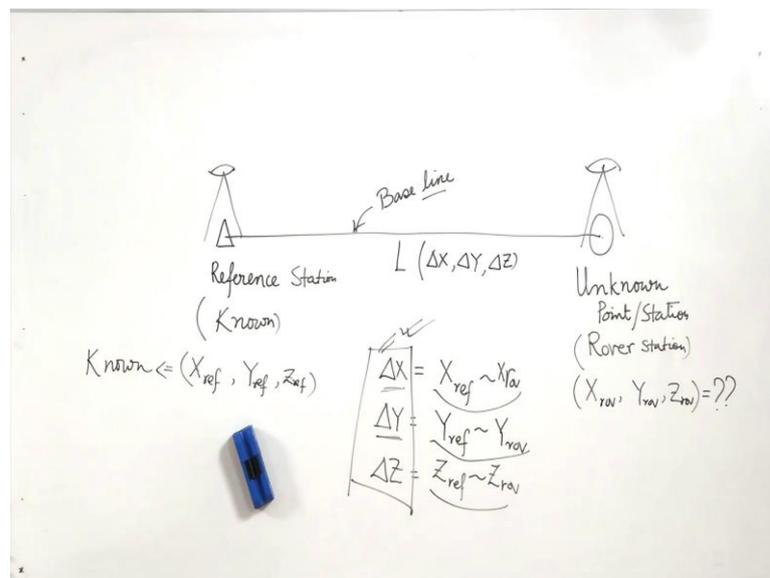


Digital Land Surveying & Mapping(DLS&M)
Dr. Jayanta Kumar Ghosh
Department of Civil Engineering
Indian Institute of Technology, Roorkee

Lecture - 14
GPS Data Processing

Welcome students, today is a 14th lecture, in this lecture I will be discussing on baseline processing this is required for GPS data processing in GPS surveying baseline is the basic unit. What is baseline? Actually baseline is a line joining 2 stations of this one station is known as reference station; that means, known point station and the other station which is called rover station or the station whose position is to be determined.

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Now, in case you have planned in your relative positioning that one of the station is known as reference station which is known and the other location whose position is to be determined unknown points for unknown station and it is known as rover station.

Now, in case of relative positioning you have seen that we do setup a instrument GPS receiver on reference another station we do setup on rover. Now the line joining these 2 points or these 2 station this is this line is called baseline. Now in case of GPS surveying it is fundamentally we do try to find out the baseline; that means, we do consider that the reference station is known reference Y reference Z reference these the location of the reference station and this is known. Now from these known station point we will take a

line up to the point whose position is to be determined and this line is known as baseline. So, if this is the baseline suppose baseline is L it can be defined by ΔX , ΔY and ΔZ where ΔX is X reference X rover ΔY ; Y reference difference between.

So, the difference between the X coordinates of the reference and rover station is ΔX and the difference between the Y coordinate of reference and rover station is ΔY and similarly for Z so; that means, we are taking X rover Y rover Z rover as the location of the rover station which we do not know. So, we have to find out this; that means, now in through a baseline processing actually we do determine this value. So, that is what it is written in and this is considered in the Cartesian coordinate system now this baseline can be determined this components can be determined by making use of code pseudo; code pseudo range or code observable or carrier phase observable or carrier phase pseudo range either single code observable or multi code observable or code observable followed by phase observable. So, whatever is the observable we make use of to determine these baseline components actually the fundamentals or the idea behind determining the baseline is same.

So, in this class today I will take up the codes pseudo range observable as the observable to be used for to determine the component of baseline.

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The image shows a handwritten derivation of the pseudo range observable. It starts with two equations for the pseudo range of a satellite i at time t for a reference station and a rover station. The reference station pseudo range is $PR_{ref}^i(t) = \rho_{ref}^i(t) + c \Delta t_{ref} - c \delta t^i + I_{ref}^i(t) + T_{ref}^i(t) + c \cdot dt_{ref} + c \cdot \delta t_{ref}^i + e_{ref}^i(t)$. The rover station pseudo range is $PR_{rov}^i(t) = \rho_{rov}^i(t) + c \Delta t_{rov} - c \delta t^i + I_{rov}^i(t) + T_{rov}^i(t) + c \cdot dt_{rov} + c \cdot \delta t_{rov}^i + e_{rov}^i(t)$. The difference between these two, labeled as the observable $K_{ref,rov}$, is $PR_{ref,rov}^i(t) = PR_{ref}^i(t) - PR_{rov}^i(t) = \rho_{ref}^i(t) - \rho_{rov}^i(t) + c(\Delta t_{ref} - \Delta t_{rov}) + [I_{ref}^i(t) - I_{rov}^i(t)] + [T_{ref}^i(t) - T_{rov}^i(t)] + c[dt_{ref} - dt_{rov}] + c[\delta t_{ref}^i - \delta t_{rov}^i] + [e_{ref}^i(t) - e_{rov}^i(t)]$. A boxed equation shows $PR_{ref,rov}^i(t) = \rho_{ref}^i(t) - \rho_{rov}^i(t) + E(t)$, where $E(t)$ represents the sum of the remaining terms.

