Digital Land Surveying and Mapping (DLS&M)
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Lecture – 13
Point Positioning

Welcome students, this is the lesson number 13 under which I will like to discuss on GPS point positioning. So, in this class actually I will like to show you the mathematical deduction of the GPS data how we arrived at the position of a point.

So, now this lesson as will be consisting of introduction and GPS positioning and I will like to take up the pseudo range observable of a point and the from that I will deduct the mathematical relations, how it conclude to the GPS point position and the same way actually we can make use of single code, multi code or single frequency GPS phase observables or multi frequency phase observables that the idea of (Refer Time: 01:36) arriving at GPS positioning is same. So, I have taken up as a C A code pseudo range observable for deriving the position of point.

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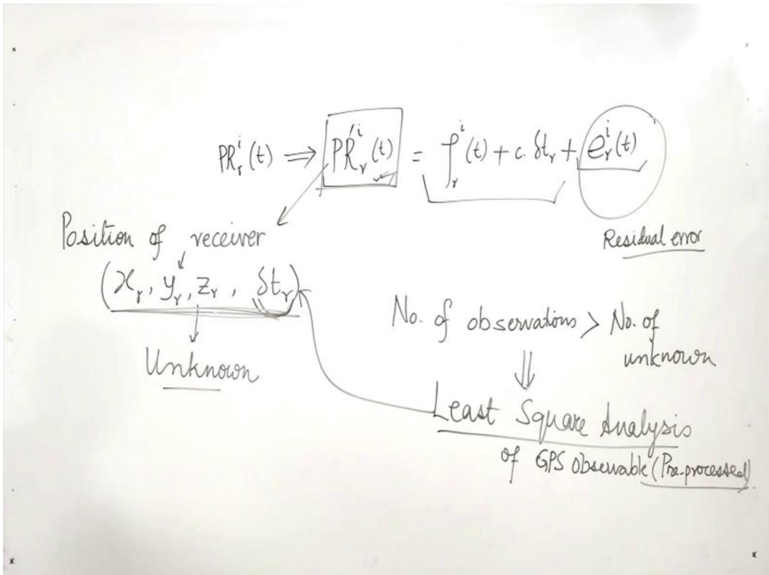

Now, as we know already we have discussed that original pseudo range from satellite i to receiver r at any instant of time t. In the last class we have discussed that the pseudo range will be pre-possessed and after pre-possessing, we will get some pseudo range value. Suppose this is dash pseudo range dashed; that means now we have removed some of the errors and this is the new value of the pseudo range observable and it will consists of the geometric range plus error due to receiver clock and other errors.

So, this is a suppose I have written a defined e so that you understand that the errors that were available like your satellite clock error, ionosphere error, troposphere error then other many other errors that has been removed many of that error has been removed or minimized; ultimately some error is there which is I can say as residual error. So, now this is the pseudo range which we have pseudo range value and this pseudo range value consists of this, now this is the residual error of this thing.

Now, from here we have to find out the position of receiver position of the receiver from which we have taken these observable. So, let us assume that the position of the receiver be X r, Y r and Z r, so to get the GPS position we need to find out these four parameters. So, these four parameters are the unknown which has to be determined by using this value.

Now, as I have already told in GPS point positioning that to and also you know that to determine four unknown, we will be in need to have at least for say 4 satellites; that means, signal from at least 4 satellites and usually we take data from more than 4 satellites. So, we go for now when the number of observations are more than the number of unknown; number of observations are more than number of unknown then we have to apply some statistical we may apply many other methods generally we use least square analysis; least square analysis method.

So, we have to go for to determine the position of the receiver and the clock receiver clock error these four unknowns we will make use of least square analysis. Now least square analysis of GPS data of GPS observables which are pre-processed, pre-processed means we have removed and many of the errors and also we have minimized the amount of error and the observables now will have only some residual errors. So, and with these assumptions we will start, now suppose this is the unknowns and as you know that the signals are coming.

