$ , then  $\mu + \sigma$  and  $\mu - \sigma$  if I take a range between this, and if a measurement falls within this range, then we will say that the measurement is having a chance to become true value of 68.5 percent. So, and other way we say that the uncertainty of the measurement is 68.5 percent. So, in that way we may define different ranges to find out the degree of uncertainty associated with the measurement or the reliability of the measurement uncertainty also defines the reliability of the measurement.

So, with these I like to conclude this class. Now I like to summarize that a physical parameter is represented by its measurement by making a measurement, like if we want to say that distance between 2 points, it is the physical thing distance between and you measure the distance that is the measurement and most of the measurements are fought with errors. And this error may be gross error systematic error and random error and gross error and systematic error. We can take out quite comfortably easily and once it is done then we should look in to the random errors associated with the measurements. And we find that random error in the measurement is very random in nature. So, that is not follow any mathematical rule. So, we go by statistical method of analysis.

The quality of the of a set of measurement is actually defined by its accuracy precision relative precision. And the precision refers to the behavior of random errors. And a we generally make use of standard error to define all these precision accuracy and relative precision as well as weight worthiness of the observed a computed value of the measurement can be tasted by using standard error and indicator for measurement of reliability. With these I like to conclude today's class and in the next class I will like to discuss how these error gets propagated in our computed value. Because we have now discussed about the measured value now we will go for computed value in surveying we will go for computation. So,

we will like to I will like to discussed how the measured error in measurements propagate to the computed value.

Thank you.