Now, we know already you have learnt you have learnt from your previous classes that the pseudo range observable of the reference station from station I at any time T will be

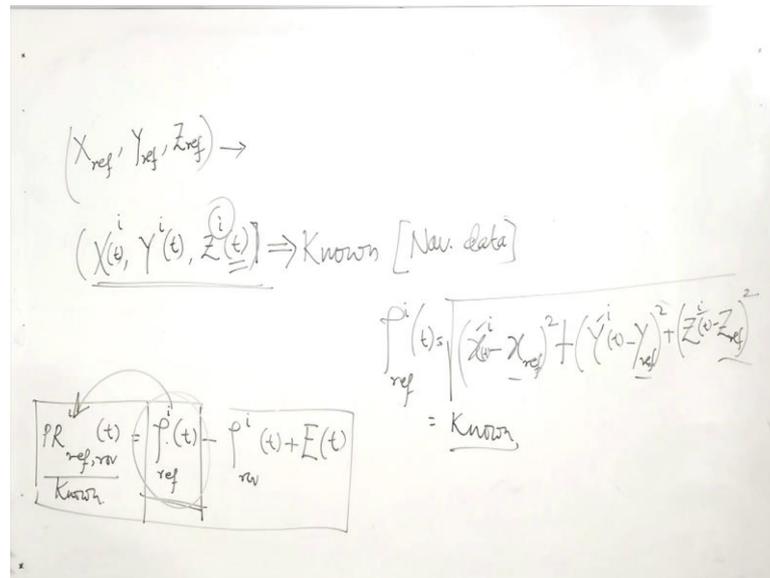
the geometrical range of the reference station from the satellite I plus visible clock error satellite clock error and ionospheric error tropospheric error visible hardware error satellite hardware error this is the multipath error and random error. So, this is the thing already you have learnt. So, when we will take the observation observable from our receiver reference receiver we will get the value of this. So, this is the value which we will get from reference receiver observables and this will be containing this components

Now, if we take next in relative positioning in baseline processing we do take the observable from reference receiver as well as rover receiver which has been observed simultaneously now the similarly if we take the observable from rover receiver from the same satellite from the same instant of time we have to take and we will get the same type of expression. That means, geometrical distance of the rover receiver from the satellite I at time T this is reference. So, this is the rover receiver clock error DTI rover tropospheric error rover receiver hardware error multipath error with the respective rover receiver and the random error.

So, this is also known. So, if we take the relative positioning or the difference in these 2 then we will get this is nothing, but the difference between these values which is known this is known then we will get the geometric range difference between reference or rover then the difference in clock error of the reference on rover receiver. Then these 2 atom will get cancelled and also this will get cancelled and these deference in ionospheric error then deference in tropospheric error. Then we will get the receiver clock receiver hardware error or reference and rover deference than the multipath error difference and the random error difference.

So, now from here you can see that the errors will get reduced errors will get reduced anyway so, the amount of error that will be now we can this for this you can see the amount of error will be reduced and anyway, but some error will be there. So, we can write it that p ; that means, the difference in Seder on (Refer Time: 12:11) between reference and rover is equal to plus some error at time T, now you in this expression this is the expression we get.

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Now, in this expression this is the geometric range of the reference receiver as I told you that the location of the reference receiver is known; that means, the location of the reference receiver suppose this is this is known as well as you know that we can get the location of the satellite at that instant of time from the navigational data.

Suppose X^i , Y^i and Z^i this is the location of the satellite I at time T and this is also known from navigational data. So, you will get this from navigational data. So, once it is known and also this is known; that means, this is now a geometric distance we know that the geometric distance of the receiver reference receiver from the satellite I can be obtained by using the relation by using this relation. So, in this relation this is known this is these are the and also these are known. So, this is known. So, this is known. So, and also this is known.