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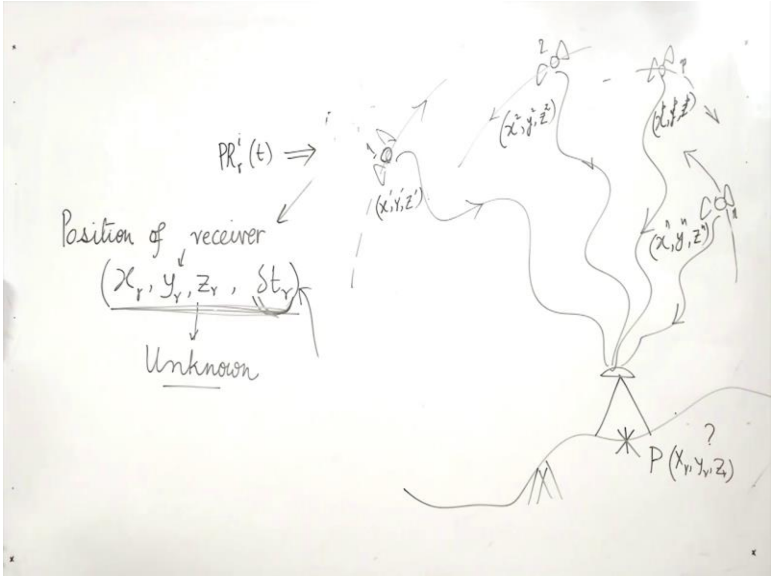

So suppose this is the parent and this is the point P where whose position X r, Y r, Z r we need to determine. So, at this position what we will do we will setup a GPS receiver and that GPS receiver will receive signal from GPS satellites. So, let us take at least 4 satellites for 1, 2, P, n. So, now as we know as I have already told; already it was told in this class that each of these satellite will have some co-ordinates or location and these co-ordinates or location of the receiver will be available from navigational data and suppose x to the power p, y to p subscript z p subscript; these are the superscripts shows the coordinate of the particular satellite x to the x n, y n, z n these are the location of the satellites which will be available from GPS data.

Now, if we know this thing then we will be able to get suppose this is 1 and receiver R. So, you will have the equation x 1 minus x r, y superscript 1 y r, z z r. So, all of you know this is the distance equation now, but we do not know what is this value. So we will not be able to know what is this value, so there lies the problem. Now how we resolve it, as I told you we will go for least square analysis to find out these values, but least square analysis require linear relation to resolve, but this is a non-linear equation you can see this is to the power one this is to the power half, so this non-linear relation first has to be linearized.

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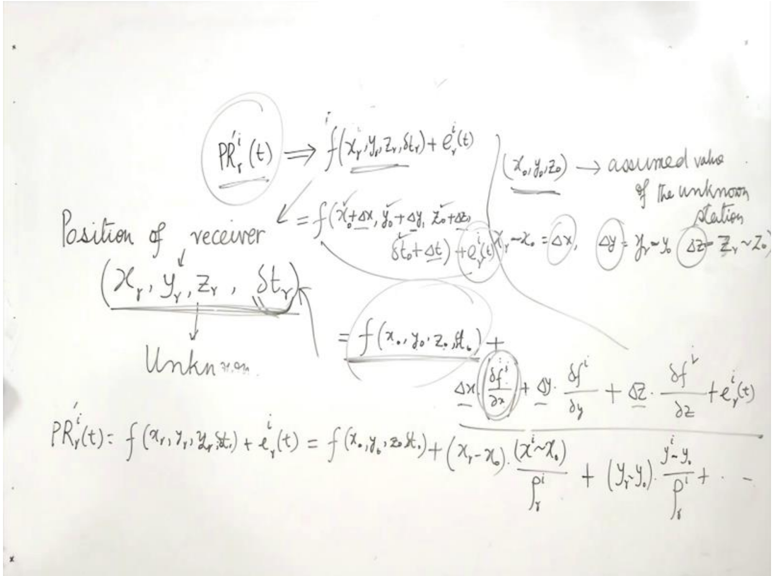