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$(X_{ref}, Y_{ref}, Z_{ref}) \rightarrow$
 $PR_{ref,rov}^i(t) - \phi_{ref}^i(t) = -\int_{rdv}^i(t) + E(t) \rightarrow$
 linearisation of
 BL prior data
 $\begin{bmatrix} \delta x \\ \delta y \\ \delta z \end{bmatrix} = -\begin{bmatrix} \frac{\partial f^i}{\partial x} & \frac{\partial f^i}{\partial y} & \frac{\partial f^i}{\partial z} \end{bmatrix}^{-1} \begin{bmatrix} X_{ref} + \delta x \\ Y_{ref} + \delta y \\ Z_{ref} + \delta z \end{bmatrix} + E(t) -$
 Residual observables for Baseline
 Between Reference & Rover
 $(X_{ref} + \delta x, Y_{ref} + \delta y, Z_{ref} + \delta z)$ Known

So, we can take this part to this side; that means I want to make the right hand; left hand side known. So, we can say that pseudo range reference rover station I T minus this part is known now is equal to T.

Now, you can see this is a part this is an equation this is an equation which we have seen our point positioning equivalent to this equation 13.2.

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Single Code Positioning.....

Let (x_r, y_r, z_r) represents the true receiver position and δ_r , the receiver clock error which are required to be determined by method of least square estimation. Then, Eq. (13.1a) can be represented as

$$PR_r^i(t) = f_r^i(x_r, y_r, z_r, \delta_r) + e_r^i(t) \quad \text{Eq. (13.2)}$$

Let the approximate value of the unknown parameters be $(x_0, y_0, z_0, \delta_0)$ and differs by $(\Delta x, \Delta y, \Delta z, \Delta \delta_r)$ from their true values. Thus, $f_r^i(x_r, y_r, z_r, \delta_r)$ can be expressed as $f_r^i(x_0 + \Delta x, y_0 + \Delta y, z_0 + \Delta z, \delta_0 + \Delta \delta_r)$. Now, applying Taylor's theorem and neglecting second and higher order terms from the expansion,

$$f_r^i(x_r, y_r, z_r, \delta_r) \approx f_r^i(x_0, y_0, z_0, \delta_0) + (x_r - x_0) \frac{\partial f_r^i}{\partial x} + (y_r - y_0) \frac{\partial f_r^i}{\partial y} + (z_r - z_0) \frac{\partial f_r^i}{\partial z} + (\delta_r - \delta_0) \frac{\partial f_r^i}{\partial \delta_r}$$

$$= f_{computed}^i + \frac{\partial f_r^i}{\partial x} \Delta x + \frac{\partial f_r^i}{\partial y} \Delta y + \frac{\partial f_r^i}{\partial z} \Delta z + \frac{\partial f_r^i}{\partial \delta_r} \Delta \delta_r \quad \text{Eq. (13.3)}$$

Assuming, the position of satellite at epoch t as (x^i, y^i, z^i) ,

$$f_{computed}^i = \sqrt{(x^i - x_0)^2 + (y^i - y_0)^2 + (z^i - z_0)^2} c \delta_0 = \rho_0^i + c \delta_0$$

$$\frac{\partial f_r^i}{\partial x} = -\frac{(x^i - x_0)}{\rho_0^i} \frac{(x_0 - x^i)}{\rho_0^i}, \quad \frac{\partial f_r^i}{\partial y} = -\frac{(y^i - y_0)}{\rho_0^i} \frac{(y_0 - y^i)}{\rho_0^i}, \quad \frac{\partial f_r^i}{\partial z} = -\frac{(z^i - z_0)}{\rho_0^i} \frac{(z_0 - z^i)}{\rho_0^i}, \quad \frac{\partial f_r^i}{\partial \delta_r} = c.$$

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You can see this is the equation which is equivalent because this is the equation which is nothing, but a function of this is rover I is a function of X rover Y rover Z rover plus del

T rover plus we can write also this is this equation also I can write like this and this is the equation which is equivalent to this. So, and also you know that this X rover Y rover Z rover ΔT rover can be written as X reference plus ΔX Y reference plus ΔY and Z reference plus ΔZ if we this.