So, how to do it now as per our assumption our pseudo range this is equal to this; this is the function of x r, y r, z r, del t r plus this r t. Now to linearize it actually we have to assume some value for the GPS receiver, now suppose x 0, y 0 and z 0. Assume value of the unknown station, now then x r minus x 0 will be del x, del y will be y r minus y zero differences I should say difference and del z is the z 0, z r, z 0. Now this del x, del y and del z of the error between the assumption and the actual value. So, now the one if we assume instead of this if we assume this then we can write this function as f of x 0 plus del x y 0 plus del y z 0 plus del z and del t 0 plus del t, so this is the thing we can write.

Now in writing this now these unknown, but these are unknown del x, del y, del z, del t. Now, the problem is not to determine x r, y r but to determine del x, del y, so this is the thing when we go for GPS observation we generally need to give the approximate value of the location; that means, we have to give the value of x 0, y 0, z 0, del t 0 and from there a they compute iteratively the del x, del y, del z and then that has been; from there we do determine this position.

Now we still now the relation of that modified pseudo range is can be this and now we can make use of tailor series, all of you know that this will be x 0, y 0, z 0, t 0, del t 0; that means, in this relation; we will instead of x r, we will if we put these values then we will be able to get approximate geometric range which is known plus now from tailor series taking the first partial derivative we can write del x into d f i by del x plus del y d f i by del y plus del z del f i plus del z and then this error plus e to the power this.

So, in this can be computed unknown this is known and this is known and this is the thing which is unknown for del x, del y this is the unknown. Now and this so we can write that p r dashed i r t is equal to f of x r, y r, z r, del t i plus e i r t is equal to; now this x 0, y 0, z 0, del t 0 plus x del x means x r minus x 0. Now d f i; d x i, What is this? This is the gradient of the function this with respect to x which is nothing, but direction cosine of the line joining the satellite to the receiver, it is the direction coming from direction of x. So, it is we can write like this x we have taken i, so x i value minus x 0 divided by rho i; i. So, this is what is the thing we get here. I can take either way this is the difference actually, so similarly I can write y r minus difference paste is the direction cosine y i minus y 0 i; so like that.

So, this is what is the relation now this one is computed we know it, we can compute this thing and these are the parameters we can show like this these are the thing. So, this is from only one satellite, so similar; so modified residual observation from the satellite I to receiver r at any time t that is nothing, but p r dashed; i r t minus f f 0, y 0, z 0 and del t 0 and this is equal to del x; d f i; d x, del y; d f i, d y; del z, d f i; d z plus e i; r t. So, finally we go the expression now when you are choosing linear you can see now this is a linear relation. So, we have done the linearization and this is from only one satellite i, so if we get observation from n satellites; so we will get this type of linearization from n satellite.

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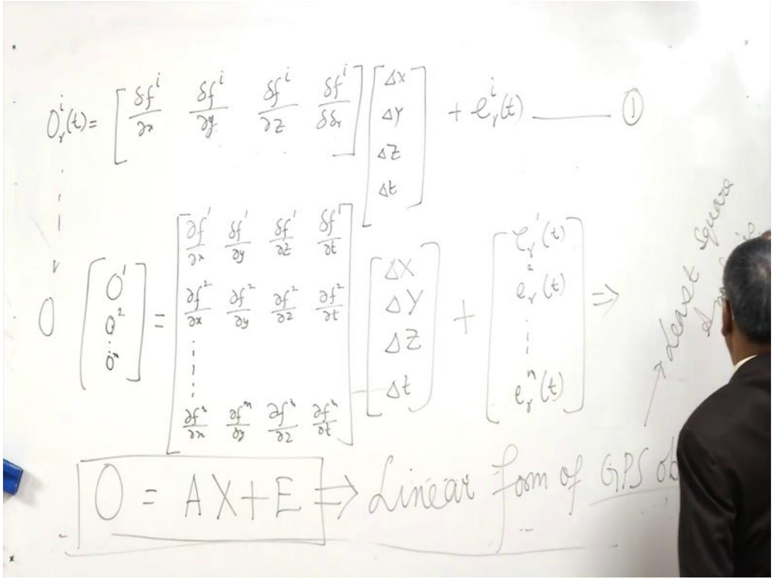

So, this is what we can again we can write these thing in a metric form o i r t is equal to d f i; d x; d f i; d y, d y, d f i; d z, d f i; del d i; del x, del y, del z, del t. So, we can write this thing in this way oh one of the I have missed one del t, del f i; d t remains t; so i r t. So, this is the expression we are getting from only one satellite, so if we take observation from n satellites so we will get n number of this expression.

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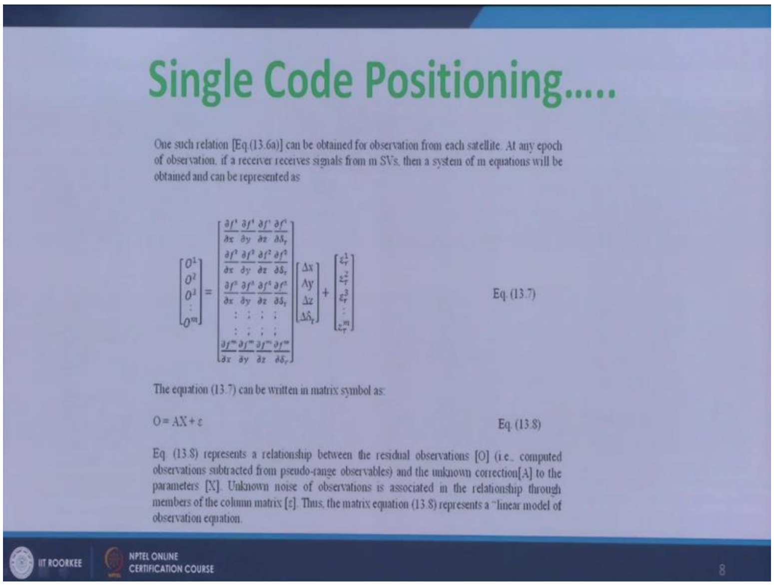

And we can write this thing in a matrix form O 1, O 2 like that O n; n numbers of satellites. So, and that can be written as like this d f 1; d x d f 1; del y, d f 1, del z; d f 1, del t. So, this is from the first satellite and from the satellite d f 2; d x; d f 2; d y; d f 2; d i; d f 2 d t this is from the second satellite and in that way n numbers of satellites we can go for and then multiply by the unknown parameters del x, del y, del z, del t and again error matrix 1, 2 like that n 1 t.

So, this is the matrix form of the error, so now if we write this in matrix form in, so instead of this O like this; that is the O matrix, this is the A matrix, this is the unknown matrix x and this is the error matrix. So, this is the final linear form of GPS observables which now we can solve this equation linear form by least square analysis. Now in this we have assumed first to start with we have to assume some value of the receiver and through iteration, we will find out the error matrix x and once this that means we have to start with x 0 and we have the error suppose del x, we will some value and these value has to be changed n numbers of times to big del x to 0.