Now in this X reference Y reference Z reference are known these are known now you can make use of the same concept these case here you can say (Refer Time: 15:41) is equal to; that means, in our case now in this rover, rover, rover, rover. So, it is the reference, reference, reference, reference. So, we can make use of the same concept that has been applied here and we can get the same expression as from this only difference is that in this ΔX ΔY ΔZ in our in the in base line processing case these are the components of baseline, but in case of point positioning ΔX ΔY ΔZ are the between the assumed value and the exact value

So, only the idea is same and, but the meaning is different because here ΔX ΔY ΔZ in point positioning it is the difference between the assumed value or the iterated value and the exact value in case of baseline processing it is the deference between these ΔX ΔY ΔZ are the components of the baseline anyway. So, now, here we have seen that this is the thing. So, for reference these again we can compute the function f . So, for whatever we have got for reference receiver that can be substituted and for ΔX ΔY ΔZ equivalent to this which is in this case so, similar to this.

And in this case we have seen that this is the in point positioning this is the thing we got an observation; that means, this is the known which will be this one for in this case and also it will come some values from here to this side.

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Single Code Positioning.....

One such relation [Eq (13.6)] can be obtained for observation from each satellite. At any epoch of observation, if a receiver receives signals from m SVs, then a system of m equations will be obtained and can be represented as:

$$\begin{bmatrix} \rho^1 \\ \rho^2 \\ \vdots \\ \rho^m \end{bmatrix} = \begin{bmatrix} \frac{\partial \rho^1}{\partial x} & \frac{\partial \rho^1}{\partial y} & \frac{\partial \rho^1}{\partial z} & \frac{\partial \rho^1}{\partial \delta_1} \\ \frac{\partial \rho^2}{\partial x} & \frac{\partial \rho^2}{\partial y} & \frac{\partial \rho^2}{\partial z} & \frac{\partial \rho^2}{\partial \delta_2} \\ \vdots & \vdots & \vdots & \vdots \\ \frac{\partial \rho^m}{\partial x} & \frac{\partial \rho^m}{\partial y} & \frac{\partial \rho^m}{\partial z} & \frac{\partial \rho^m}{\partial \delta_m} \end{bmatrix} \begin{bmatrix} \Delta x \\ \Delta y \\ \Delta z \\ \Delta \delta_1 \end{bmatrix} + \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \vdots \\ \epsilon_m \end{bmatrix} \quad \text{Eq. (13.7)}$$

The equation (13.7) can be written in matrix symbol as:

$$O = AX + \epsilon \quad \text{Eq. (13.8)}$$

Eq. (13.8) represents a relationship between the residual observations [O] (i.e., computed observations subtracted from pseudo-range observables) and the unknown correction [X] to the parameters [X]. Unknown noise of observations is associated in the relationship through members of the column matrix [ε]. Thus, the matrix equation (13.8) represents a "linear model of observation equation."

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So, these will be the observable and these are the thing which we will get corresponding to here and (Refer Time: 18:53) in single point positioning we have done this observable equation which is a linear equation.

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Single Code Observables Base line Processing

In relative positioning, pseudo-range code observable from rover is being considered with respect to reference receiver. Thus,

$$PR_{ref,rover}^i(t) = PR_{ref}^i(t) - PR_{rover}^i(t) \quad [\text{Known}]$$

or, $PR_{ref,rover}^i(t) = \rho_{ref}^i(t) - \rho_{rover}^i(t) + c\delta t_{ref,rover} + I_{ref,rover}^i(t) + T_{ref,rover}^i(t) + dt_{ref,rover} + d_{ref,rover}^i(t)$

$$\text{or, } [PR_{ref,rover}^i(t) - \rho_{ref}^i(t)] = -\rho_{rover}^i(t) + E_{ref,rover}^i(t) \quad \text{Eq. (14.2)}$$

Eq. (14.2) is similar to Eq. (13.2)

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So, similar to this in baseline processing also, 14.2 is similar to 13.2 as I have told you already and 13.2 I have shown you.

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**Single Code Observables
Base line Processing**

The coordinates of the rover station $(x_{rov}, y_{rov}, z_{rov})$ is being considered in terms of reference station coordinates $(x_{ref}, y_{ref}, z_{ref})$ and the base line components $(\delta X, \delta Y, \delta Z)$ between the reference and rover stations, so that

$$(x_{rov}, y_{rov}, z_{rov}) = (x_{ref} + \delta X, y_{ref} + \delta Y, z_{ref} + \delta Z)$$

Thus, Eq. 14.2 can be expressed as

$$[PR_{ref,rov}(t) - \beta_{ref}^j(t)] = f_{iov}^j(\delta X, \delta Y, \delta Z, \delta \delta) + E_{ref,rov}^j(t) \quad \text{Eq. (14.3)}$$

The observation equations can be expressed in the form $O = AX + E$, a linear relation where known parameters are included in the O matrix, having dimension $d \times 1$, where d is the number of linearly independent data.

the estimated parameters can be obtained using

$$\hat{X} = (A^T W A)^{-1} A^T W O \quad \text{Eq. (14.4)}$$

where, W is the data weight matrix, to be derived later on, and O is a vector containing the residual observations.