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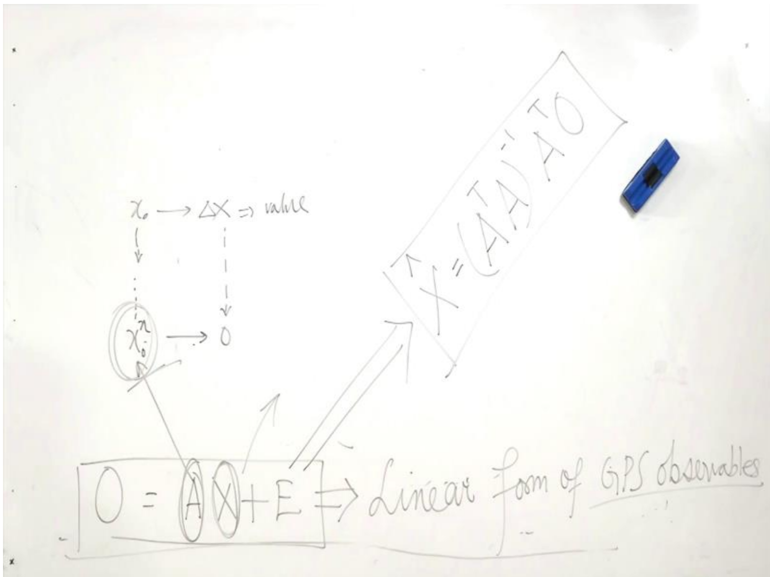

So, this is the what is inside being we will go on iterating this till we make this del x fixed del x, del y, del z all these will be 0 and the value for which this del x matrix, del x this x (Refer Time: 23:23) will be 0; that is and for that whatever design matrix that value will be there that is the position of the unknown location.

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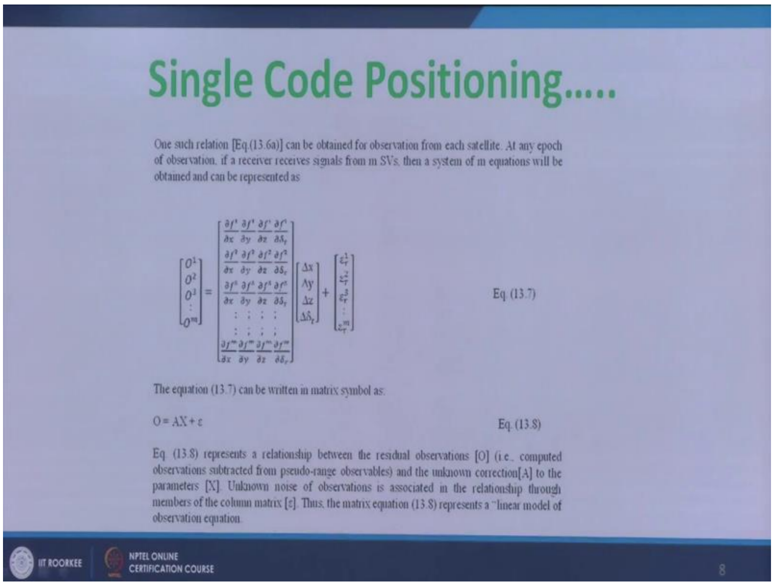

So, that is what is done in this ah means GPS positioning and that is; that means, from this through this least square analysis, we get the x is equal to del plus a whole inverse; inverse A transpose O, so this is the solution for this; that means, this should be equal to 0 finally and then what for whatever value of this; this will be go on changing, this x 0 will be go on changing and some value suppose x 0 1, similarly y 0 will be y 0 n, z will be z 0 n like this and for whatever value that will be this will be lead to 0 that is what is the position of the GPS receiver.

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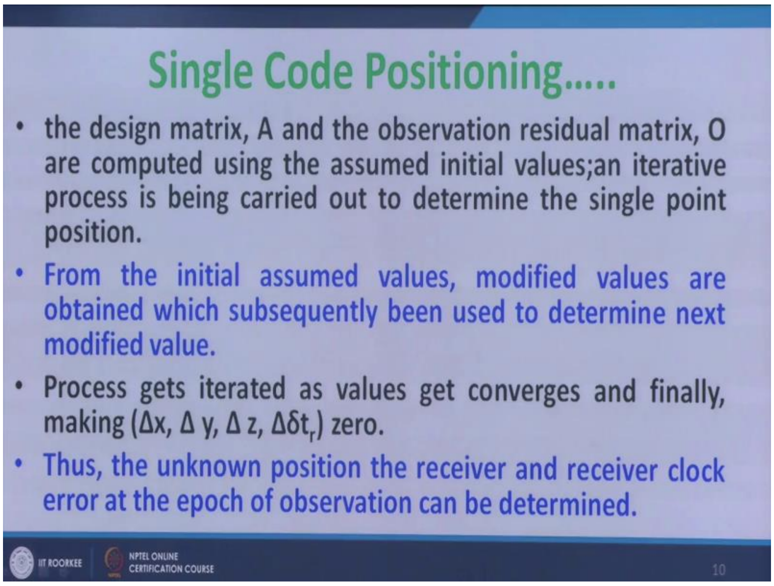

So, in that way we compute the position of GPS through iteration method. So, that is what is written the design matrix A and the observation residual matrix O are computed using the assumed initial value; as I told you x 0, y 0, z 0 of the assume initial values. An iterative process is being carried out to determine the single point position; iteratively we do go on doing go on doing so that the del x, del y, del z, del t is 0. From the initial values, the modified values are obtained which subsequently been used to determine next modified value that is what it is told, we will assume some value we will make del x will get some value of del x and then we will change this value in such a way that del x value go on decreasing.

Process gets iterated as values get converge and finally, making del x, del y, del z, del t is 0 that is what we have told. Thus the unknown position; that means, for whatever value of this; this will be equal to 0 that then that is the value of the receiver position; the as an unknown position of the receiver and receiver clock error at the epoch (Refer Time: 26:11) can be determined.

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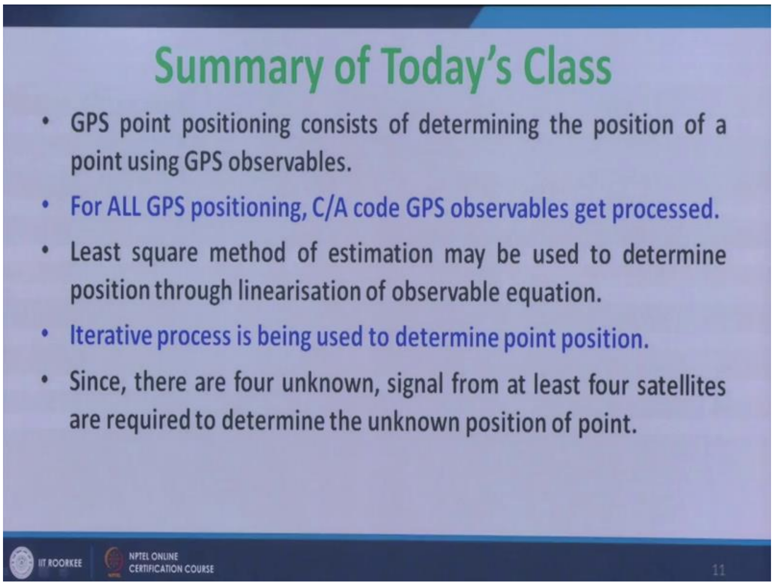

So, that is what is in short about the GPS positioning mathematics, so summary of today's class GPS positioning consist of determining the position of a point using GPS observable. All GPS positioning C A co GPS obvious get processed and generally we get an approximate location, from that approximate location more precise location we do go for through phase observables. In finding out the position of GPS one of the methods is the list is least square analysis method, which we have discussed in today's class it is; an iterative and iterative process through which we determine the point position. Since there are four unknowns signal from at least four satellites are required to determine the unknown position of point.

With this I will like to conclude today's class and this is a question we should and for further reading you can go through this book and in the next class I will like to go for establishment of GPS or how GPS based then processing is been carried out, that will be shown mathematically. So, with this I want to conclude today's class.

Thank you.