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And the location of the rover receiver considered as the reference plus δX reference Y reference equal to Y reference plus δY Z reference plus δZ where δX δY δZ are the components of the baseline vector between the reference on the rover station. So, it will come to this expression which is equivalent to our previous point positioning expression only the meaning of the parameter are different

So, this equation can be converted to O plus $A X$ plus E of course, this O , I should see right baseline O design matrix O baseline this. So, it is better to have this expression rather than this expression because this expression may confuse you along with this respective point positioning now this is the observable; that means, residual observables residual observables for baseline and between the reference and the between reference and rover. So, and this is the design matrix which will be default from point design matrix and this design matrix is for baseline design matrix for baseline this is the unknown parameter for baseline though it looks like same as point positioning δX δY δZ , but this δX δY δZ is the baseline, baseline, baseline; it is better to write like this and this is the error that will be obtained due to the processing already I had shown.

Now, this is the linearization of this is the fundamental equation; that means, the linearization of baseline BL baseline processing data now which is the fundamental for further a mathematical analysis. So, once you will get this linear relation from this linear

relation as I told you there are. So, many mathematical way how we can analyze one of the way how we can do it is the least square analysis.

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$X_{rw} = X_{ref} + \Delta X_A$; $Y_{rw} = Y_{ref} + \Delta Y$; $Z_{rw} = Z_{ref} + \Delta Z$

Least Square Analysis

→ Weighted L.S.A

Linearization of BL prior data

$$O = AX + E$$

(Residual observables for Baseline Between Reference & Rover)

$$X = (A^T W A)^{-1} A^T W O_{BL}$$

$\begin{bmatrix} \Delta X_{BL} \\ \Delta Y_{BL} \\ \Delta Z_{BL} \end{bmatrix}$

And again under least square analysis least square analysis under least square analysis they are maybe different methods one is that weighted least square analysis in the positioning case we have considered that the weight of all the observed data are of same nature or equal; that means, the amount of errors that has been associated with different observables are considered to be equal in case of point positioning.

Now, if we consider the weight of all the observations in case of baseline is not equal then we will go for weighted least square analysis now under weighted least square analysis the solution of this can be obtained as this is the solution; that means, the baseline component this is this X matrix means del X del Y del Z baseline. So, this is this is the solution where w I should write it like baseline to make it make it different from what we have learnt in point positioning because these design matrix this observable matrix this matrix will be different from what we have got in point positioning, but its name is same and though this matrix looks same, but its component represents the baseline components.

So, in this way we can get the we can resolve the component of the baseline and once you resolve the component of the baseline then actually the location of the rover station we get it from X reference plus del x. So, once you know the baseline. So, from the

known baseline components we from the known position of the reference we can get the location of the rover stations. So, in this way, Y_{rover} is equal to $Y_{\text{reference}}$ plus ΔY Z_{rover} reference plus ΔZ . So, now, you can see the coordinate of the rover station has been determined.

So, this is the idea behind all GPS processing to find out the position of unknown stations with this I want to complete these analysis and the let me summarize in GPS surveying baseline is the most fundamental parameter object which is required to be estimated and baseline processing determines the unknown rover position with respect to reference station. So, as I told you that by finding out the component of baseline we can make use of component of the baseline with the reference location and find out the location of the rover station the estimation of baseline components may be based on methods of observation as well as on the type of observable baseline processing depends.

There are different methods for baseline solution of which least square method is widely (Refer Time: 27:08) and has been discussed for single code observables in this class, in baseline processing solution consists of estimation of defines parameter for the baseline as I told you ΔX ΔY ΔZ are the components which we do determine in baseline processing and from there by using this relation we can we do find out the unknown location of the rover station. So, and they are about this topic something more elaboration has been given in this textbook if you want you can get a copy from this and in the next class I will discuss on network adjustment another important GPS data processing work to be done for during GPS surveying, next class on network adjustment see you.

Thank you.